QFT during Inflation: Issues & Results

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The Geometry of Inflation

- $ds^2 = -dt^2 + a^2(t) d\vec{x} \cdot d\vec{x}$
  - $H(t) \equiv \frac{\dot{a}}{a}$, $\epsilon(t) \equiv -\frac{\dot{H}}{H^2}$ (q = - $\frac{\ddot{a}}{aH}$ = -1 + ϵ)
- Inflation is defined as
  - $H(t) > 0$ and $\epsilon(t) < 1$
- Different possibilities for $\epsilon(t)$
  1. de Sitter → $\epsilon(t) = 0$
  2. Constant ϵ → $\epsilon(t) = \epsilon_1$
  3. Realistic → $\dot{\epsilon}(t) \neq 0$

What the CMB data says

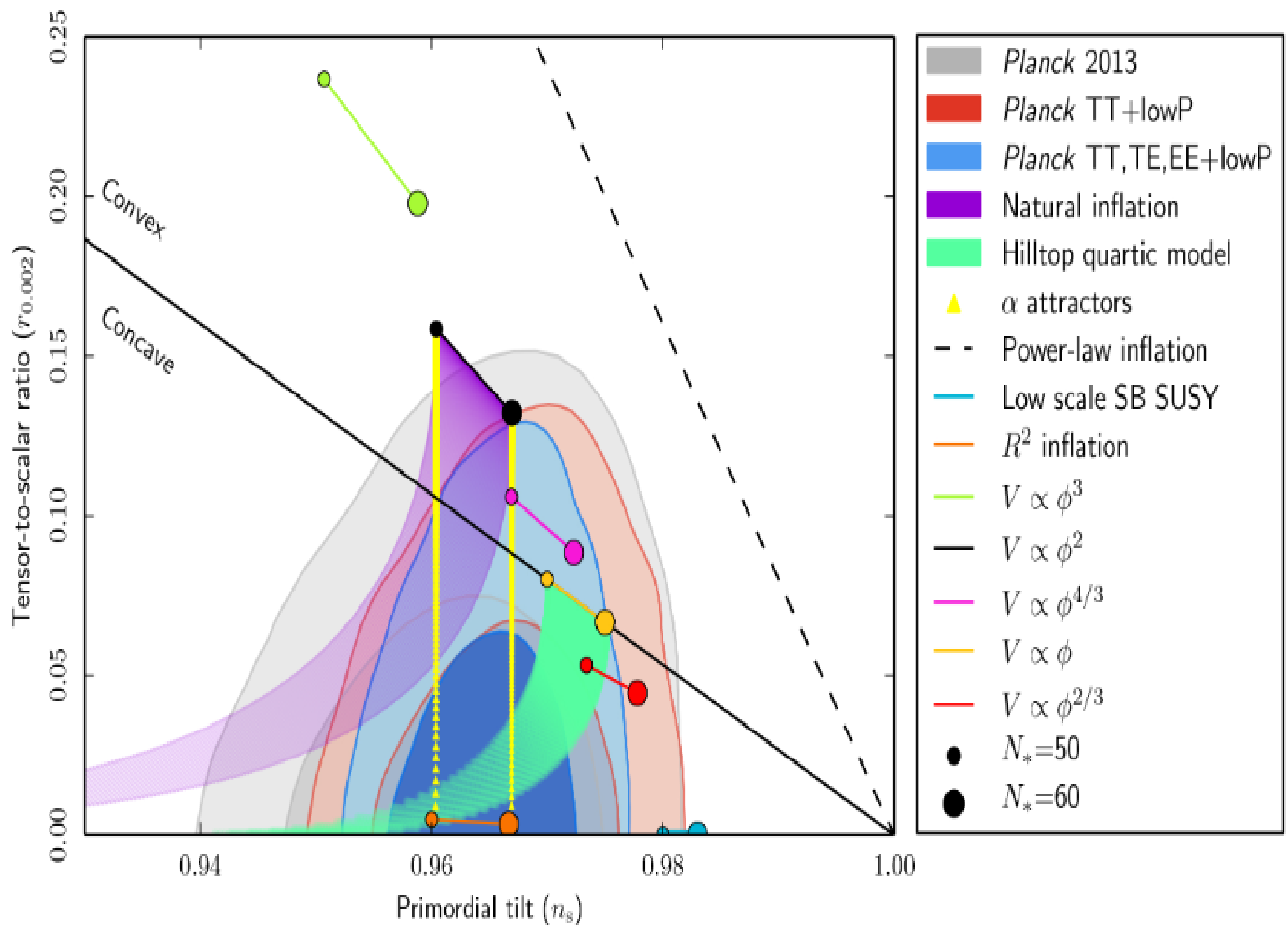
1. $\epsilon(t) \ll 1$

   Single-φ ➔ $\epsilon < 0.006$

   ∴ ~ de Sitter, except for $\frac{1}{\epsilon}$

2. $\dot{\epsilon}(t) \neq 0$

   ∴ exact results impossible



Why QFT from primordial inflation is observable

1. Produces super-Hubble MMC scalars & gravitons
   - $\Delta t \Delta E < 1$ ➔ $\int_t^{t+\Delta t} dt' \frac{k}{a(t')} < 1$
2. Hubble parameter is large
   - $G H_0^2 \sim 10^{-122}$ ➔ $G H_i^2 \sim 10^{-10}$
   - Quantum gravity perturbative but not negligible
3. Scalar & graviton perturbations fossilize
   - $\ddot{u} + 3H\dot{u} + \frac{k^2}{a^2} u = 0$ $\qquad u\dot{u}^* - \dot{u}u^* = \frac{i}{a^3}$
   - $\ddot{v} + \left(3H + \frac{\dot{\epsilon}}{\epsilon}\right)\dot{v} + \frac{k^2}{a^2} v = 0$ $\qquad v\dot{v}^* - \dot{v}v^* = \frac{i}{\epsilon a^3}$

$$N\left(t,\vec{k}\right) = \frac{\pi\Delta_h^2(k)}{64Gk^2} \times a^2(t)$$

- $\mathcal{L} = -\frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi g^{\mu\nu}\sqrt{-g} = \frac{1}{2}a^3\dot{\varphi}^2 - ½a\vec{\nabla}\varphi\cdot\vec{\nabla}\varphi$

  $$L \equiv \int d^3x\mathcal{L} = \tfrac{1}{2}a^3(t)\int\frac{d^3k}{(2\pi)^3}\left[\left|\dot{\psi}(t,\vec{k})\right|^2 + \frac{k^2}{a^2(t)}\left|\psi(t,\vec{k})\right|^2\right]$$

- Each $\vec{k}$ is a harmonic oscillator with
  - $m(t) \propto a^3(t)$ and $\omega(t,k) = \frac{k}{a(t)}$
- $E\left(t,\vec{k}\right) = \frac{1}{2}a^3(t)\left[\dot{q}^2\left(t,\vec{k}\right) + \frac{k^2}{a^2(t)}q^2\left(t,\vec{k}\right)\right] = \frac{k}{a(t)}\left[½ + N(t,\vec{k})\right]$
  - $q\left(t,\vec{k}\right) = u(t,k)\,\alpha\left(\vec{k}\right) + u^*(t,k)\,\alpha^\dagger(\vec{k})$
- $\left\langle\Omega\middle|E(t,\vec{k})\middle|\Omega\right\rangle = \frac{1}{2}a^3(t)\left[|\dot{u}(t,k)|^2 + \frac{k^2}{a^2(t)}|u(t,k)|^2\right] = \frac{k}{a(t)}\left[½ + N(t,\vec{k})\right]$
  - $u(t,k) = \frac{H}{\sqrt{2k^3}}[1 - ix]e^{ix}$, $x = \frac{k}{Ha(t)}$, $N(t,\vec{k}) = [\frac{1}{2x}]^2$
- $\Delta_h^2(k) = \frac{k^3}{2\pi^2} \times 32\pi G \times 2 \times \lim_{t\gg t_k}|u(t,k)|^2$

Types of Inflationary QFT Effects

1. Driven by MMC scalars & gravitons
   - Loop corrections to the power spectra
   - Changes in particle kinematics
   - Changes in EM and GR forces
2. Driven by other particles
   - On the force of gravity
   - On the inflaton effective potential

Linearized Effective Field Eqns

- Scalar self-mass $-iM^2(x;x')$

$$\partial_\mu[\sqrt{-g}g^{\mu\nu}\partial_\nu\phi(x)] - \int d^4x' M^2(x;x')\phi(x') = 0$$

- Fermion self-energy $-i[{}_i\Sigma_j](x;x')$

$$\sqrt{-g}e^\mu_a\gamma^a_{ij}\big[\partial_\mu - ½A^{bc}_\mu J_{bc}\big]_{jk}\Psi_k - \int d^4x'[{}_i\Sigma_j](x;x')\Psi_j(x') = 0$$

- Vacuum polarization $+i[{}^\mu\Pi^\nu](x;x')$

$$\partial_\mu\big[\sqrt{-g}g^{\mu\rho}g^{\nu\sigma}F_{\rho\sigma}(x)\big] + \int d^4x'[{}^\mu\Pi^\nu](x;x')A_\nu(x') = J^\mu(x)$$

- Graviton self-energy $-i[{}^{\mu\nu}\Sigma^{\rho\sigma}](x;x')$

$$\sqrt{-g}\mathcal{L}^{\mu\nu\rho\sigma}h_{\rho\sigma}(x) - \int d^4x'[{}^{\mu\nu}\Sigma^{\rho\sigma}](x;x')h_{\rho\sigma}(x') = 8\pi G T^{\mu\nu}_{lin}(x)$$

Effects from MMC Scalars on nondynamical de Sitter

1. MMC + $\lambda\varphi^4$
   - $\langle\Omega|T_{\mu\nu}|\Omega\rangle$ & $-iM^2(x;x')$ at 2 loops
   - $0 < \rho \rightarrow +\#H^4$ ➔ super-acceleration
2. SQED
   - $\langle\Omega|T_{\mu\nu}|\Omega\rangle$ & $\langle\Omega|F_{\mu\nu}F_{\rho\sigma}|\Omega\rangle$ at 2 loops
   - $i[{}^{\mu}\Pi^{\nu}](x;x')$ & $-iM^2(x;x')$ at 1 loops
   - $M_\gamma \rightarrow \#H$ ➔ EM screening , $0 > \rho \rightarrow -\#H^4$
3. Yukawa
   - $\langle\Omega|T_{\mu\nu}|\Omega\rangle$, $-i[{}_{i}\Sigma_{j}](x;x')$ & $-iM^2(x;x')$ at 1 loop
   - M_Ψ grows w/o bound & $0 > \rho \rightarrow -\infty$
4. GR
   - $-i[{}^{\mu\nu}\Sigma^{\rho\sigma}](x;x')$ at 1 loop
   - No effect on gravitons, but secular screening of gravity force

Effects from Gravitons on de Sitter

1. On MMC scalars
   - $-iM^2(x;x')$ at 1 loop
   - No significant effect because only coupling is $\frac{k}{a(t)}$
2. On electromagnetism
   - $i[^{\mu}\Pi^{\nu}](x;x')$ at 1 loop (in general gauges)
   - Secular excitation of photons (through spin) & EM force
3. On fermions
   - $-i[_{i}\Sigma_{j}](x;x')$ at 1 loop (for small mass)
   - Secular excitation of fermions through spin
4. On gravity
   - $-i[^{\mu\nu}\Sigma^{\rho\sigma}](x;x')$ at 1 loop (but not with dim. reg.)
   - Secular excitation of gravitons, no result yet for force

Effects of Ordinary Matter on Inflatons & Gravitons

1. On the inflaton effective potential
   - On de Sitter from scalars, fermions & photons
   - $V_{eff} = \pm H^4 f\left(\frac{c^2\varphi^2}{H^2}\right)$ NOT PLANCK SUPPRESSED
     - Coleman-Weinberg for large φ (small H)
     - Complicated for small φ
   - "H" not constant, or even local ➔ cannot subtract
   - How much does this disturb inflation?
2. On the force of gravity
   - $-i[^{\mu\nu}\Sigma^{\rho\sigma}](x;x')$ at 1 loop from EM
   - No effect on gravitons
   - Secular enhancement of gravitational force

Need a controlled framework of approximations

- "No one has ever done a complete 1-loop computation on a realistic background"
  - Don't know $u(t, k)$ and/or $v(t, k)$ for realistic $\epsilon(t)$!
- Loop counting parameter is $GH^2(t)$
  - But at what time t?
  - At early times it's big, at late times it's smaller
- The ζ propagator has a factor of $\frac{1}{\epsilon(t)}$
  - But at what time t?
  - At early times it's big, at late times it's smaller

$u(t,k)$ & $v(t,k)$ for general $\epsilon(t) < 1$

- $M(t,k) \equiv |u(t,k)|^2 \quad , \quad N(t,k) \equiv |v(t,k)|^2$
  - $\ddot{M} + 3H\dot{M} + \frac{2k^2}{a^2}M = \frac{\dot{M}^2}{2M} + \frac{1}{2Ma^6}$
- $M(t,k) = M_0(t,k) \times Exp[-½h(n,k)]$
  - $h'' - \frac{\omega'}{\omega}h' + \omega^2 h = S + ¼h'^2 + \omega^2[1 - h - e^h]$
- Frequency: $\omega(n,k) \equiv [H(t)a^3(t)M_0(t,k)]^{-1}$
  - Huge for horizon crossing, near zero afterwards
- Source: $S \equiv 2\left[\frac{M_0''}{M_0} - ½\left(\frac{M_0'}{M_0}\right)^2 + (3-\epsilon)\frac{M_0'}{M_0}\right] + \frac{4k^2}{H^2a^2} - \omega^2$
  - Driven by $\dot{\epsilon}$ and $\ddot{\epsilon}$
- Bottom line:
  - $n < n_k - 2$ ➔ Instantaneously constant ϵ valid (corrections local)
  - $n > n_k + 2$ ➔ Known constant

Much remains to be done

- Functional dependence on $H(t)$ & $\epsilon(t)$ known for $u(t,k)$ & $v(t,k)$
  - But need to find associated propagators
  - Also need for other fields
- Loops integrate vertices times propagators!
  - Which time dominates?
  - How large are the corrections?
- Accuracy of approximations
  - de Sitter (after extracting factors of $\frac{1}{\epsilon}$)
  - Constant $\epsilon(t)$

Need a definition of Observables

1. IR finite
   - Independent of IR cutoff
2. UV renormalizable (at least BPHZ)
   - Controlling IR no good if we lose the UV
3. Gauge independent
   - Independent of choice of field variable
4. Correspond to observations
   - In principle, not necessarily detectable now

What VEV's give $\Delta^2_{\mathcal{R}}(k)$ & $\Delta^2_h(k)$?

- At tree order

$$\Delta^2_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2}\int d^3x\, e^{-i\vec{k}\cdot\vec{x}}\langle\Omega\big|\zeta(t,\vec{x})\zeta(t,\vec{0})\big|\Omega\rangle$$

$$\Delta^2_h(k) = \frac{k^3}{2\pi^2}\int d^3x\, e^{-i\vec{k}\cdot\vec{x}}\langle\Omega\big|h_{ij}(t,\vec{x})h_{ij}(t,\vec{0})\big|\Omega\rangle$$

- IR cutoff enters loop corrections

  ➔ Must be different operators

- $\vec{x} \to \vec{\chi}[\zeta, h](\vec{x})$ works . . . but
  - New, uncontrollable UV divergences appear
  - Disrupts pattern of ϵ-suppression

How do we observe the other effects?

E.g., secular changes of EM force

- Release $+Q$ at rest from $x = 0$
- Release neutral & $-q$ & at rest from $x = L$
- Measure separation between neutral & $-q$

E.g., secular changes of GR force

- Release (M, Q) at rest from $x = 0$
- Release (m, q) & $(m, 0)$ at rest from $x = 0$
- Measure separation between $(m, 0)$ & (m, q)

But how does a late time observer see anything?

- Changes in 0-point fluctuations ➔ decay rates?
- Time variation in constants?

Need to understand secular effects

- Factors of $\ln[a(t)]$ in de Sitter
  - From $\int \frac{d^3k}{(2\pi)^3} \, |u(t,k)|^2 = \frac{1}{64\pi G} \int_{k_i}^{Ha} \frac{dk}{k} \Delta_h^2(k)$
- $\Delta\mathcal{L} = C \times (\varphi, h)^N \times (\Psi, A, \partial\varphi, \partial h)$ ➔ $C^2[\ln(a)]^N$
  - $\lambda\varphi^4$ ➔ $\lambda[\ln(a)]^2$
  - SQED $(e^2\varphi^*\varphi A^2)$ ➔ $e^2 \ln(a)$
  - Yukawa $(f\varphi\bar{\Psi}\Psi)$ ➔ $f^2 \ln(a)$
  - GR $(\sqrt{G} h\partial h\partial h)$ ➔ $G \ln(a)$
- Perturbation theory eventually breaks down!

Nonperturbative resummations

- Solved for scalar potential models
  - Starobinsky & Yokoyama 1994
- Also SQED & Yukawa
  - Integrate out $\overline{\Psi}\Psi$ & A_μ → scalar potential
- Derivative interactions unsolved
  - GR
  - Nonlinear σ-models
- Check beliefs (& prejudices) against explicit computations
  - Dirac + GR → Miao 2005-6
  - Maxwell + GR → Glavan,Leonard,Miao,Prokopec,Wang 2013-16
  - Transformation ansatz disproven, arXiv:1606.02417

Initial State Corrections

- $\ln\left[\frac{a(t)}{a_i}\right]$ ➔ Must start with finite time t_i
  ➔ must perturbatively correct initial state
- Not doing this leads to divergences at time $t = t_i$

$$\partial_\mu\left[\sqrt{-g}\,g^{\mu\rho}g^{\nu\sigma}F_{\rho\sigma}(x)\right] + \int d^4x' \left[{}^{\mu}\Pi^{\nu}\right](x;x')A_\nu(x') = J^\mu(x)$$

- $H = H_0 + H_1 \;\;,\;\; H_0|n\rangle = E_n|n\rangle \;,\; H|\Omega\rangle = E|\Omega\rangle$

$$E - E_0 = \langle 0|H_1|0\rangle \quad , \quad \langle n|\Omega\rangle = \frac{\langle n|H_1|0\rangle}{E_0 - E_n}$$

- But there are no eigenstates!
  - Can do it for flat space
  - Absorb surface terms (certain partial integrations cry out to be done)
- Try to avoid dogmatism

Conclusions: Virgin territory for Mathematical Physics

- Many QFT effects from φ & h_{ij} & on them
- Need controlled approximations
  - No Feynman procedure for loop integrals
- Need a definition of observables
  - No S-matrix, no Borcher's Theorem, no LSZ
- Need to understand secular effects
  - Best chance for big results
- Need to understand initial state corrections
  - Failure of usual trick taking free vacuum as $t \to \pm\infty$
- A chance to play at being Feynman & Schwinger!
  - Why is everyone ignoring this?