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"Dynamics of oscillating scalar field in thermal environment"

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Dynamics of Oscillating Scalar Field in Thermal Environment

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Based on **1208.3399** with **Kazunori Nakayama**

Introduction

Introduction

- At the early universe, a scalar field may condensate homogeneously w/ a far from equilibrium value.
- It begins to oscillate coherently at H ~ m and behaves as "Matter".
  - Examples: Inflaton, Moduli, Curvaton, Affleck-Dine field...
- To avoid the "overclosure", the scalar condensate should decay to the "Radiation" sector at an appropriate epoch.

➡ **Interaction** btw the scalar and radiation

Introduction

- Such **interactions** btw scalar and radiation induce various effects that make the scalar dynamics complicated:

  - Thermal Effects
    - -Thermally modified effective potential of the scalar field
    - -Dissipation of scalar condensate into the radiation sector

    e.g., [J. Yokoyama; M. Drewes; A. Berera et al.]

  - Non-perturbative particle production

    e.g., [L. Kofman, A. Linde, A. Starobinsky]

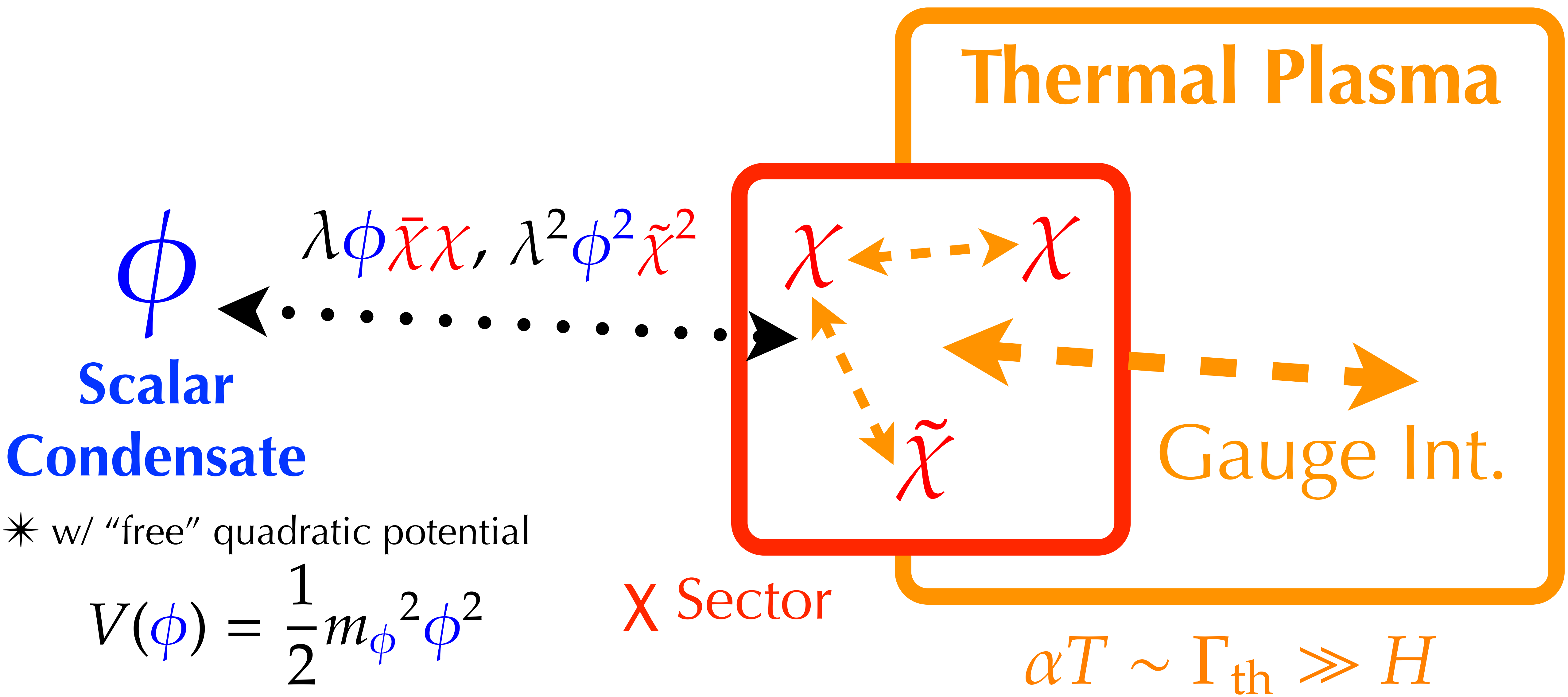
  - (possible) non-topological soliton formation

    e.g., [E. Copeland et al.; S. Kasuya et al.]

➡We included **these effects** simultaneously.

Set Up

■ Let us consider the following simplest set up:



Outline

- Introduction
- Closed Time Path Formalism
- Thermal Effects
- Non-perturbative Particle Production
- Numerical Results

Thermal Effects

Thermal Effects

- Typically, there are two effects from thermal plasma.
  - Force from thermal plasma
  - Dissipation to thermal plasma

✳If the Bkg plasma remains in thermal equilibrium.

➡Reduced to Coarse-Grained EOM of Φ:

$$0 = \frac{\delta S}{\delta \phi} + \frac{\delta \tilde{\Gamma}}{\delta \phi} = \frac{\delta S}{\delta \phi} - \frac{\partial \mathcal{F}}{\partial \phi} - \Pi_{\rm ret} * \delta\phi + \cdots$$

$$\simeq \frac{\delta S}{\delta \phi} - \frac{\partial \mathcal{F}}{\partial \phi} - \Gamma_\phi \dot{\phi}; \quad \Gamma_\phi \simeq \lim_{\omega \to 0} \frac{\Pi_J(\omega, \mathbf{0})}{2\omega}.$$

e.g., [A. Berera et al., hep-ph/9803394]

Thermal Effects

- Typically, there are two effects from thermal plasma.
  - Force from thermal plasma:

$$\ddot{\phi} + (3H + \Gamma_\phi)\dot{\phi} + m_\phi^2 \phi = -\frac{\partial \mathcal{F}}{\partial \phi}$$

Free Energy of thermal plasma w/ the background ϕ.

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Free Energy of thermal plasma w/ the background ϕ.

-Small amplitude ⇒ Thermal mass

$$\mathcal{F} \propto \lambda^2 T^2 \phi^2; \;\; \lambda\phi \ll T$$

-Large amplitude ⇒ Thermal log

$$\mathcal{F} \propto \alpha^2 T^4 \ln(\lambda^2\phi^2/T^2); \;\; \lambda\phi \gg T$$

[A. Anisimov, M. Dine]

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-Small amplitude: processes including χ.

$$\Gamma_\phi \sim \lambda^2 \alpha T$$

-Large amplitude: multiple scattering by gauge bosons.

$$\Gamma_\phi \sim \frac{\alpha^2}{\ln \alpha^{-1}} \frac{T^3}{\phi^2}$$ [D. Bodeker; M. Laine]

Non-perturb. Particle Production

Non-perturb. Production

- The adiabaticity of χ particles can be broken down when the scalar passes through the origin.
  - Adiabaticity is broken if $\epsilon \gg 1$.

$$\epsilon := \left| \frac{\dot{\omega}_\chi}{\omega_\chi^2} \right| ; \quad \omega_\chi = \sqrt{\mathbf{k}^2 + m_{\rm eff}^\chi(T)^2 + \lambda^2 \phi^2(t)}$$

➡Efficient χ production can occur at Φ~0.

[L. Kofman, A. Linde, A. Starobinsky, hep-ph/9704452]

Non-perturb. Production

- If Φ oscillates w/ the finite T potential (free-energy),

$$\begin{cases} \lambda \ll \alpha : \text{No efficient prod.} \\ \lambda \gg \alpha : \text{Complicated...} \end{cases}$$

- If Φ oscillates w/ the zero T potential, the efficient particle production occurs @ $|\phi| \lesssim \left[m_\phi \tilde{\phi}/\lambda\right]^{1/2}$.

$$n_\chi \sim \frac{k_*^3}{(2\pi)^3}; \;\; k_* = \sqrt{\lambda \tilde{\phi} m_\phi}$$

[$\tilde{\phi}$: amplitude]

Non-perturb. Production

- The produced χ may decay @ $t_{\rm dec}\,\Gamma_\chi(\phi(t_{\rm dec})) \sim 1.$

  [Otherwise the parametric resonance may occur]

- The Φ's energy is partially converted to the radiation, and it is estimated as

$$\frac{\delta\rho_\phi}{\rho_\phi} \sim \frac{\lambda^2}{4\pi^3\alpha^{1/2}}; \text{ @ every one oscillation.}$$

Here we assumed that the decay rate of χ is given by

$$\Gamma_\chi \sim \alpha\, m_\chi(\phi).$$

[Felder, Kofman, Linde, "Instant preheating",hep-ph/9812289]

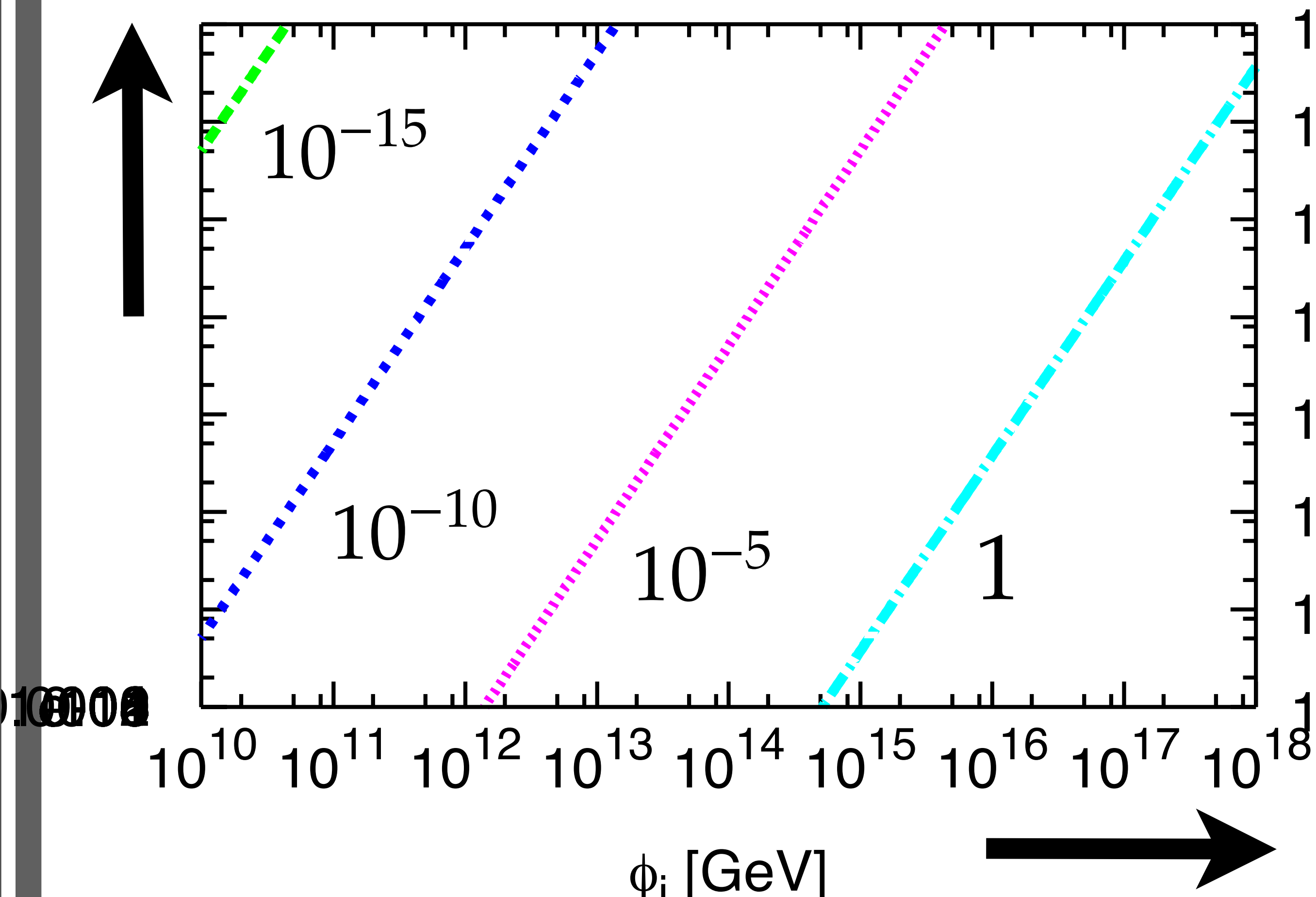
Numerical Results

Numerical Results

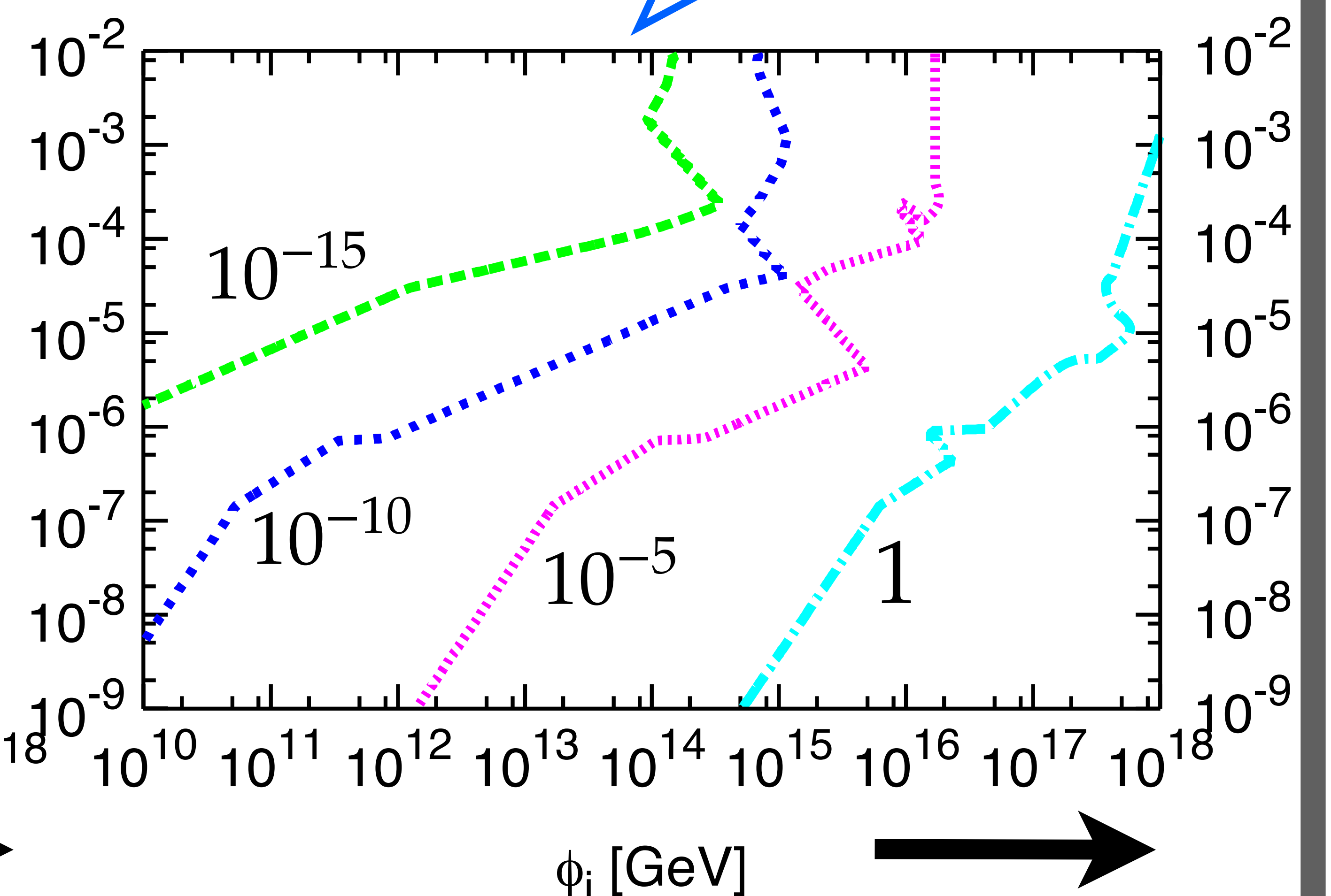
- Contour plot of $\left.\dfrac{m_\phi^2 \tilde{\phi}^2/2}{\rho_{\rm rad} + \rho_{\rm inf}}\right|_{H=\Gamma_\phi}$. $[\tilde{\phi}$: amplitude$]$

$\alpha = 0.05;$
$T_R = 10^9$ GeV;
$m_\phi = 1$ TeV.

Coupling
btw Φ & radiation: λ



Initial amplitude: ϕ_i [GeV]



Initial amplitude: ϕ_i [GeV]

Summary

- **Interactions** btw scalar and radiation are often introduced to avoid overclosure.
- Such **interactions** induce various effects that make the dynamics of scalar condensate complicated.
- We take into account all these effects properly, and show that such effects can change the abundance of scalar condensate by orders in a broad range of parameters.

Back Up

CTP formalism

CTP Formalism

- Closed Time Path (CTP) formalism gives us useful tools to study an **evolution of expectation value**.

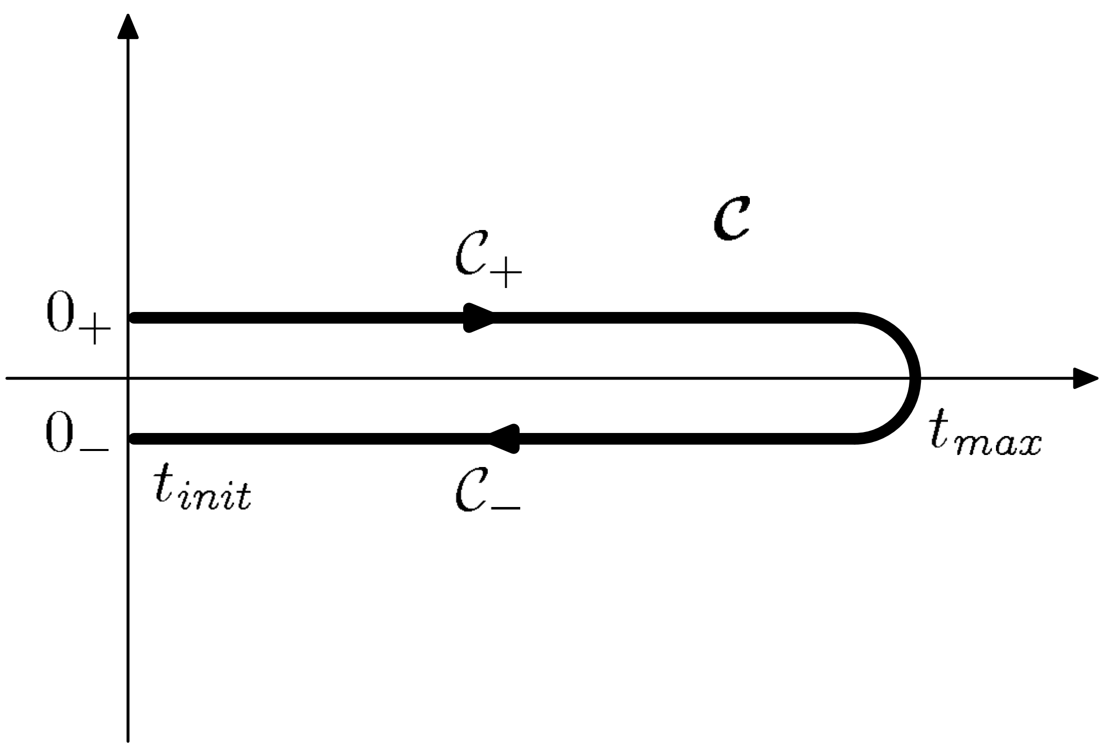


Fig. from M. Garny et al., 0904.3600

▶An expectation value can be obtained from CTP integral:

$$\langle \hat{O}_H(t) \rangle = \mathrm{tr}\left[\hat{\rho}\, T_C \exp\left(-i \int_C dt' H_I(t') \right) \hat{O}_I(t) \right]$$

T_C : contour C ordering

ρ : density matrix

$\hat{O}_H(t)$: Heisenberg Picture

$\hat{O}_I(t)$: Interaction Picture

e.g., [J. Berges, hep-ph/0409233]

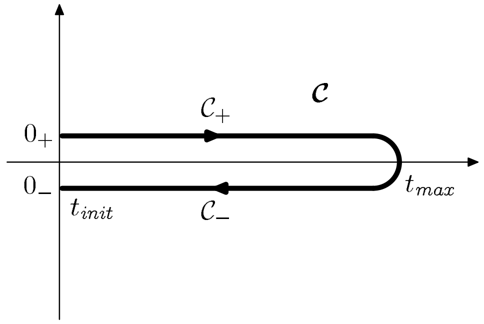
CTP Formalism

- It is useful to consider n-point functions on CTP.

$$\phi = \mathrm{tr}\left[\hat\rho\hat\phi\right]$$

$$G = \mathrm{tr}\left[\hat\rho\, T_C\, \hat\phi\hat\phi\right]^{\mathrm{con}}$$

$$V_n = \mathrm{tr}\left[\hat\rho\, T_C\, \hat\phi\cdots\hat\phi\right]^{\mathrm{con}}$$



T_C : contour C ordering
[con. stands for connected part]

- **Kadanoff-Baym**: Follow the evolution of **Φ** and G self-consistently.

➡Schwinger Dyson equations on CTP [w/ skeleton diagram expansion]

$$G^{-1} = G_0^{-1} + \Pi[\phi, G]; \qquad \Pi = -2i\frac{\delta\tilde\Gamma[\phi,G]}{\delta G}$$

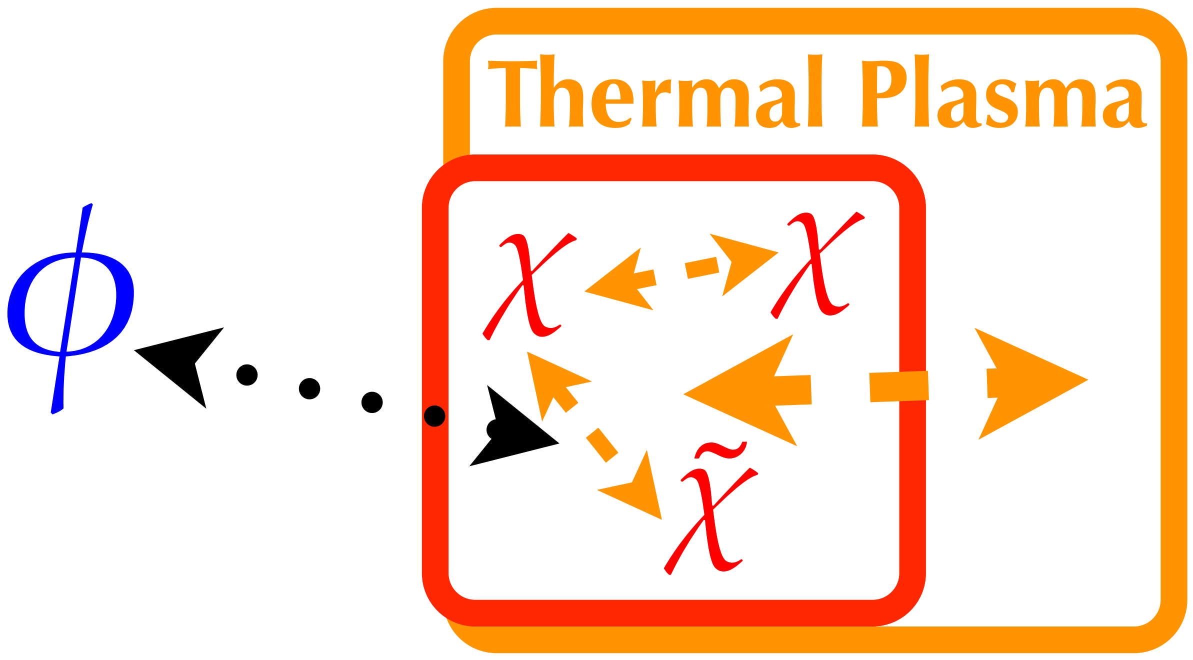
$$0 = \frac{\delta S}{\delta\phi} + \frac{\delta\tilde\Gamma[\phi,G]}{\delta\phi}$$

[Cornwall, Jackiw, Tomboulis,1974]

CTP Formalism

- Top-down approach:

Kadanoff-Baym eqs.



- Some fields may be kept in thermal equilibrium during the course of Φ's dynamics.
- Propagators of these fields

→ **Thermal Propagators**
-Thermal mass
-Thermal width

Reduce

Coarse-grained eq. for Φ

Coherent Oscillation

- Scalar condensate is obtained from one point func.
- If the scalar oscillates adiabatically w.r.t. thermal plasma...
  - Propagators in thermal bath → thermal ones
  - ➡Reduced to Coarse-Grained EOM of Φ:

$$0 = \frac{\delta S}{\delta \phi} + \frac{\delta \tilde{\Gamma}}{\delta \phi} = \frac{\delta S}{\delta \phi} - \frac{\partial \mathcal{F}}{\partial \phi} - \Pi_{\rm ret} * \delta\phi + \cdots$$

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e.g., [A. Berera et al., hep-ph/9803394]

Coleman-Weinberg potential

CW potential

- In non-SUSY, there exists **Coleman Weinberg (CW) potential:**

$$\mathcal{V}_{\rm CW} = \sum_F \epsilon_F \frac{m_\chi^4(\phi)}{64\pi^2}\left[\ln\frac{m_\chi^2(\phi)}{\mu^2} - \frac{3}{2}\right]$$

$$\epsilon_F := \begin{cases} +1 & \text{for real scalar} \\ -2 & \text{for Weyl fermion} \end{cases} .$$

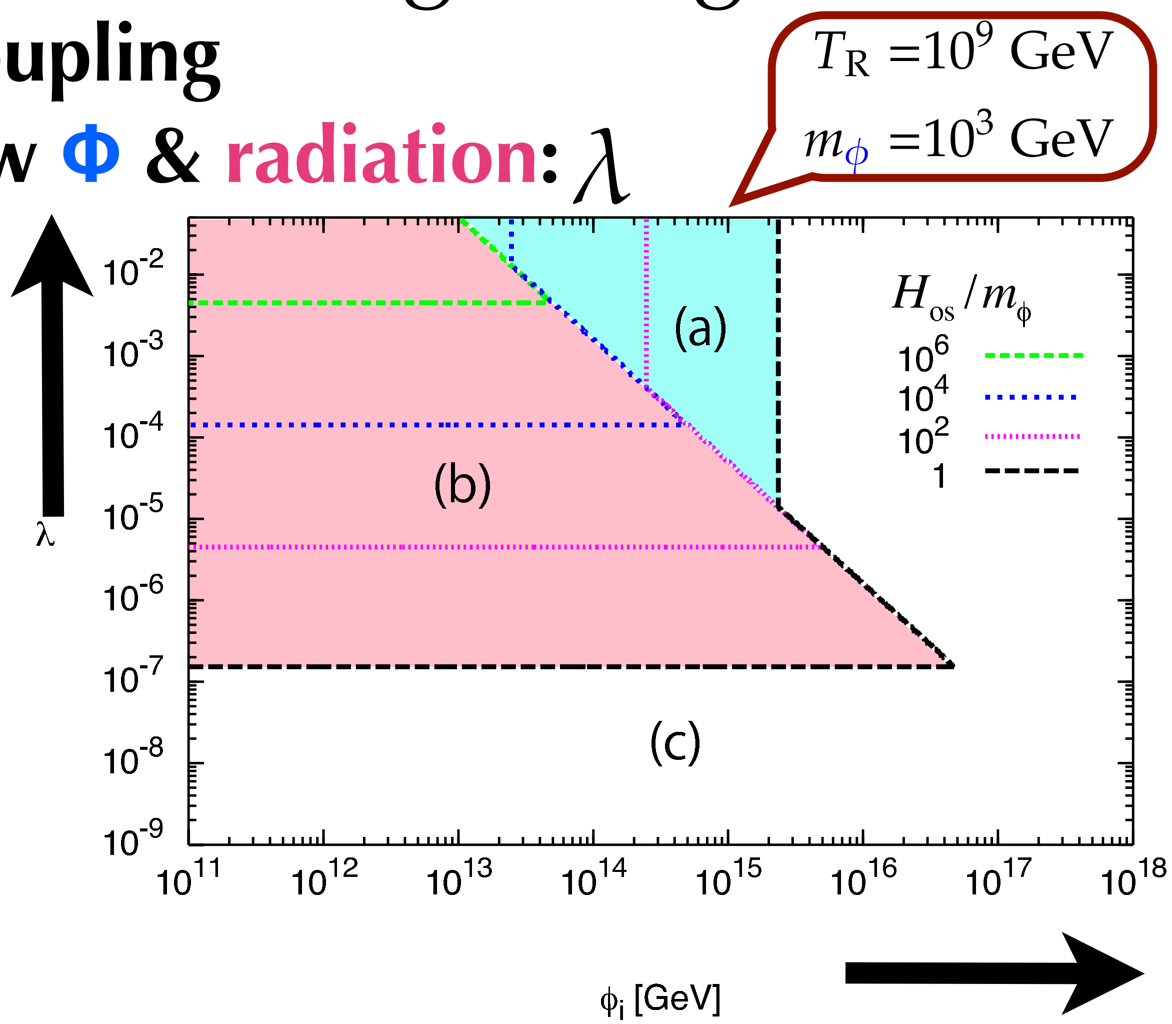
- In SUSY, such radiative corrections to flat directions are suppressed and remain only small logs due to the SUSY breaking effects.

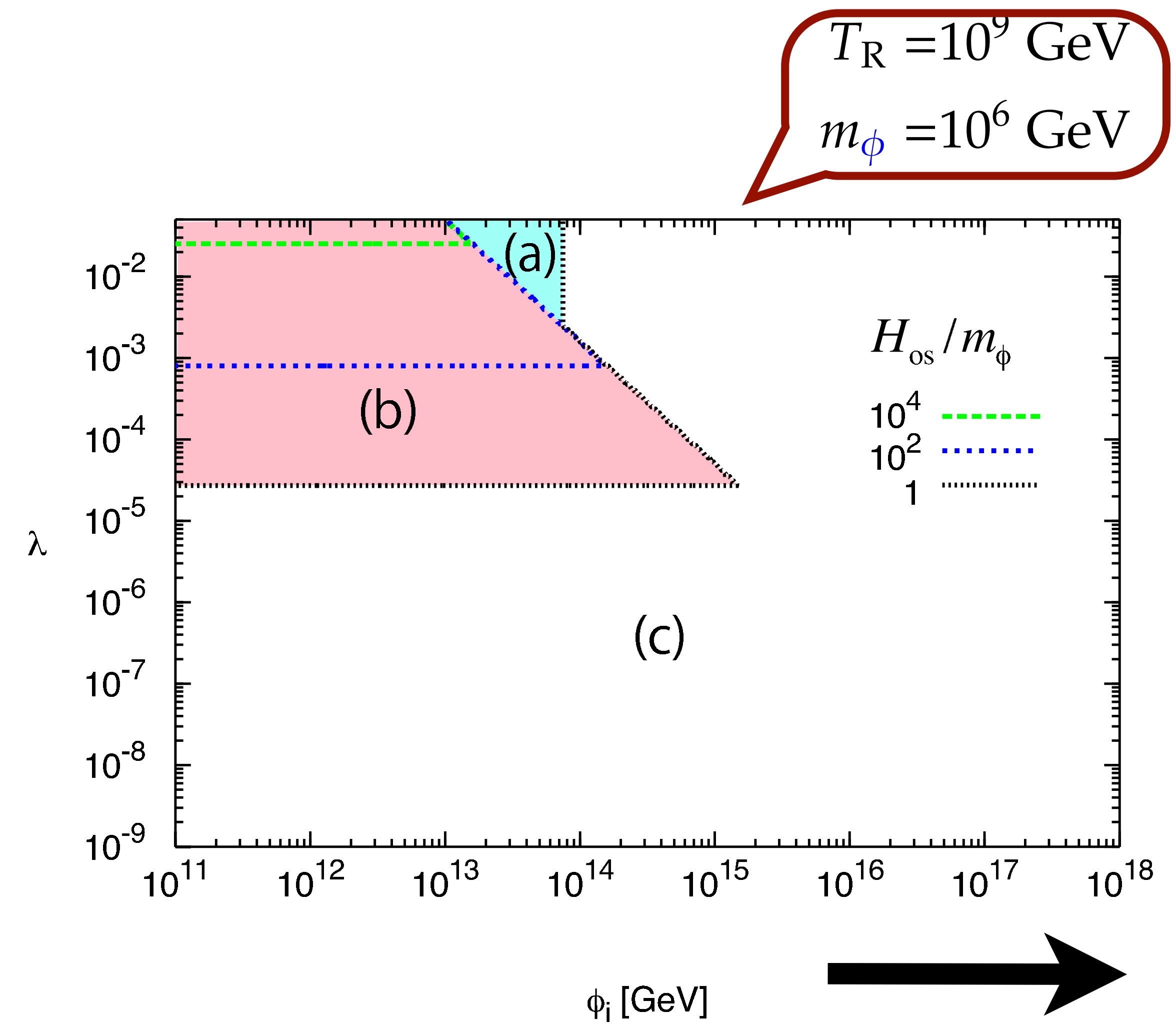
Numerical Results

Numerical Results

■ The beginning of oscillation:

Coupling btw Φ & radiation: λ





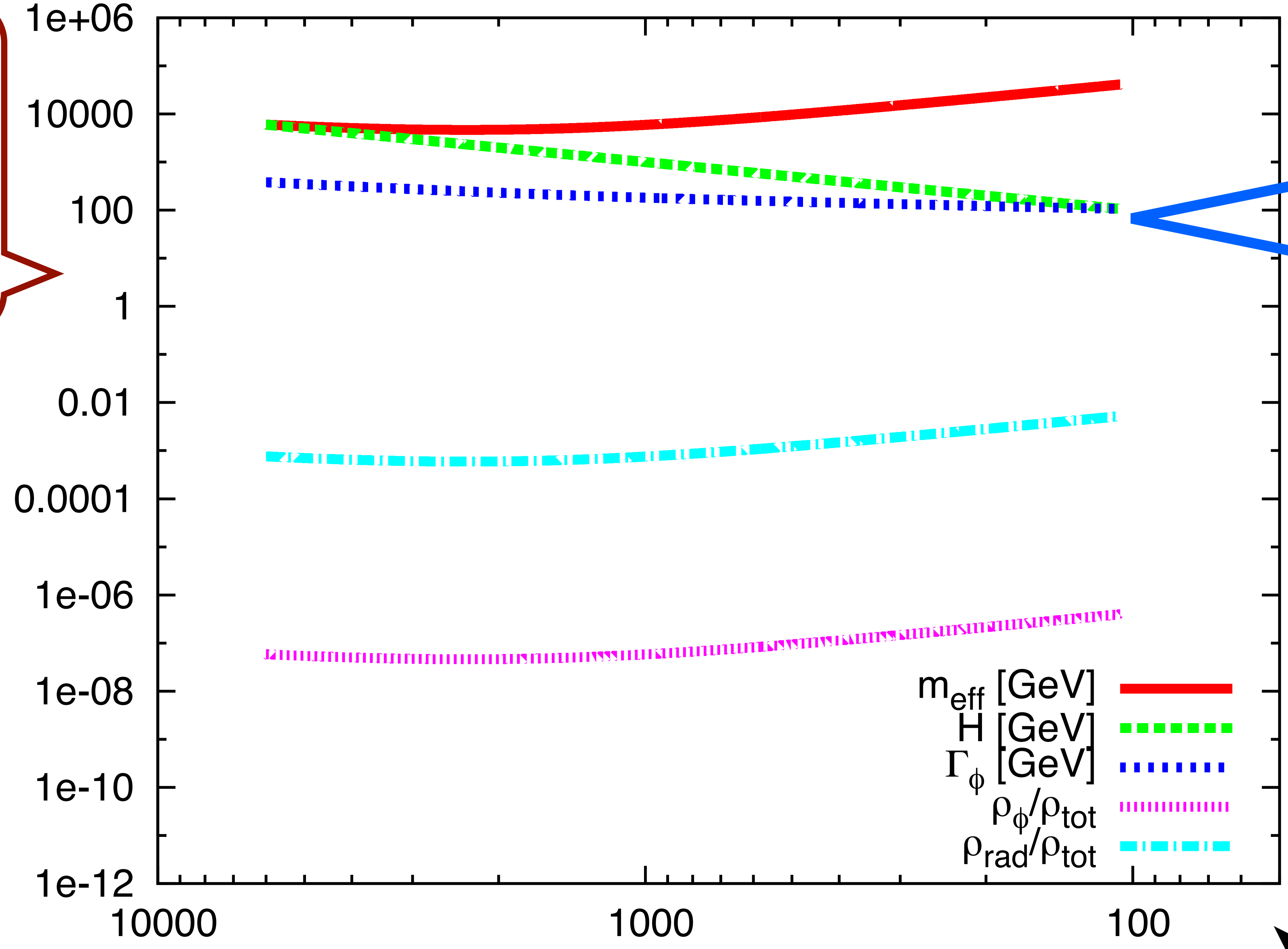
Initial amplitude: ϕ_i [GeV] **Initial amplitude:** ϕ_i [GeV]

(a): thermal log, (b): thermal mass, (c): zero T mass

Numerical Results

- Oscillation w/ thermal log:

$m_\phi = 1\,\mathrm{TeV}$
$T_R = 10^9\,\mathrm{GeV}$
$\lambda = 2\times10^{-3}$
$\phi_i = 10^{15}\,\mathrm{GeV}$

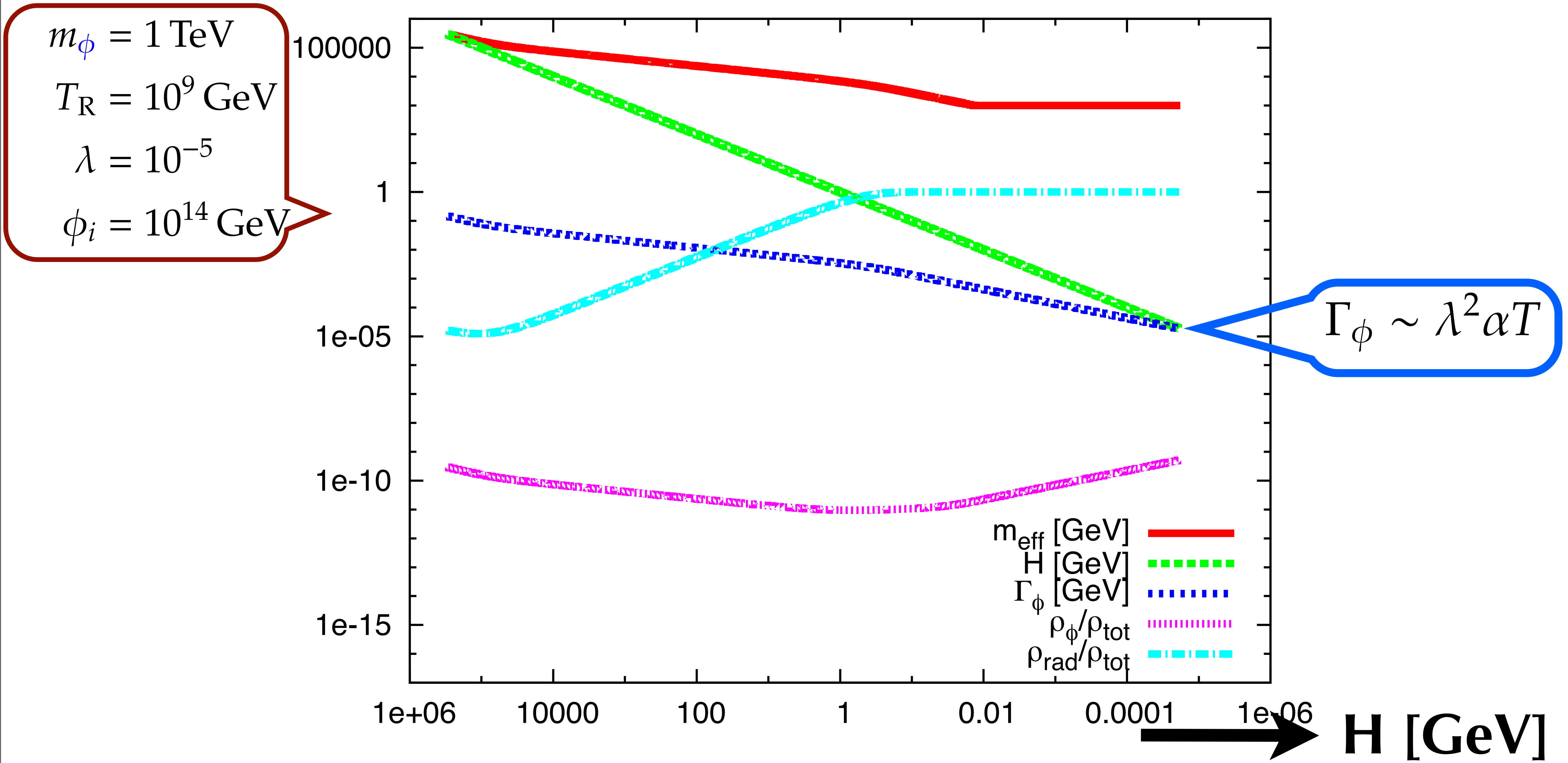


$\Gamma_\phi \sim \alpha^2 \frac{T^3}{\phi^2}$

H [GeV]

Numerical Results

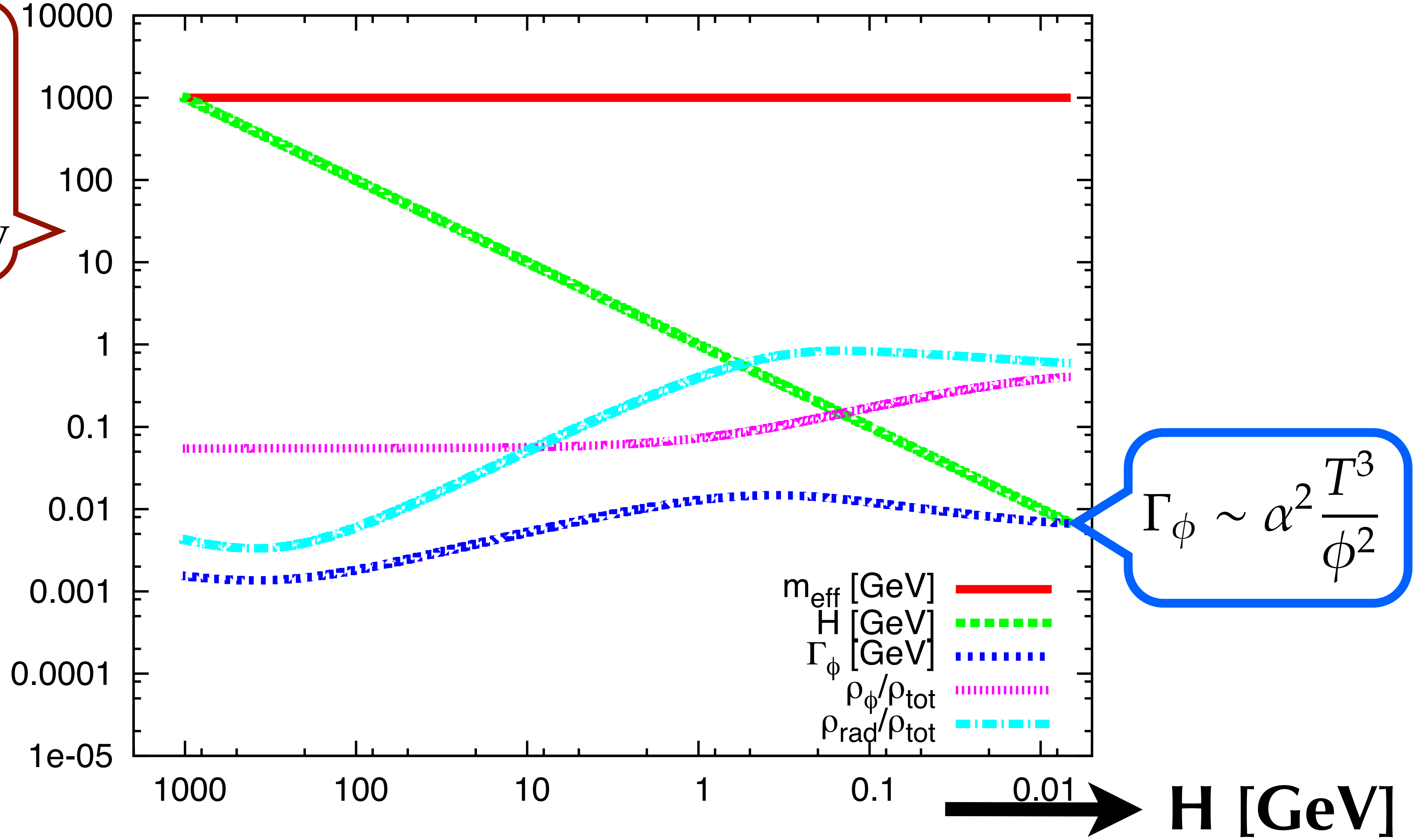
- Oscillation w/ thermal mass:



Numerical Results

- Oscillation w/ zero T mass:

$$m_\phi = 1\,\mathrm{TeV}$$
$$T_\mathrm{R} = 10^9\,\mathrm{GeV}$$
$$\lambda = 10^{-2}$$
$$\phi_i = 10^{18}\,\mathrm{GeV}$$



$$\Gamma_\phi \sim \alpha^2 \frac{T^3}{\phi^2}$$

Oscillon

Oscillon (I-ball)

- A coherently oscillating scalar with a potential flatter than the quadratic one has an instability and may fragment into classical lumps.
- Even if there is no conserved charge, their stability is guaranteed by the adiabatic invariant.

  [Kasuya, Kawasaki, Takahashi, hep-ph/0209358]
- Such a non-topological soliton is dubbed as oscillon or I-ball.

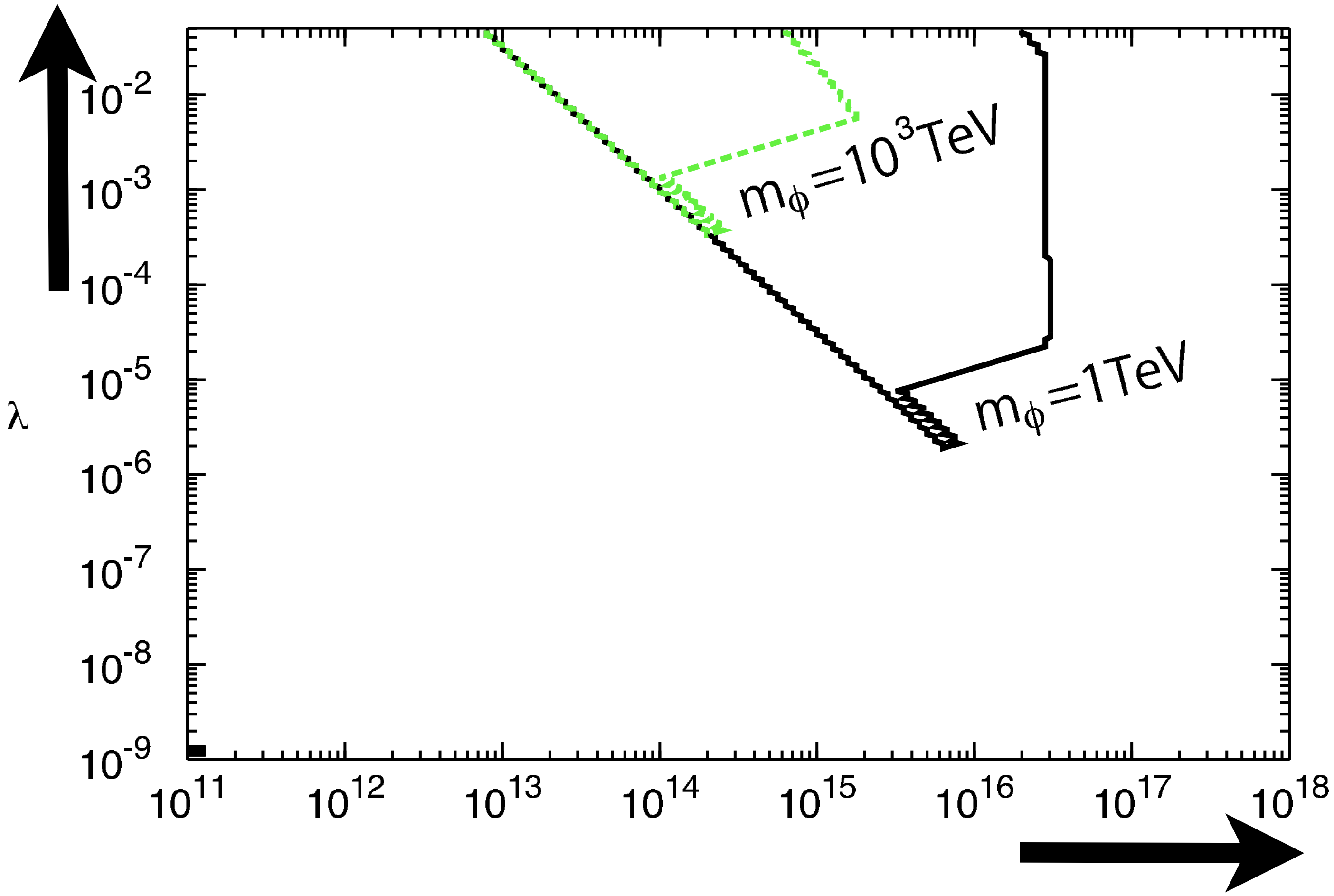
  e.g., [Copeland, Gleiser, Muller, hep-ph/9503217]

  [Kasuya, Kawasaki, Takahashi, hep-ph/0209358]

Oscillon (I-ball)

- The region where the I-ball may be formed.

Coupling
btw Φ & radiation: λ



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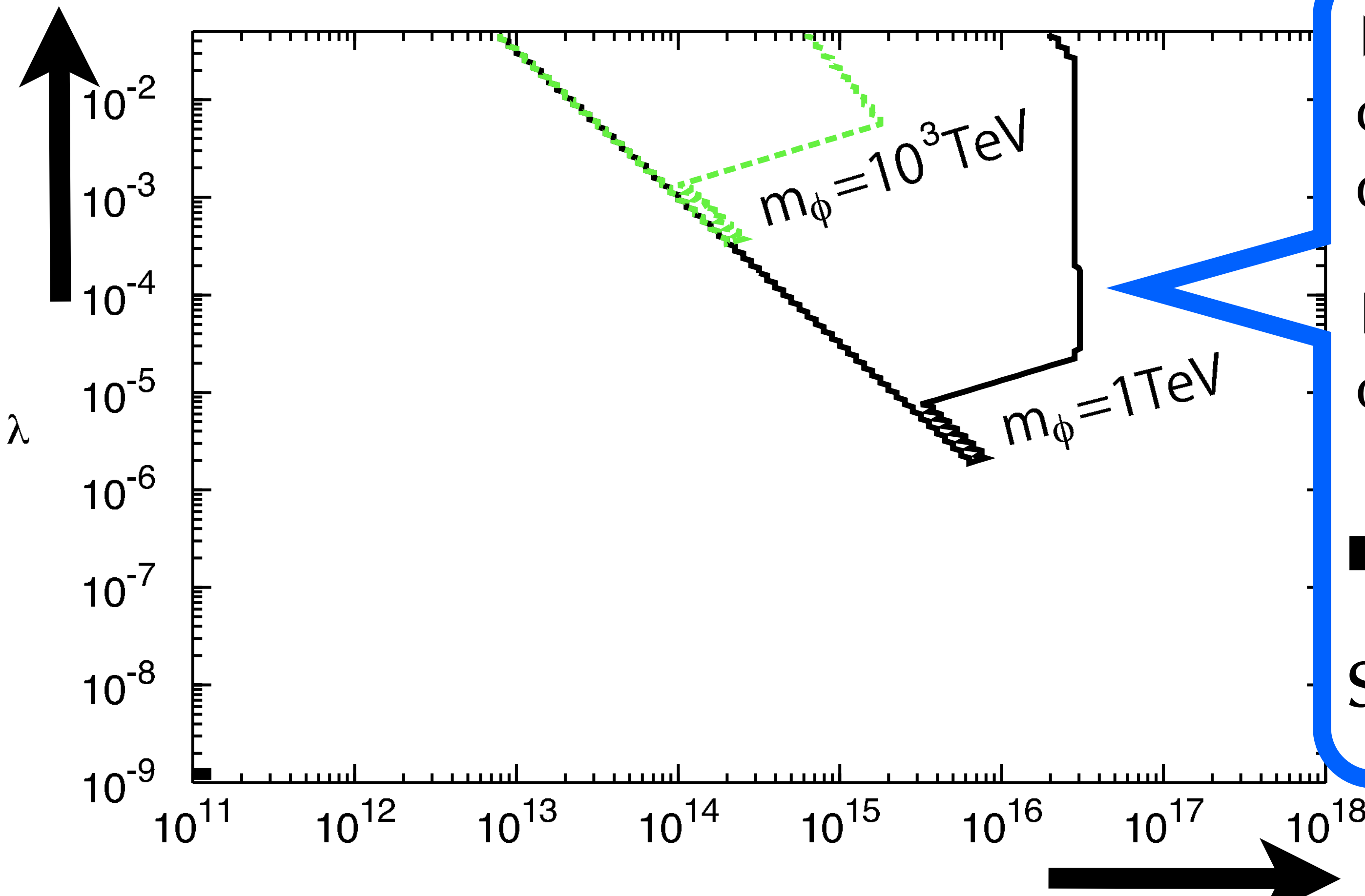

Initial amplitude: ϕ_i [GeV]

- It is also possible that the scalar condensation evaporates due to the dissipation before the formation of oscillon.
- Even if this is the case, the delayed type oscillon may be formed.

➡ Further study is needed to say something conclusively.

Bulk Viscosity

Bulk Viscosity

- The dissipation rate at large amplitude regime is directly related to the bulk viscosity of Yang-Mills plasma.

$$\Gamma_\phi = \lim_{\omega\to 0} \frac{\Pi_J(\omega, \mathbf{0})}{2\omega}$$

$$= \lim_{\omega\to 0} \frac{1}{2\omega} \int d^4x\, e^{-i\omega t} \langle [\hat{O}(t,\mathbf{x}), \hat{O}(0)] \rangle; \quad \hat{O}(x) = \frac{A}{8\pi^2\phi} F^{a\mu\nu}(x) F^a_{\mu\nu}(x)$$

[D. Bodeker; M. Laine]

Bulk Viscosity: $\zeta = \frac{1}{9} \int d^4x\, e^{-i\omega t} \langle [T^\mu{}_\mu(t,\mathbf{x}), T^\nu{}_\nu(0,\mathbf{0})] \rangle$

$\zeta \sim \frac{\alpha^2 T^3}{\ln[1/\alpha]}$; @ weak coupling

[Arnold, Dogan. Moore, hep-ph/0608012]