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“The pre-inflationary dynamics of loop quantum cosmology:

Confronting quantum gravity with observations”

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# RESCEU SYMPOSIUM ON

# GENERAL RELATIVITY AND GRAVITATION

# JGRG 22

**November 12-16 2012**

**Koshiba Hall, The University of Tokyo, Hongo, Tokyo, Japan**

# The pre-inflationary dynamics of LQC
# Confronting quantum gravity with observations

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Dedicated with Pleasure to

**Hideo Kodama, Misao Sasaki & Toshi Futamase**

Friends, Colleagues and Creative Scientists,

Who Have Enriched us With So Many Insights!

**RESCEU Symposium on GRG, 12-16 November, 2012**

# Organization

1. Introduction: Successes and Limitations of Inflation

Overcoming the limitations:

2. Singularity Resolution in Loop Quantum Cosmology
3. Cosmological Perturbations on quantum FLRW Space-times
4. Extracting Physics: Power Spectra & Non-Gaussiantity.
5. Summary and Discussion

Understanding emerged from the work of many researchers, especially:

Agullo, Barrau, Bojowald, Cailleatau, Campiglia, Corichi, Grain,
Kaminski, Lewandowski, Mielczarek, Nelson, Pawlowski, Singh, Sloan, ...

For summary, see AA, Agullo & Nelson 1209.1609 PRL (at Press); More complete references: AA, Agullo & Nelson 1211:1354; AA Sloan GRG (2011), PLB (2009), GRG (2011); AA, Corichi & Singh PRD (2008); Pawlowski, Singh, PRL & PRD (2006).

# Inflationary Paradigm

• Major success: Prediction of inhomogeneities in CMB which serve as seeds for structure formation. Observationally relevant wave numbers in the range $\sim (k_o, 2000k_o)$ (radius of the observable CMB surface $\sim \lambda_o$).

• Rather minimal assumptions:

1. Some time in its early history, the universe underwent a phase of accelerated expansion during which the Hubble parameter $H$ was nearly constant.

2. Starting from this phase till the CMB era, the universe is well-described by a FLRW background with linear perturbations. Only matter: inflaton in a suitable potential.

3. At the onset of this 'slow roll inflationary phase' Fourier modes of quantum fields describing perturbations were in the Bunch-Davis vacuum (at least for co-moving wave numbers in the range $\sim (k_o, 2000k_o)$); and,

4. Soon after a mode exited the Hubble radius, its quantum fluctuation can be regarded as a classical perturbation and evolved via linearized Einstein's equations.

• Then QFT on FLRW space-times (and classical GR) implies the existence of tiny inhomogeneities in CMB seen by the 7 year WMAP data.

**All large scale structure emerged from vacuum fluctuations!**

# Inflationary Paradigm: Incompleteness

**Particle Physics Issues:**

• Where from the inflaton? A single inflaton or multi-inflatons? Interactions between inflatons? How are particles/fields of the standard model created during 'reheating' at the end of inflation? ...

**Quantum Gravity Issues:**

• Big bang singularity also in the inflationary models (Borde, Guth & Vilenkin). Is it resolved by quantum gravity as has been hoped since the 1970's? What is the nature of the quantum space-time that replaces Einstein's continuum in the Planck regime?

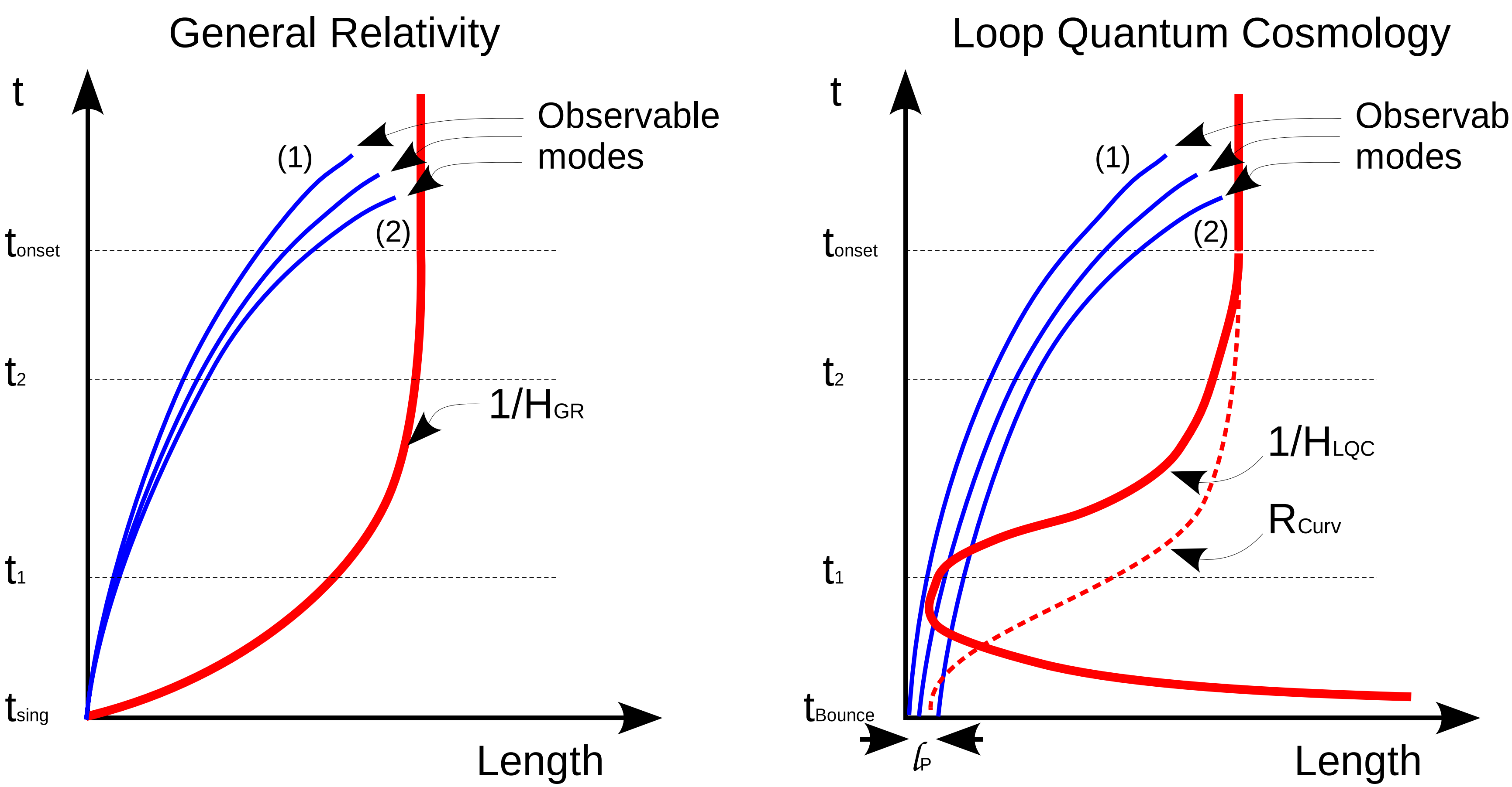
• Does the slow-roll inflation used to explain the WMAP data naturally arise from natural initial conditions 'at the Beginning' that replaces the big bang in quantum gravity?

• In classical GR, if we evolve the modes of interest back in time, they become trans-Planckian. Is there a QFT on quantum cosmological space-times needed to adequately handle physics at that stage?

• Can one arrive at the Bunch-Davis vacuum (at the onset of the WMAP slow roll) from more fundamental considerations?

# ‘Standard’ View & its limitations

Why Planck scale physics could affect the scenario?



# Inflationary Scenario: Incompleteness

**Quantum Gravity Issues:**

• Big bang singularity also in the inflationary models (Borde, Guth & Vilenkin). Is it resolved by quantum gravity as has been hoped? Nature of the quantum space-time that replaces Einstein's continuum in the Planck regime?

> "One may not assume the validity of field equations at very high density of field and matter and one may not conclude that the beginning of the expansion should be a singularity in the mathematical sense." A. Einstein, 1945

• Does the slow-roll inflation used to explain the WMAP data naturally arise from natural initial conditions 'at the Beginning' that replaces the big bang in quantum gravity?

• In classical GR, if we evolve the modes of interest back in time, they become trans-Planckian. Is there a QFT on quantum cosmological space-times needed to adequately handle physics at that stage?

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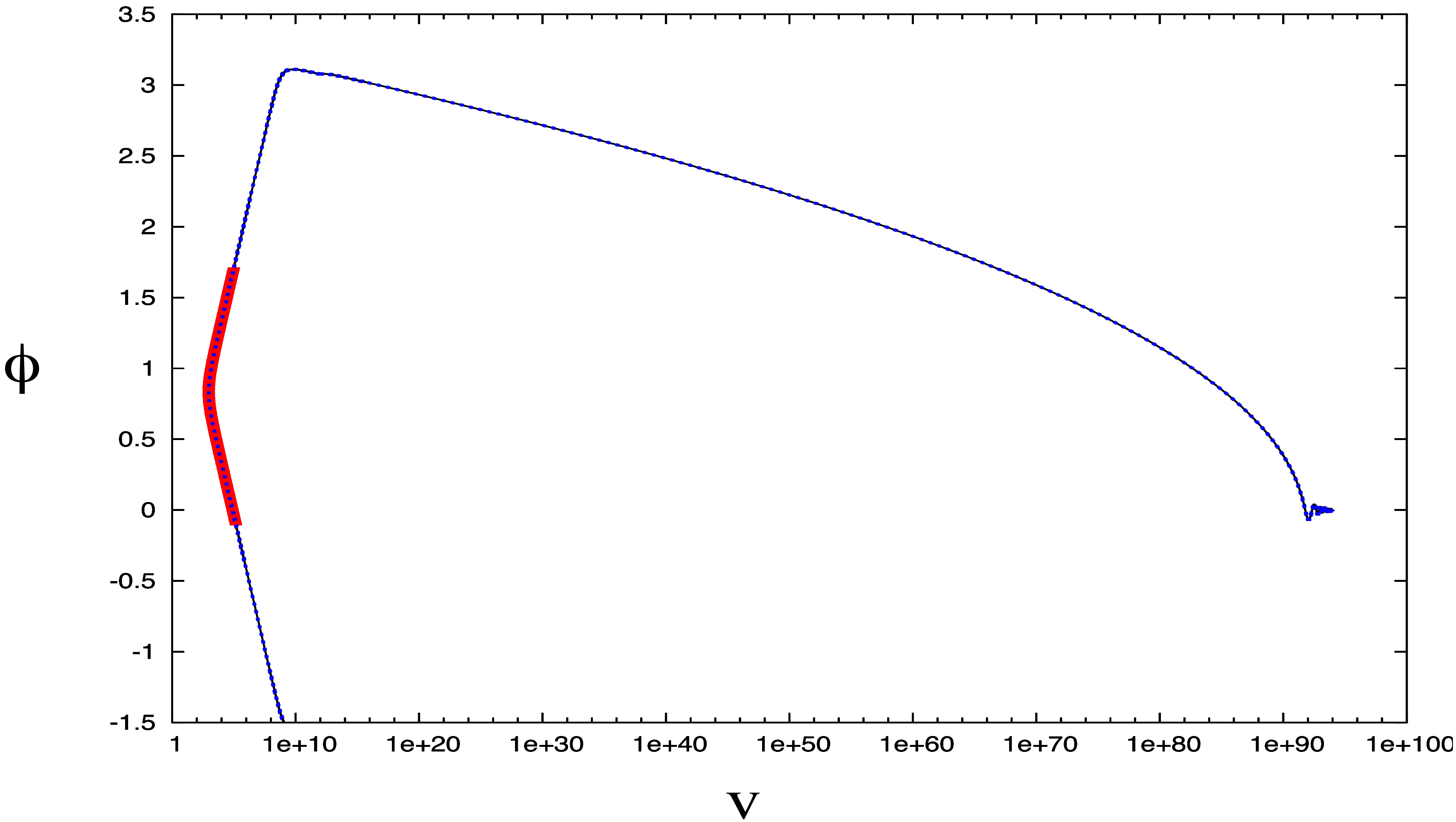
# 2. Singularity Resolution




Expectations values and dispersions of $\hat{V}|_\phi$ for a massive inflaton $\phi$ with phenomenologically preferred parameters (AA, Pawlowski, Singh). The Big Bang is replaced by a Big Bounce. Similar resolution in a wide class of cosmological models.

# What is behind singularity resolution?

• In full LQG, we have a mathematically rigorous kinematical framework uniquely selected by the requirement of background independence (Lewandowski, Okolow, Sahlmann, Thiemann; Fleishchhack). This descends to LQC in a well defined manner (AA, Campiglia).

• This kinematics is distinct from the Schrödinger representation used in the WDW theory. In particular, the differential operator of the WDW equation, $\partial^2\Psi_o(v,\phi)/\partial v^2 = \ell_P^2 \hat{H}_\phi \Psi_o(v,\phi)$ fails to be well-defined on the LQC Hilbert space and is naturally replaced by a difference operator:

$$C^+(v)\,\Psi_o(v+4,\,\phi) + C^o(v)\,\Psi_o(v,\phi) + C^-(v)\Psi_o(v-4,\,\phi) \;=\; \ell_P^2\;\hat{H}_\phi\Psi_o(v,\phi)$$

(The Step size is determined by the area gap of Riemannian quantum geometry underlying LQG)

• Good agreement with the WDW equation at low curvatures but drastic departures in the Planck regime precisely because the WDW theory ignores quantum geometry (the area gap).

# Singularity Resolution in LQC: k=0

- No unphysical matter. All energy conditions satisfied. But the left side of Einstein's equations modified because of quantum geometry effects (discreteness of eigenvalues of geometric operators.)

- Effective Equations: To compare with the standard Friedmann equation, convenient to do an algebraic manipulation and move the quantum geometry effect to the right side. Then:

$$(\dot{a}/a)^2 = (8\pi G\rho/3)[1 - \rho/\rho_{\rm crit}] \quad \text{where} \quad \rho_{\rm crit} \sim 0.41\rho_{\rm Pl}.$$

Big Bang replaced by a quantum bounce. *Effective equations are surprisingly effective even in the Planck regime.*

- Observables: The matter density operator $\hat{\rho}$ has an absolute upper bound on the physical Hilbert space (AA, Corichi, Singh):

$$\rho_{\rm sup} = \sqrt{3}/16\pi^2\gamma^3 G^2\hbar \approx 0.41\rho_{\rm Pl}!$$

Provides a precise sense in which the singularity is resolved.

- Mechanism: Quantum geometry creates a brand new repulsive force in the Planck regime, neatly encoded in the difference equation. Replaces the big-bang by a quantum bounce.

# Inflationary Paradigm: Incompleteness

**Quantum Gravity Issues:**

- Big bang singularity also in the inflationary models (Borde, Guth & Vilenkin). Is it resolved by quantum gravity as had been long hoped? What is the nature of the quantum space-time that replaces Einstein's continuum in the Planck regime?

- Does the slow-roll inflation used to explain the WMAP data naturally arise from natural initial conditions 'at the Beginning' that replaces the big bang in quantum gravity?

- In classical GR, if we evolve the modes of interest back in time, they become trans-Planckian. Is there a QFT on quantum cosmological space-times needed to adequately handle physics at that stage?

- In the more complete theory, is the Bunch-Davis vacuum at the onset of the slow roll compatible with WMAP generic or does it need enormous fine tuning?

# 3. Cosmological Perturbations on Quantum Space-times

• **LQG Strategy:** Focus on the appropriate truncation/sector of classical GR and pass to quantum gravity using LQG techniques. Sector of interest for inflation: Linear Perturbations off FLRW background with an inflaton $\phi$ in a suitable potential as matter. Includes inhomogeneities, but as perturbations. In detailed calculations, $V(\phi) = (1/2)m^2\phi^2$.

Truncated Phase Space $\ni$ $\{(v, \phi;\, \delta h_{ab}(x), \delta\phi(x))$ and their conjugate momenta$\}$
Quantum Theory: Start with $\mathbf{\Psi}(v, \phi;\, \delta h_{ab}(x), \delta\phi(x))$ and proceed to the quantum theory using LQG techniques.

• Test field approximation: $\mathbf{\Psi} = \Psi_o(v, \phi) \otimes \psi(\delta h_{ab}, \phi)$. Linearized constraints $\Rightarrow \psi(\delta h_{ab}, \phi) = \psi(T^{(1)}, T^{(2)}, \mathcal{R};\, \phi)$, where $T^{(1)}, T^{(2)}$ are the tensor modes and $\mathcal{R}$ the scalar mode. In the Planck regime of interest, $\phi$ serves as the 'internal/relational time'.

• Idea: Choose $\Psi_o(v, \phi)$ to be sharply peaked at an effective LQC solution $g^o_{ab}$. Such 'coherent states' exist. $\psi$ propagates on the quantum geometry determined by $\Psi_o$: QFT on QST a la AA, Lewandowski, Kaminski.

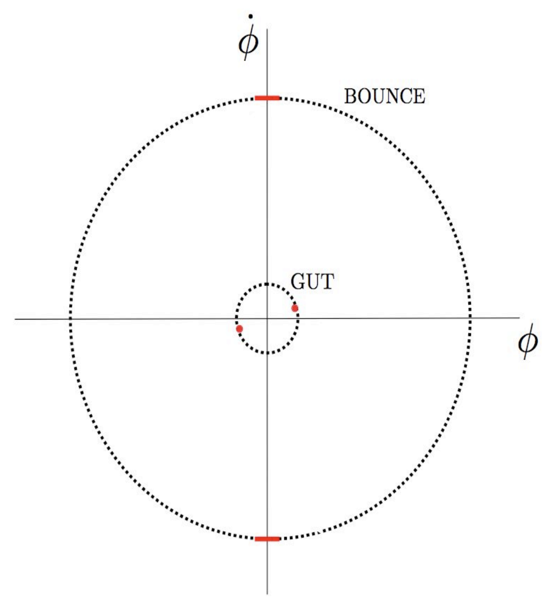
# Choice of the Cosmological Background $\Psi_o$

- Let us start with generic data at the bounce **in the effective theory** and evolve. Will the solution enter slow roll at energy scale $\rho \approx 7.32 \times 10^{-12} m_{\rm Pl}^4$ determined from the 7 year WMAP data ? Note: 11 orders of magnitude from the bounce to the onset of the desired slow roll!

- Answer: **YES**. In LQC, $|\phi_{\rm B}| \in (0, 7.47 \times 10^5)$. If $|\phi_{\rm B}| \geq 1.05 m_{\rm Pl}$, the data evolves to a solution that encounters the slow roll compatible with the 7 year WMAP data sometime in the future. In this sense, 'almost every' initial data at the bounce evolves to a solution that encounters the desired slow roll sometime in the future. (AA & Sloan, Corichi & Karami)

- Result much stronger than the 'attractor' idea because it refers to the slow roll compatible with WMAP.

- Hence, for the background quantum geometry, we can choose a 'coherent' state $\Psi_o$ sharply peaked at a 'generic' effective trajectory at the bounce and evolve using LQC. Slow roll phase ensured!



# Inflationary Paradigm: Incompleteness

**Quantum Gravity Issues:**

- Big bang singularity also in the inflationary models (Borde, Guth & Vilenkin). Is it resolved by quantum gravity as had been long hoped? What is the nature of the quantum space-time that replaces Einstein's continuum in the Planck regime?

- In the systematic evolution from the Planck regime in the more complete theory, does a slow roll phase compatible with the WMAP data arise generically or is an enormous fine tuning needed?

- In classical GR, if we evolve the modes of interest back in time, they become trans-Planckian. Is there a QFT on quantum cosmological space-times needed to adequately handle physics at that stage? **Yes!** Can the basic ideas in AA, Lewandowski, Kaminski be developed to obtain well-defined stress energy operators in this new theory? Can one justify the truncation procedure even in the Planck regime?

- Can one arrive at the Bunch-Davis vacuum (at the onset of the WMAP slow roll) from more fundamental considerations?

# 4. Extracting Physics

First, thanks to the background quantum geometry, trans-Planckian modes pose no problem, provided the test field approximation holds: $\rho_{\rm Pert} \ll \rho_{\rm BG}$ all the way from the bounce to the onset of slow roll.

- Second, surprisingly, truncated dynamics of $\hat{T}^{(1)}, \hat{T}^{(2)}, \hat{\mathcal{R}}$ on the quantum geometry of $\Psi_o$ is mathematically equivalent to that of $\hat{T}^{(1)}, \hat{T}^{(2)}$, $\hat{\mathcal{R}}$ as quantum fields on a smooth space-time with a 'dressed' effective, c-number metric $\bar{g}_{ab}$ (whose coefficients depend on $\hbar$):

$$\bar{g}_{ab}dx^a dx^b = \bar{a}^2(-d\bar{\eta}^2 + d\vec{x}^2)$$

with

$$d\bar{\eta} = \langle \hat{H}_o^{-1/2}\rangle\, [\langle \hat{H}_o^{-1/2} \hat{a}^4 \hat{H}_o^{-1/2}\rangle]^{1/2}\, d\phi; \qquad \bar{a}^4 = (\langle \hat{H}_o^{-1/2}\, \hat{a}^4 \hat{H}_o^{-1/2}\rangle)/\langle \hat{H}_o^{-1}\rangle$$

where $H_o$ is the Hamiltonian governing dynamics of $\Psi_o$. Analogy with light propagating in a medium.

- Because of this, the mathematical machinery of adiabatic states, regularization and renormalization of the Hamiltonian can be lifted to the QFT on cosmological QSTs under consideration. Result: Mathematical control to compute the CMB power spectrum, and spectral indices starting from the bounce.

# Initial conditions

• Hilbert space: $\mathcal{H}$ of perturbations $\psi$ on the quantum geometry $\Psi_o$ is spanned by 4th adiabatic order states on the smooth Friedmann metric $\bar{g}_{ab}$. Excellent control.

• Symmetries: $\Psi_o$ (and hence $\bar{g}_{ab}$) homogeneous and isotropic. Preferred states: $\psi \in \mathcal{H}$ also invariant under translations and rotations. Far from unique. However, Can narrow it down by:

• Physical Considerations: State $\Psi_o \otimes \psi$ at the bounce $\Rightarrow$ **Initial quantum homogeneity!** Can be qualitatively justified as follows. Because of inflation, the observable universe has size of $\leq 10\ell_{\rm Pl}$ at the bounce. The repulsive force of quantum geometry dilutes all inhomogeneities at this scale. So universe is as homogeneous and isotropic as the uncertainty principle allows it to be!

• Renormalized energy density is well-defined on these states. For $\hat{T}^{(1)}$, $\hat{T}^{(2)}$, and $\hat{\mathcal{R}}$ to be test fields on the quantum geometry $\Psi_o$ we also require that the stress energy in $\psi$ be negligible compared that in $\Psi_o$ at the bounce time.

# Key Questions

1. Does the back-reaction remain negligible as $\psi$ evolves all the way to the onset of the slow roll compatible with WMAP (so that our truncation strategy is justified by self-consistency)?
Answer: **YES**

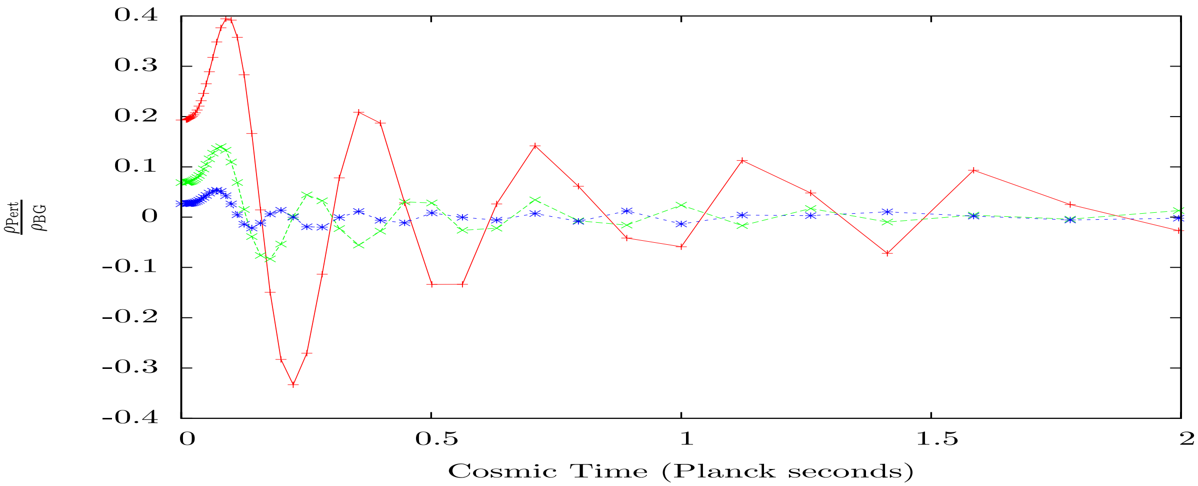
2. At the end of the WMAP compatible slow roll, do we recover the observed power spectrum: $\Delta^2_{\mathcal{R}}(k, t_{k^\star}) \approx \frac{H^2(t_{k^\star})}{\pi m^2_{\rm Pl}\epsilon(t_{k^\star})}$? ($t_{k^\star}$ is the time the reference mode $k^\star \approx 8.58 k_o$ exits the Hubble horizon during slow roll)

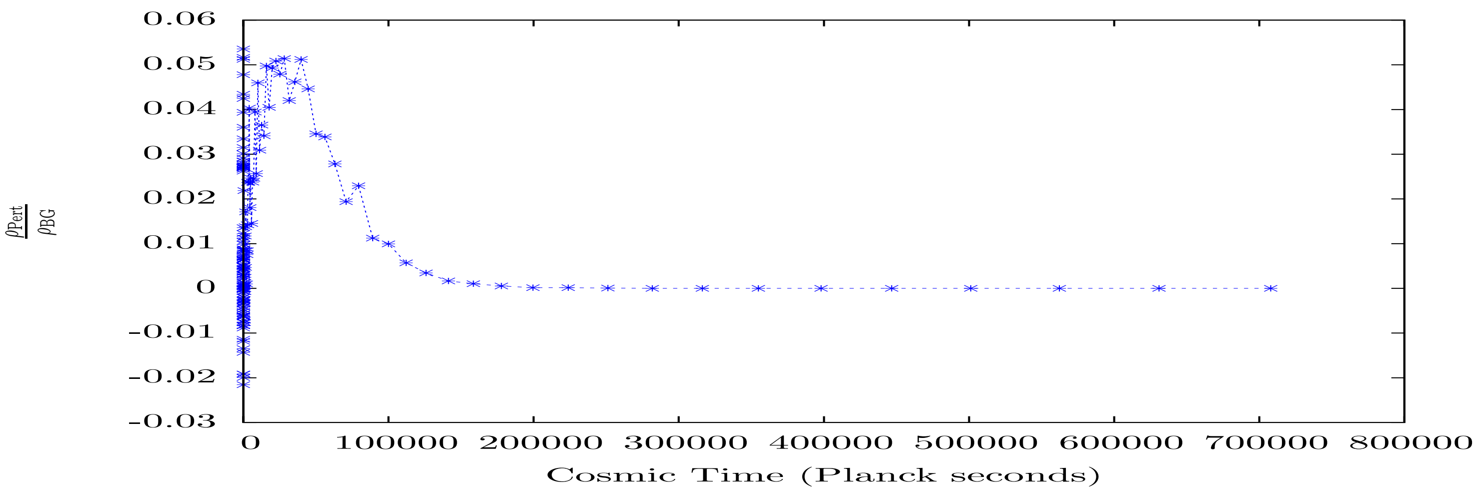
Answer: **YES** provided $\phi_{\rm B} \geq 1.14 m_{\rm Pl}$. Thus, we have arrived at a quantum gravity completion of the inflationary paradigm.

3. Does $\psi(T^{(1)}_{\bar{k}}, T^{(2)}_{\bar{k}}, \mathcal{R}_{\bar{k}};\, \phi_{\rm B})$ evolve to a state which is indistinguishable from the Bunch Davis vacuum at the onset of slow roll or are there deviations with observable consequences for more refined future observations (e.g. non-Gaussianitities in the bispectrum)?
(Agullo & Shandera; Ganc & Komatsu)

Answer: There **ARE** deviations if $\phi_{\rm B}$ lies in a small window just after $\phi_{\rm B} = 1.14 m_{\rm Pl}$.

# Self-consistency of Truncation: $\rho_{\rm Pert}/\rho_{\rm BG}$ vs time




Renormalized energy density in $\psi$ is negligible compared to that in $\Psi_o$ all the way from the bounce to the onset of slow roll. Here $\phi_{\rm B} = 1.15 m_{\rm Pl}$.

# Key Questions

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Answer: **YES**

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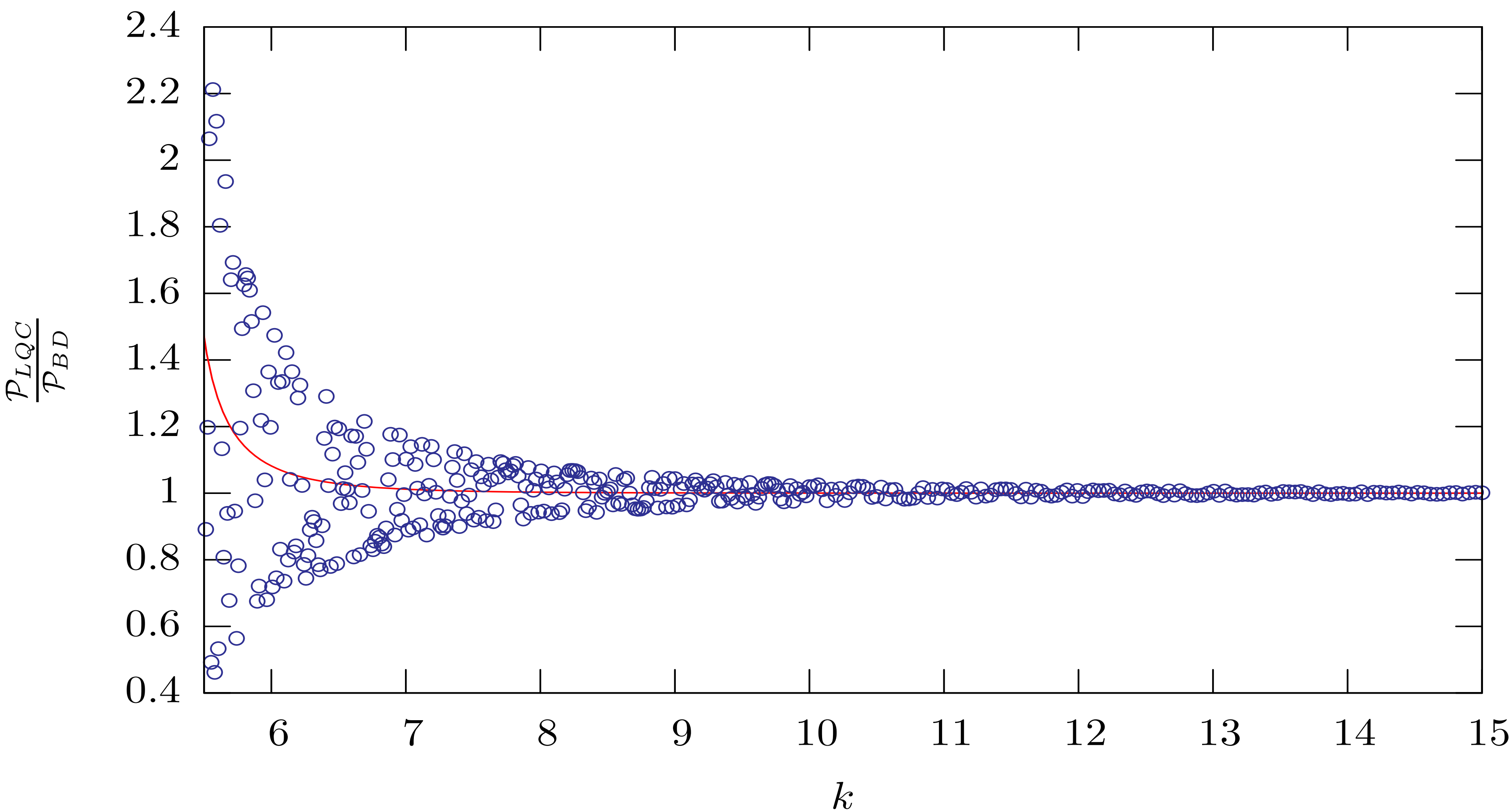
(Agullo & Shandera; Ganc & Komatsu)
Answer: There **ARE** deviations if $\phi_{\rm B}$ lies in a small window just after $\phi_{\rm B} = 1.14 m_{\rm Pl}$.

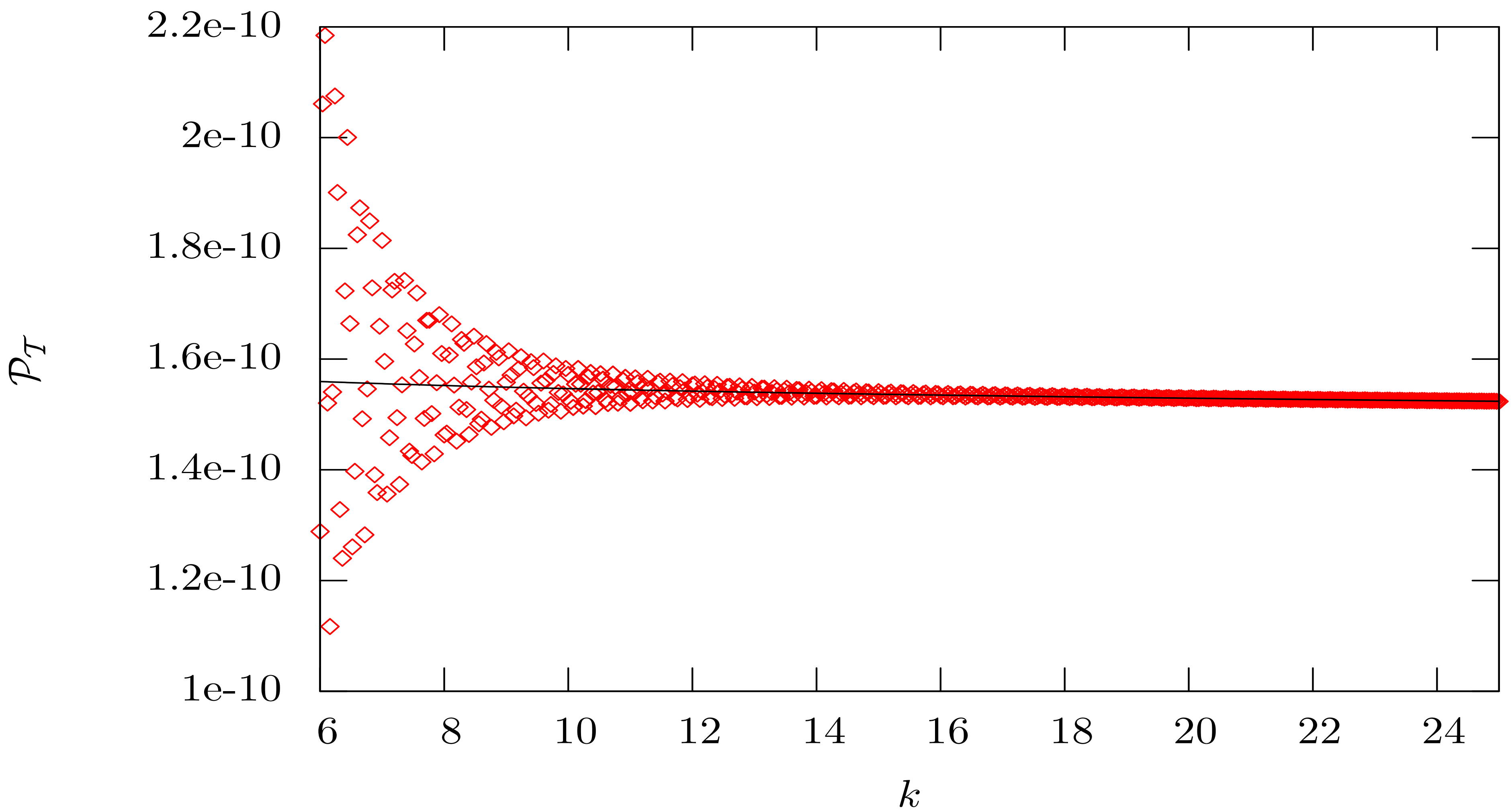
# The Scalar Power spectrum: Ratio $(\mathcal{P}_{\rm LQG}/\mathcal{P}_{\rm BD})$




Ratio of the LQC and the standard BD power spectrum for the scalar mode. Blue: Raw data points. Red: Average. LQC prediction is within observational errors for $\phi_{\rm B} \geq 1.14 m_{\rm Pl}$.

For $\phi_{\rm B} = 1.2 m_{\rm Pl}$, WMAP $k_{\rm min} = 9 m_{\rm Pl}$. Complete agreement with BD vacuum for $\phi_{\rm B} \geq 1.2 m_{\rm Pl}$. For $\phi_{\rm B} < 1.2 m_{\rm Pl}$: Certain non-Gaussiainities for future observations.

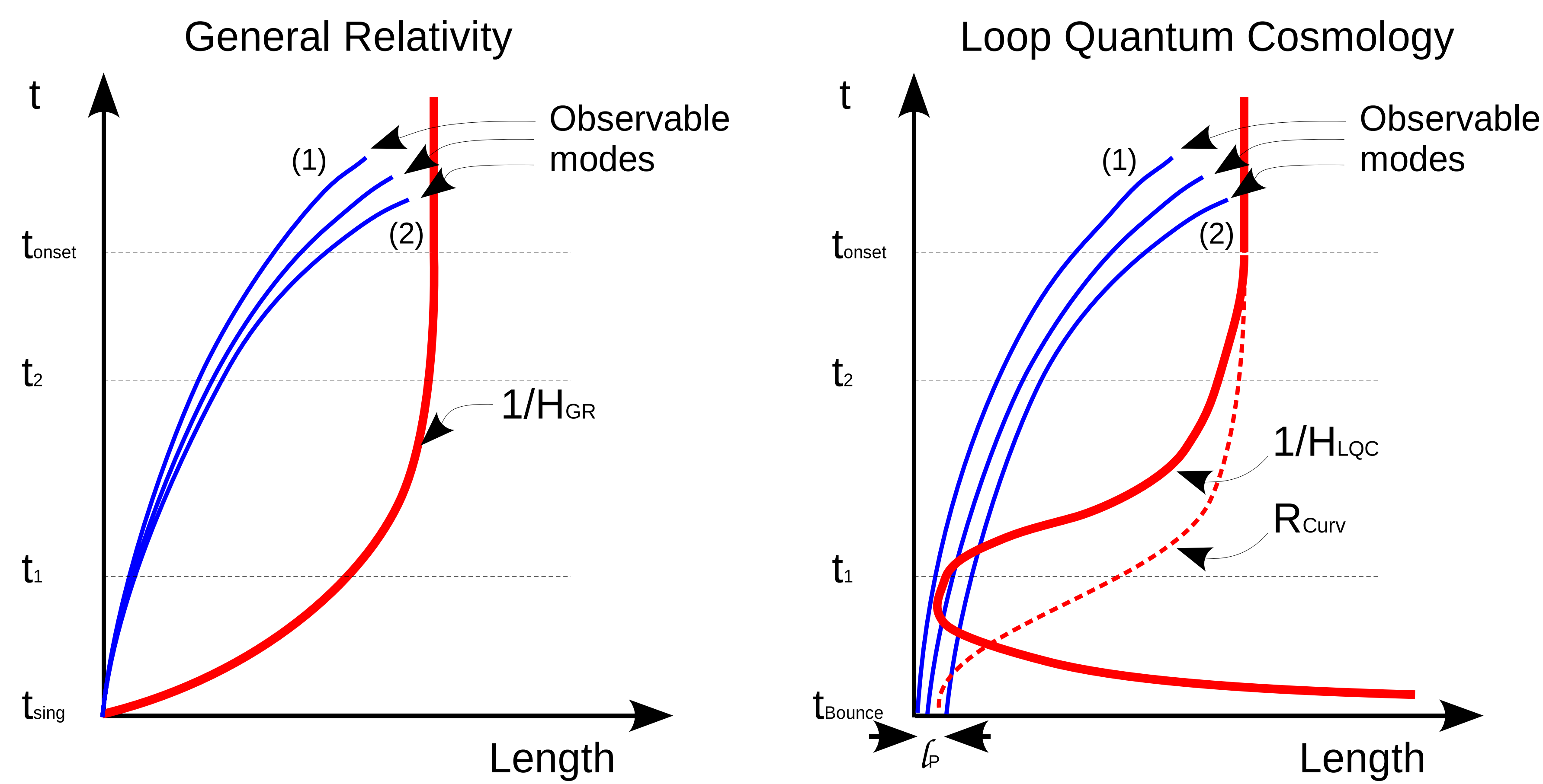
# The LQC Tensor Power spectrum




Predicted power spectra for the tensor mode. Black: Average. Red: Raw data points.

# Understanding the Power Spectrum

Modes with $\lambda > R^{-1/2}$, the curvature radius, in the pre-inflationary era are excited and populated at the onset of inflation. Can occur in a narrow window for $\phi_{\rm B} \leq 1.2 m_{\rm Pl}$.



# 5. Summary: Framework

• The early universe provides an ideal setting to test quantum gravity ideas. The inflationary paradigm provides a telescope to peer into the Planck regime. Furthermore, it has been extremely successful with structure formation. Can one provide a quantum gravity completion of the inflationary paradigm thereby overcoming some of its limitations?

In LQG the answer is **YES** in the following precise sense:

• **Background geometry:** Singularity Resolution and precise quantum geometry for the Planck regime. $\surd$

• **Perturbations:** Since they propagate on quantum geometry, using QFT on cosmological quantum geometries (AA, Lewandowski, Kaminski), trans-Planckian issues can be handled systematically provided the test field approximation holds. Analyzed in detail using the renormalized stress-energy of $\hat{T}^{(1)}, \hat{T}^{(2)}, \hat{\mathcal{R}}$ on the quantum geometry of $\Psi_o$. Detailed numerics show that the approximation does hold if $\phi_B > 1.14 m_{\rm Pl}$. (Agullo, AA, Nelson)

# Summary: Implications

- Extension: If $\Phi_{\rm B} > 1.14 m_{\rm Pl}$ & $\psi$ is a 'permissible vacuum,' modes of observational interest are all in the Bunch Davis vacuum at the onset of the WMAP slow roll $\Rightarrow$ Predictions of the standard inflationary scenario for the power spectra, spectral indices & ratio of tensor to scalar modes are recovered starting from the deep Planck era. (Agullo, AA, Nelson)

- Non-Gaussianity: There is a small window in $\phi_{\rm B}$ for which at the onset of inflation $\psi$ has excitations over the Bunch-Davis vacuum. These give rise to specific 3-point functions ('bi-spectrum') which are important for the 'halo bias'. Could be observed in principle: Link between observations and the initial state! A window to probe the Planck era around the LQC bounce. (Agullo, AA, Nelson, Shandera, Ganc, Komatsu)

- Note: LQG does not imply that inflation must have occurred because it does not address particle physics issues. The analysis simply assumes that there is an inflaton with a suitable potential. But it does show concretely that many of the standard criticisms (e.g. due to Brandenberger) of inflation can be addressed in LQG by facing the Planck regime squarely.