

Jinn-Ouk Gong, JGRG 22(2012)111302

“Towards more precise estimates of the primordial bispectrum”

**RESCEU SYMPOSIUM ON
GENERAL RELATIVITY AND GRAVITATION**

JGRG 22

November 12-16 2012

Koshiba Hall, The University of Tokyo, Hongo, Tokyo, Japan



Towards more precise estimates of the primordial bispectrum

Jinn-Ouk Gong

APCTP, Pohang 790-784, Korea

RESCEU Symposium on General Relativity and Gravitation
(JGRG22)

University of Tokyo, Tokyo, Japan

13th November, 2012

Celebrating 60th birthday of T. Futamase, H. Kodama and M. Sasaki

Based on

- C. T. Byrnes and [JG](#), arXiv:1210.1851 [astro-ph.CO]
- A. Achucarro, [JG](#), G. A. Palma and S. P. Patil, to appear
- [JG](#), K. Schalm and G. Shiu, to appear

Outline

1 Introduction

2 Effects of non-trivial speed of sound

3 Bispectrum in general slow-roll

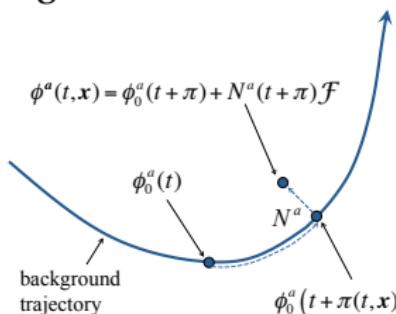
4 Running of f_{NL}

5 Summary

General single field inflation

$$S = \int d^4x \sqrt{-g} \left[\frac{m_{Pl}^2}{2} R + P(X, \phi) \right] \quad \text{with} \quad X \equiv -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

Originated from **multi-field setup**: light \mathcal{R} and heavy \mathcal{F}



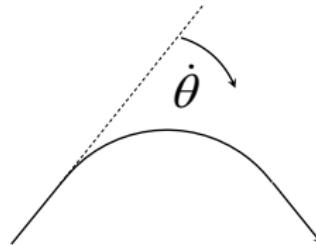
- Trajectory along the **lightest** direction
- Effects of heavy physics in curved traj
- Can we find **universal features** of “heavy” physics?

- ① Write the action in terms of \mathcal{R} (along traj) and \mathcal{F} (off traj)
- ② Integrate out \mathcal{F} : $e^{S_{\text{eff}}[\mathcal{R}]} = \int [D\mathcal{F}] e^{S[\mathcal{R}, \mathcal{F}]}$
 $\left[= \text{equiv to plugging linear sol: } (-\square + M_{\text{eff}}^2) \mathcal{F} = -2\dot{\theta}(\dot{\phi}_0/H)\dot{\mathcal{R}} \right]$
- ③ Effective single field action $S_{\text{eff}}[\mathcal{R}]$

Effects of heavy physics as non-trivial c_s

Effects of heavy physics in “speed of sound”

$$c_s^{-2} \equiv 1 + \frac{4\dot{\theta}^2}{M_{\text{eff}}^2} \quad (\dot{\theta}: \text{angular velocity of traj})$$



Single field theory with non-trivial c_s^2 : Footprint of heavy physics

(Achucarro et al. 2012a)

\mathcal{F} borrows kinetic energy of $\mathcal{R} \rightarrow$ propagation speed c_s reduced

- EFT in \square/M_{eff}^2 : universal footprint of heavy physics
- Many scalar fields in BSM, e.g. moduli
- New observables poorly constrained → to be tested in next decades

Splitting canonical action

EFT = canonical ($c_s = 1$) + (occasional) departure from $c_s = 1$

$$\begin{aligned} S &= \underbrace{\int d^4x a^3 \epsilon m_{\text{Pl}}^2 \left[\frac{\dot{\mathcal{R}}^2}{c_s^2} - \frac{(\nabla \mathcal{R})^2}{a^2} \right]}_{=S_2, \text{ "free" part}} + S_3 + \dots \\ &= \underbrace{S_{2,\text{canonical}}}_{c_s=1 \text{ part}} + \underbrace{\int d^4x a^3 \epsilon m_{\text{Pl}}^2 \left(\frac{1}{c_s^2} - 1 \right) \dot{\mathcal{R}}^2}_{\equiv S_{2,\text{int}}} + S_3 + \dots \end{aligned}$$

- Well known, accurate Green's function

(For example, [JG](#) & Stewart 2001, Choe, [JG](#) & Stewart 2004)

- Interaction valid for a limited interval (c.f. Chen & Wang 2010)

c.f. Using $dy \equiv c_s d\tau = c_s dt/a$, $q^2 \equiv a^2 \epsilon / c_s$ and $v = \sqrt{2} q \mathcal{R}$ (Baumann, Senatore & Zaldarriaga 2011)

$$S_2 = \int d^4x \frac{m_{\text{Pl}}^2}{2} \left[(v')^2 - (\nabla v)^2 + \frac{q''}{q} v^2 \right]$$

But see later parts of this presentation

Features in the power spectrum

Interaction Hamiltonian at quadratic order

$$H_{\text{int}}^{(2)}(t) = \int d^3x \left(\frac{\partial \mathcal{L}_{\text{int}}^{(2)}}{\partial \dot{\mathcal{R}}} \dot{\mathcal{R}} - \mathcal{L}_{\text{int}}^{(2)} \right) = \int d^3x a^3 \epsilon m_{\text{Pl}}^2 \underbrace{\left(\frac{1}{c_s^2} - 1 \right)}_{\equiv -u(t)} \dot{\mathcal{R}}^2$$

Features in the power spectrum

$$\begin{aligned} \langle \hat{\mathcal{R}}_{\mathbf{k}}(\tau) \hat{\mathcal{R}}_{\mathbf{q}}(\tau) \rangle &= -i \int_{\tau_{\text{in}}}^{\tau} a(\tau') d\tau' \langle 0 | [\hat{\mathcal{R}}_{\mathbf{k}}(\tau) \hat{\mathcal{R}}_{\mathbf{q}}(\tau), H_{\text{int}}^{(2)}(\tau')] | 0 \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k} + \mathbf{q}) \frac{2\pi^2}{k^3} \Delta \mathcal{P}_{\mathcal{R}} \\ &\rightarrow \frac{\Delta \mathcal{P}_{\mathcal{R}}}{\mathcal{P}_{\mathcal{R}}} = \kappa \int_0^\infty dt u(t) \sin(2\kappa t) \quad \text{with} \quad \mathcal{P}_{\mathcal{R}} = \frac{H^2}{8\pi^2 m_{\text{Pl}}^2 \epsilon}, \quad t \equiv \frac{\tau}{\tau_*}, \quad \kappa \equiv \frac{k}{k_*} \end{aligned}$$

Inverting this relation to write u in terms of observable $\Delta \mathcal{P}_{\mathcal{R}} / \mathcal{P}_{\mathcal{R}}$

$$u(t) = \frac{2i}{\pi} \int_{-\infty}^{\infty} \frac{d\kappa}{\kappa} \frac{\Delta \mathcal{P}_{\mathcal{R}}}{\mathcal{P}_{\mathcal{R}}} \left(\frac{\kappa}{2} \right) e^{i\kappa t}$$

Correlated bispectrum and power spectrum: $B_{\mathcal{R}} = \int (\cdots \Delta \mathcal{P}_{\mathcal{R}} / \mathcal{P}_{\mathcal{R}})$

Leading bispectrum for varying c_s

Leading order action in terms of $u(t)$

$$S_3 \supset \int d^4x a^3 m_{Pl}^2 \epsilon \left[3u \dot{\mathcal{R}}^2 \mathcal{R} - (u + 2s) \mathcal{R} (\nabla \mathcal{R})^2 \right] \quad \left(s \equiv \frac{\dot{c}_s}{H c_s} \right)$$

Assumption: H , ϵ and η_\parallel approximately **constant** ($K \equiv k_1 + k_2 + k_3$)

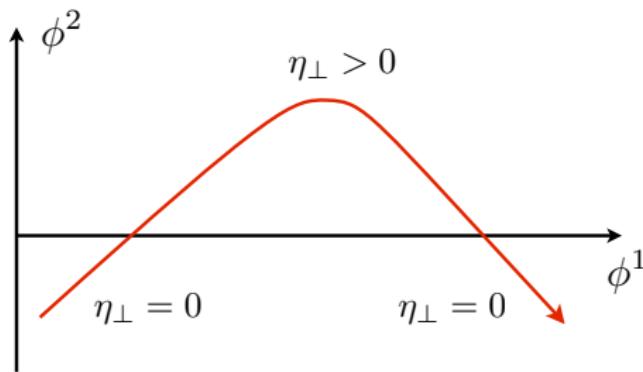
$$\begin{aligned} B_{\mathcal{R}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) &= 2\Re \left\{ 2i\widehat{\mathcal{R}}_{k_1}(0)\widehat{\mathcal{R}}_{k_2}(0)\widehat{\mathcal{R}}_{k_3}(0) \left[3\epsilon \frac{m_{Pl}^2}{H^2} \int_{-\infty}^0 d\tau \frac{u}{\tau^2} \frac{d\widehat{\mathcal{R}}_{k_1}^*(\tau)}{d\tau} \frac{d\widehat{\mathcal{R}}_{k_2}^*(\tau)}{d\tau} \widehat{\mathcal{R}}_{k_3}^*(\tau) + 2 \text{ perm} \right. \right. \\ &\quad \left. \left. + \epsilon \frac{m_{Pl}^2}{H^2} (\mathbf{k}_1 \cdot \mathbf{k}_2 + 2 \text{ perm}) \int_{-\infty}^0 d\tau \frac{u+2s}{\tau^2} \widehat{\mathcal{R}}_{k_1}^*(\tau) \widehat{\mathcal{R}}_{k_2}^*(\tau) \widehat{\mathcal{R}}_{k_3}^*(\tau) \right] \right\} \\ &= \frac{(2\pi)^4 \mathcal{P}_{\mathcal{R}}^2}{(k_1 k_2 k_3)^3} \left[\frac{3}{2} (k_1 k_2)^2 \left\{ \frac{1}{K} \frac{\Delta \mathcal{P}_{\mathcal{R}}}{\mathcal{P}_{\mathcal{R}}} \left(\frac{K}{2k_\star} \right) - k_3 \frac{d}{dk} \left[\frac{1}{k} \frac{\Delta \mathcal{P}_{\mathcal{R}}}{\mathcal{P}_{\mathcal{R}}} \left(\frac{k}{2k_\star} \right) \right] \right\|_{k=K} + 2 \text{ perm} \right. \\ &\quad \left. + \frac{1}{2} (\mathbf{k}_1 \cdot \mathbf{k}_2 + 2 \text{ perm}) \left\{ \frac{K^2 - (k_1 k_2 + 2 \text{ perm})}{K} \frac{\Delta \mathcal{P}_{\mathcal{R}}}{\mathcal{P}_{\mathcal{R}}} \left(\frac{K}{2k_\star} \right) + k_1 k_2 k_3 \frac{d}{dk} \left[\frac{1}{k} \frac{\Delta \mathcal{P}_{\mathcal{R}}}{\mathcal{P}_{\mathcal{R}}} \left(\frac{k}{2k_\star} \right) \right] \right\|_{k=K} \right. \\ &\quad \left. - (k_1 k_2 + 2 \text{ perm}) \frac{d}{dk} \left[\frac{\Delta \mathcal{P}_{\mathcal{R}}}{\mathcal{P}_{\mathcal{R}}} \left(\frac{k}{2k_\star} \right) \right] \right\|_{k=K} + k_1 k_2 k_3 \frac{d^2}{dk^2} \left[\frac{\Delta \mathcal{P}_{\mathcal{R}}}{\mathcal{P}_{\mathcal{R}}} \left(\frac{k}{2k_\star} \right) \right] \right\|_{k=K} \right] \end{aligned}$$

(Achucarro, [JG](#), Palma & Patil, to appear)

Correlation between spectra is manifest!

Modeling curvilinear trajectory

A **cosh turn** in otherwise straight trajectory in 2-field system

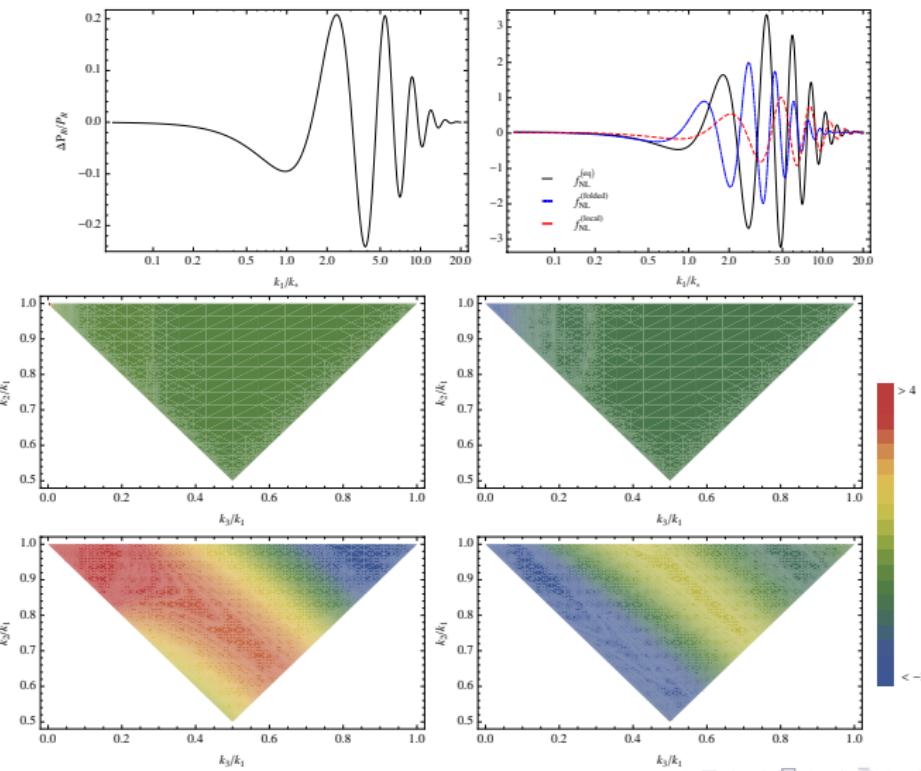


$$\eta_{\perp} = \frac{\dot{\theta}}{H} = \frac{\eta_{\perp,\max}}{\cosh^2 [2(N - N_{\star})/\Delta N]}$$

(Equations of motion: see Achucarro et al. 2011)

Features from smooth curvilinear trajectory

$\eta_{\pm\max}=5$, $\Delta N=0.5$, $H^2/M^2=1/300$



General slow-roll approximation

- $\hat{\mathcal{R}}_k(\tau) = \text{de Sitter piece} + \text{higher order corrections}$
- **No guarantee** for the hierarchy between slow-roll parameters
- Up to 1st order corrections in the standard SR known (Chen et al. 2007)
- Consistent account in more general contexts

Mode equation: $z^2 \equiv 2a^2 m_{Pl}^2 \epsilon, y \equiv \sqrt{2k} z \hat{\mathcal{R}}_k, dx \equiv -k c_s dt/a, f \equiv 2\pi xz/k$

$$\underbrace{\frac{d^2y}{dx^2} + \left(1 - \frac{2}{x^2}\right)y}_{\text{de Sitter solution}} = \underbrace{\frac{1}{x^2} \frac{f'' - 3f'}{f} y}_{\begin{array}{c} \equiv g(\log x) \\ \text{departure from dS} \end{array}} \quad \left(f' \equiv \frac{df}{d\log x}\right) \rightarrow y_0(x) = \left(1 + \frac{i}{x}\right) e^{ix}$$

Green's function solution ([JG](#) & Stewart 2001)

$$\begin{aligned} y(x) &= y_0(x) + \frac{i}{2} \int_x^\infty \frac{du}{u^2} g(\log u) [y_0^*(u)y_0(x) - y_0^*(x)y_0(u)] y(u) \\ &\equiv y_0(x) + L(x, u) y(u) \\ &= y_0(x) + L(x, u) y_0(u) + L(x, u) L(u, v) y_0(v) + \dots \end{aligned}$$

3rd order action reprocessed

$\dot{\mathcal{R}}^3$ and $\dot{\mathcal{R}}^2 \mathcal{R}$: cumbersome to compute with many derivatives

$$\int \dot{\mathcal{R}}^3 \sim \int (\dot{y}_0 + \dot{L}y_0 + L\dot{y}_0 + \dots)^3 \sim \textcircled{S}$$

Using partial int and linear eq to **reduce the # of derivatives**

$$\frac{\delta L}{\delta \mathcal{R}} \Big|_1 \equiv \frac{a^3 \epsilon}{c_s^2} \left\{ \ddot{\mathcal{R}} + \underbrace{\left[\frac{c_s^2}{a^2 \epsilon} \frac{d}{dt} \left(\frac{a^2 \epsilon}{c_s^2} \right) + H \right] \dot{\mathcal{R}} - \frac{\Delta}{a^2} \mathcal{R} }_{\equiv C = H(3+\eta-2s)} \right\}$$

$$\int A \dot{\mathcal{R}}^3 = \int \frac{\ddot{A} - 3\dot{A}C - 2A\dot{C} + 2AC^2}{2} \frac{d(\mathcal{R}^3)}{dt} \Big|_3 + \dots + \frac{\delta L}{\delta \mathcal{R}} \Big|_1 \frac{c_s^2}{a^3 \epsilon} \left(\frac{\ddot{A} - 2AC}{2} \mathcal{R}^2 + \dots \right)$$

$$\int B \dot{\mathcal{R}}^2 \mathcal{R} = \int \frac{-\ddot{B} + BC}{2} \frac{d(\mathcal{R}^3)}{dt} \Big|_3 + \dots + \frac{\delta L}{\delta \mathcal{R}} \Big|_1 \frac{c_s^2}{a^3 \epsilon} \frac{B}{2} \mathcal{R}^2$$

Field redefinition with more terms involved [\(JG, Schalm & Shiu, to appear\)](#)

$$S_3 = \int d\tau d^3x \underbrace{\frac{m_{Pl}^2}{3} \frac{a^2 \epsilon}{c_s} \left[-c_s a H \left(3s + \frac{\epsilon \eta}{2} + \epsilon s + 9us - 2s^2 \right) - \frac{1}{2} \frac{d}{d\tau} \left(\frac{\eta}{c_s^2} \right) \right]}_{\equiv \mathfrak{C}} \frac{d}{d\tau} (\mathcal{R}^3) + \text{(higher SR terms)}$$

1st order bispectrum in GSR

“Source” for the bispectrum

$$g_B(\log \tau) = \frac{c_s}{a^2 m_{Pl}^2 \epsilon} \frac{-\tau}{f} \mathfrak{C} = \frac{1}{f} \left[c_s a H \left(3s + \frac{\epsilon \eta}{2} + \epsilon s + 9us - 2s^2 \right) + \frac{1}{2} \frac{d}{d \log \tau} \left(\frac{\eta}{s} \right) \right]$$

Window functions constructed from homogeneous solution

$$y_0(k_1 \tau) y_0(k_2 \tau) y_0(k_3 \tau) = W_B(k_1, k_2, k_3; \tau) + i X_B(k_1, k_2, k_3; \tau)$$

$$y_0(k_1 \tau) y_0(k_2 \tau) y_0^*(k_3 \tau) = W_B(k_1, k_2, -k_3; \tau) + i X_B(k_1, k_2, -k_3; \tau) \equiv W_{B3} + i X_{B3}$$

Bispectrum up to 1st correction [i.e. $\mathcal{O}(g)$] (c.f. Adshead et al. 2011)

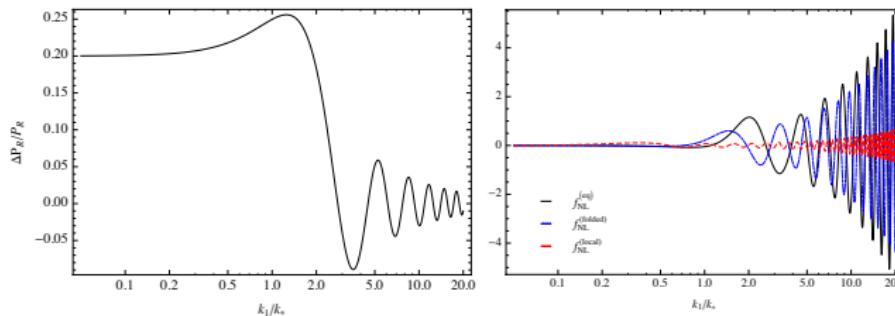
$$\begin{aligned} B_{\mathcal{R}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) &= \frac{(2\pi)^4}{4} \frac{\sqrt{\mathcal{P}_{\mathcal{R}}(k_1)}}{k_1^2} \frac{\sqrt{\mathcal{P}_{\mathcal{R}}(k_2)}}{k_2^2} \frac{\sqrt{\mathcal{P}_{\mathcal{R}}(k_3)}}{k_3^2} \int_0^\infty \frac{d\tau}{\tau} g_B(\log \tau) \\ &\times \left\{ \left(d_\tau - 3 \frac{f'}{f} \right) W_B + \frac{1}{3} d_\tau (X_B + X_{B3}) \int_0^\infty \frac{d\tilde{\tau}}{\tilde{\tau}} g(\log \tilde{\tau}) X(k_3 \tilde{\tau}) + 2 \text{ perm} \right. \\ &- \frac{1}{3} d_\tau W_{B3} \int_\tau^\infty \frac{d\tilde{\tau}}{\tilde{\tau}} g(\log \tilde{\tau}) W(k_3 \tilde{\tau}) - \frac{1}{3} d_\tau X_{B3} \int_0^\tau \frac{d\tilde{\tau}}{\tilde{\tau}} g(\log \tilde{\tau}) X(k_3 \tilde{\tau}) + 2 \text{ perm} \\ &\left. - \frac{1}{2} d_\tau (X_B + X_{B3}) \int_\tau^\infty \frac{d\tilde{\tau}}{\tilde{\tau}} g(\log \tilde{\tau}) \left(\frac{1}{k_3 \tilde{\tau}} + \frac{1}{k_3^3 \tilde{\tau}^3} \right) + 2 \text{ perm} \right\} \quad \left(d_\tau \equiv \frac{d}{d \log \tau} + 3 \right) \end{aligned}$$

Example: Starobinsky model

Starobinsky model: linear $V(\phi)$ + sudden slope change (Starobinsky 1992)

$$V(\phi) = V_0 \times \begin{cases} \left[1 - A(\phi - \phi_0)\right] & \text{for } \phi < \phi_0 \\ \left[1 - (A + \Delta A)(\phi - \phi_0)\right] & \text{for } \phi > \phi_0 \end{cases}$$

de Sitter approx: $\frac{f'}{f} = -\frac{\ddot{\phi}}{H\dot{\phi}}$, $g = -3\frac{V''}{V}$, $g_B = \frac{1}{f} \frac{d}{d\log\tau} \left(\frac{\dot{\phi}}{H\dot{\phi}} \right)$ (Choe, JG & Stewart 2004)



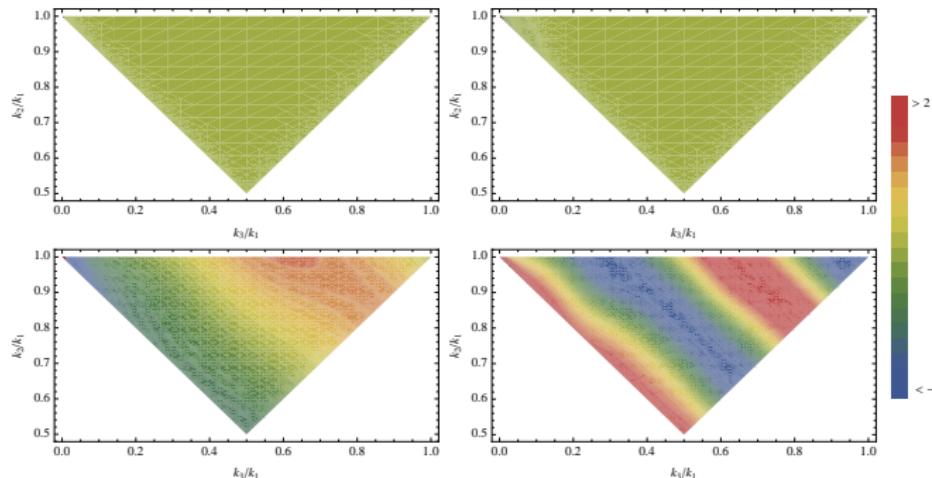
(cf. Arroja & Sasaki 2012)

Example: Starobinsky model

Starobinsky model: linear $V(\phi)$ + sudden slope change (Starobinsky 1992)

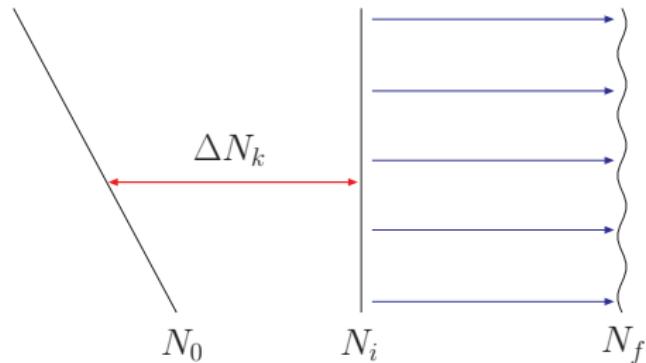
$$V(\phi) = V_0 \times \begin{cases} \left[1 - \textcolor{red}{A}(\phi - \phi_0) \right] & \text{for } \phi < \phi_0 \\ \left[1 - (\textcolor{red}{A} + \Delta A)(\phi - \phi_0) \right] & \text{for } \phi > \phi_0 \end{cases}$$

de Sitter approx: $\frac{f'}{f} = -\frac{\ddot{\phi}}{H\dot{\phi}}$, $g = -3\frac{V''}{V}$, $g_B = \frac{1}{f} \frac{d}{d\log\tau} \left(\frac{\dot{\phi}}{H\dot{\phi}} \right)$ (Choe, JG & Stewart 2004)



(cf. Arroja & Sasaki 2012)

Evolution of field fluctuations



- ① N_i : initial slice (flat) for the δN formalism, $\delta\phi_{\text{flat}}^a \equiv Q^a$
- ② N_f : final slice (comoving) for the δN formalism
- ③ N_0 : horizon crossing of a mode k

$Q^a(N_0)$ = Gaussian $\rightarrow Q^a(N_i = N_0 + \Delta N_k)$ = non-Gaussian

$$\Delta N_k = \log\left(\frac{a_i}{a_0}\right) \approx \log\left[\frac{(aH)_i}{k}\right] \quad \rightarrow \quad k\text{-dependence}$$

Power spectrum and its running

Evolution equation of Q^a on large scales (Elliston, Seery & Tavakol 2012)

$$D_N Q^a = w^a{}_b Q^b + \frac{1}{2} w^a{}_{bc} Q^b Q^c + \dots$$

$$w_{ab} = u_{(a;b)} + \frac{R_{c(ab)d}}{3} \frac{\dot{\phi}_0^c}{H} \frac{\dot{\phi}_0^d}{H} \quad \left(u_a = -\frac{V_{;a}}{3H^2} \right)$$

$$w_{abc} = u_{(a;bc)} + \frac{1}{3} \left[R_{(a|de|b;c)} \frac{\dot{\phi}_0^d}{H} \frac{\dot{\phi}_0^e}{H} - 4 R_{a(bc)d} \frac{\dot{\phi}_0^d}{H} \right]$$

$$Q^a (\textcolor{red}{N_i = N_0 + \Delta N_k}) = Q^a (N_0) + \Delta N_k \left(w^a{}_b Q^b + \frac{1}{2} w^a{}_{bc} Q^b Q^c + \dots \right) + \dots$$

Power spectrum and the spectral index

$$\langle \mathcal{R}_{\mathbf{k}}(t_f) \mathcal{R}_{\mathbf{q}}(t_f) \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k} + \mathbf{q}) \frac{2\pi^2}{k^3} \mathcal{P}_{\mathcal{R}}(\mathbf{k}) = N_a(t_i) N_b(t_i) \langle Q_{\mathbf{k}}^a(t_i) Q_{\mathbf{q}}^b(t_i) \rangle$$

$$\langle Q_{\mathbf{k}}^a(t_i) Q_{\mathbf{q}}^b(t_i) \rangle = \langle Q_{\mathbf{k}}^a(t_0) Q_{\mathbf{q}}^b(t_0) \rangle + 2\Delta N_k w^a{}_c \langle Q_{\mathbf{k}}^b(t_0) Q_{\mathbf{q}}^c(t_0) \rangle$$

$$\langle Q_{\mathbf{k}}^a Q_{\mathbf{q}}^b \rangle = \frac{H^2}{2k^3} \delta^{(3)}(\mathbf{k} + \mathbf{q}) (\gamma^{ab} + \epsilon^{ab})$$

$$n_{\mathcal{R}} - 1 = \frac{D \log \mathcal{P}_{\mathcal{R}}}{d \log k} = -2\epsilon - 2 \frac{N_a N_b w^{ab}}{N_c N^c} \quad (\text{Sasaki \& Stewart 1996})$$

General formula for the running of f_{NL}

$$\begin{aligned} \langle \mathcal{R}_{\mathbf{k}_1}(t_f) \mathcal{R}_{\mathbf{k}_2}(t_f) \mathcal{R}_{\mathbf{k}_3}(t_f) \rangle &= (2\pi)^3 \delta^{(3)}(\mathbf{k}_{123}) B_{\mathcal{R}}(k_1, k_2, k_3) \\ &= N_a N_b N_c \left\langle Q_{\mathbf{k}_1}^a Q_{\mathbf{k}_2}^b Q_{\mathbf{k}_3}^c \right\rangle + \frac{1}{2} \left\{ N_{ab} N_c N_d \left\langle \left[Q^a \star Q^b \right]_{\mathbf{k}_1} Q_{\mathbf{k}_2}^c Q_{\mathbf{k}_3}^d \right\rangle + 2 \text{ perm} \right\} \end{aligned}$$

- ① 1st term: NL evolution between horizon crossing & initial slice

$$\begin{aligned} &N_a(t_i) N_b(t_i) N_c(t_i) \left\langle Q_{\mathbf{k}_1}^a(t_i) Q_{\mathbf{k}_2}^b(t_i) Q_{\mathbf{k}_3}^c(t_i) \right\rangle \\ &= (2\pi)^3 \delta^{(3)}(\mathbf{k}_{123}) N_a(t_i) N_b(t_i) N_c(t_i) \frac{H^4(t_0)}{4k_1^3 k_2^3 k_3^3} w^{abc} (k_1^3 \Delta N_{k_1} + 2 \text{ perm}) \end{aligned}$$

- ② 2nd term: NL evolution between initial & final slices

$$\begin{aligned} &\frac{1}{2} N_{ab}(t_i) N_c(t_i) N_d(t_i) \left\langle \left[Q^a(t_i) \star Q^b(t_i) \right]_{\mathbf{k}_1} Q_{\mathbf{k}_2}^c(t_i) Q_{\mathbf{k}_3}^d(t_i) \right\rangle \\ &= (2\pi)^3 \delta^{(3)}(\mathbf{k}_{123}) N_{ab}(t_i) N_c(t_i) N_d(t_i) \frac{H^4(t_0)}{4k_1^3 k_2^3} (\gamma^{ac} \gamma^{bd} + 2 \Delta N_{k_1} w^{ac} \gamma^{bd} + 2 \Delta N_{k_2} \gamma^{ac} w^{bd}) \end{aligned}$$

$$\frac{6}{5} f_{\text{NL}} = \frac{N_{ab} N^a N^b}{(N_c N^c)^2} \underbrace{\left\{ 1 - \Delta N_k \left[-\frac{N_a N_b N_c w^{abc}}{N_{de} N^d N^e} + 4 w^{ab} \left(\frac{N_a N_b}{N_d N^d} - \frac{N_{ac} N_b N^c}{N_{de} N^d N^e} \right) \right] \right\}}_{\equiv n_{f_{\text{NL}}}}$$

Summary

- General single field inflation
 - ① From multi-field setup: by integrating out heavy field
 - ② Non-trivial c_s : footprint of heavy physics
- Features in the power spectrum ($S_{2,int}$) and bispectrum (S_3)
 - ① Heavy quanta extract kinetic energy
 - ② Non-trivial, oscillatory, correlated features
- General slow-roll scheme
 - ① Terms with more derivatives → field redefinition
 - ② More complete 1st order bispectrum
- Running of f_{NL}
 - ① Sensitive probe of early universe physics
 - ② Non-trivial evolution after horizon crossing

WE WANT YOU!!!

APCTP cosmology groups (Ki Young Choi, [JG](#) and Arman Shafieloo) are looking for **up to 5 post-doctoral fellows** to start in 2013.

CV, list of publications, research statements + 3 reference letters to
cosmopd2012@apctp.org

For further information, please consult APCTP homepage
(<http://www.apctp.org>).