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Self-gravity effects of blackfold

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Abstract

Blackfold approach can be used as a perturbative method to construct higher dimensional black hole solutions. We develop this blackfold approach to higher order and investigate the physical effects of higher order solutions such as self-gravity effects.

1 Introduction

The blackfold approach has been at first proposed as the worldvolume theory for the dynamical black branes by Emparan et. al [1], and it is supposed that the blackfold approach can be also used to construct the higher dimensional black hole solutions perturbatively. The construction of the higher dimensional black hole by the blackfold approach is based on the fact that the black hole solutions in $d \geq 6$ dimensions can have two different scales, $R \gg r_0$. Here R is the largest curvature scale of the black hole and r_0 is a thickness of the black holes. The black hole with the large angular momentum can have much bigger curvature R than its horizon thickness of r_0. Thus we can expect that the blackfold represents the black holes with large angular momentum in higher dimensions. From this speculation, Emparan et.al. constructed the higher dimensional black ring solutions with a large angular momentum perturbatively [2] and discussed phase diagrams of the various higher dimensional black holes [3]. To obtain further insight into the higher dimensional black holes, we should investigate the blackfold approach in detail and develop to higher order perturbation solutions.

In the blackfold approach, black hole solutions are constructed by matching two perturbation solutions in two different regions, far and near region. In Ref. [2], the Newtonian solutions corresponding to higher dimensional black ring was constructed and computed the first order correction to the metric in the near region by matching the Newtonian solution. In Ref [3], the Newtonian solutions corresponding to the various higher dimensional black holes were constructed and phase diagram of each solutions were proposed. In these researches, the properties of the blackfold were derived from the linear perturbation solutions. But, the effect of its backreaction has not been considered. In this sense it seems to be fair to say that we have not obtained the definite properties of higher dimensional black holes and not understood the essential property of them. In this paper we consider backreaction effects, that is, self-gravity effects, of the blackfold and investigate the properties of the higher dimensional black holes more accurately.

To study the self-gravity effects precisely, we must solve the higher order perturbation equation. The linear solution has the singular behavior on the source. The higher order perturbation equation contains the linear order solution in the source term. Thus, to solve the higher order perturbation equation, we should perform the appropriate regularization and specify the proper boundary conditions on the source. We use the technique of matched asymptotic expansion (MAE) to treat this problem. The MAE was used to compute the first correction to the metric around the near region in the original paper of the blackfold [2]. In the MAE method, we can calculate the self-gravity effects of the blackfold. The purpose of this paper is to clarify the matching ladder structure of the blackfold approach and compute the self-gravity effects.

2 Matching ladder structure

The blackfold has two separated scales $R \gg r_0$. Thus there are two different regimes of the geometry of the blackfold. In the far region $r \gg r_0$, the geometry of the blackfold can be well approximated with

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the perturbations by the energy-momentum tensor of the blackfold around the flat spacetimes. In the near region $R \gg r$, we can regard the geometry as the perturbation around the black p-branes. These two perturbation solutions are matched in the matching region $R \gg r \gg r_0$ and we obtain the geometry produced by the blackfold. In the MAE method, we can obtain the self-gravity corrections not only to the geometry in the far region, but also to the metric in the near region. Thus, we can check the regularity of the perturbative black hole solution including the self-gravity corrections. In this paper, we consider the blackfold which describes the d dimensional black ring solution. It is a simple task to extend our analysis to other blackfold solutions.

To clarify the matching ladder structure, we should study solutions in far and near region. In the far region, the geometry can be treated as the Newton theory and the corresponding Newton potential is produced by the energy-momentum tensor of the blackfold. In the black ring case, we find the solution at far region as

$$ds^2 = (-1+\Psi)dt^2 - 2Adtd\phi + \left(1+\frac{\Psi}{d-3}\right)ds_E^2, \tag{1}$$

where ds_E^2 is the ring coordinate as

$$ds_E^2 = dr_1^2 + r_1^2 d\Omega_{d-3}^2 + dr_2^2 + r_2^2 d\phi^2. \tag{2}$$

Ψ and A can be written as

$$\Psi = Rr_0^{d-4}\int_0^{2\pi}\frac{d\phi}{(r_1^2+(R\cos\phi - r_2)^2 + R^2\sin^2\phi)^{d-3/2}}, \tag{3}$$

and

$$A = \frac{Rr_0^{d-4}}{\sqrt{d-3}}\int_0^{2\pi}\frac{r_2\cos\phi d\phi}{(r_1^2+(R\cos\phi - r_2)^2 + R^2\sin^2\phi)^{d-3/2}}. \tag{4}$$

This Newton solution is produced by the following energy-momentum tensor

$$T_{tt} = \frac{d-2}{16\pi G}Rr_0^{d-4}\delta(r)\,,\, T_{t\phi} = \frac{\sqrt{d-3}}{16\pi G}R^2 r_0^{d-4}\delta(r). \tag{5}$$

$r = 0$ represents the location of a blackfold. This energy-momentum tensor comes from Brown-York energy-momentum tensor of boosted black string. A boost parameter is determined by the blackfold equation.

This far region solution determines boundary conditions on the perturbation at near region. The geometry at near region is described by perturbations around $d-1$ dimensional boosted black string. The perturbation around this black string can be decomposed by spherical harmonics on S^{d-3}. By expanding te far region solution Eq. (1) at matching region $R \gg r \gg r_0$, we can see that the expansion parameter is r_0/R and the l mode perturbation with respect to spherical harmonics on S^{d-1} has $(r_0/R)^l$ order. Therefore the near region geometry $g_{\mu\nu}^{\rm near}$ can be represented as

$$g_{\mu\nu}^{\rm near} = g_{\mu\nu}^{\rm string} + h_{\mu\nu}, \tag{6}$$

where $g_{\mu\nu}^{\rm string}$ is a metric of boosted black string, background geometry. $h_{\mu\nu}$ is a perturbation metric which can be written as

$$h_{\mu\nu} = \sum_l \left(\frac{r_0}{R}\right)^l Y^l C_{\mu\nu}^l, \tag{7}$$

where Y^l is a spherical harmonics on S^{d-3}. The amplitude of the perturbation $C_{\mu\nu}^l$ is determined by the far region solution. Furthermore, using this corrected near region geometry as a boundary condition, we can compute the correction to the far region solution Eq. (1). This matching structure can be shown in Figure.1 up to self-gravity order. Here we call the correction to mass, angular momentum and area of black ring self-gravity effects. These corrections can be read from the corrections to Brown-York energy momentum tensor. As seen in Fig.1, there are two contribution to the self-gravity corrections. One is coming from $l = 0$ mode perturbation on black string. Another is from the non linear contribution of $l > 0$ mode perturbation. For instance, in $d = 6$ dimensions case, self-gravity corrections have $(r_0/R)^2$ order. $l = 1$ mode perturbation has r_0/R order and its quadratic contribution can be source of $l = 0$ mode perturbation with $(r_0/R)^2$ order.

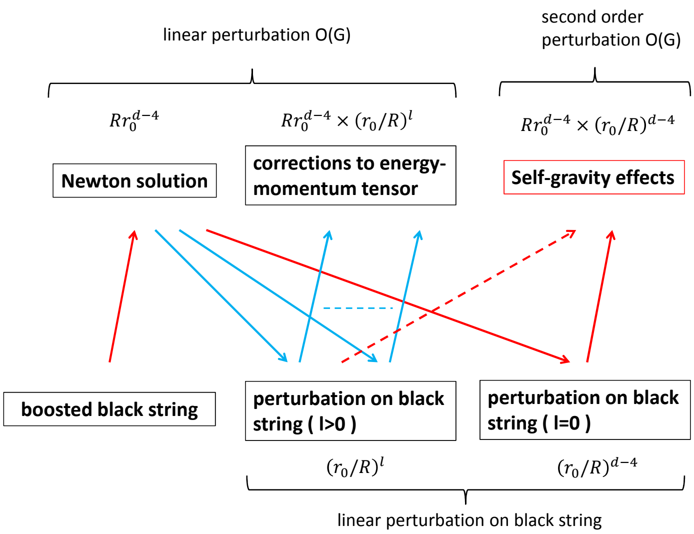

Figure 1: The matching ladder structure up to self-gravity order is shown. The red line represents the ladder of $l = 0$ mode perturbation. The blue line corresponds to $l > 0$ mode perturbation matching. The dashed red line describes the non linear contribution of $l > 0$ mode perturbation to $l = 0$ mode perturbation.

3 Self-gravity effects

To see self-gravity effects of blackfold, we study the second order solution at far region. The Newton solution Eq. (1) is the first order solution. The metric $g^{\rm far}_{\mu\nu}$ at far region can be written up to the second order as

$$g^{\rm far}_{\mu\nu} = \eta_{\mu\nu} + h^{(1)}_{\mu\nu} + h^{(2)}_{\mu\nu}, \tag{8}$$

where $\eta_{\mu\nu}$ is a flat metric and $h^{(1)}_{\mu\nu}$ is the first order solution. $\eta_{\mu\nu} + h^{(1)}_{\mu\nu}$ corresponds to the Newton solution Eq. (1). Then, the equation for the second order metric $h^{(2)}_{\mu\nu}$ can be derived from the Einstein equation as

$$\begin{aligned}\frac{1}{2}\Delta h^{(2)}_{\mu\nu} &= \frac{1}{2}h^{(1)\rho\sigma}\nabla_\mu\nabla_\nu h^{(1)}_{\rho\sigma} + \frac{1}{4}\nabla_\mu h^{(1)\rho\sigma}\nabla_\nu h^{(1)}_{\rho\sigma} + \frac{1}{2}(\nabla^\rho h^{(1)\sigma}{}_\mu - \nabla^\sigma h^{(1)\rho}{}_\mu)\nabla_\rho h^{(1)}_{\sigma\nu} \\ &\quad - \frac{1}{2}h^{(1)\rho\sigma}(\nabla_\rho\nabla_\mu h^{(1)}_{\sigma\nu} + \nabla_\rho\nabla_\nu h^{(1)}_{\sigma\mu} - \nabla_\rho\nabla_\sigma h^{(1)}_{\mu\nu}),\end{aligned} \tag{9}$$

where ∇_μ is a covariant derivative with respect to $\eta_{\mu\nu}$. Solving this second order equation, we can obtain the corrections to the mass, angular momentum and are of black ring solutions. The boundary conditions for the second order solutions can be read from the perturbed metric at near region (see Fig. 1). It is useful for understanding the physical meaning of corrections to draw the phase diagram of black ring solutions. To do this, we define the normalized area s and angular j momentum by the mass as

$$j^{d-3} = \frac{\Omega_{d-3}}{2^{d+1}}\frac{(d-2)^{d-2}}{(d-3)^{d-3/2}}\frac{J^{d-3}}{GM^{d-2}}, \tag{10}$$

$$s^{d-3} = \frac{\Omega_{d-3}}{2(16\pi)^{d-3}}(d-2)^{d-2}\left(\frac{d-4}{d-3}\right)^{d-3/2}\frac{S^{d-3}}{(GM)^{d-2}}. \tag{11}$$

Here M, J and S is a mass, angular momentum and are of black ring solution respectively.

In $d = 6$ dimensional case, the phase diagram of black ring up to the second order solutions is described in Fig. 2. Compared with the first order solutions, the area of the second order solutions is

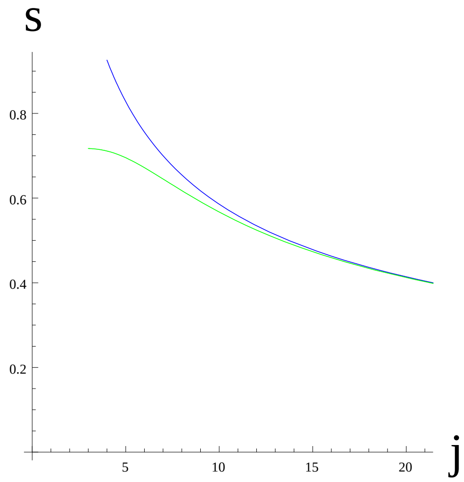

Figure 2: The blue line is the phase diagram up to first order, and the green line represents the phase diagram up to the second order solutions. This diagram shows the self-gravity effects of blackfold works attractively and its form is shrinking.

decreasing. This is because the self-gravity effects work attractively and the solutions becomes shrinking. This behavior does not depend on its dimensions, and in more higher dimensions $d > 6$, same behavior of the second order solutions can be confirmed.

4 Summary

The blackfold approach is very useful approach to investigate the physical properties of higher dimensional black holes. To develop the idea of blackfold, we should understand more precisely how black hole solutions can be constructed by this approach. In this paper, we clarify the matching ladder structure of the blackfold approach. Using this matching ladder, self-gravity effects of black ring solutions can be computed, and its attractiveness is shown explicitly.

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