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"Critical exponents of gravity with quantum perturbations"

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Critical exponents of gravity with quantum perturbations

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Prime ref: H Zhang (张宏升), X Li, arXiv:1208.0106

Outline

- 1. Introduction to thermo dynamical gravity
- 2. Critical exponents of RN-AdS black hole
- 3. Quantum perturbations included

Introduction to thermo dynamical gravity

- Temperature: $1/(8\pi M)$
- Entropy: $A/4$
- Energy: M
- Event horizon: $r_+=2M$, $A=4\pi r_+$

Van der Waals-Maxwell gas-liquid system

Equation of state

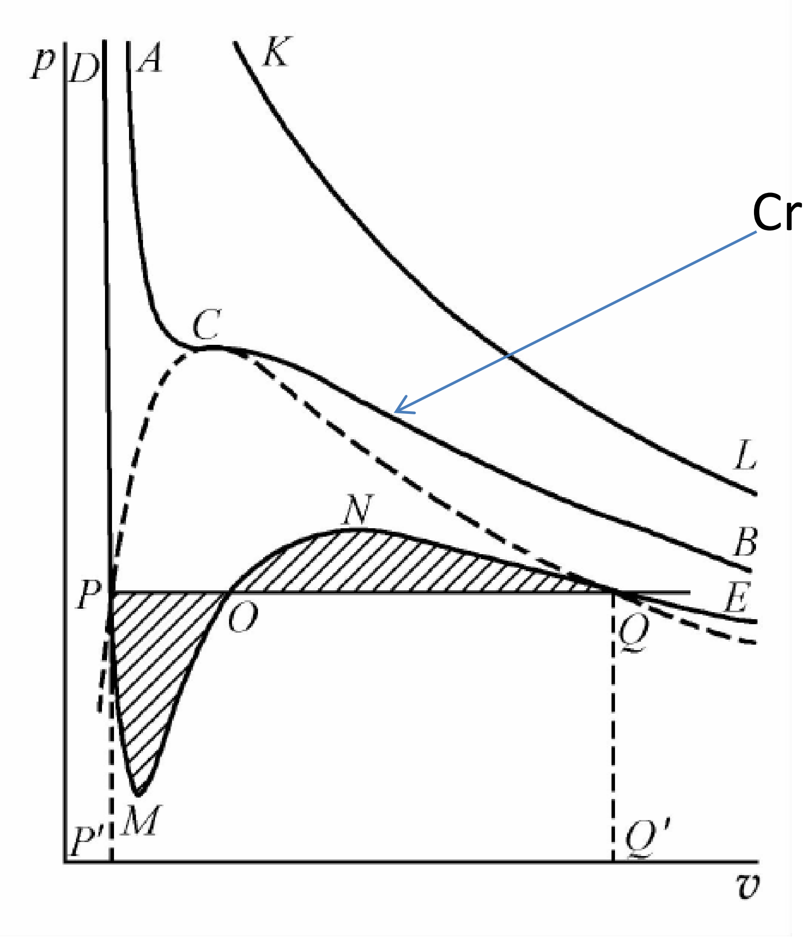
$$p = \frac{RT}{v - b} - \frac{a}{v^2}$$

Critical point

$$\left(\frac{\partial p}{\partial v}\right)_T = 0,$$

$$\left(\frac{\partial^2 p}{\partial v^2}\right)_T = 0$$

Critical isotherm



Critical isotherm

- Critical exponents of RN black hole

RN-AdS space time

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2$$

$$f(r) = 1 - \frac{2M}{r} + \frac{q^2}{r^2} + \frac{r^2}{l^2}$$

$$2M = r_+ + r_- + \frac{r_+^4 - r_-^4}{l^2(r_+ - r_-)}$$

$$q^2 = r_+ r_- \left(1 + \frac{r_+^3 - r_-^3}{l^2(r_+ - r_-)}\right).$$

Entropy, temperature, and potential

$$S = \frac{1}{4}A = \pi r_+^2$$

$$T = \left(\frac{\partial M}{\partial S}\right)_q = \frac{(r_+ - r_-)(l^2 + 3r_+^2 + 2r_+r_- + r_-^2)}{4\pi l^2 r_+^2}$$

$$\phi = \left(\frac{\partial M}{\partial q}\right)_S = \frac{\sqrt{r_+r_-[1 + (r_+^2 + r_+r_- + r_-^2)/l^2]}}{r_+} = \frac{q}{r_+}$$

Equation of state of RN-AdS

$$4\pi T q\phi = \phi^2 - \phi^4 + \lambda q^2$$

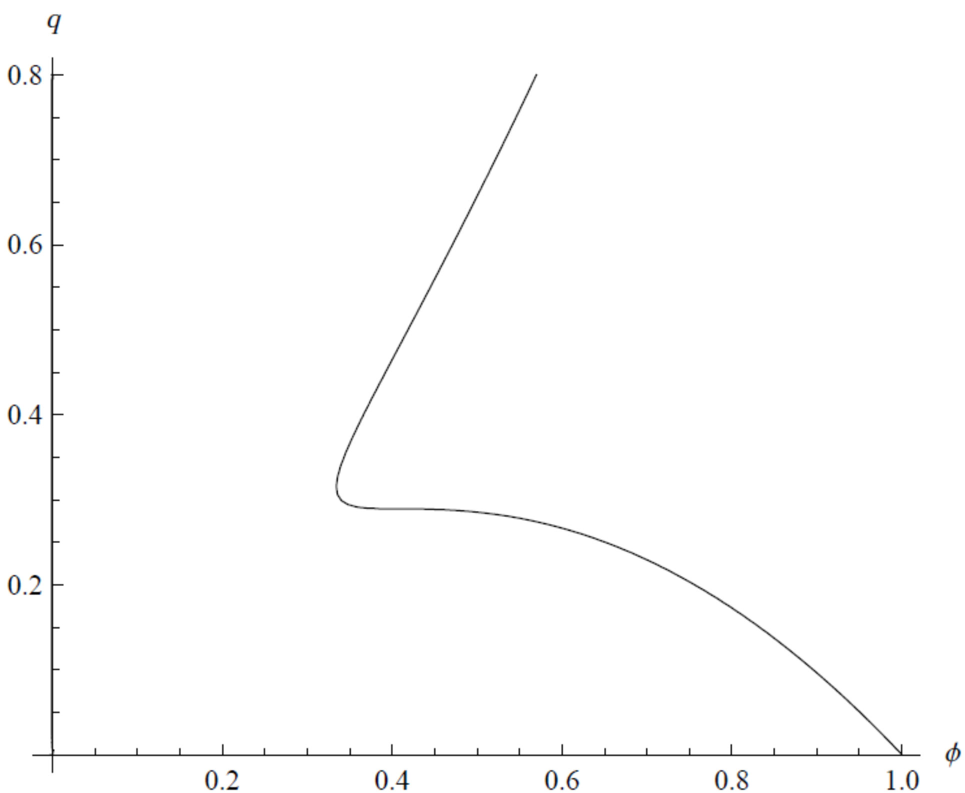
$$\left(\frac{\partial q}{\partial \phi}\right)_T = 0$$

$$\left(\frac{\partial^2 q}{\partial \phi^2}\right)_T = 0$$

$$q \longleftrightarrow p(\text{pressure}), \qquad \phi \longleftrightarrow v(\text{volume})$$

Critical point

$$T_c = \frac{\sqrt{2\lambda}}{3\pi}, \quad q_c = \frac{1}{\sqrt{12\lambda}}, \quad \phi_c = \frac{1}{\sqrt{6}}.$$



Critical exponents

Definition

$$
\begin{aligned}
&(1) \quad q - q_c \sim |\phi - \phi_c|^{\delta} \quad (T = T_c),\\
&(2) \quad \phi - \phi_c \sim |T - T_c|^{\beta} \quad (q = q_c),\\
&(3) \quad C_q \sim |T - T_c|^{-\alpha} \quad (q = q_c),\\
&(4) \quad \kappa_T \sim |T - T_c|^{-\gamma} \quad (q = q_c).
\end{aligned}
$$

$$\delta = 3, \quad \beta = 1/3, \quad \alpha = 2/3, \quad \gamma = 2/3.$$

N-dimensional case

- For n-dim RN-AdS black hole, the critical exponents take the same values as that of 4-dim case, ie,

$$\delta = 3, \quad \beta = 1/3, \quad \alpha = 2/3, \quad \gamma = 2/3.$$

C Niu, Y Tian, X Wu, Phys.Rev.D85:024017,2012

- This is a distinctive property of MFT(mean field theory).

- Quantum perturbations included

Temperature correction of Hawking radiation

$$T_{qu} = T\left(1 - \frac{k}{M^2}\right)$$

This equation includes the total effects of the quantum perturbations to all orders.

R. Banerjee and B. R. Majhi, JHEP 0806, 095 (2008)

$$T_{qu} = \left(1 - \frac{k}{M^2}\right)\left(\phi^2 - \phi^4 + \lambda q^2\right)$$

Critical point for corrected temperature

$$\left(\frac{\partial q}{\partial \phi}\right)_T = 0, \qquad \left(\frac{\partial^2 q}{\partial \phi^2}\right)_T = 0$$

$$\frac{q\left\{q^2\left(-\phi^2+3\phi^4+\lambda q^2\right)\left(3\phi^2+3\phi^4+\lambda q^2\right)^3-36k\phi^6\left[-6\phi^6+15\phi^8+14\lambda\phi^2q^2+7\lambda^2q^4+\phi^4\left(3-6\lambda q^2\right)\right]\right\}}{\phi q^2\left(-\phi^2+\phi^4+\lambda q^2\right)\left(3\phi^2+3\phi^4+\lambda q^2\right)^3+36k\phi^7\left[3\phi^8-14\lambda\phi^2q^2-7\lambda^2q^4-\phi^4\left(3+4\lambda q^2\right)\right]}=0$$

Values of critical quantities

- A special method to solve the above equation.
- The essential condition is that two roots of degenerates to one. Under this condition, and

$$\phi^2 \equiv x$$

$$(x - x_c)^2(c_0 + ... + c_5 x^5 + x^6) = 0$$

The simplified equation

$$c_0 x_c^2 - \frac{\lambda^4 q^8}{81} = 0,$$

$$(-2c_0 xc + c_1 x_c^2) - \frac{8}{81}\lambda^3 q^6 = 0,$$

$$\left(c_0 - 2c_1 x_c + c_2 x_c^2\right) - \frac{\left(18\lambda^2 q^6 + 12\lambda^3 q^8\right)}{81q^2} = 0,$$

$$(c_1 - 2c_2 x_c + c_3 x_c^2) - \frac{(180k\lambda^2 q^4 + 72\lambda^2 q^6)}{(81q^2)} = 0,$$

The simplified equation

$$(c_2 - 2c_3x_c + c_4x_c^2) - \frac{(-27q^2 + 360k\lambda q^2 + 108\lambda q^4 + 54\lambda^2 q^6)}{81q^2} = 0,$$

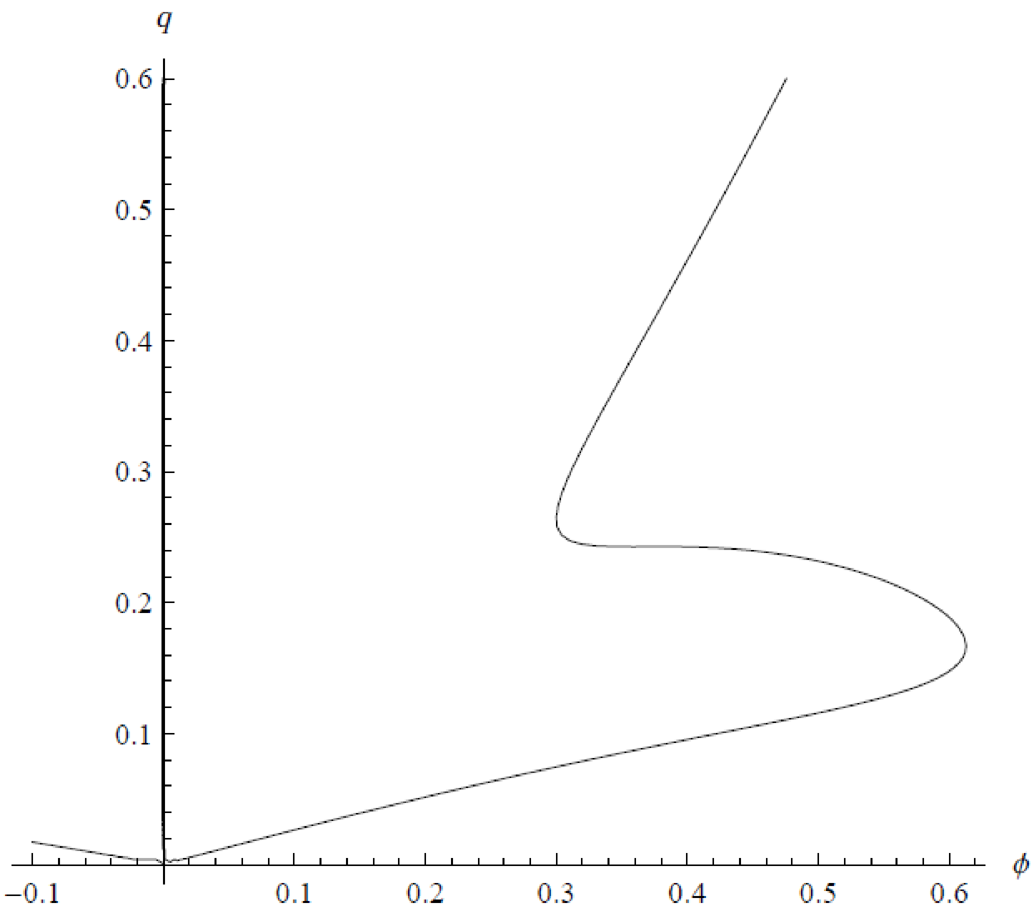
$$(c_3 - 2c_4x_c + c_5x_c^2) - \frac{(324k - 648k\lambda q^2 + 216\lambda q^4)}{81q^2} = 0,$$

$$(c_4 - 2c_5x_c + x_c^2) - \frac{(-648k + 162q^2 + 108\lambda q^4)}{81q^2} = 0,$$

$$(c_5 - 2x_c) - \frac{(-108k + 216q^2)}{81q^2} = 0.$$

8 variables in the above set $x_c, q, \; c_0, \; c_1 \ldots, \; c_5$

The critical isotherm



Result of critical exponents

$$\delta = 3, \quad \beta = 1/3, \quad \alpha = 2/3, \quad \gamma = 2/3.$$

Physical interpretations

- For ordinary matter, MFT and RGT present different critical exponents. Theoretically, MFT omits the perturbations around the critical point, while RGT carefully considers the perturbation effects at the critical point. In RGT, the whole system at the critical point is length scale free, that is, there is no special length scale in this system. In a gravity system, there is an inherent length scale *G^(-1/2), which* makes the RGT cannot do its work in a gravity system. A popular result is that the *G^(-1/2) with a length scale* hinders us to renormalize gravity. Here it hinders us to apply RGT in gravity, which makes the perturbed gravity and unperturbed gravity share the same critical exponents, though the perturbation shifts the critical point.

- *Thank you for your attention.*