Ivan Arraut, JGRG 22(2012)111221

"Dark matter and dark energy as a single manifestation of a fundamental length scale"

RESCEU SYMPOSIUM ON

GENERAL RELATIVITY AND GRAVITATION

JGRG 22

November 12-16 2012

Koshiba Hall, The University of Tokyo, Hongo, Tokyo, Japan

Dark Matter and Dark Energy as a possible manifestation of a fundamental scale.

Ivan Arraut

(Osaka University)

Motivation

Question in GR:

1). Why is the cosmological constant so small?

2). Why the velocity of rotation in galaxies is bigger than what is normally expected and what about the Gravitational Lenses?

→ The project

- Find some intermediate interesting scales.
- Analyze the importance of such scales.
- Find the Einstein's equations in a new formalism (in process, 3 different paths).
- For now the evolution of the parameters is not my bussiness.

Standard GR with Λ

- The Einstein's eqs. with a Cosmological Constant.

$$R_{\mu\nu} = -8\pi G S_{\mu\nu} - \Lambda g_{\mu\nu}$$

•The vacuum solution:

$$ds^2 = -e^{\nu(r)}dt^2 + e^{-\nu(r)}dr^2 + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2$$

$$e^{\nu(r)} = 1 - \frac{2r_s}{r} - \frac{r^2}{3r_\Lambda^2}$$

•With an effective potential:

$$U_{eff}(r) = -\frac{r_s}{r} - \frac{1}{6}\frac{r^2}{r_\Lambda^2} + \frac{r_l^2}{2r^2} - \frac{r_s r_l^2}{r^3}$$

$$U_{eff}(r) \approx -\frac{r_s}{r} - \frac{r^2}{6r_\Lambda^2} + \frac{r_l^2}{2r^2}$$

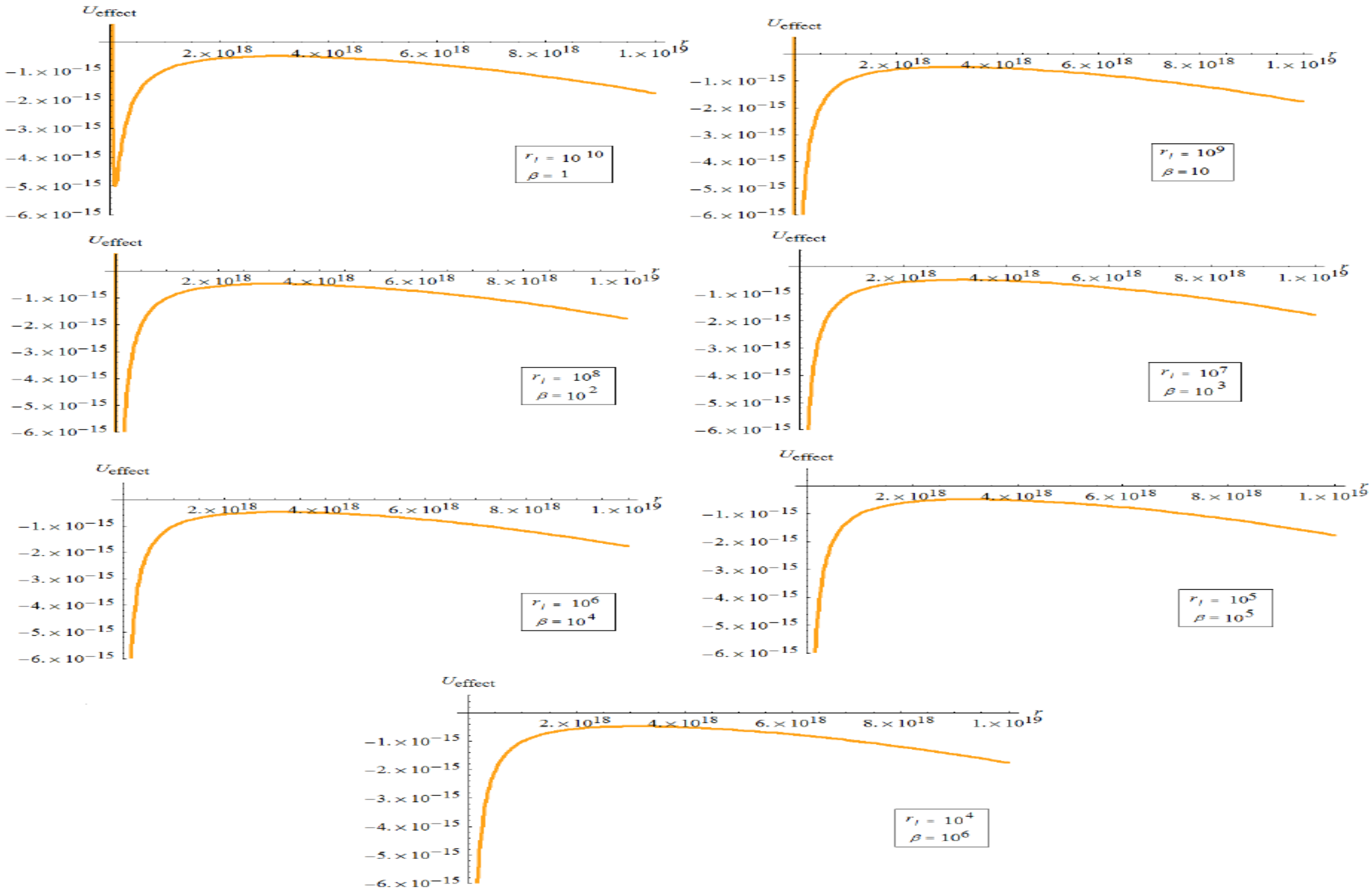


Figure 2.3: Effective potential for values of the distance near to the final critical point and $r_l >> r_s$.

•Special scale:

$$r_g = \left(\frac{3}{2} r_s r_\Lambda^2\right)^{1/3}$$

•The same scale used by Bousso and Hawking for the time-like Killing vector normalization (Hawking radiation). This is not a new scale at all!!!

•What about Dark Matter effects?
Absent until now. What can we do?

•MOND has something to tell us:
The fundamental scale in MOND is:

$$a_0^2 \sim \Lambda,$$

•Only one fit parameter.

•One interesting prediction of MOND:

•Low surface brightness galaxies shows higher discrepancy in the mass content.

→ Problems with MOND

- Violation of energy-momentum conservation.
- It does not predict gravitational lenses effects.
- Problems with the cluster of galaxies.

•What can we do?

→ Possible Solution

- Relativistic version derivable from an action principle (many candidates).
- Among the possibilities we have:

A). Non-localities which can screen the Cosmological Constant but also can create Dark Matter effects. (Sasaki and colleagues. Mashoon).

B). But locality is relative in agreement with Amelino-Camelia, Lee Smolin, Freidel and Kowalzky. Should we pay attention to it?

C). But if there is Relative Locality, there should be Relative Co-locality which is a manifestation of the spacetime curvature (Arraut 2012, paper to be submitted soon).

D). Another path can be taken with gravity as a gauge theory (in process).

- The MOND formula is:

$$\tilde{\mu}(|\mathbf{a}|/a_0)\mathbf{a} = \mathbf{g}_N.$$

$$\tilde{\mu}(x) = \frac{x}{1+x}.$$

- In the full MONDIAN regime, we have:

$$\frac{GM}{r^2} - \frac{1}{3}\frac{r}{r_\Lambda^2} = \sqrt{\frac{GM}{r_\Lambda}}\frac{1}{r}$$

•I am including the repulsive effect due to Λ in order to find the bound for the rotation curve.

•We can translate the problem. The MOND fit parameter is equivalent to say that the Dark matter scale is:

$$r_* \approx (GMr_\Lambda)^{1/2} = \left(\frac{r_s r_\Lambda}{2}\right)^{1/2}$$

•Additionally, it is equivalent to say that the Tully-Fisher law in its mass version is valid:

$$V_c^2 = \left(\frac{GM}{r_\Lambda}\right)^{1/2} - \frac{1}{3}\left(\frac{GM}{r_\Lambda}\right) \quad \Rightarrow \quad v^4 \sim M$$

$$\Downarrow$$

$$r_{max} = \left(\frac{3}{2} r_s r_\Lambda^2\right)^{1/3}$$

•If: $r_s << r_\Lambda$

•What I propose: Is there any formalism where you can introduce at least 2 scales, one very large and another very small, such that those scales become dual? Rta: q-Bargmann Fock (Quantum groups).

•In the simplest case, the Bosonic algebra is deformed in agreement with:

$$\bar{\eta} := \frac{1}{2L}x - \frac{i}{2K}p \qquad \partial_{\bar{\eta}} := \frac{1}{2L}x + \frac{i}{2K}p$$

⇓

$$[x,p] = i\hbar + i\hbar(q^2-1)\left(\frac{x^2}{4L^2} + \frac{p^2}{4K^2}\right)$$

⇓

$$\Delta X \Delta P \geq \frac{\hbar}{2}\left(1 + (q^2-1)\left(\frac{(\Delta X)^2}{4L^2} + \frac{(\Delta P)^2}{4K^2}\right)\right)$$

•If we impose as an UV cut-off the Planck scale and as an IR cut-off the Cosmological Constant scale, then:

Minimum, maximum and mix scale

- The minimum scales in this model is: Kempf 1994.

$$\Delta X_{min} = L\sqrt{1-q^{-2}} \qquad \Delta P_{min} = K\sqrt{1-q^{-2}}$$

$$q \approx 1 + \frac{l_{pl}}{r_\Lambda} \qquad KL = \frac{(q^2+1)\hbar}{4}$$

$$f(\Delta X, \Delta P) := \Delta X \Delta P - \frac{\hbar}{2}\left(1 + (q^2-1)\left(\frac{(\Delta X)^2 + <X>^2}{4L^2} + \frac{(\Delta P)^2 + <P>^2}{4K^2}\right)\right)$$

- The total extremal condition is:

$$df(\Delta X, \Delta P) = 0 \qquad \Delta P \approx \frac{1}{(l_{pl} r_\Lambda)^{1/2}} = \frac{1}{l_0} \qquad \Delta X \approx (l_{pl} r_\Lambda)^{1/2} = l_0$$

- IR-UV mix scale.

The same but taking into account Relative Locality.

$$q \approx 1 + \frac{1}{r_\Lambda}\sqrt{\frac{\hbar|x|}{m_{pl}c}} \quad \Rightarrow \quad \Delta P_{mix} \approx \left(\frac{\hbar}{2}\right)^{1/2}\left(\frac{\hbar m_{pl}c}{|x|r_\Lambda^2}\right)^{1/4}$$

$$\Delta X_{mix} \approx \left(\frac{\hbar}{2}\right)^{1/2}\left(\frac{|x|r_\Lambda^2}{\hbar m_{pl}c}\right)^{1/4}$$

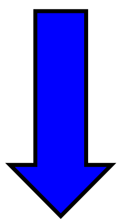
- Extending the relative locality to the momentum space (the fundamental scale in momentum space taken as Λ:

$$\Delta X_{min} = L\sqrt{\frac{q^2-1}{q^2}} \approx \sqrt{\frac{\hbar|x|}{m_{pl}c}} \quad \Leftrightarrow \quad \Delta P_{min} = K\sqrt{\frac{q^2-1}{q^2}} \approx \sqrt{\frac{\hbar|p|}{r_\Lambda}}$$

Relative Co-locality (extending the Majid ideas).

$$q \approx 1 + \sqrt{\frac{|p||x|}{r_\Lambda m_{pl} c}} + \ldots$$

$$\Delta P_{mix} \approx \left(\frac{\hbar}{2}\right)^{1/2} \left(\frac{|p| m_{pl} c}{|x| r_\Lambda}\right)^{1/4}$$



$$\Delta X_{mix} \approx \left(\frac{\hbar}{2}\right)^{1/2} \left(\frac{|x| r_\Lambda}{|p| m_{pl} c}\right)^{1/4}$$

•The correction to the observed momentum looks like the Tully Fisher law explained before. Can be Dark Matter only an UV-IR mix effect when we extend the Relativity principle to the phase space with a minimum scale in position and momentum?

Open problem and Conclusions.

- Still we have to verify if this is in reality a manifestation of the Tully-Fisher law and not a mere coincidence.
- Nex Step (in process). Derive the Einstein’s equations inside this formalism. You should obtain: The standard GR with a Cosmological Constant + some contribution for DM due to UV-IR effect. For now it looks promising.

Acknowledgements

- 1). Lee Smolin for the bibliograpy suggestions and discussion.
- 2). Laurent Freidel for the discussion at the P.I about Relative Locality.
- 3). The friends of the Yukawa Institute, including Prof. Sasaki, Prof. Shiromizu for the kind weekly discussion about different topics.
- 4). Achim Kempf for his suggestions during the conference in Taipei “Quantum Horizons”.
- 5). Hagen Kleinert, for the additional submitted references. He has a different point of view.