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Research of the celestial objects by gravitational lensing

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Abstract

We will suggest two methods to detect Ellis wormholes which is an example of traversable wormholes of the Morris-Thorne class the with Einstein ring systems and with the double images. We show that, given the configuration of the gravitational lensing and the radii of the Einstein ring and relativistic Einstein rings, we can distinguish between a black hole and Ellis wormhole in principle. We also discuss the signed magnification sums of general spherical lens models including the singular isothermal sphere, the Schwarzschild lens and the Ellis wormhole. We show that the signed magnification sums are a very useful tool to distinguish exotic lens objects.

1 Comparison between wormholes and black holes with their Einstein ring and relativistic Einstein rings

The Ellis spacetime was investigated as a geodesically complete particle model by Ellis [1] and turned out to describe a wormhole connecting two Minkowski spacetimes. The Ellis wormhole spacetime is a static, spherically symmetric, asymptotically flat solution of the Einstein equation with a massless scalar field with a wrong sign as a matter field. Although such a matter field violates energy conditions, it could represent the negative energy density from the quantum effects, such as the Casimir effect. This spacetime is a typical and simplest example of wormholes proposed by Morris and Thorne [2, 3]. This is a traversable wormhole in the sense that an observer can cross this wormhole in both directions.

The deflection angle of light in the Ellis wormhole geometry was studied by Chetouani and Clement [4] and recently Nakajima and Asada [5]. The gravitational lensing on the Ellis geometry was studied by Dey and Sen [6], Abe [7] and Toki *et al.* [8] in the weak gravitational field and Perlick [9], Nandi *et al.* [10] and Tejeiro and Larranaga [11] in the strong gravitational field (see Virbhadra and Keeton [12], Virbhadra [13], Bozza [14], Bozza and Mancini [15] and references therein for the strong field limit).

Recently, Abe suggests to detect the Ellis wormholes with the light curves [7] and Toki *et al.* suggest a method to detect the Ellis wormholes with the astrometric image centroid displacements [8]. We will suggest two methods to detect them with Einstein ring systems [16] and with double images [17]. In this section, we consider the Einstein ring system.

The line element in the Ellis wormhole solution is given by

$$ds^2 = -dt^2 + dr^2 + (r^2 + a^2)(d\theta^2 + \sin^2\theta d\phi^2), \tag{1}$$

where a is a positive constant corresponding the radius of the wormhole throat at $r = 0$. The photon is scattered if $|b| > a$, while reaches the throat if $|b| \le a$, where b is the impact parameter of the photon. Since we are interested in the scattering problem, we assume $|b| > a$. Chetouani and Clement [4] derived the exact deflection angle α of light on the Ellis wormhole geometry as follows:

$$\alpha = 2K\left(\frac{a}{b}\right) - \pi, \tag{2}$$

where K is the complete elliptic integral of the first kind. The deflection angle is diverging in the limit $|b| \to a$, while it is approximately given in the weak-field regime $|b| \gg a$ by $\alpha \approx \pm\frac{\pi}{4}\left(\frac{a}{b}\right)^2$.

Now we will consider the case that both the observer and the source object are far from the lensing object, or $D_l \gg b$ and $D_{ls} \gg b$, where D_l and D_{ls} are the separations between the observer and lens and between the lens and source, respectively. The configuration of the gravitational lensing is given in FIG 1.

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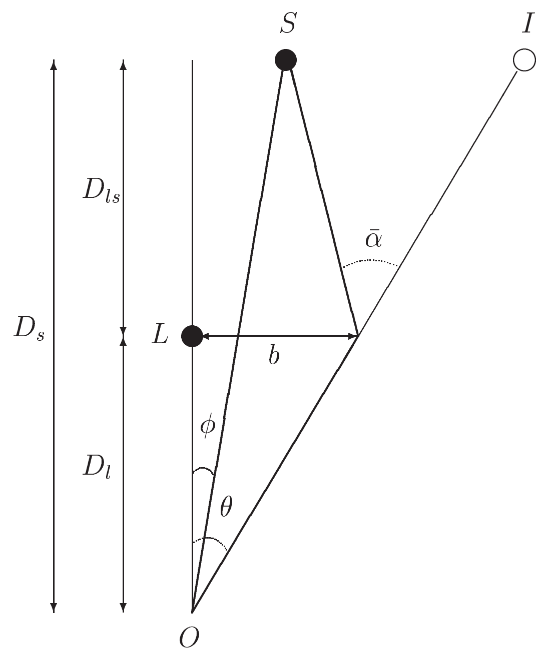

Figure 1: The configuration of the gravitational lensing. The light rays emitted by the source S are deflected by the lens L (a wormhole or a black hole) and reach the observer O with the angle of the lensed image θ, instead of the real angle ϕ. b and $\bar{\alpha}$ are the impact parameter and the effective deflection angle, respectively. D_l and D_{ls} are the separations between the observer and the lens and between the lens and the source, respectively.

The lens equation is given by

$$D_{ls}\bar{\alpha} = D_s(\theta - \phi), \tag{3}$$

where $\bar{\alpha} = (\alpha \bmod 2\pi)$ is the effective deflection angle, θ and ϕ are the angles of the lensed image and the real image from the observer, respectively, and $D_s = D_l + D_{ls}$ is the separation between the observer and source. Note that we have assumed $|\bar{\alpha}| \ll 1$, $|\theta| \ll 1$ and $|\phi| \ll 1$. The deflection angle can be expressed $\alpha = \bar{\alpha} + 2\pi N$, where N is a non-negative integer, denoting the winding number of the light ray. The ring image corresponds to the image angle θ for vanishing real angle $\phi = 0$. By the symmetry, the image is necessarily a ring with the diameter angle θ.

Since $b = D_l\theta$, we find that the ring image is given by $\theta_N = a/(D_l k_N)$, where $k_N \in (0, 1)$ is a unique root of the transcendental equation

$$2K(k) - \frac{\eta}{k} = (2n+1)\,\pi, \tag{4}$$

where $\eta = (D_s a)/(D_l D_{ls})$. We should note that $2K(k) - \eta/k$ is monotonically increasing with respect to k and changes from $-\infty$ to ∞ as k increases from 0 to 1. The uniqueness of the root follows from the monotonicity. Moreover, we can conclude that k_N monotonically increases and approaches 1 as $N \to \infty$ and hence the image angle θ_N monotonically decreases and approaches a/D_l.

In the weak-field regime $|b| \gg a$, the winding number N should be $N = 0$. Using the deflection angle $\alpha \approx \pm\frac{\pi}{4}\left(\frac{a}{b}\right)^2$, we can solve the equation (4) approximately and get the diameter angle of the Einstein ring

$$\theta_0 \simeq \left(\frac{\pi}{4}\frac{D_{ls}}{D_s D_l^2}a^2\right)^{\frac{1}{3}} \simeq 2.0\,\text{arcsecond}\left(\frac{D_{ls}}{10\text{Mpc}}\right)^{\frac{1}{3}}\left(\frac{20\text{Mpc}}{D_s}\right)^{\frac{1}{3}}\left(\frac{10\text{Mpc}}{D_l}\right)^{\frac{2}{3}}\left(\frac{a}{0.5\text{pc}}\right)^{\frac{2}{3}}. \tag{5}$$

This approximation is good for $D_l \gg a$ and $D_{ls} \gg a$. The relative error is $\sim 10^{-2}$ for $a = 0.5$pc and $D_l = D_{ls} = 10$ Mpc.

In the especially strong-field regime, where the winding number N becomes $N \geq 1$, we can easily check that $a \simeq b$ or $k_N \simeq 1$ satisfies the transcendental equation (4) in numerical calculations. Physically this means that the light rays which wind around the wormhole nearly on the photon sphere make the relativistic Einstein rings [9, 18]. Then the diameter angles of the relativistic Einstein rings are approximately given by

$$\theta_{N\geq 1} \simeq \frac{a}{D_l} \simeq 1.0 \times 10^{-2}\,\text{arcsecond} \left(\frac{10\text{Mpc}}{D_l}\right)\left(\frac{a}{0.5\text{pc}}\right). \tag{6}$$

If we are given the distance D_s to the source from the observer, the distance D_l to the lens from the observer and the radius θ_0 of the Einstein ring, we can determine the radius of the throat a from Eq. (5). Then, we can use $\theta_{N\geq 1}$ (6) to test the assumption that the lens object is a wormhole. From Eqs. (5) and (6) we obtain the relation between θ_0 and $\theta_{N\geq 1}$ by

$$\theta_{N\geq 1} \simeq \left(\frac{4}{\pi}\frac{D_s}{D_{ls}}\right)^{\frac{1}{2}} \theta_0^{\frac{3}{2}}. \tag{7}$$

This relation generally holds in astrophysical situations, as long as $a \ll D_l$ and $a \ll D_{ls}$ are satisfied.

The relation between θ_0 and $\theta_{N\geq 1}$ for the Schwarzschild spacetime (, see [19, 20, 21],)

$$\theta_{N\geq 1} \simeq \frac{3\sqrt{3}}{4}\frac{D_s}{D_{ls}}\theta_0^2 \tag{8}$$

is different from that on the Ellis spacetime (7). FIG. 2 shows the angle of the relativistic Einstein ring $\theta_{N\geq 1}$ versus the angle of the Einstein ring θ_0 for $D_l = D_{ls} = 10$Mpc. Thus, we can distinguish between black holes and wormholes in principle if we are given D_s/D_l, θ_0 and $\theta_{N\geq 1}$. We consider the experimental

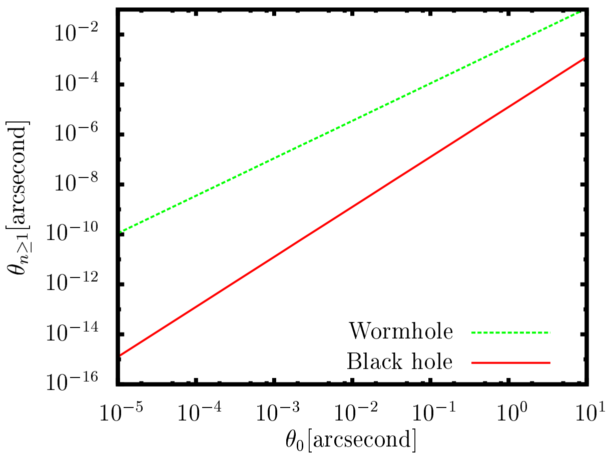

Figure 2: The angle of the relativistic Einstein ring $\theta_{N\geq 1}$ versus the angle of the Einstein ring θ_0 for $D_l = D_{ls} = 10$Mpc. The broken (green) and solid (red) lines plot the cases where the lens objects are a wormhole and a black hole, respectively.

situation where we know the separation D_s between the observer and the source and the separation D_l between the observer and the lens. We assume that we do not know whether the lens object is a black bole or a wormhole and do not its parameter, i.e., the mass M or the radius a of the throat in advance.

We need at least two observable quantities to determine whether the lens object is a black hole or wormhole since the lens system has one parameter in this situation. First, we observe an Einstein ring and determine the parameter for both possibilities. Second, we observe relativistic Einstein rings and tell the wormhole from the black hole. If the predicted relativistic ring angles by the black hole and by the wormhole were of similar size, we could not discern the difference. However, Eqs. (7), (8) and Fig. 2 show that we do not confuse them.

We conclude that we can detect the relativistic Einstein rings by wormholes which have $a \simeq 0.5$pc at a galactic center with the distance $D_l = D_{ls} = 10$Mpc and which have $a \simeq 10$AU in our galaxy with the distance $D_l = D_{ls} = 10$kpc using the most powerful modern instruments which have the resolution of 10^{-2}arcsecond such as a 10-meter optical-infrared telescope. Note that the corresponding black holes which have the Einstein rings of the same size are galactic supermassive black holes with $10^{10} M_{\odot}$ and $10^7 M_{\odot}$, respectively, and that the relativistic Einstein rings by these black holes are too small to measure with the current technology.

2 The signed magnification sums of the general spherical lens

In this section, we will show that the signed magnification sum would be a powerful tool to research the lens objects as well as the total magnification and the magnification ratio if we observe a multiple image. In particular, we will show that one can distinguish between the Ellis wormhole lens and the Schwarzschild lens with the signed magnification sums.

We consider the general spherical lens model with the deflection angle in the weak field approximation, parametrized by

$$\alpha = \pm C b^{-n} = \pm \frac{C}{D_l^n} \theta^{-n}, \tag{9}$$

where C is a positive constant and n is a non-negative integer and we have used the relation $b = D_l \theta$. If n is odd, then the sign is only the upper one, while if n is even, then the sign is the upper one for $\theta > 0$ and the lower one for $\theta < 0$. Thus, we have to treat two lens equations when n is even. This lens model describes the singular isothermal sphere, the Schwarzschild lens and the Ellis wormhole for $n = 0$, 1 and 2, respectively. The case where $n \geq 3$ would describe some exotic lens objects and the gravitational lens effect of modified gravitational theories. We do not consider the case $n = 0$ below for simplification. The following discussion does not depend on the value of C.

The lens equation is given by

$$\hat{\theta}^{n+1} - \hat{\phi}\hat{\theta}^n \mp 1 = 0, \tag{10}$$

where $\hat{\theta} \equiv \theta/\theta_0$ and $\hat{\phi} \equiv \phi/\theta_0$ and $\theta_0 \equiv \left(\frac{D_{ls}C}{D_s D_l^n}\right)^{\frac{1}{n+1}}$ is the Einstein ring angle. The lens equation (10) has symmetry with respect to the point $\hat{\phi} = \hat{\theta} = 0$, We can concentrate ourselves on the case where the source angle ϕ is positive for symmetry. The solutions $\hat{\theta}_1, \hat{\theta}_2, \cdots, \hat{\theta}_{n+1}$ of the lens equation (10) of $(n+1)$-th degree satisfy

$$\sum_{i=1}^{n+1} \frac{\hat{\theta}_i}{\hat{\phi}} \frac{d\hat{\theta}_i}{d\hat{\phi}} = 1. \tag{11}$$

Note that these solutions $\hat{\theta}_i$ may be complex and not all the magnifications are always physical and that Eq. (11) is satisfied regardless of the sign of Eq. (10). We realize the number of the real solutions ($\hat{\theta}_+ > 0$ and $\hat{\theta}_- < 0$) is always two , regardless of the value of n. Thus, the magnification invariant (11) is a observable quantity only when $n = 1$.

The signed magnifications of the images in the weak field limit are given by

$$\mu_{0\pm}(\hat{\phi}) \equiv \frac{\hat{\theta}_\pm(\hat{\phi})}{\hat{\phi}} \frac{d\hat{\theta}_\pm}{d\hat{\phi}}(\hat{\phi}). \tag{12}$$

Figure 3 shows that one can distinguish the general spherical lens models with their signed magnification sums $\mu_{0+} + \mu_{0-}$ which are less than unity. The lower bound of the total magnification $|\mu_{0+}| + |\mu_{0-}|$ is given by

$$\frac{2}{1+n} \leq \mu_{0+} + \mu_{0-} \leq |\mu_{0+}| + |\mu_{0-}|. \tag{13}$$

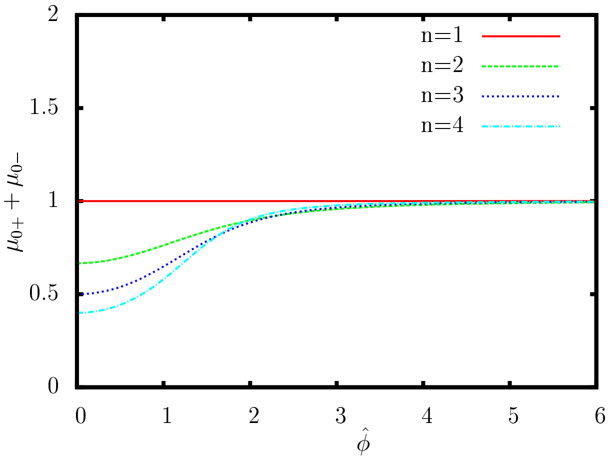

Figure 3: The singed magnification sums of some general spherical lens models. The solid, broken, dot and dot-dashed lines are the general spherical lens models for $n = 1$, 2, 3 and 4, respectively. This shows that we can distinguish each models from the others.

Therefore, gravitational lensing necessarily gives amplified light curves for $n = 1$, while it does not necessarily for $n > 1$. Recently, Kitamura *et al.* investigated the demagnified light curves [22].

The signed magnification sum is a powerful tool to find exotic lens objects because it only depends on the deduced source angle $\hat{\phi}$ and n and we just have to observe the images for $\hat{\phi} \lesssim 1$ and for $\hat{\phi} \gg 1$ to determine the signed magnification sum. However, we need a high resolution to observe the double images. We would also distinguish the lens objects with the ratio of magnifications of the double images and the total magnification. If we also measure the difference $\theta_+ - \theta_-$ of the image angles, one can determine the Einstein ring angle θ_0 and the source angle $\phi = \theta_0\hat{\phi}$.

Our method with the signed magnification sums is complementary to the methods to detect exotic lens objects with the light curves [7, 22] and the astrometric image centroid displacements [8]. To observe double images are much more feasible than to observe relativistic Einstein rings [16] because relativistic images are faint and small and because relativistic rings are rare sights.

References

[1] H. G. Ellis, J. Math. Phys. **14**, 104 (1973).

[2] M. S. Morris and K. S. Thorne, Am. J. Phys. **56**, 395 (1988).

[3] M. S. Morris, K. S. Thorne and U. Yurtsever, Phys. Rev. Lett. **61**, 1446 (1988).

[4] L. Chetouani and G. Clément, Gen. Relativ. Gravit. **16**, 111 (1984).

[5] K. Nakajima and H. Asada, Phys. Rev. D **85**, 107501 (2012).

[6] T. K. Dey and S. Sen, Mod. Phys. Lett. A, **23**, 953 (2008).

[7] F. Abe, Astrophys. J. **725**, 787 (2010).

[8] Y. Toki, T. Kitamura, H. Asada and F. Abe, Astrophys. J. **740**, 121 (2011).

[9] V. Perlick, Phys. Rev. D **69**, 064017 (2004).

[10] K. K. Nandi, Y. Z. Zhang and A. V. Zakharov, Phys. Rev. D **74**, 024020 (2006).

[11] J. M. Tejeiro and E. A. Larranaga, Rom. J. Phys. **57**, 736 (2012).

[12] K. S. Virbhadra and C. R. Keeton, Phys. Rev. D **77**, 124014 (2008).

[13] K. S. Virbhadra, Phys. Rev. D **79**, 083004 (2009).

[14] V. Bozza, Gen. Relativ. Gravit. **42**, 2269 (2010).

[15] V. Bozza and L. Mancini, Astrophys. J. **753**, 56 (2012).

[16] N. Tsukamoto, T. Harada and K. Yajima, Phys. Rev. D **86**, 104062 (2012).

[17] N. Tsukamoto and T. Harada, Phys. Rev. D **87**, 024024 (2013).

[18] V. Bozza, Phys. Rev. D **66**, 103001 (2002).

[19] S. Frittelli, T. P. Kling and E. T. Newman, Phys. Rev. D **61**, 064021 (2000).

[20] K. S. Virbhadra and G. F. R. Ellis, Phys. Rev. D **62**, 084003 (2000).

[21] V. Bozza, S. Capozziello, G. Iovane and G. Scarpetta, Gen. Relativ. Gravit. **33**, 1535 (2001).

[22] T. Kitamura, K. Nakajima and H. Asada, Phys. Rev. D **87**, 027501 (2013).