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“Non-gaussian bubbles from tunneling in the inflationary era”

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**RESCEU SYMPOSIUM ON  
GENERAL RELATIVITY AND GRAVITATION**

**JGRG 22**

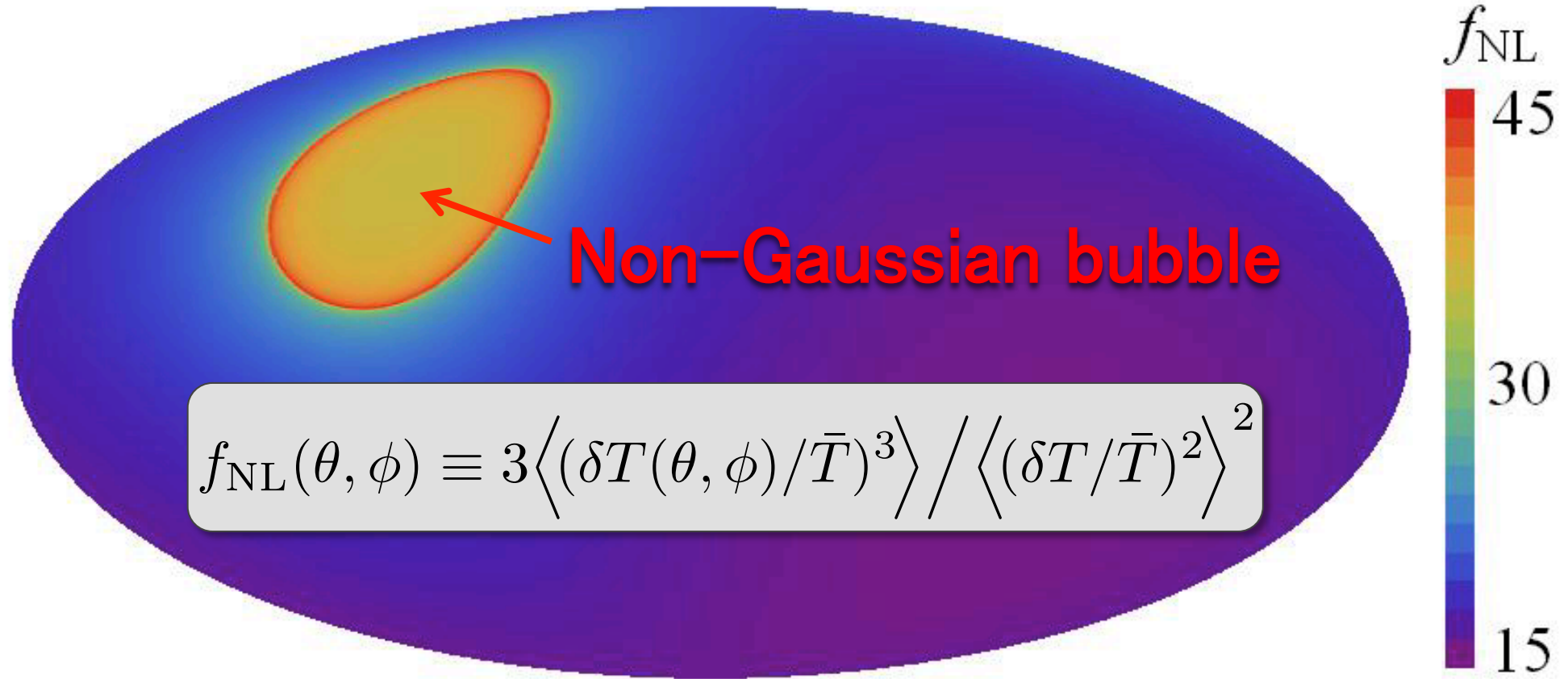
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Koshiba Hall, The University of Tokyo, Hongo, Tokyo, Japan



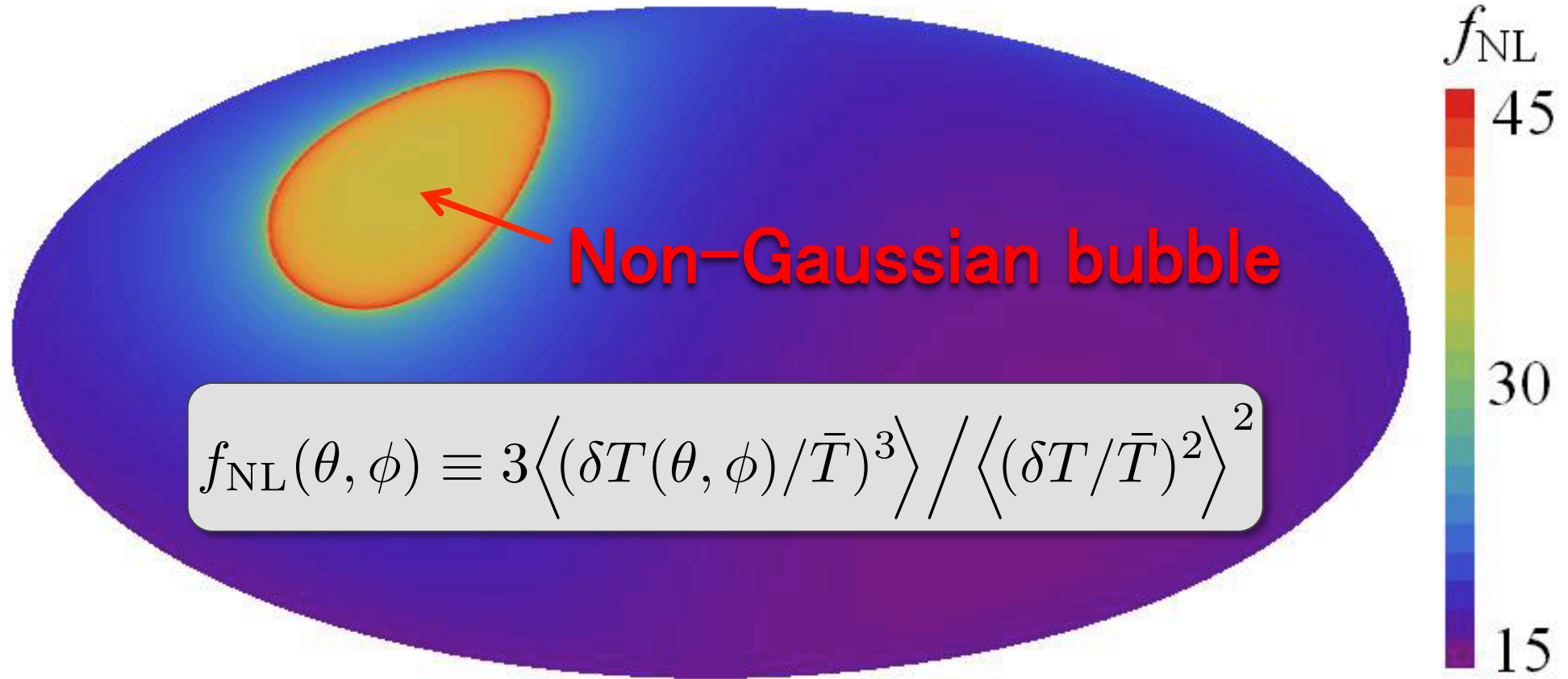


# Non-Gaussian bubbles in the sky





# Non-Gaussian bubbles in the sky





# Introductions

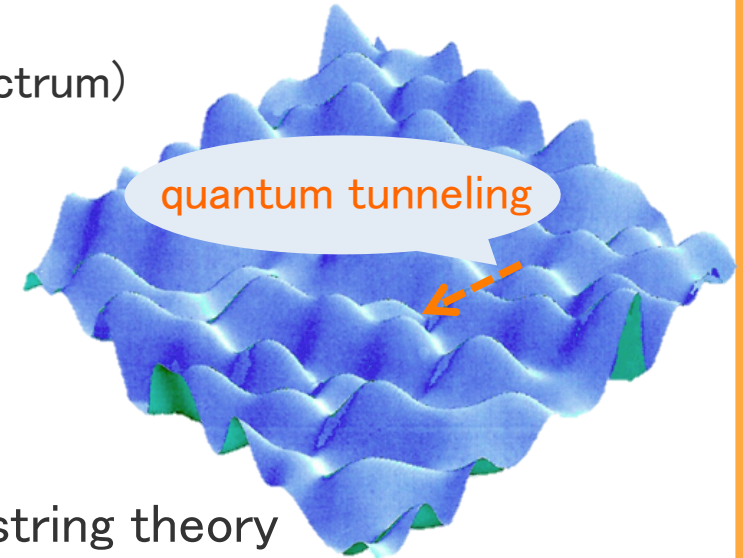


# Motivation

<http://journalofcosmology.com>

## □ Inflation (flatness, homogeneity, power spectrum)

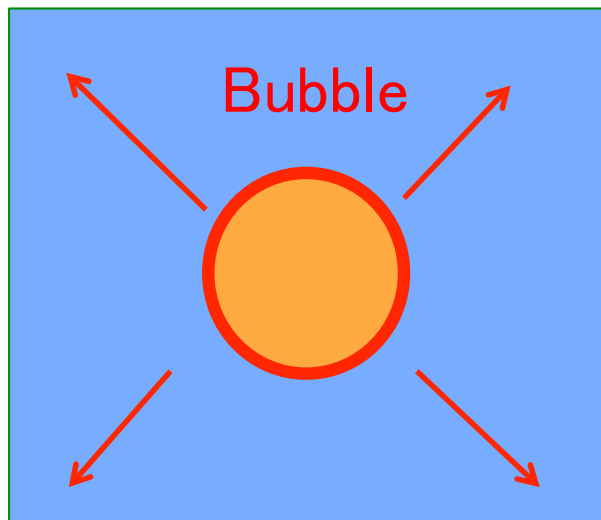
- good consistency with observation
- What is the physical origin of inflation?



## □ String landscape (Susskind, 2003)

- attempt to explain the origin of inflation with string theory
- many scalar fields & local potential minima

potential for scalar fields



bubble nucleation

## □ Bubble nucleation (= quantum tunneling) (= 1<sup>st</sup> order phase transition)

- tunneling between local minima
- scalar-field bubble is nucleated during inflation



# Bubbles in the sky

## After inflation

- all scalar fields, including bubbles, decay
- but, signature of bubbles may be seen as some bubble feature in the sky

## Non-Gaussianity $f_{\text{NL}}$ (Komatsu and Spergel, 2001)

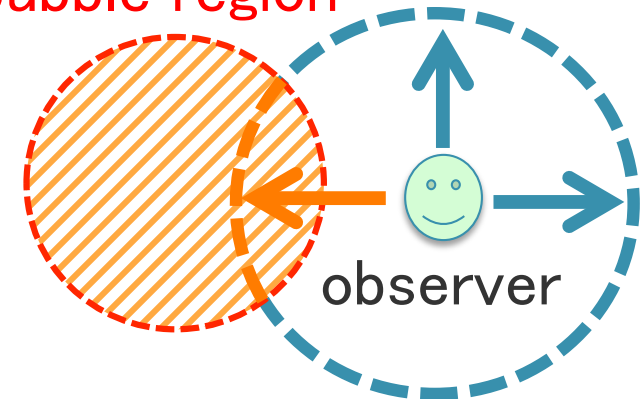
$$\phi(x) = \phi_{\text{L}}(x) + f_{\text{NL}} \left( \phi_{\text{L}}^2(x) - \langle \phi_{\text{L}}^2 \rangle \right)$$

- $f_{\text{NL}}$  is one way to parameterize deviation from Gaussian statistics
- small for the simplest inflation model (Maldacena 2002) ( $\phi_{\text{L}}$ : Gaussian variable)
- suitable for detection of deviation from the simplest model

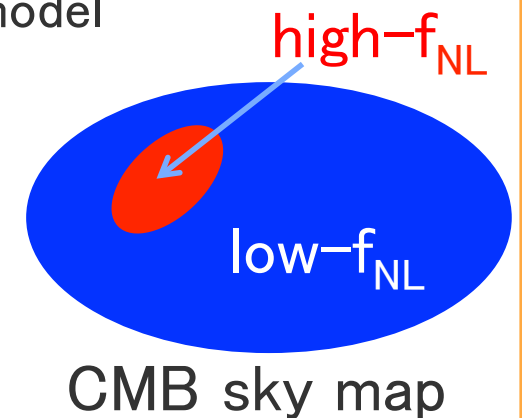
## Non-Gaussian bubbles

- bubble signature may be seen in non-Gaussianity
- $f_{\text{NL}}$  can be different for each CMB sky patch
- we show how it happens using a toy model

last scattering surface  
ex-bubble region

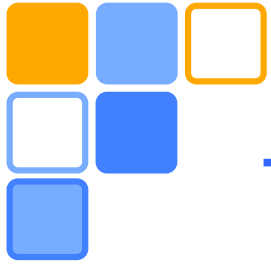


(in comoving coordinate)





# Toy Model



# Three scalar fields

Name

Role

Energy density/Hierarchy

□ Inflaton:  $\Psi$

slow-roll inflation

high

□ Tunneling field:  $\sigma$

bubble nucleation

middle

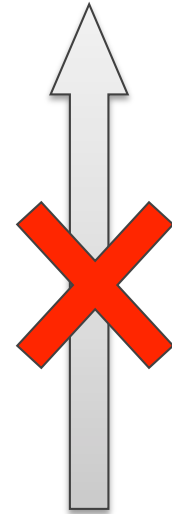
□ Curvaton:  $\phi$

curvature pert. generation

low

(Lyth, Ungarelli & Wands;  
Enqvist & Sloth; Moroi & Takahashi)

no backreaction approximation



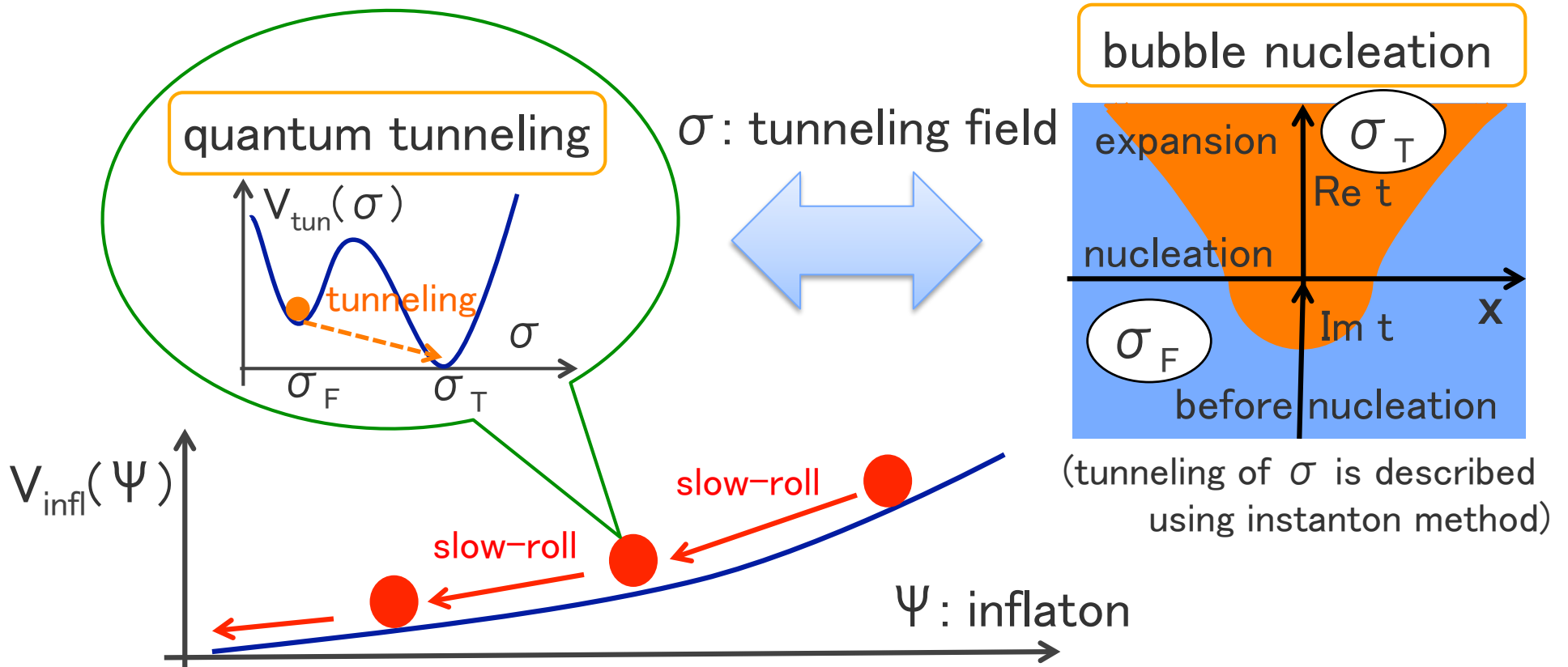
(**Curvaton scenario**: Curvaton affects neither slow-roll inflation nor bubble nucleation. But it decays later than inflaton and tunneling field. Then, its energy density becomes relatively higher and it creates curvature pert. when it decays.)

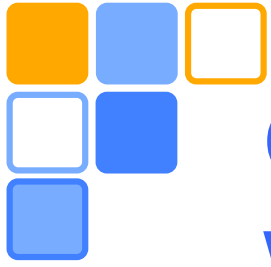




# Bubble nucleation during slow-roll inflation

- Slow-roll inflation is not affected by bubble nucleation
- Bubble nucleates at one moment of slow roll inflation  
(for simplicity, we consider single nucleation case)





# Curvaton evolution in the universe with a bubble

- Original potential for curvaton  $\phi$  has interaction with tunneling field

$$V(\phi) = \frac{m^2}{2} \phi^2 + V_{\text{int}}(\sigma, \phi)$$

- Effective potential for  $\phi$  in the universe with bubble of  $\sigma$

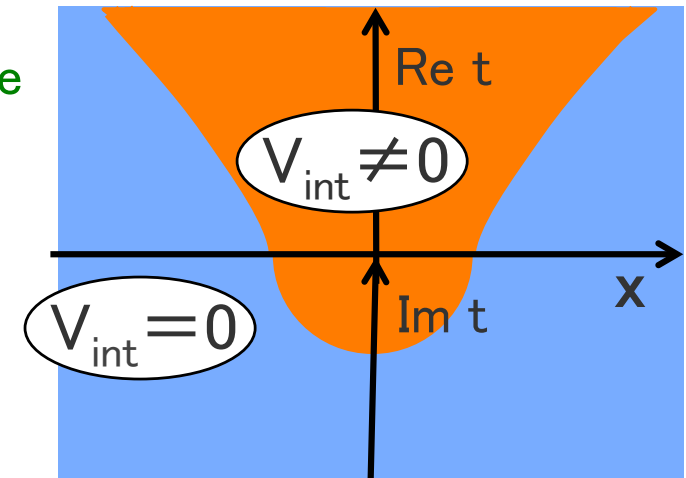
$$V_{\text{int}}^{(\text{eff})}(\phi; x) := V_{\text{int}}(\bar{\sigma}(x), \phi)$$

substituting background bubble

- Non-linear self interaction

is assumed to vanish at false vacuum

$$V_{\text{int}}(\sigma, \phi) = \lambda(\sigma) \phi^3 \quad (\lambda(\sigma_F) = 0)$$



Non-Gaussianity is generated only inside bubble



# Method



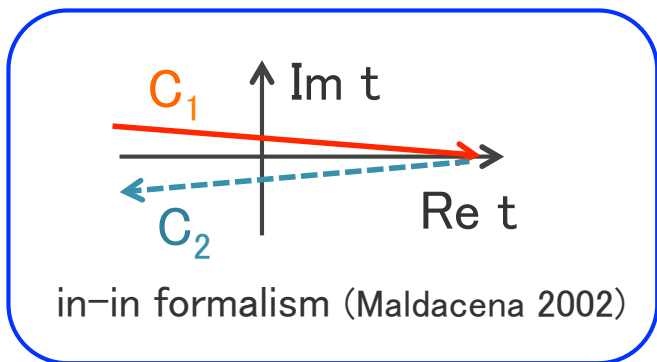
# How to calculate 3pt function in the universe with a bubble

- Background bubble of  $\sigma$  is described by CDL instanton  
(Coleman and De Luccia(CDL), 1980)
  - imaginary time evolution describes nucleation process
  - real time evolution describes expansion afterwards

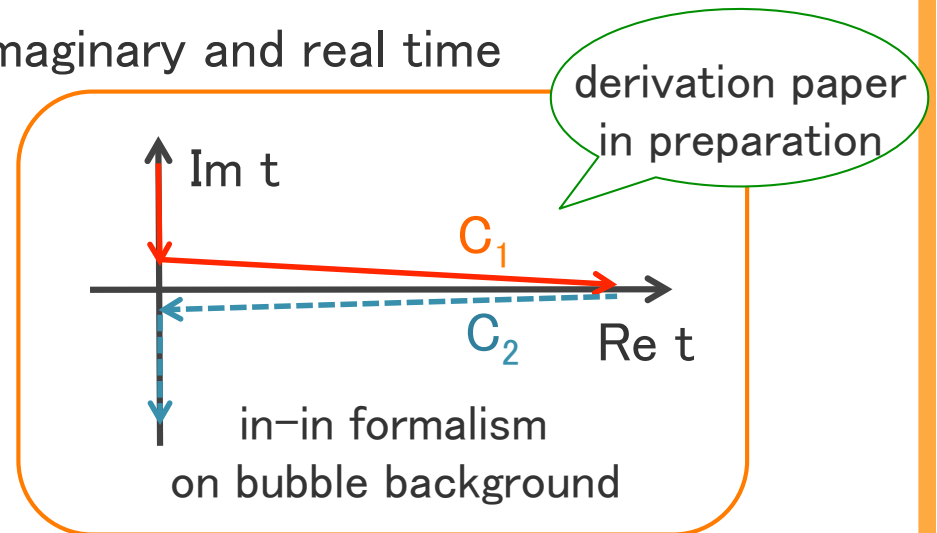
## skewness of $\phi$ on bubble background

$$\langle \phi^3(x) \rangle = \left\langle 0 \left| P \left( \phi^3(x) \exp \left[ -\frac{i}{\hbar} \int_{C_1+C_2} dt \int d^3 \mathbf{x} \sqrt{-g} V_{\text{int}}^{(\text{eff})}(\phi(x); x) \right] \right) \right| 0 \right\rangle$$

- extension of in-in formalism to the case with bubble
- in-in time path consists of both imaginary and real time



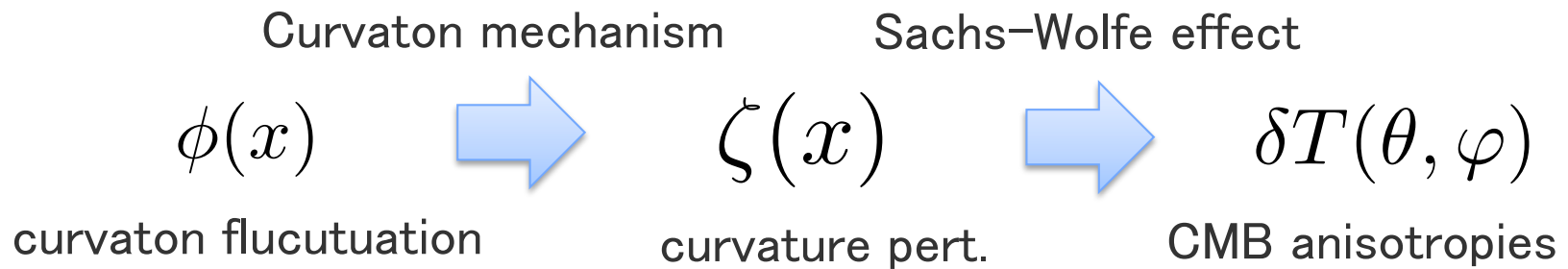
extension





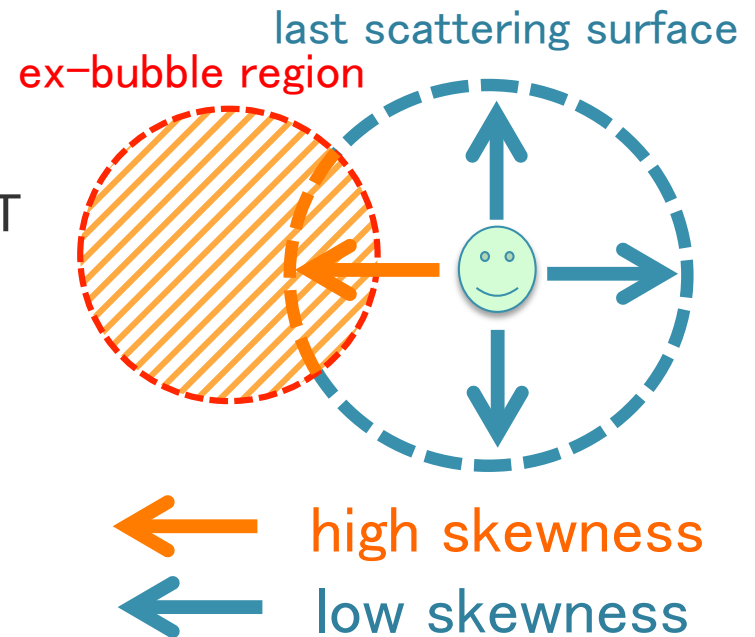
# From curvaton fluctuation to $\delta T$

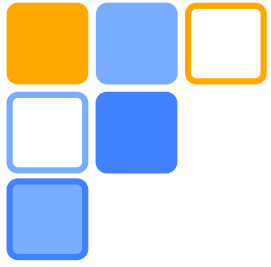
## How fluctuation evolves



## CMB skewness: $\langle \delta T^3(\theta, \varphi) \rangle$

- skewness of  $\phi$  becomes skewness of  $\delta T$
- observer sees high skewness spot  
(= Non-Gaussian bubble)



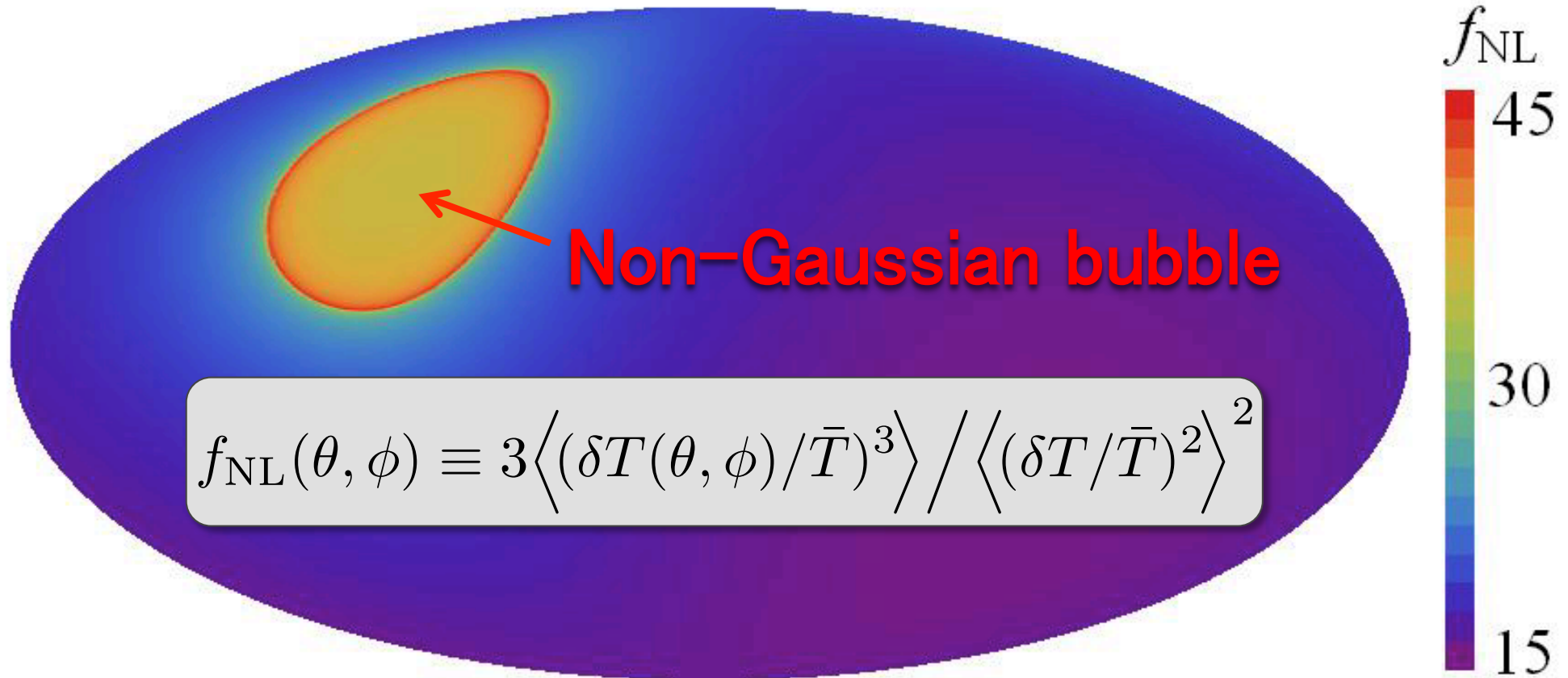


# Result and Conclusion



# Result

- calculated non-Gaussian bubble in the CMB sky



cf. PLANCK' s expected precision(2013 March)  
(for homogeneous non-Gaussianity)

$$|\Delta f_{\text{NL}}| \sim \mathcal{O}(5)$$



# Conclusions

non-Gaussian bubble



□ We have shown that a bubble-shaped high-skewness spot in CMB anisotropies may be generated if a bubble is nucleated during inflationary era by using a toy model.

□ Usual analysis using statistically homogeneous templates will miss non-Gaussian bubbles even if they exist. However, you may find them by making special analysis targeting them.  
(future work)

□ If you find non-Gaussian bubbles in near-future observations, it might be the first observational signature of string theory!





# Appendix



# Model parameters

## Thin-wall instanton

$$\bar{\sigma}(\chi) = \begin{cases} \sigma_T & \text{for } 0 \leq \chi < HR_W, \\ \sigma_F & \text{for } HR_W < \chi \leq \pi \end{cases}$$

(O(4) symmetric instanton  
depends only on coordinate  $\chi$ )

## Effective Lagrangian for $\phi = \phi_0 + \delta \phi$

$$\mathcal{L}_I = -\sqrt{-g} \lambda H \delta (\chi - HR_W) \delta \phi^3$$

effective coupling const.

$V_{\text{int}}$  is assumed to be large  
only around bubble wall

## Model parameters for the example

$$H/\phi_0 = 0.001 \quad \lambda = 0.005, \quad m/H = 0.3, \quad HR_W = 0.2\pi$$

$$r_\phi = 0.1, \quad |\mathbf{x}_0| = r_* = 2, \quad Ht_e = 50$$

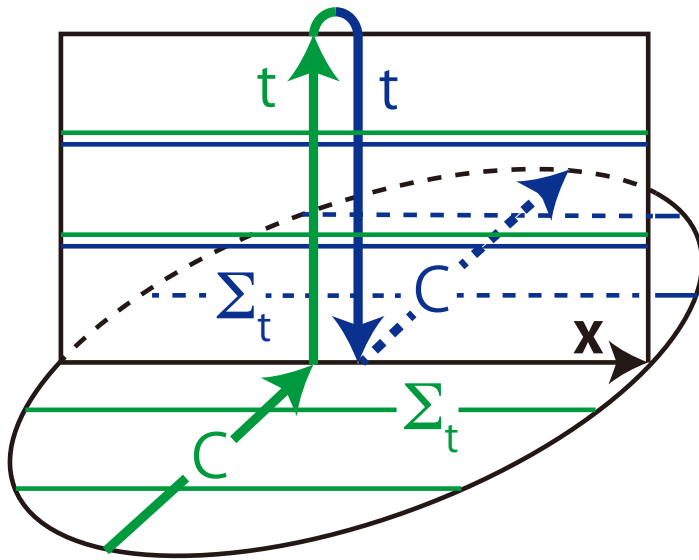
(parameters are chosen so that curvaton doesn't affect power spectrum,  
which is assumed to be generated by inflaton's fluctuation)

## Parameter dependence of $f_{\text{NL}}$ at the center of bubble

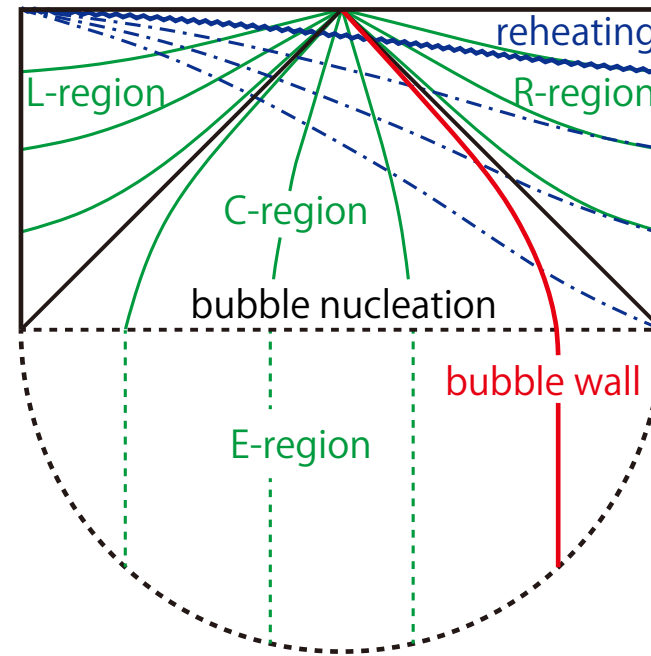
$$f_{\text{NL}}^{(\text{cen})} \approx \frac{3 \times 10^{-4} \lambda r_\phi^3 \sin^3(HR_W)}{A_\zeta^4 \exp\left(\left(\frac{m}{H}\right)^2 Ht_e\right)} \left(\frac{H}{m}\right)^6 \left(\frac{H}{\phi_0}\right)^3.$$



# Graphical description of in-in formalism with bubble



in-in time path for in-in formalism with bubble



Penrose-like diagram of the bubble nucleating universe

## □ N-point function

$$\langle \delta\phi(x_1)\delta\phi(x_2)\cdots\delta\phi(x_N) \rangle = \frac{\langle 0 | P \delta\phi(x_1)\delta\phi(x_2)\cdots\delta\phi(x_N) e^{i \int_{C \times \Sigma_t} dt d^3\mathbf{x} \mathcal{L}_I} | 0 \rangle}{\langle 0 | P e^{i \int_{C \times \Sigma_t} dt d^3\mathbf{x} \mathcal{L}_I} | 0 \rangle}$$



# curvaton mechanism and $\langle \delta\phi^3 \rangle$ to $f_{NL}$

(Lyth, Ungarelli & Wands;  
Enqvist & Sloth; Moroi & Takahashi)

## Curvature pert. at the decay of curvaton

(conserves outside horizon after that)

$$\zeta = (1 - r_\phi)\zeta_r + r_\phi\zeta_\phi$$

- curvature pert. from inflaton

- ratio of curvaton contribution

- curvature pert. from curvaton

$$\zeta_\phi = \frac{1}{3} \left. \frac{\delta\rho_\phi}{\rho_\phi} \right|_{t_e} = \frac{1}{3} \left( 2 \frac{\delta\phi}{\phi_0} + \frac{\delta\phi^2}{\phi_0^2} \right) \Big|_{t_e}$$

inflation end

subdominant

## Sachs-Wolfe effect

$$(\delta T/T)(\hat{n}) = (1/5) \zeta(\mathbf{x}_0 + r_*\hat{n}, t_*)$$

## $f_{NL}$ as a function of $\langle \delta\phi^3 \rangle$

$$f_{NL}(\mathbf{n}) \equiv 3 \frac{\langle (\delta T(\mathbf{n})/\bar{T})^3 \rangle}{\langle (\delta T/\bar{T})^2 \rangle^2}$$

$$\approx \frac{40r_\phi^3}{81\langle \zeta^2 \rangle^2} \frac{\langle \delta\phi^3(|\mathbf{x}_0 + r_*\hat{n}|, t_e) \rangle}{\phi_0^3}$$

$$\langle \zeta^2 \rangle = A_\zeta^2 \equiv 6.25 \times 10^{-10} \text{ (obs.)}$$

