Forecast constraints on cosmic strings from future CMB, pulsar timing and gravitational wave direct detection experiments

Sachiko Kuroyanagi
Univ. of Tokyo
RESCEU
(RESearch Center for the Early Universe)

12 Nov 2012

Based on
S. Kuroyanagi, K. Miyamoto, T. Sekiguchi, K. Takahashi, J. Silk,
Cosmic string?

One dimensional topological defect generated in the early universe

**Generation mechanism**

1: Phase transition

2: Cosmic superstrings

Cosmological size D-strings or F-strings remains after inflation

→ could provide some insight into fundamental physics
Evolution of cosmic strings

The energy density of cosmic strings \( \propto a^{-2} \)?

**Scaling law**

Cosmic String Networks approach a self-similar solution, which always looks same at the Hubble scale.

Cosmic strings become loops via **reconnection**.

Loops lose energy by emitting **gravitational waves**.

![Diagram](image)
Observational Probe

1. Direct detection
   Ground: Advanced-LIGO, KAGRA, Virgo, IndIGO (2017-)
   Space: eLISA/NGO (2022?), DECIGO (2027)

2. Pulsar timing: SKA (2020)

3. CMB temperature fluctuation + B-mode polarization: Planck, CMBpol
Current constraints on cosmic string parameters

3 parameters to characterize cosmic string

- $G\mu (=\mu/M_{pl}^2)$: tension (line density)
- $\alpha$: initial loop size $L \sim \alpha H^{-1}$
- $p$: reconnection probability

Phase transition origin: $p=1$
Cosmic superstring: $p<<1$

- CMB temperature fluctuation: $G\mu <\sim 10^{-7}$
- Gravitational lensing: $G\mu <\sim 10^{-6}$
- Gravitational waves
  - Pulsar timing: $G\mu <\sim 10^{-9}$
  - Direct detection (LIGO GWB): $G\mu <\sim 10^{-6}$

What about future constraints?
Gravitational wave signals

Strong GW emission from singular points called **kinks** and **cusps**

**Rare Burst:** GWs with large amplitude coming from close loops → direct detection

**Gravitational wave background (GWB):** superposition of small GWs coming from the early epoch → direct detection + pulsar timing
How many cosmic string bursts are coming to the earth per year? (plotted as a function of the amplitude for the fixed frequency $@220\text{Hz}$)

$$G\mu = 10^{-7}, \alpha = 10^{-16}, p=1$$

\[ \text{rate (per year)} \sim \frac{dR}{d\log h} \]

\[ h f \sim \text{amplitude} \]
How many cosmic string bursts are coming to the earth per year? (plotted as a function of the amplitude for the fixed frequency @220Hz)

\[ G \mu = 10^{-7}, \alpha = 10^{-16}, p=1 \]

LIGO\( \sim \)220Hz

220 oscillations per second

\[ = 7 \times 10^9 \text{ oscillations per year} \]
How many cosmic string bursts are coming to the earth per year? (plotted as a function of the amplitude for the fixed frequency @220Hz)

Gμ = 10^{-7}, α = 10^{-16}, p = 1

LIGO ∼ 220Hz

220 oscillations per second = 7 \times 10^9 oscillations per year

rate (per year)

\[ \frac{dR}{d \log h} \]

small amplitude but numerous

GWB
How many cosmic string bursts are coming to the earth per year? (plotted as a function of the amplitude for the fixed frequency @220Hz)

\[ G \mu = 10^{-7}, \, \alpha = 10^{-16}, \, p = 1 \]

\[ h \approx 10^{-25} @ f \approx 220Hz \]
How many cosmic string bursts are coming to the earth per year? (plotted as a function of the amplitude for the fixed frequency @220Hz)

\[ G\mu = 10^{-7}, \alpha = 10^{-16}, p=1 \]

LIGO
\[ h \sim 10^{-25} \text{ @ } f \sim 220\text{Hz} \]

rate (per year) \[ \text{d}R/\text{d}\log h \]

amplitude \[ hf \]

GWB

rare burst
How many cosmic string bursts are coming to the earth per year? (plotted as a function of the amplitude for the fixed frequency @220Hz)

- GWB
- LIGO
  - rare burst
  - near (new)
  - distant (old)

Gμ = 10^{-7}, α = 10^{-16}, p = 1

\[ h \sim 10^{-25} @ f \sim 220\text{Hz} \]
Parameter dependences of the rate

\( G \mu \uparrow \) \text{ amplitude of GWs } \uparrow \text{ lifetime } \downarrow \text{ number density } \downarrow

\( \text{GW power } \quad P = \Gamma G\mu^2 \)

\( \Gamma: \) numerical constant \( \sim 50-100 \)

\text{Lifetime of loops } = \frac{\text{(initial loop energy)}}{\text{(energy release rate per time)}} = \frac{\mu \alpha t}{\Gamma G\mu^2} = \frac{\alpha t}{\Gamma G\mu}
Parameter dependences of the rate

The parameter dependences of the large burst (rare burst) and small burst (GWB) are different because they are looking at different epoch of the Universe → give different information on cosmic string parameters
Spectrum of the GWB

- **$G \mu$**
  - $G\mu = 10^{-6}, 10^{-10}, 10^{-12}, 10^{-14}, 10^{-16}$
  - $\alpha = 10^{-1}, p = 1$

- **$\Omega_{GW}$**

- **$\alpha$**

- **$\Omega_{GW}$**
  - $G\mu = 10^{-8}, p = 1$
  - $\alpha = 10^{-1}, 10^{-5}, 10^{-9}, 10^{-13}, 10^{-17}$
Advanced-LIGO can detect both rare bursts and GWB.

\[ G_{\mu} = 10^{-7}, \ \alpha = 10^{-16}, p = 1 \]
Constraint from direct detection experiments

different parameter dependence = different constraints on parameters

Example: $G\mu = 10^{-7}$, $\alpha = 10^{-16}$, $p=1$

Adv-LIGO 3-year

Kuroyanagi et. al. PRD 86, 023503 (2012)

black : Burst only
red : Burst + GWB

$\log_{10} G\mu$ vs $\log_{10} p$

$\log_{10} \alpha$ vs $\log_{10} p$
Constraint from direct detection experiments

Before marginalized over

Strong degeneracy seen in constraint from GWB since the observable is only $\Omega_{GW}$

Example: $G\mu = 10^{-7}$, $\alpha = 10^{-16}$, $p=1$

Adv-LIGO 3year

Kuroyanagi et. al. PRD 86, 023503 (2012)

black : Burst only
dotted: GWB only
red : Burst + GWB
Signals in the CMB

temperature fluctuation

B-mode polarization

\[ G \mu \]

\[ p \]

\[ C_{l}^{TT,\text{str}} \text{ for } G_{\mu} = 10^{-0.5} \]
\[ G_{\mu} = 10^{-7} \]
\[ G_{\mu} = 10^{-7.5} \]

\[ C_{l}^{TT,\text{priv}} \]

Planck noise

\[ \text{CMB} \text{ polarization noise} \]

\[ C_{l}^{BB,\text{str}} \text{ for } p = 1 \]
\[ p = 10^{-0.5} \]
\[ p = 10^{-1} \]

\[ C_{l}^{BB,\text{sim}} \]
\[ C_{l}^{BB,\text{skn}} \]
If we combine CMB constraints...

G\mu = 10^{-7}, \alpha = 10^{-16}, p=1

Adv-LIGO 3year + CMB B-mode

Kuroyanagi et al. arXiv:1210.2829

black : LIGO Burst only
red : LIGO Burst + GWB
blue: LIGO +Planck
green: LIGO+CMBpol
orange: CMB pol only
Constraints from pulsar timing and space direct detection mission

- Observing GWs from different epochs

- CMB
- Pulsar timing
- Direct detection
- KAGRA
- LIGO
- SKA
- eLISA
- DECIGO

- Planck
- CMBpol

- Inflation
- Cosmic string

- Cusps on loops
- Kinks on infinite strings

- Frequency [Hz]

Old versus new
Pulsar timing (SKA) + Advanced-LIGO burst search

$G \mu = 10^{-9}, \alpha = 10^{-9}, p = 1$

- dotted: Burst
- solid: GWB

Current LIGO
- Adv. LIGO
- eLISA
- BBO

NANOGrav
- SKA
- BBN
- CMB
Direct detection + Pulsar timing

$G \mu = 10^{-9}, \alpha = 10^{-9}, p=1$

Adv-LIGO 3year (burst only) + SKA 10year

Kuroyanagi et al. arXiv:1210.2829
Parameter constraint by eLISA

$G \mu = 10^{-9}$, $\alpha = 10^{-9}$, $p = 1$

eLISA 3-year
(burst only)

Kuroyanagi et al. arXiv:1210.2829
• Future CMB and GW experiments can be a powerful tool to probe cosmic strings.

• If signals are detected, it would determine cosmic string parameters, which can provide us with hints of fundamental physics such as particle physics or superstring theory.

• Two different kinds of GW observation (rare burst and GWB) provide different constraints on cosmic string parameters and lead to better accuracy in determining parameters.

• Combination of different experiments (CMB, Pulser timing, direct detection) also helps to get stronger constraints.

• Space GW missions are more powerful to prove cosmic strings.
Estimation of the GW burst rate

**Initial number density of loops**

\[
\bar{n}_L = \frac{\text{(length of infinite string discarded to loops)}}{\text{(initial length of loops) } = \alpha t_i}
\]

Depends on \(\alpha\) and \(p\)

**Evolution of infinite strings**

- Velocity-dependent one-scale model

\[
2\frac{dL}{dt} = 2HL(1 + v^2) + cv
\]

*Energy conservation*

\[
\frac{d\rho_{\text{inf}}}{dt} \bigg|_{\text{loop}} = cv\frac{\rho_{\text{inf}}}{L}
\]

For small \(p\): \(c \rightarrow cp\)

**Energy discarded to loops**

**Damping due to the expansion**

\[
\frac{dv}{dt} = (1 - v^2) \left( \frac{k}{R} - 2Hv \right)
\]

Acceleration due to the curvature of the strings

Momentum parameter: \(k = \frac{2\sqrt{2}}{\pi} \left( \frac{1 - 8v^6}{1 + 8v^6} \right)\)
Estimation of the GW burst rate

Loop evolution depends on $G\mu$ and $\alpha$

**Initial loop length**

$$l(t_i) = \alpha t_i$$

t$_i$ : time when the loop formed

**GW power**

$$P = \Gamma G\mu^2$$

$\Gamma$: numerical constant $\sim 50-100$

From the energy conservation law

(energy of loop at time $t = \mu l$)

$$= \text{(initial energy of the loop) } \mu \alpha t_i \text{ } - \text{ (energy released to GWs) } P \Delta t$$

**Loop length at time $t$**

$$l(t, t_i) = \alpha t_i - \Gamma G\mu (t - t_i)$$

**Lifetime of the loop**

$$\text{Lifetime of the loop} = \frac{\text{(initial loop energy)}}{\text{(energy release rate per time)}}$$

$$= \frac{\mu \alpha t_i}{\Gamma G\mu^2} = \frac{\alpha t_i}{\Gamma G\mu}$$
Estimation of the GW burst rate

GW burst rate emitted at $t \sim t + dt$ from loops formed at $t_i \sim t_i + dt_i$

$$\frac{dR}{dtdt_i} = \frac{1}{4} \theta_m(f, z, l)^2 \frac{2c}{(1 + z)l(t, t_i)} \frac{dn}{dtt_i}(t, t_i) \frac{dV}{dt} dtdt_i \times \Theta(1 - \theta_m(f, z, l))$$

**Beaming**

**Time interval of GW emission**

$\propto (\text{loop length at } t)^{-1}$

**Loop number**

$$N = \frac{p^{-1}t}{\alpha t} = \frac{1}{p\alpha}$$

**$\theta_m$**

$\propto (\text{loop length at } t)^{1/3}$

$$l(t, t_i) = \alpha t_i - \Gamma G \mu (t - t_i)$$

GW amplitude from loop of length $l$

$$h(f, z, l) \simeq 2.68 \frac{G \mu l}{((1 + z)fl)^{1/3} r(z) f}$$
Generation mechanism 1: phase transition

The Universe has experienced symmetry breakings.

If you consider U(1) symmetry breaking...

High energy vacuum remains at the center

Tension $G \mu \sim$ the energy scale of the phase transition
Generation mechanism 2: Cosmic superstrings

Cosmological size D-strings or F-strings remains after inflation in superstring theory

**Difference from phase transition origin**

- low reconnection probability \( p \) because of the extra dimension
  
  Phase transition origin: \( p=1 \)

- broad values of \( G\mu \) depending on the inflation scale and the extra internal degrees of freedom

\( D\text{-string:} \quad p=0.1-1 \)

\( F\text{-string:} \quad p=10^{-3}-1 \)

\( \cdot \) Y-junction

\( \cdot \) mixed strings with different \( G\mu \)

→ Cosmic strings could give some insight into fundamental physics
Evolution of cosmic strings

**Scaling law**

Cosmic strings become loops when they collide and form a network composed by loops and infinite strings.

Loops lose energy by emitting **gravitational waves** and shrink.

- Increase of infinite string length by the Hubble expansion
- Loss of infinite string length by generation of loops
- Higher reconnection rate
- More efficient generation of loops
- More energy release by the emission of GWs
Evolution of cosmic strings

energy density

\[ \propto a^{-4} \]
\[ \propto a^{-2} \]
\[ \propto a^{-3} \]

\( a: \) scale factor

The network keeps \( \mathcal{O}(1) \) number of infinite strings in the Hubble horizon → cosmic strings does not dominate the energy density of the Universe.

\[ \alpha : \text{initial loop size } L \sim \alpha H^{-1} \]

\( p : \) reconnection probability
Estimation of the GW burst rate

Initial number density of loops

Loop number generated per unit time
To satisfy the scaling law, infinite strings should lose $O(1)$ Hubble length per 1 Hubble time. So they should reconnect $O(1)$ times per Hubble time.

To reconnect $O(1)$ times per Hubble time, number of infinite strings per Hubble volume should be $\sim p^{-1}$

$\rightarrow$ total length of infinite strings $\sim p^{-1}H^{-1}\sim p^{-1}t$

**Number of loops**

$$\text{Number of loops} = \frac{\text{(length to lose)}}{\text{(initial length of loops)}} = \frac{p^{-1}t}{\alpha t} = \frac{1}{p\alpha}$$
Constraint on parameters

Fisher information matrix

$$F_{ij} = -\left< \frac{\partial^2 \ln L}{\partial p_i \partial p_j} \right>$$

If the likelihood shape is sensitive to the parameter = easy to estimate the parameter

Burst Observable: amplitude vs number

N is predictable by the rate $dR/dh$

$$F_{ij} \propto \frac{\partial (dR/dh)}{\partial p_i} \frac{\partial (dR/dh)}{\partial p_j}$$
Constraint on parameters

**Fisher information matrix**

\[
F_{ij} = -\langle \frac{\partial^2 \ln L}{\partial p_i \partial p_j} \rangle
\]

log(Likelihood)

If the likelihood shape is sensitive to the parameter = easy to estimate the parameter

**GWB Observable**: \( \Omega_{GW} \)

\[
F_{ij} \propto \frac{\partial \Omega_{GW}}{\partial p_i} \frac{\partial \Omega_{GW}}{\partial p_j}
\]