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“On the vacua of maximal gauged supergravity”

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On the vacua of maximal gauged supergravity

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Cowork with Hideo Kodama (KEK)

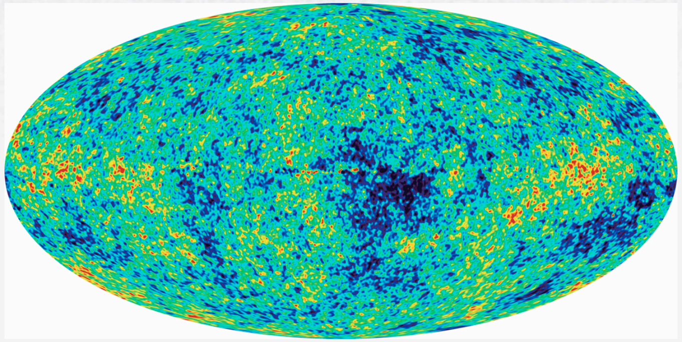
Based on

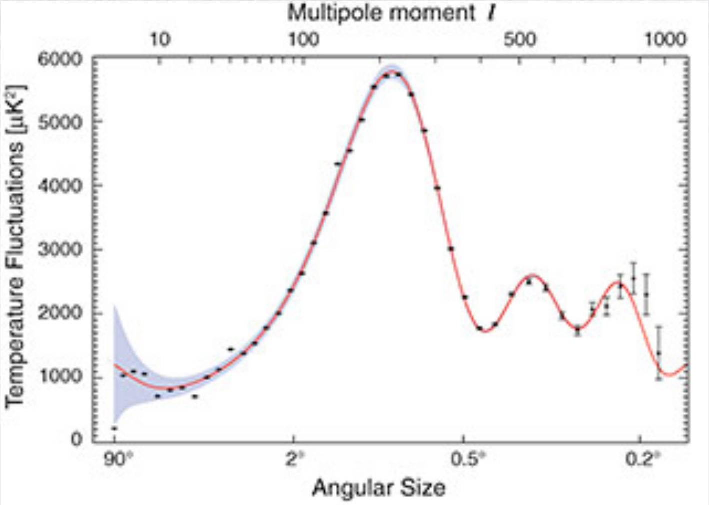
H.Kodama-M.N, arXiv:1210.4238

talk at JGRG22

Inflation in string theory

Precision Cosmology





CMB data strongly implies **inflation**

Challenge: account for inflation from fundamental theories

String theory: candidate for unifying all gauge interactions

- based on supersymmetry
- formulated in 10/11 dimensions

not fully successful due to moduli stabilization, η problem...

KKLT, KKLMMT, axion monodromy,...

Why N=8 supergravity

N=8 supergravity *fails* to describe unified gauge interactions, but...

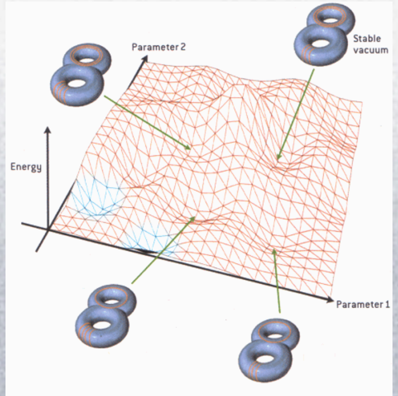
- *N*=8 SUGRA is the *restrictive* than *N*<8 theories
  - UV finite? (consensus: up to 7 loops)

    creditable guide toward quantum gravity

- direct relation to fundamental theories
  - IIA, IIB, M-theory : *N*=8

    ideal playground for exploring string theory dynamics independent of SUSY breaking objects

- Effective theory of M2branes
- AdS/CMP applications
- Landscape for string vacua



"The Landscape" (Picture from *Scientific American*)

Our work

We want to reveal *full* moduli space structure of N=8 supergravity

- Focus:
  - classifying vacua (Mink/AdS/dS)
  - stability of vacua
- Motivations:
  - more realistic construction of string inflation
  - extract universal features of gravity sector of string theory
- Key:
  - embedding tensor formalism
  - homogeneous scalar manifold $E_{7(7)}/SU(8)$
- Abstract of our results:
  - an exhaustive list of SL(8,**R**)-type vacua
  - an analytic expression of full mass spectra

N=8 Supergravity

- gauged supergravity
- embedding tensor

N=8 ungauged supergravity

- Maximal (N=8) *ungauged* supergravity Cremmer & Julia 78

  - N=8 is the *maximal # of SUSYs* for spin≦2
  - unique multiplet: 2^8=128 bosons+128 fermions

	graviton	gravitino	vector	gaugino	scalar
# of fields	1	8	28	56	70
dof	2	2x8=16	2x28=56	2x56=112	1x70

- 70 scalars parametrize $E_{7(7)}/SU(8)$

  coset representative: 56x56 matrix $i,j,\ldots$= **8** of SU(8)

$$L(\phi)_{\underline{M}}{}^{\underline{N}} = \exp \begin{pmatrix} 0 & \phi_{ijkl} \\ \phi^{ijkl} & 0 \end{pmatrix} , \quad \phi_{ijkl} = \phi_{[ijkl]} = \eta(\star\bar{\phi})_{ijkl} , \quad \eta = \pm 1$$

- obtained via T^7 compactification of M-theory
- no scalar potential, no nonabelian gauge fields

Gauged supergravity

- N=8 *gauged* supergravity *a la* *embedding tensor formalism*

deWit-Samtleben-Trigiante 03

Gauging: enhances *global symmetry* $G \subset E_{7(7)}$ to *local symmetry*

$$\partial_\mu \;\to\; D_\mu = \partial_\mu - g A_\mu{}^M X_M \, .$$

g: gauge coupling

X_M: generators of G
t_α: generators of $E_{7(7)}$

$$X_M = \Theta_M{}^\alpha t_\alpha$$

α=1,...,133: adjoint of $E_{7(7)}$
M=1,...,56: fundamental of $E_{7(7)}$

Θ: embedding tensor

- Consistency

(i) Gauge group G satisfies dimG≦28

✓simplest ex: SO(8) gauging de Wit & Nicolai 81

(ii) G must form a closed subgroup

$$\Omega^{MN}\Theta_M{}^\alpha\Theta_N{}^\beta = 0$$

(iii) SUSY

$$\Theta_M{}^\alpha \in \mathbf{912} \subset \mathbf{56} \times \mathbf{133}$$

Ω: Sp(56,R)⊃$E_{7(7)}$ metric

✓$O(g)$: SUSY transformation rules for fermions

✓$O(g)$: *mass terms for fermions*

✓$O(g^2)$: *scalar potential*

Scalar potential

Mink/AdS/dS ⇒ Einstein-70scalar system $E_{7(7)}$/SU(8)

$$L = \frac{1}{2}R + \frac{1}{8}\mathrm{Tr}(\partial^\mu M \partial_\mu M^{-1}) - V(\phi)\,,$$

$M_{MN} = (L^T L)_{MN}$

L=$L(\phi_{ijkl})$: coset rep. of $E_{7(7)}$/SU(8)

$$V = \frac{g^2}{672}\left(X_{MN}{}^P X_{PQ}{}^S M^{MP} M^{NQ} M_{RS} + 7 X_{MN}{}^Q X_{PQ}{}^N M^{MP}\right)\,.$$

- unbounded from below and above, and nonlinear fun. of ϕ_{ijkl}
- we can move critical pts to origin of scalar mfd   Dall'Agata-Inverso'11
  - homogeneity of scalar mfd $E_{7(7)}$/SU(8)
  - V is invariant under $L(\phi') = UL(\phi)h^{-1}(\phi,\phi')$ + $\Theta' = U\Theta$

At the origin, ϕ_{ijkl}=0, $L(\phi)$=$\mathbf{1}_{56}$

$$\partial_\rho V \propto t_{\rho M}{}^N \Theta_M{}^\alpha \Theta_N{}^\beta (\delta_{\alpha\beta} + 7\eta_{\alpha\beta}) + \Theta_M{}^\alpha \Theta_M{}^\beta f_{\rho\beta}{}^\gamma \delta_{\alpha\gamma}$$

combined with closure condition $\Omega^{MN}\Theta_M{}^\alpha \Theta_N{}^\beta = 0$

→ just solve a set of quadratic eqs. for Θ

SL(8,R) vacua

Dall'Agata-Inverso considered gaugings ⊂SL(8,**R**)⊂$E_{7(7)}$
for which Θ∈**912**=**36**+**36'**+~~**420**~~+~~**420'**~~

$$\Theta_{ab}{}^{c}{}_{d} = \delta_{[a}{}^{c}\theta_{b]d}\,, \qquad \Theta^{abc}{}_{d} = \delta^{[a}{}_{d}\xi^{b]c}\,,$$ (θ,ξ): 8x8 matrices

- assumed ansatz on (θ,ξ) and found several vacua (Mink/AdS/dS)
- mass spectra coincide if residual gauge symmetries are equivalent

✓diagonalized 70x70 mass matrix by computers

gauging	residual sym.	V_c	$m^2/\lvert V_c\rvert$
SO(8) SO(7, 1) SO(7) ⋉ T^7 SO(6)× SO(1, 1) ⋉ T^{12}	SO(6)	AdS	$2^{(2)}, -1^{(20)}, -\frac{1}{4}^{(20)}, 0^{(28)}$
SO(7, 1) SO(6)× SO(1, 1) ⋉ T^{12}	SO(5)	AdS	$2^{(3)}, -\frac{4}{3}^{(14)}, \frac{2}{3}^{(5)}, 0^{(48)}$
SO(3, 5)	SO(3) × SO(5)	dS	$-2^{(1)}, 4^{(5)}, 2^{(30)}, \frac{4}{3}^{(14)}, -\frac{2}{3}^{(5)}, 0^{(15)}$

Scanning Vacua

- all SL(8,$\mathbf{R}$) type vacua
- stability of vacua

SL(8,R) vacua

We can list *all* possible SL(8,**R**) vacua by obtaining all 8x8 matrices (θ,ξ) satisfying

Kodama-MN '12

$$2(\theta^2 - \xi^2) - (\theta \mathrm{Tr}\theta - \xi \mathrm{Tr}\xi) = 2a\mathbb{I}_8 \,, \qquad \theta\xi = c\mathbb{I}_8$$

a, c : constants

- SL(8,**R**) vacua are *exhausted* by list of Dall'agata-Inverso

Gauging	$G_{\rm reg}$	Λ
$\mathrm{SO}(4,4)$	$\mathrm{SO}(4)\times\mathrm{SO}(4)$	dS
$\mathrm{SO}(5,3)$	$\mathrm{SO}(5)\times\mathrm{SO}(3)$	dS
$\mathrm{SO}(8)$ $\mathrm{SO}(7,1)$ $\mathrm{SO}(7)\ltimes\mathbb{T}^7$	$\mathrm{SO}(7)$	AdS
$\mathrm{SO}(8)$ $\mathrm{SO}(7,1)$ $\mathrm{SO}(7)\ltimes\mathbb{T}^7$ $\mathrm{SO}(6)\times\mathrm{SO}(1,1)\ltimes\mathbb{T}^{12}$	$\mathrm{SO}(6)$	AdS
$\mathrm{SO}(7,1)$ $\mathrm{SO}(6)\times\mathrm{SO}(1,1)\ltimes\mathbb{T}^{12}$	$\mathrm{SO}(5)$	AdS
$\mathrm{SO}(6,2)$	$\mathrm{SO}(2)^4$	Mink.
$\mathrm{SO}(4)\times\mathrm{SO}(2,2)\ltimes\mathbb{T}^{16}$	$\mathrm{SO}(2)^4$	Mink.
$\mathrm{SO}(2)\times\mathrm{SO}(2)\ltimes\mathbb{T}^{20}$	$\mathrm{SO}(2)^2$	Mink.

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- Minkowski vacua are parametrized by continuous parameters

- For some vacua ($\Lambda \neq 0$), a single parameter remains

  e.g., dS vacua with SO(4,4)→SO(4)xSO(4) gauging

$$\theta = \lambda \mathbb{I}_4 \oplus (-\lambda)\mathbb{I}_4 \,, \qquad \xi = \theta^{-1} \,, \qquad V = \frac{g^2}{4}\left(\lambda^2 + \lambda^{-2}\right) \,,$$

implying 1parameter *deformation of the theory*

c.f. Dall'Agata-Inverso-Trigiante '12 for SO(8)

SL(8,R) vacua

Mass spectra are *analytically* determined Kodama-MN '12

$$(M^2)_{\rho\sigma} := \frac{g^2}{168}\left[(s_{(\rho}s_{\sigma)})_M{}^N \mathrm{Tr}(X_M{}^T X_N + 7X_M X_N) + 2(s_{(\rho})_M{}^N \mathrm{Tr}(s_{\sigma)}[X_{(M}, {}^T X_{N)}]) - \mathrm{Tr}([s_{(\rho}, X_M][s_{\sigma)}, {}^T X_M])\right] .$$

$$s_\rho = \frac{1}{2}(t_\rho + {}^T t_\rho) = \begin{pmatrix} 2S_\rho \wedge \mathbb{I} & U_\rho \\ \star U_\rho & -2S_\rho \wedge \mathbb{I} \end{pmatrix}$$

$$\Theta_{ab}{}^c{}_d = \delta_{[a}{}^c \theta_{b]d} , \qquad \Theta^{abc}{}_d = \delta^{[a}{}_d \xi^{b]c} ,$$

decompose into SL(8,**R**) irrep →

S_{ab}:**35** scalars $S = {}^T S , \quad \mathrm{Tr}(S) = 0 ,$

U_{abcd}:**35** pseudoscalars $U = \star U .$

→ kinetic term= $\frac{1}{2}\mathrm{Tr}((\partial S)^2) + \frac{1}{12}\partial U \cdot \partial U$

e.g. electric gaugings (ξ=0) & $V_c \neq 0$: cosmological constant

$$\theta = \begin{pmatrix} \lambda_1 \mathbb{I}_{n_1} & 0 \\ 0 & \lambda_2 \mathbb{I}_{n_2} \end{pmatrix} \qquad S = \begin{pmatrix} A_{11} & A_{12} \\ {}^T A_{12} & A_{22} \end{pmatrix} \qquad A_{11} = \frac{1}{n_1}\mathrm{Tr}(A_{11})\mathbb{I}_{n_1} + \hat{A}_1 ,$$

n_1+n_2(=8) split

SL(8,R) vacua

Mass spectra for Λ≠0 vacua

Kodama-MN '12

$$M^2_{(1)} = \frac{8}{n_1 n_2} m^2_{0(\mathbf{1},\mathbf{1})} \mathrm{Tr}(A_{11})^2 + m^2_{1(\mathbf{N}_1,\mathbf{1})} \mathrm{Tr}(\hat{A}_1^2) + m^2_{2(\mathbf{1},\mathbf{N}_2)} \mathrm{Tr}(\hat{A}_2^2) + 2m^2_{*(\mathbf{n}_1,\mathbf{n}_2)} \mathrm{Tr}({}^T A_{12} A_{12}) \,,$$

SO(n_1,n_2)→ SO(n_1)xSO(n_2) $\quad \theta = \begin{pmatrix} \lambda_1 \mathbb{I}_{n_1} & 0 \\ 0 & \lambda_2 \mathbb{I}_{n_2} \end{pmatrix}$

(n_1, n_2) correspond to the unbroken gauge symmetries

$$m^2_{0(\mathbf{1},\mathbf{1})} = -2V_c \,, \qquad m^2_{1(\mathbf{N}_1,\mathbf{1})} = \frac{4V_c}{n_1 - 2} \,,$$

m_*=0: NG directions

$m^2_{(\mathbf{p},\mathbf{q})}$: (**p**,**q**) irrp of SO(n_1)xSO(n_2) $\quad N_1 = \frac{1}{2}(n_1 - 1)(n_1 + 2)$

Mass of pseudoscalars can be obtained similarly

$$m^2_{U[\ell]} = \frac{2[2\ell^2 - 2n_1\ell + (n_1 - 2)^2]}{(n_1 - 6)(n_1 - 2)} V_c \,.$$

l: labeling index

Mass spectra are dependent only on residual gauge symmetries

⇒ insensitive to original gaugings & deformation parameters

SL(8,R) vacua

Minkowski vacua

SO(6,2), SO(4)xSO(2,2)$\ltimes T^{16}$, SO(2)xSO(2)$\ltimes T^{20}$ admit Minkowski vacua

e.g., 35 scalar mass eigenvalues for SO(6,2)→SO(2)^4 gauging

$\theta = r\mathbb{I}_2 \oplus s\mathbb{I}_2 \oplus t\mathbb{I}_2 \oplus \left(-\frac{1}{rst}\right)\mathbb{I}_2 \,,$:specified by 3 continuous parameters

$$m^2_{(\mathbf{1},\mathbf{1},\mathbf{1})(\times 3)} = 0\,, \qquad m^2_{*(\times 24)} = 0 \quad : (\mathbf{2},\mathbf{2},\mathbf{1},\mathbf{1}) + \cdots \,,$$

$$m^2_{i(\times 8)} = \frac{g^2}{16r^2s^2t^2} \times \begin{cases} 4st(r-s)(r-t)(1+r^2st)\,, & : (\mathbf{2},\mathbf{1},\mathbf{1},\mathbf{1}) \\ 4rt(s-r)(s-t)(1+rs^2t)\,, & : (\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1}) \\ 4rs(r-t)(s-t)(1+rst^2)\,, & : (\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{1}) \\ 4(1+r^2st)(1+rs^2t)(1+rst^2)\,, & : (\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2}) \end{cases} .$$

Unless (*r*,*s*,*t*) are pairwise equal, m^2 will be negative

→ generic Minkowski vacua are unstable

SL(8,R) vacua

Exhaustive list of SL(8,**R**) type vacua & mass spectra

Gauging	G_{reg}	Λ	m_S^2	m_U^2
$SO(4,4)$	$SO(4)\times SO(4)$	dS	$-2_{(\times 1)},\ 2_{(\times 18)},\ 0_{(\times 16)}$	$2_{(\times 18)},\ 1_{(\times 16)},\ -2_{(\times 1)}$
$SO(5,3)$	$SO(5)\times SO(3)$	dS	$-2_{(\times 1)},\ \frac{4}{3}_{(\times 14)},\ 4_{(\times 5)},\ 0_{(\times 15)}$	$2_{(\times 30)},\ -\frac{2}{3}_{(\times 5)}$
$SO(8)$ $SO(7,1)$ $SO(7)\ltimes \mathbb{T}^7$	$SO(7)$	AdS	$2_{(\times 1)},\ -\frac{4}{5}_{(\times 27)},\ 0_{(\times 7)}$	$-\frac{2}{5}_{(\times 35)}$
$SO(8)$ $SO(7,1)$ $SO(7)\ltimes \mathbb{T}^7$ $SO(6)\times SO(1,1)\ltimes \mathbb{T}^{12}$	$SO(6)$	AdS	$-1_{(\times 20)},\ 2_{(\times 2)},\ 0_{(\times 13)}$	$0_{(\times 15)},\ -\frac{1}{4}_{(\times 20)}$
$SO(7,1)$ $SO(6)\times SO(1,1)\ltimes \mathbb{T}^{12}$	$SO(5)$	AdS	$-\frac{4}{3}_{(\times 14)},\ 2_{(\times 3)},\ 0_{(\times 18)}$	$\frac{2}{3}_{(\times 5)},\ 0_{(\times 30)}$
$SO(6,2)$	$SO(2)^4$	Mink.	Eq. (4.54)	Eq. (4.55)
$SO(4)\times SO(2,2)\ltimes \mathbb{T}^{16}$	$SO(2)^4$	Mink.	Eq. (4.68)	Eq. (4.69)
$SO(2)\times SO(2)\ltimes \mathbb{T}^{20}$	$SO(2)^2$	Mink.	Eq. (4.70)	Eq. (4.71)

- mass eigenvalues are insensitive to a deformation parameter
- all vacua break SUSY completely
- (A)dS vacua are always unstable

Summary

What we have done

- *complete classification* of vacua for gauge group contained in SL(8,**R**)
  - ✓ many vacua ($\Lambda \neq 0$) contain deformation parameter
  - ✓ except N=8 AdS vacua, all SUSYs are broken
- obtained *analytic expressions* of mass eigenvalues
  - ✓ insensitive to underlying gaugings & deformation parameter
  - ✓ all vacua are generically unstable

Future prospects

- explore other gaugings $\not\subset$ SL(8,**R**)
- construct inflationary models
- find higher dimensional flux vacua description