Upper limits of particle emission from high-energy collision and reaction near a maximally rotating Kerr black hole

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“Kerr BHs as particle accelerators” (Bañados, Silk & West 2009): Collision with an arbitrarily high centre-of-mass (CM) energy near the horizon of a maximally rotating BH. Implication to DM particles pair annihilation.

Critical comments: Berti et al. 2009, Jacobson & Sotiriou 2010

Astrophysical relevance: Harada & Kimura 2011abc
Can we observe new physics?

- Particle collision with extremely high CM energy might produce an exotic particle. Can we observe it?
- If a high-energy and/or super-heavy particle is to be emitted from the collision of ordinary particles, we need energy extraction from the BH.
- This is possible in general for a rotating BH, as is well known.
Collisional Penrose Process

Energy can be extracted from a rotating BH due to the negative energy orbit in the ergoregion.

Collisional Penrose process (Piran, Shaham & Katz 1975)

Jacobson & Sotiriou (2010) argue that no energy extraction occurs through the BSW collision.
Maximally rotating BH

Maximally rotating Kerr BH
- Boyer-Lindquist coordinates: \((t, r, \theta, \phi)\)
- \(a = M\): \(r_H = M\), \(\Omega_H = 1/(2M)\), \(\kappa_H = 0\)
- Ergoregion: \(M < r < M(1 + \sin \theta)\)

Geodesic motion in the equatorial plane
- 1D potential problem

\[
\frac{1}{2} (p^r)^2 + V(r) = 0, \quad \text{or} \quad p^r = \sigma \sqrt{-2V(r)}, \quad \text{where} \quad p^r = \frac{dr}{d\lambda},
\]

where \(\lambda\) is the affine parameter,

\[
V(r) = -\frac{Mm^2}{r} + \frac{L^2 - a^2(E^2 - m^2)}{2r^2} - \frac{M(L - aE)^2}{r^3} - \frac{E^2 - m^2}{2},
\]

and \(E\) and \(L\) are conserved.
- Forward-in-time condition: \(p^t = dt/d\lambda > 0\)
- This implies \(2E - \tilde{L} \geq 0\) in the limit \(r \rightarrow r_H\), where \(\tilde{L} = L/M\). We define a critical particle as a particle satisfying \(2E - \tilde{L} = 0\).
Collision and reaction: $1 + 2 \rightarrow 3 + 4$

- CM energy: $E_{cm}^2 = -(p_1^a + p_2^a)(p_1^a + p_2^a) = -(p_3^a + p_4^a)(p_3^a + p_4^a)$
- Conservation: $E_1 + E_2 = E_3 + E_4$ and $\tilde{L}_1 + \tilde{L}_2 = \tilde{L}_3 + \tilde{L}_4$
- Radial momentum conservation: $p_1^r + p_2^r = p_3^r + p_4^r$

BSW collision: particle 1 is critical ($2E_1 - \tilde{L}_1 = 0$), while particle 2 is subcritical ($2E_2 - \tilde{L}_2 > 0$). If the two particles collide at $r = M/(1 - \epsilon)$ ($0 < \epsilon \ll 1$) with $p^r < 0$,

$$E_{cm} \approx \sqrt{2(2E_1 - \sqrt{3E_1^2 - m_1^2})(2E_2 - \tilde{L}_2)}$$

$E_{cm} \rightarrow \infty$ as $\epsilon \rightarrow 0$. 
Particle motion near the horizon

- Let $\tilde{L} = 2E(1 + \delta)$, $\delta = \delta_{(1)}\epsilon + \delta_{(2)}\epsilon^2 + O(\epsilon^3)$.
- The forward-in-time condition at $r = M/(1 - \epsilon)$ yields $\delta < \epsilon + O(\epsilon^2)$.
- Turning points of the potential

$$r_{t,\pm}(e) = M\left(1 + \frac{2e}{2e \mp \sqrt{e^2 + 1}}\delta_{(1)}\epsilon\right) + O(\epsilon^2), \quad \text{where} \quad e = E/m.$$ 

![Graph of potential with turning points](image)

- To escape to infinity from $r = M/(1 - \epsilon)$, we need $e \geq 1$ and
  - (a) $\delta_{(1)} < 0$ and $\sigma = 1$
  - (b) $\delta_{(1)} > 0$ and $r \geq r_{t,+}(e)$ or $0 < \delta_{(1)} \leq \delta_{(1),\text{max}} = (2e - \sqrt{e^2 + 1})/(2e)$.
Collision and reaction near the horizon

Let us consider a collision at $r = \frac{M}{1 - \epsilon}$.

Let $\tilde{L}_3 = 2E_3(1 + \delta)$, $\sigma_3 = \pm 1$ and $\sigma_4 = -1$.

The forward-in-time condition is taken into account.

The radial momentum conservation: $p_r^1 + p_r^2 = p_r^3 + p_r^4$.

- Expand $p_r^i$ ($i = 1, 2, 3, 4$) in terms of $\epsilon$.
- The radial momentum conservation implies at $O(\epsilon)$

\[
(2E_1 - \sqrt{3E_1^2 - m_1^2}) + 2E_3(\delta(1) - 1) = \sigma_3 \sqrt{E_3^2(3 - 8\delta(1) + 4\delta^2(1)) - m_3^2}.
\]

- It implies at $O(\epsilon^2)$ an equation including $m_4$. With this equation, we can check whether $m_4^2 \geq 0$ is satisfied or not.
The energy of the escaping particle

- The radial momentum conservation implies at $O(\epsilon)$

$$
(2E_1 - \sqrt{3E_1^2 - m_1^2}) + 2E_3(\delta_1 - 1) = \sigma_3 \sqrt{E_3^2(3 - 8\delta_1 + 4\delta_1^2) - m_3^2}. \tag{1}
$$

- Squaring the both sides of Eq. (1) yields the following quadratic equation for $E_3$.

$$
4A_1 E_3(1 - \delta_1) = A_1^2 + (E_3^2 + m_3^2), \tag{2}
$$

where $A_1 = 2E_1 - \sqrt{3E_1^2 - m_1^2} > 0$.

- Solving Eq. (2) for $\delta_1$ and substituting it into Eq. (1) yields

$$
A_1^2 - (E_3^2 + m_3^2) = 2\sigma_3 A_1 \sqrt{E_3^2(3 - 8\delta_1 + 4\delta_1^2) - m_3^2}. \tag{3}
$$
We assume $E_1 \geq m_1$ so that particle 1 is initially at infinity.

(i) $\sigma_3 = 1$: Eq. (3) immediately implies $E_3 \leq \sqrt{A_1^2 - m_3^2} < E_1$, i.e., no energy extraction.

(ii) $\sigma_3 = -1$ and $0 < \delta_{(1)} \leq \delta_{(1),\text{max}}$: $E_3 = 2.186E_1$ is possible.

- Eq. (2) immediately implies $\lambda_- \leq E_3 \leq \lambda_+$, where
  
  $\lambda_{\pm} = 2A_1 \pm \sqrt{3A_1^2 - m_3^2}$ and the equality holds for $\delta_{(1)} = 0$.

- This implies that $E_3/E_1$ takes a maximum $(2 - \sqrt{2})/(2 - \sqrt{3}) \approx 2.186$ for $E_1 = m_1$, $m_3 = 0$ and $\delta_{(1)} = +0$. 

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Energy extraction is possible only with bounce ($\sigma_3 = -1$).
Energy gain efficiency

The upper limit of the energy gain efficiency $\eta = E_3/(E_1 + E_2)$ can be further studied based on $O(\epsilon^2)$ equation.

The upper limit of the efficiency for $E_3 = E_B$ is given by 146.6 % for any BSW collision.

The upper limits are 117.6 % for perfectly elastic collision, 137.2 % for inverse Compton scattering and 109.3 % for pair annihilation.

Our result agrees with a numerical work by Bejger, Piran, Abramowicz & Hakanson (2012) and contradicts a simplistic argument by Jacobson & Sotiriou (2010).

On the other hand, the efficiency is not very high but modest at most.
The rotational energy of a maximally rotating BH can be extracted through a BSW collision, whereas the emitted particle cannot be highly energetic.

Note, however, that the BSW collision may open a new reaction channel because of high CM energy, which can leave its features on the gamma-ray spectrum (cf. Cannoni, Gomez, Perez-Garcia & Vergados 2012).