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“Effects of chameleon scalar field on rotation curves of the galaxies”

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Effects of Chameleon Scalar Field on Rotation Curves of the Galaxies

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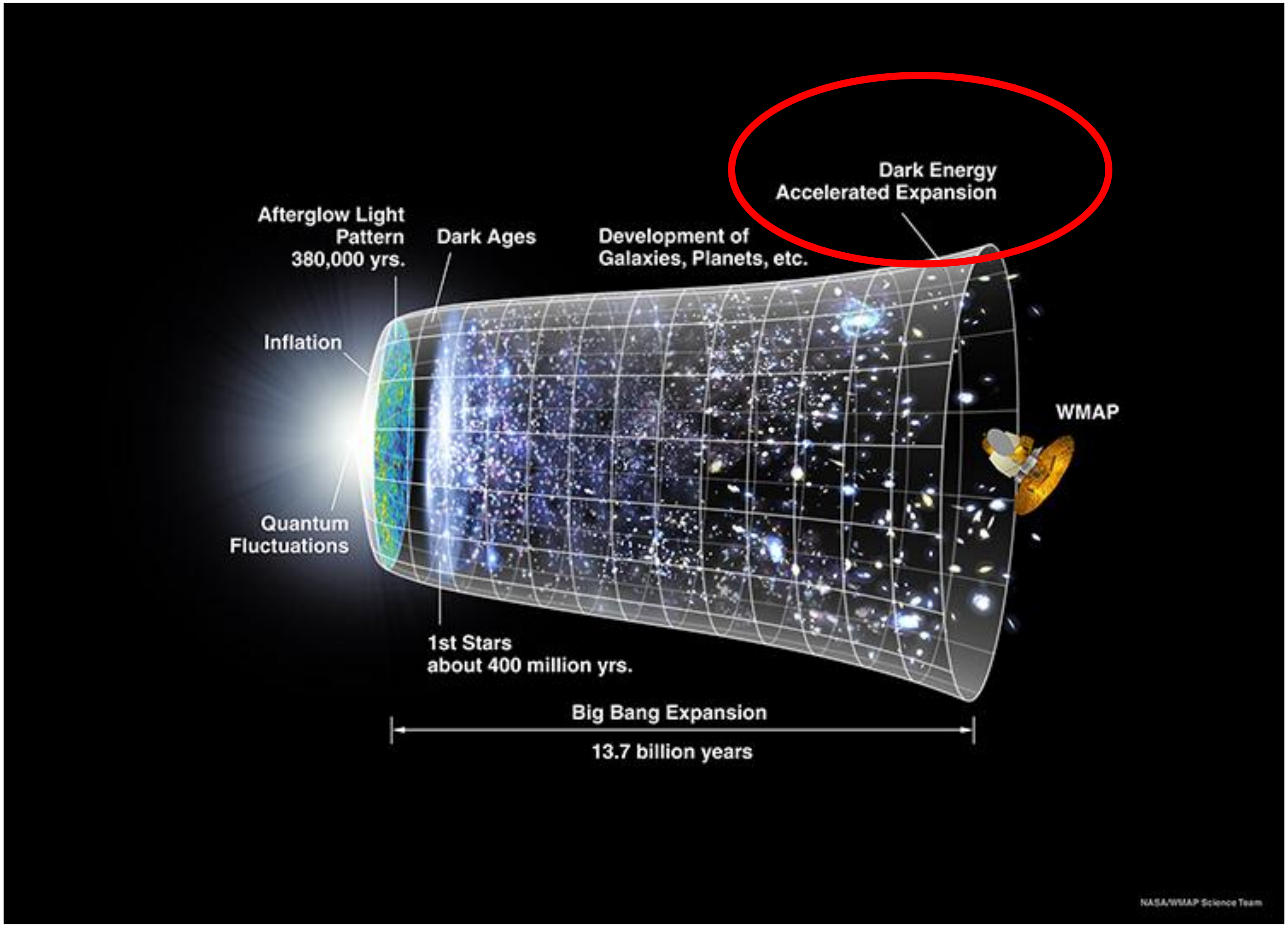
Outlines

- Introduction
- Chameleon Dark Energy Model
- Effects on a Rotation Curve
- Results
- Conclusions

Introduction



http://map.gsfc.nasa.gov/

Introduction

- From Equation of state $\boxed{P = w\rho}$

Matter
(non-relativistic particles) ⇒ $\boxed{w_{matter} = 0}$

Radiation
(relativistic particles and photons) ⇒ $\boxed{w_{radiation} = \frac{1}{3}}$

cannot be dark energy

- But, the accelerated expansion requires

$$w < -\frac{1}{3}$$

Introduction

- Quintessence

$$S = \int d^4x \sqrt{-g}\left[-\frac{1}{2}\partial^\mu \phi \partial_\mu \phi - V(\phi)\right]$$

Equation of motion

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0$$

Friction term

Driving term

Energy density and pressure density
+ Friedmann acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{8\pi G}{3}\left[\dot{\phi}^2 - V(\phi)\right]$$

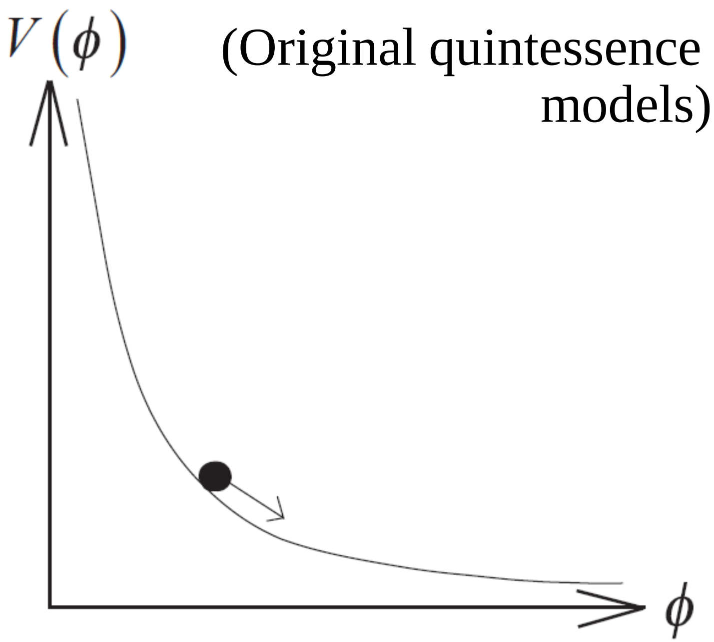
Accelerated expansion requires

$$\dot{\phi}^2 < V(\phi)$$

Flat potential
(in late time)

Introduction

- Quintessence



(Original quintessence models)

Power-law potential

$$\boxed{V(\phi) = \frac{M^{4+n}}{\phi^n}}$$

Equation of state

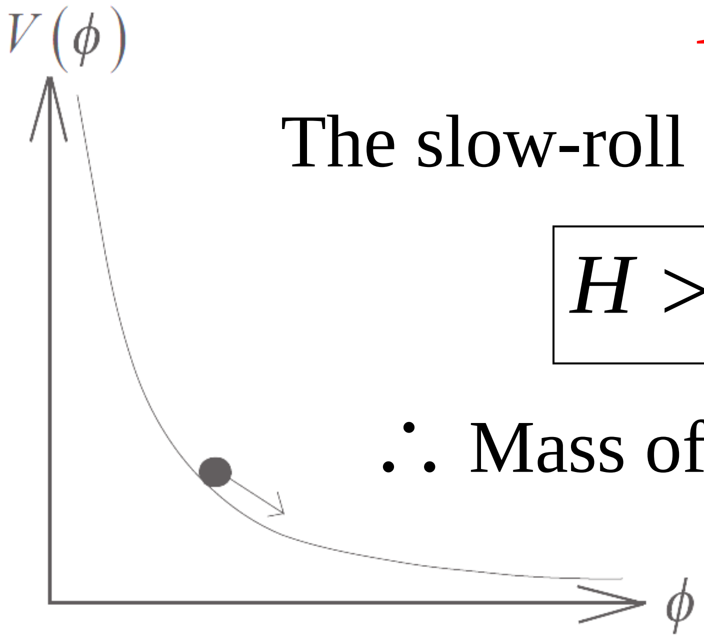
$$w_\phi = \frac{P_\phi}{\rho_\phi} = \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)}$$

Slow-roll limit : $\frac{\dot{\phi}^2}{2} \ll V(\phi)$

$$\Rightarrow \boxed{w_\phi = -1}$$

Introduction

- Quintessence



Problem

The slow-roll limit (overdamping) will occur if

$$\boxed{H > m_\phi} \quad \left(H_0 \approx 10^{-33} eV\right)$$

$\therefore$ Mass of scalar-field is very tiny

$\boxed{\text{Long range force}}$?

(Why have we never detected it before?)

Chameleon Dark Energy Model

- Action in the Einstein frame

$$S=\int d^4x\sqrt{-g}\left(\frac{M_{Pl}^2}{2}R-\frac{(\partial\phi)^2}{2}-V(\phi)\right)-\int d^4x\mathcal{L}_m\left(\psi_m^{(i)},g_{\mu\nu}^{(i)}\right)$$

matter fields couple to a metric

Equation of motion

(Cosmological scale)

$$\ddot{\phi}+3H\dot{\phi}+\frac{\partial V_{eff}}{\partial\phi}=0$$

$$\boxed{V_{eff}(\phi)=V(\phi)+\rho_m e^{\beta\phi/M_{Pl}}}$$

Conformal rescaling

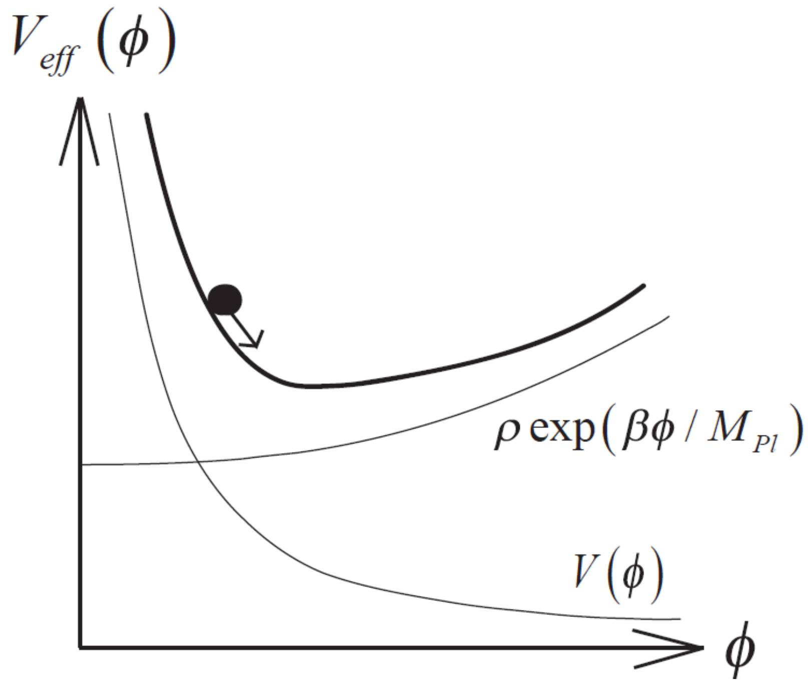
(Conformal transformation)

$$g_{\mu\nu}^{(i)}=e^{2\beta_i\phi/M_{Pl}}g_{\mu\nu}$$

Einstein frame metric

Chameleon Dark Energy Model

The effective potential



Minimum point

$$V_{,\phi}(\phi_{\min}) + \frac{\beta}{M_{Pl}} \underline{\rho_m} e^{\beta\phi_{\min}/M_{Pl}} = 0$$

Mass of scalar field

(about the minimum)

$$m^2 = V_{,\phi\phi}(\phi_{\min}) + \frac{\beta^2}{M_{Pl}^2} \underline{\rho_m} e^{\beta\phi_{\min}/M_{Pl}}$$

Large matter density ➡ Huge m_ϕ

Small matter density ➡ Tiny m_ϕ

Chameleon

Chameleon Dark Energy Model

Equation of motion (full-form)

$$\boxed{\nabla^2\phi = V_{,\phi} + \frac{\beta}{M_{Pl}}\rho_m e^{\beta\phi/M_{Pl}}}$$

In cosmological scale
(neglect the gradient term)

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V_{eff}}{\partial \phi} = 0$$

In the chameleon model

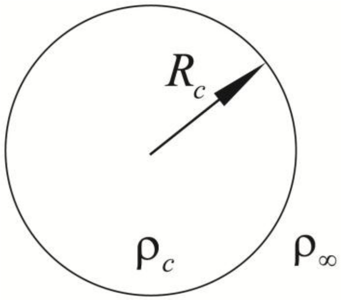
$$\boxed{m \gg H}$$

But, the slow-roll limit still occurs
(arXiv:astro-ph/0408415v2)

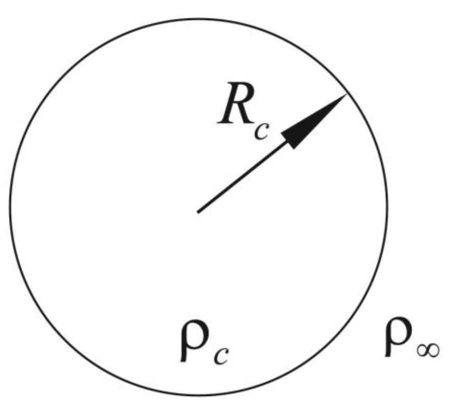
On a spherically symmetric object
(neglect the time-dependent term)

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} = V_{,\phi} + \frac{\beta}{M_{Pl}}\rho(r) e^{\beta\phi/M_{Pl}}$$

$$\boxed{\text{Profile } \phi(r)}$$



Chameleon Dark Energy Model



(Homogeneous density)

Since $\phi_{\min}$ and m_ϕ depend on local matter density

$\phi_{\min}$ and m_ϕ inside an object	$\neq$	$\phi_{\min}$ and m_ϕ outside an object

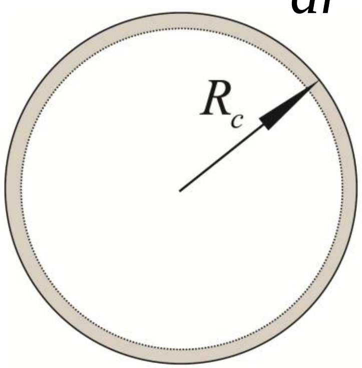
The scalar field has to roll out minimum to satisfy continuous conditions (ϕ and $\frac{d\phi}{dr}$)

2 types of profile

(Divided by size of object)

Thin-shell regime
(large object)

Thick-shell regime
(small object)



(Thin-shell regime)

Chameleon Dark Energy Model

Since scalar field couples to matter ➡ **Fifth force**

$$\boxed{\vec{F}_\phi = -\frac{\beta}{M_{Pl}} M \vec{\nabla}\phi}$$

Scalar field acts as a potential for the force

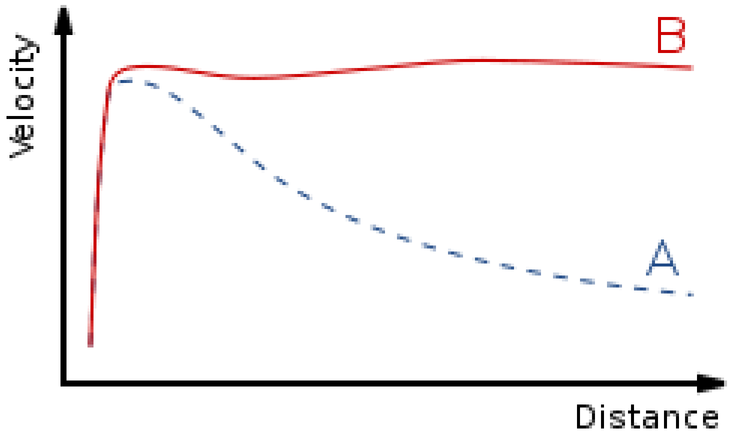
Proved by Conformal transformation + Geodesics equation

$$\ddot{x}^\mu + \Gamma^\mu_{\alpha\beta}\dot{x}^\alpha \dot{x}^\beta + \alpha_\phi \frac{\partial \phi}{\partial x_\mu} = 0$$

We have profile $\phi(r)$ from the thin-shell or thick-shell of an object.

Effects on Rotation Curves

Rotation curve



http://en.wikipedia.org/wiki/Galaxy_rotation_curve

Observational data (B)

$\neq$

Theoretical prediction (A)

We need dark matter

What will happen on a rotation curve if
we have chameleon scalar field in our universe?

Effects on Rotation Curves

- Dark matter halo profiles for investigating are as the following

NFW profile	ISO profile	Parametrized model
$\rho(r) = \dfrac{\rho_0}{\dfrac{r}{a}\left(1+\dfrac{r}{a}\right)^2}$	$\rho(r) = \dfrac{\rho_0}{1+\left(\dfrac{r}{R_s}\right)^2}$	$\rho(r) = \dfrac{\rho_0}{\left(\dfrac{r}{r_s}\right)^{\alpha}\left(1+\dfrac{r}{r_s}\right)^{3-\alpha}}$

- We suppose that dark matter halo is a spherical symmetric object

Effects on Rotation Curves

- Chameleon profile inside a dark matter halo

$$\frac{d^2\phi}{dr^2}+\frac{2}{r}\frac{d\phi}{dr}=V_{,\phi}+\frac{\beta}{M_{Pl}}\rho(r)e^{\beta\phi/M_{Pl}}$$

Numerical solution

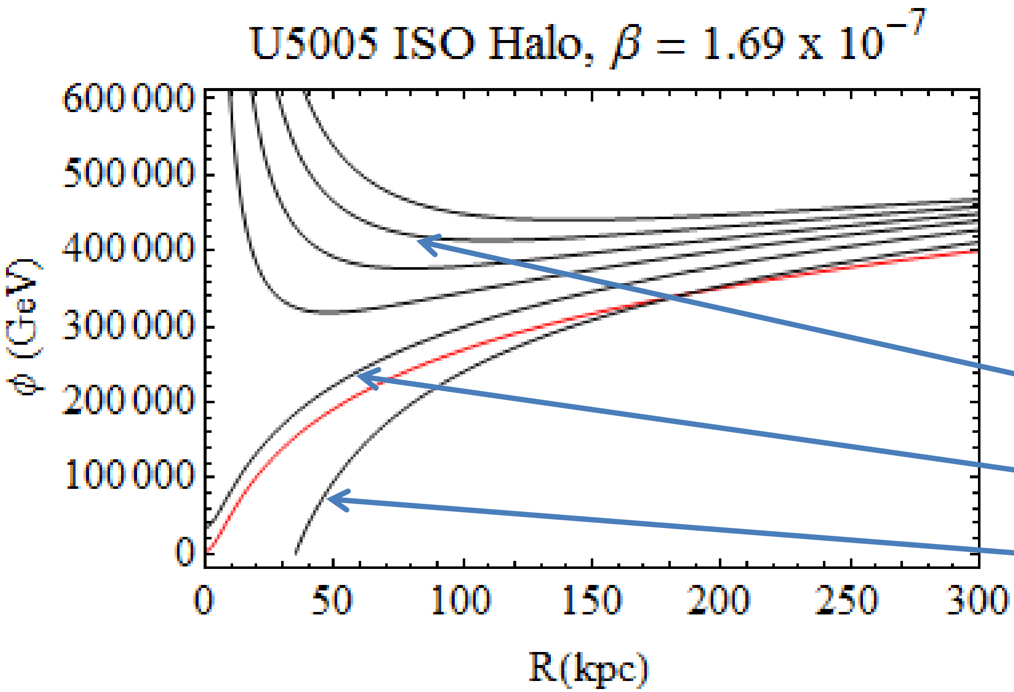
Analytic solution

  - Thick-shell regime : density of DM halo is a function of radius
  - Power-law potential : $\boxed{V(\phi)=\frac{M^{4+n}}{\phi^n}}$

Effects on Rotation Curves

- Numerical solution

$$\frac{d^2\phi}{dr^2}+\frac{2}{r}\frac{d\phi}{dr}=-n\frac{M^{4+n}}{\phi^{n+1}}+\frac{\beta}{M_{Pl}}\rho(r)e^{\beta\phi/M_{Pl}}$$



3 types of solution

- Diverge at origin
- Finite at origin
- Truncate before origin

Effects on Rotation Curves

- Analytic solution

$$e^{\beta\phi/M_{Pl}} \approx 1$$

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} \approx \frac{\beta}{M_{Pl}}\rho(r)$$

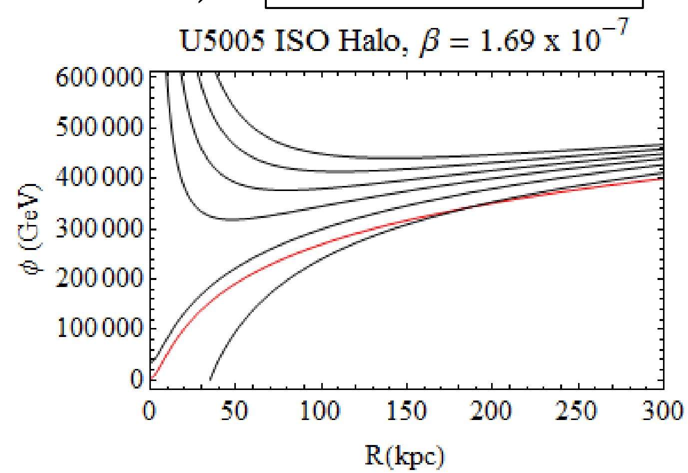
Neglect the power-law potential term

$$\phi'(r) = \frac{\beta}{4\pi M_{Pl} r^2}\left(M(r) - M_0 + \gamma\right)$$

M_0 : Total mass
$\gamma \propto$ Slope

- Diverge at origin
- Finite at origin
- Truncate before origin

$\gamma < M_0$
$\gamma = M_0$
$\gamma > M_0$



Effects on Rotation Curves

- Constraint on the matter-chameleon coupling

From EOM with approximation

$$\phi'(r) = \frac{\beta}{4\pi M_{Pl}} \left(\frac{M(r)}{r^2} \right) + \frac{1}{r^2} \left(\phi' r^2 \big|_{r=0} \right)$$

Boundary conditions
(original paper)

$$\frac{d\phi}{dr} = 0 \quad at \quad r = 0$$

$$\phi \to \phi_\infty \quad as \quad r \to \infty$$

$$\phi(r_{\max}) = \frac{\beta}{4\pi M_{Pl}} \int_0^{r_{\max}} dr \frac{M(r)}{r^2} + \phi(0)$$

$$\phi_\infty \geq \frac{\beta}{4\pi M_{Pl}} \int_0^{r_{\max}} dr \frac{M(r)}{r^2}$$

Minimum of effective pot. $\quad \phi_\infty = \left(\frac{n M^{4+n} M_{Pl}}{\rho_\infty \beta} \right)^{\frac{1}{n+1}}$

$$\beta_{\max} = \left(\frac{n M^{4+n} M_{Pl}}{\rho_\infty} \right)^{\frac{1}{n+2}} \left(\frac{4\pi M_{Pl}}{\int_0^{r_{\max}} dr \frac{M(r)}{r^2}} \right)^{\frac{n+1}{n+2}}$$

ISO U5005 1.69×10^{-7}

NFW U5005 1.76×10^{-7}

Effects on Rotation Curves

- Analytic solution (NFW profile)

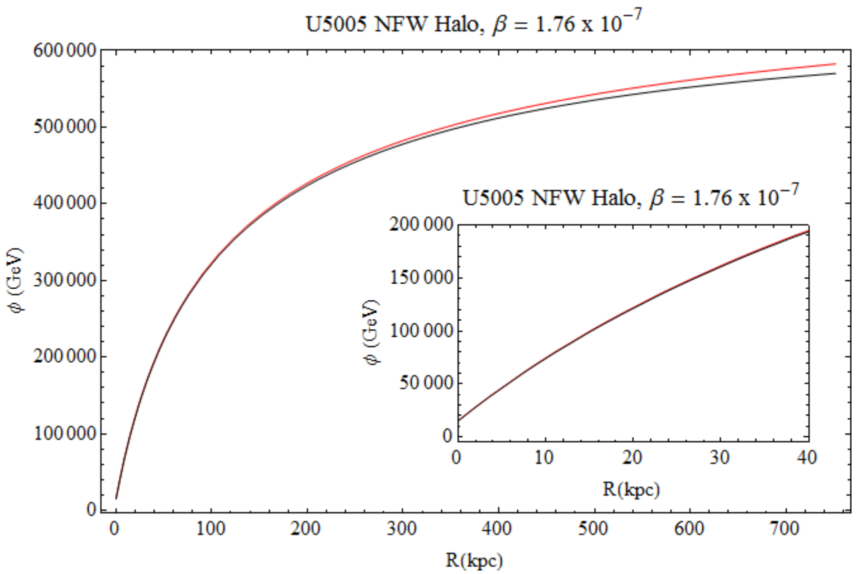
$$\rho(r) = \frac{\rho_0}{\frac{r}{a}\left(1+\frac{r}{a}\right)^2}$$

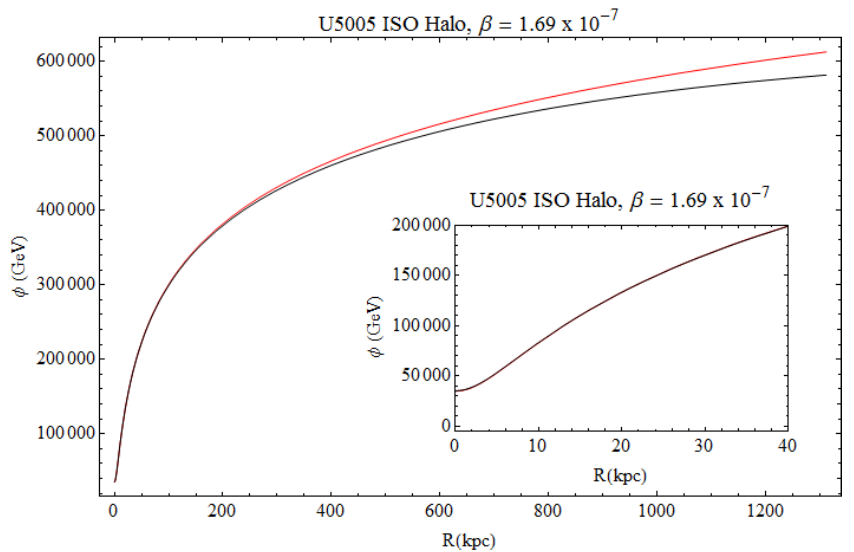
$$\boxed{\phi(r) = \frac{a^3\beta\rho_0}{M_{Pl}}\left(\frac{1}{a} - \frac{\ln(1+r/a)}{r}\right) + \phi(0)}$$

- Analytic solution (ISO profile)

$$\rho(r) = \frac{\rho_0}{1+\left(\frac{r}{R_s}\right)^2}$$

$$\boxed{\phi(r) = \frac{\beta R_s^3\rho_0}{M_{Pl}}\left(\frac{\arctan(r/R_s)}{r} + \frac{\ln(1+r^2/R_s^2)}{2R_s} - \frac{1}{R_s}\right) + \phi(0)}$$



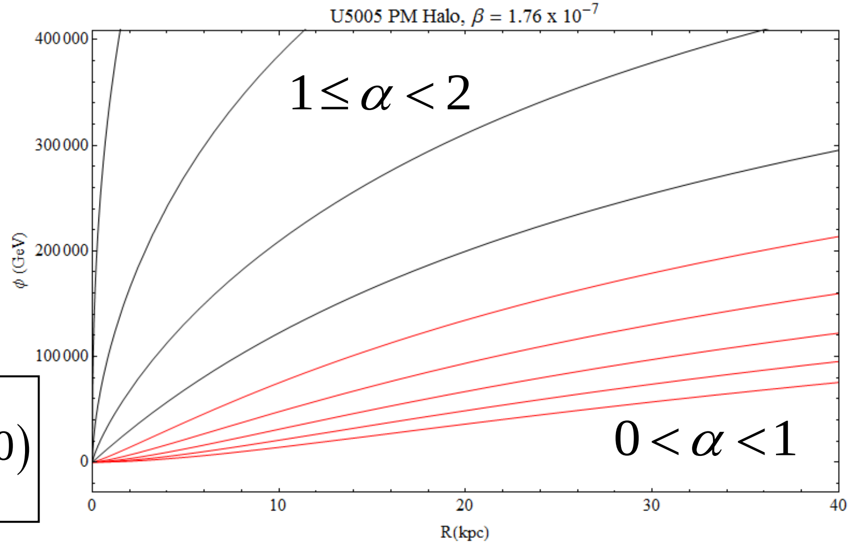


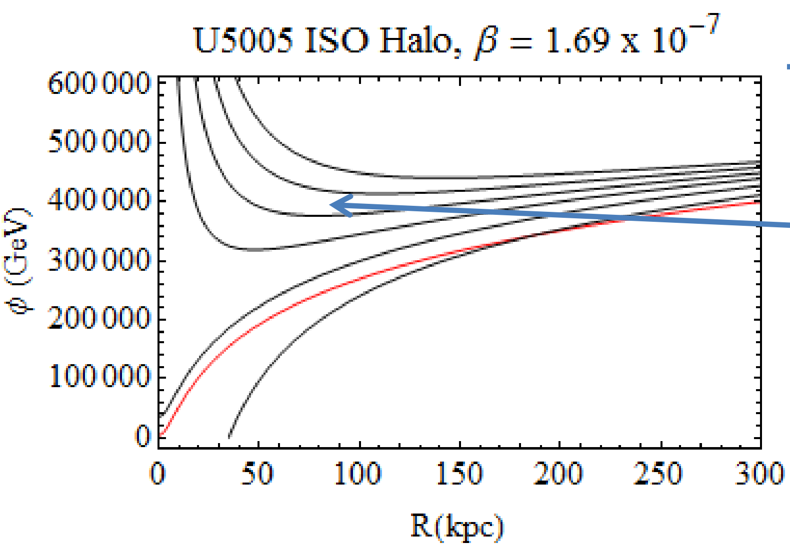
Effects on Rotation Curves

- Analytic solution (PM profile)

$$\rho(r) = \frac{\rho_0}{\left(\frac{r}{r_s}\right)^{\alpha}\left(1+\frac{r}{r_s}\right)^{3-\alpha}}$$

$$\boxed{\phi(r) = \frac{\beta\rho_0}{M_{Pl}}\frac{r^{\alpha}}{3-\alpha}\left(\frac{r^{2-\alpha}}{2-\alpha}\,{}_2F_1\left(2-\alpha, 3-\alpha, 4-\alpha, -\frac{r}{r_s}\right)\right)+\phi(0)}$$





- Diverge at origin
- ~~Finite at origin~~
- ~~Truncate before origin~~

Effects on Rotation Curves

Circular velocity + Fifth force

$$v_c(r) = \sqrt{\frac{GM(r)}{r} + \frac{\beta r}{M_{Pl}} \frac{d\phi}{dr}}$$

Only DM halo profile

Numerical solution

Late-type low surface brightness (LSB) galaxies

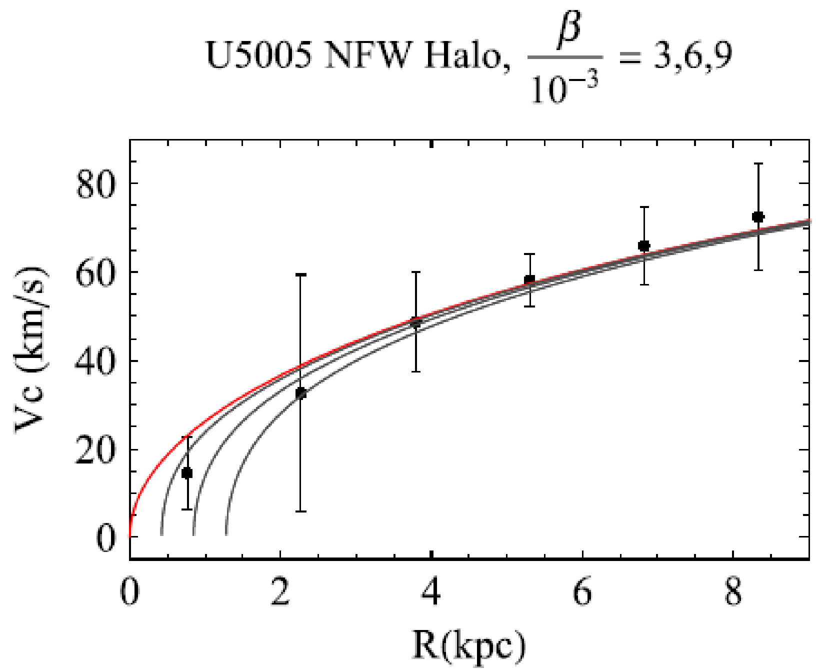
Mainly effect on rotation curve comes from dark matter halo

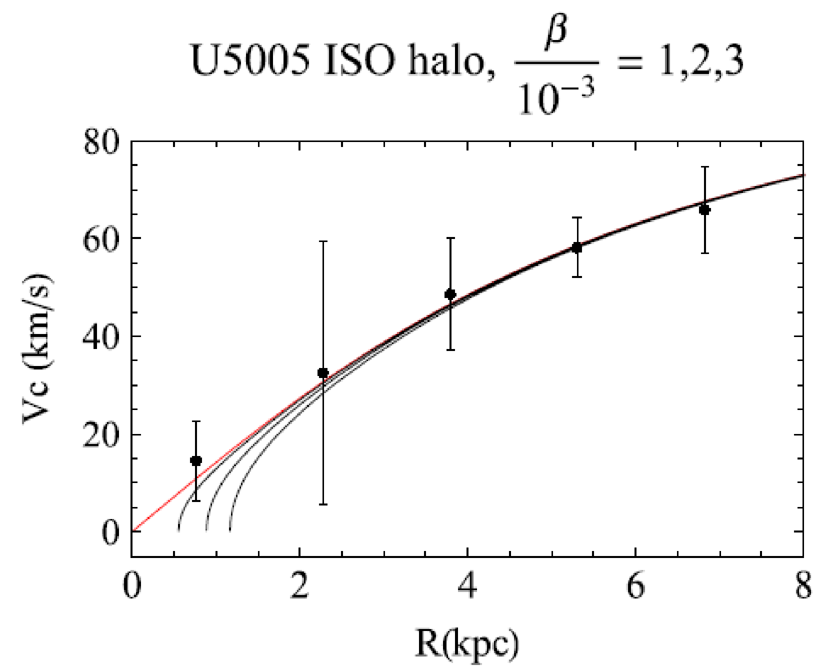
Reference:

de Blok, W.J.G., and Bosma, A. High-resolution rotation curves of Low Surface Brightness galaxies. Astron. Astrophys. 385 (2002): 816

Results

- Rotation curves of U5005

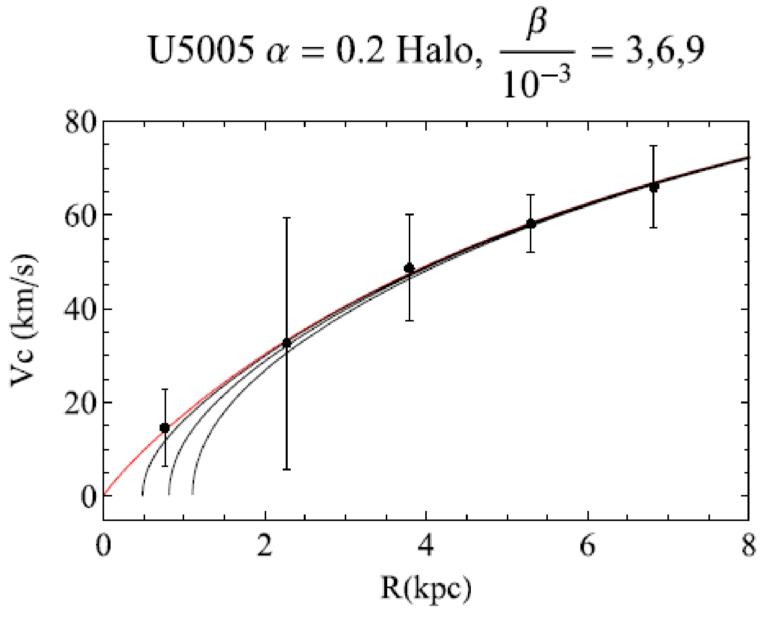


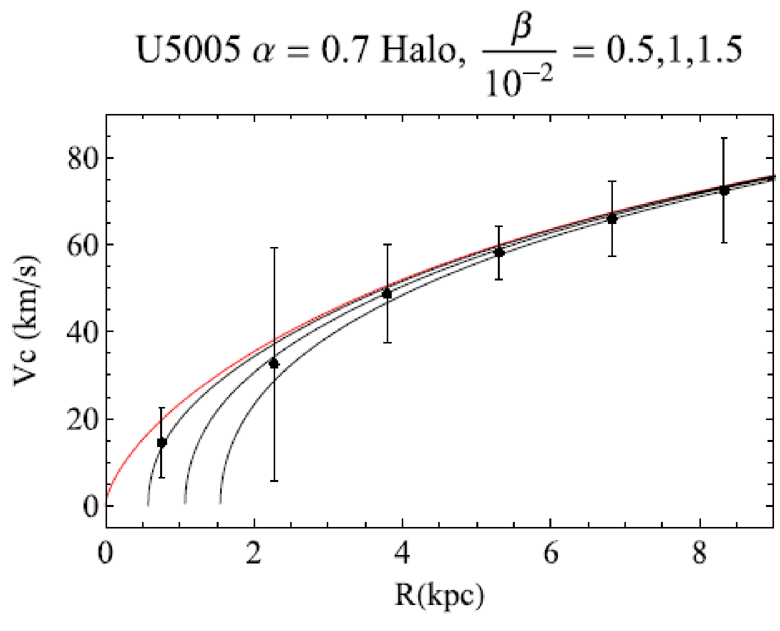


Red lines → No chameleon effect

Error bar:
http://vizier.cfa.harvard.edu/viz-bin/VizieR?-source=J/A+A/385/816/

Results





LSB galaxy	Upper bound on β at 95% C.L.
U5005 (NFW)	6×10^{-3}
U5005 (ISO)	2×10^{-3}
U5005 (PM $\alpha = 0.2$)	6×10^{-3}
U5005 (PM $\alpha = 0.7$)	9×10^{-3}

Conclusion

- We investigate effects of chameleon scalar field on rotation curves by **adding the fifth force**

- Analytic solution + Normal boundary conditions
  - ➡ Constraint on coupling constant
  - ➡ cannot occur in some DM halo profiles

- Divergent profile
  - ➡ Rotation curves are steeper around center of galaxy
  - ➡ Upper bound on coupling constant from observational data

Thank you for your attention

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