

# Higgs inflation :updates



Seongchan Park

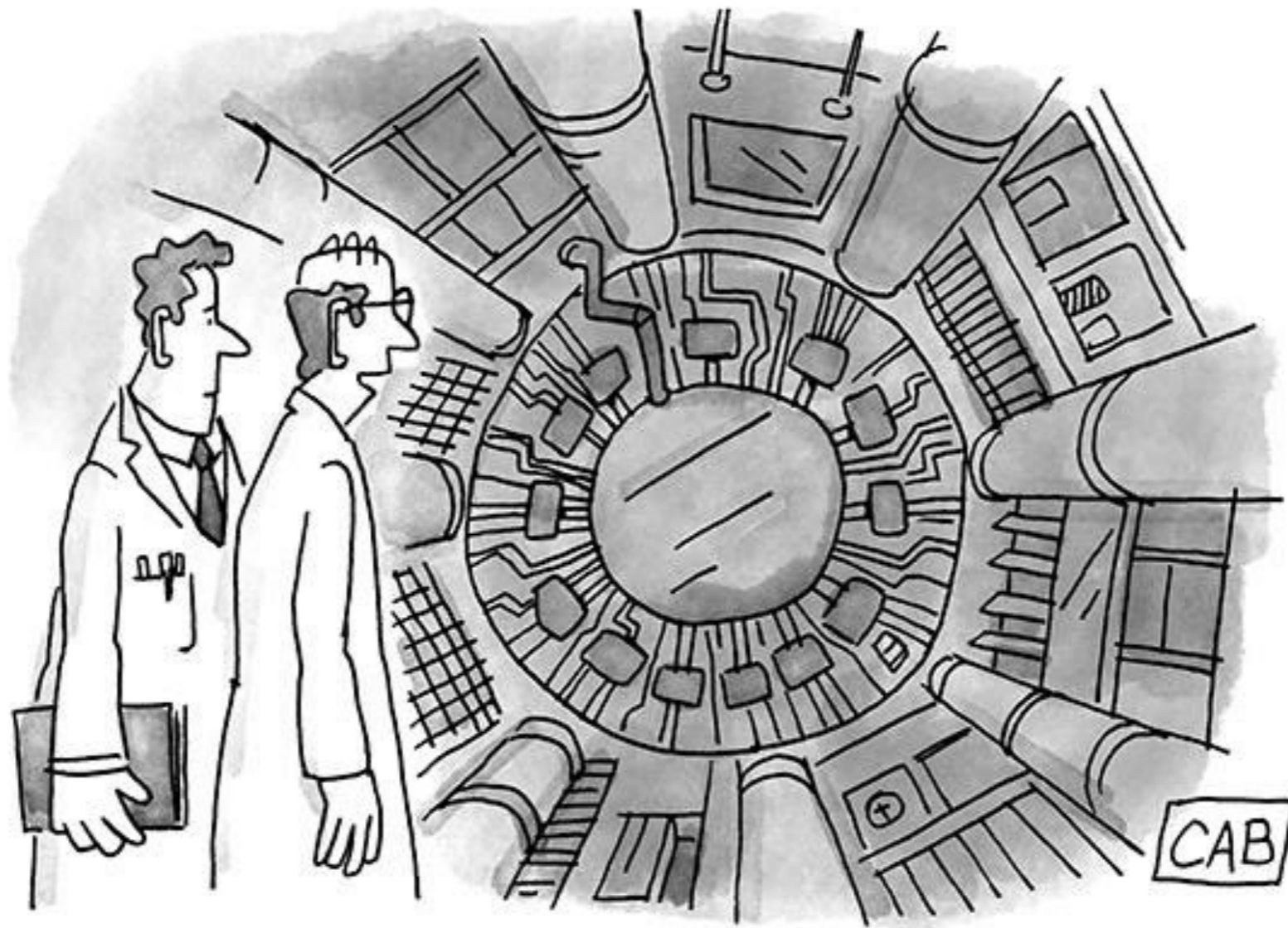


**International Symposium on  
Cosmology and Particle Astrophysics**

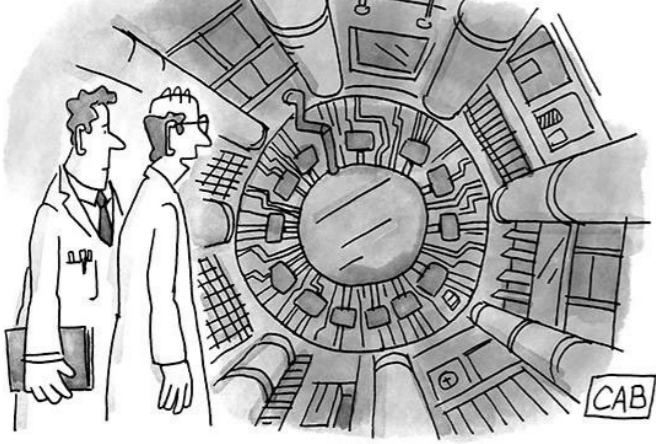
**CosPA 2017**

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**Yukawa Institute for Theoretical Physics, Kyoto University, JAPAN**



*“Once you have a collider, every problem starts to look like a particle.”*



*“Once you have a collider, every problem starts to look like a particle.”*

In particle physics, inflation takes place due to **a scalar field**

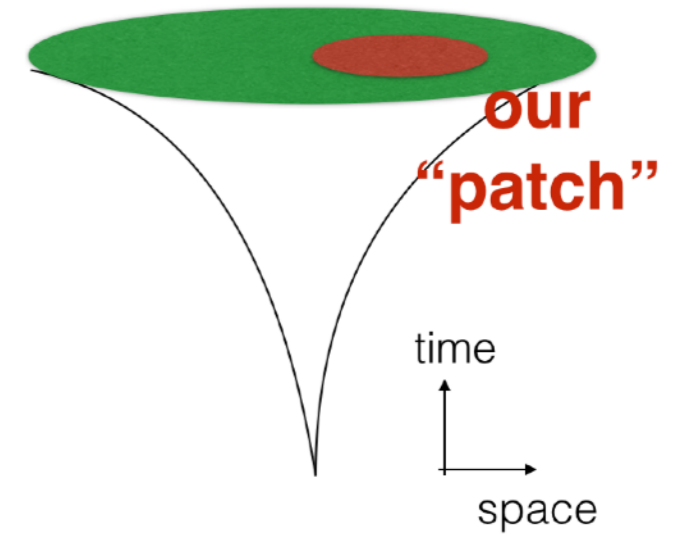
**N.B.**  $A^M = (A_5, A^\mu) = (\phi, A^\mu)$

ex) Extra natural inflation [Arcani-Hamed et. al. (2003)]

Orbifold GUT inflation [SCP (2007)]

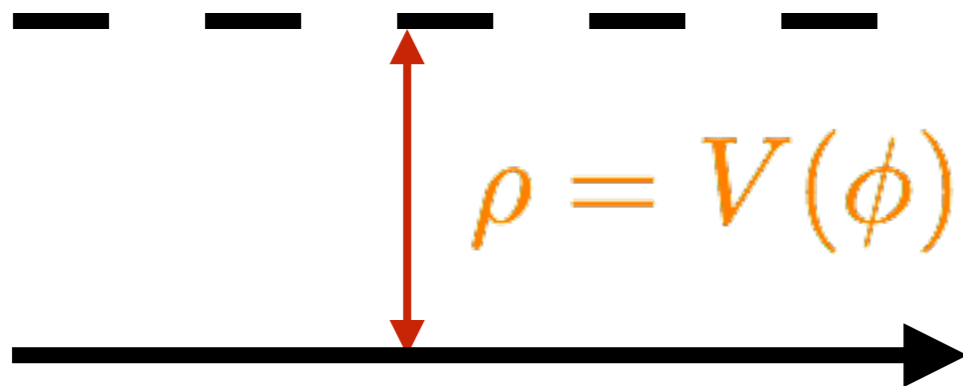
This is what we want:  $a(t) = a_0 e^{H(t-t_0)}$

$$ds^2 = dt^2 - a(t)^2 d\vec{x} \cdot d\vec{x}$$



This is the equation:  $H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho}{3M_p^2}$

It is realized if the potential is “flat”



“slow-roll conditions”

$$(V'/V)^2 \ll 1$$

$$V''/V \ll 1$$

(ex)  $V = \lambda\phi^4, \lambda \sim 10^{-12}$

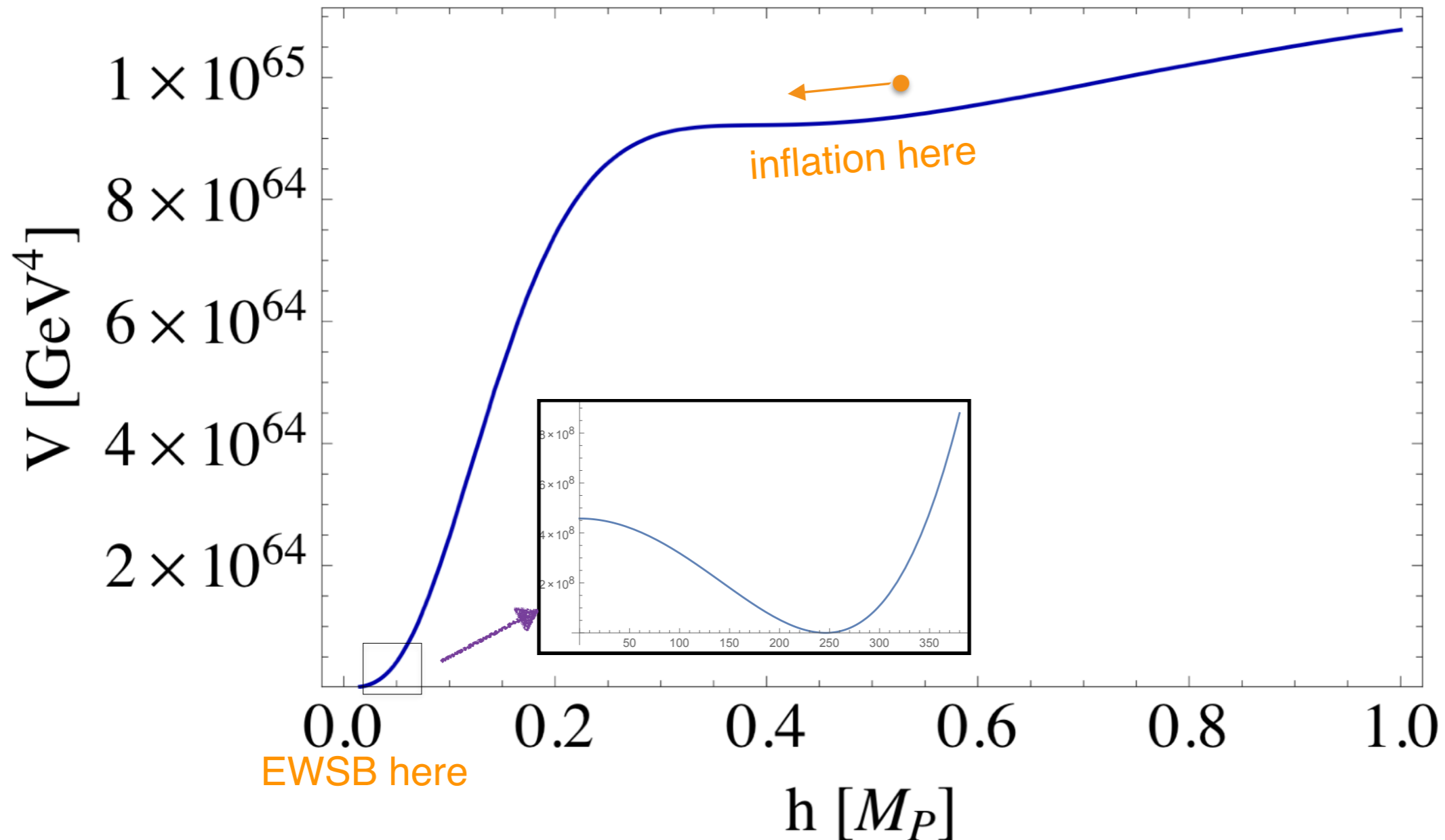
# Higgs inflation

An economical and predictive idea

**: The SM Higgs = Inflaton field**

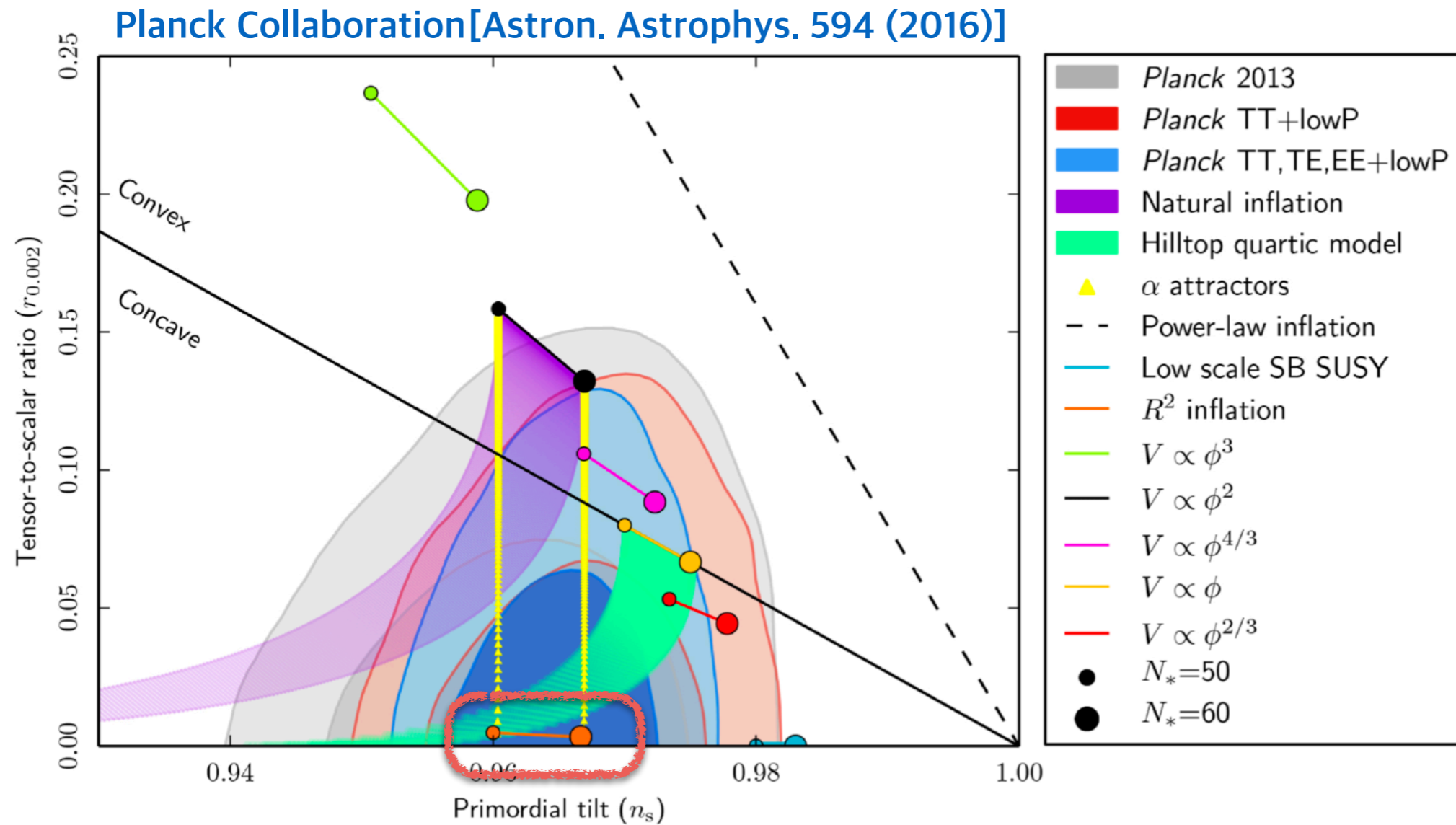
- at low scale ( $\sim 100$  GeV) responsible for EWSB
- at high scale ( $\sim 10^{17}$  GeV) responsible for cosmic inflation

# Higgs potential with RG running and with $|H|^2 R$ term



- \* Reheating process is built-in due to the Higgs Decay (as checked by LHC) :-)
- \* Link EW scale physics to cosmology!

# (the best) fit to the 'DATA'



**$R^2$  inflation  
=Higgs inflation**

# plan

- Higgs in the SM —status review
- Higgs inflation with RGE
- Discussions on consistency and further implications
- (+one more thing)



# The SM<sub>N</sub>

- Local gauge symmetry  $SU(3) \times SU(2) \times U(1)_Y$  with Higgs mechanism  $\Rightarrow SU(3) \times U(1)_{em}$  in broken phase
- $N$ -generations of Weyl spinors:  $\{Q, U, D, L, E, (n)\} \times N$
- Quiz) how many free parameters?

# Free parameters in $SM_N$

- Total  $(N^2+N+6)$  free parameters (+ neutrino masses and mixings)
  - ★ gauge couplings  $(g_3, g_2, g_1) : 3$
  - ★ Higgs self coupling + VEV : 2
  - ★ Fermion masses :  $(2N + N)$
  - ★  $N(N-1)/2$  mixing angles +  $(N-1)(N-2)/2$  complex phases
- $N=3$  (4th chiral gen. ruled out by the LHC), 18 in total
- 17 known before the LHC

# The Higgs potential

$$V(H) = \lambda \left( H^\dagger H - v^2/2 \right)^2$$

$$H = \begin{pmatrix} \phi^+ \\ \frac{v+h+i\phi^0}{\sqrt{2}} \end{pmatrix}$$

Lorentz scalar (s=0)  
(1,2,1/2) of SU(3)XSU(2)XU(1)

# The Higgs potential

$$V(H) = \lambda \left( H^\dagger H - v^2/2 \right)^2$$

The most general  
renormalizable  
potential with a  
Tachyonic mass

$$H = \begin{pmatrix} \phi^+ \\ \frac{v+h+i\phi^0}{\sqrt{2}} \end{pmatrix}$$

Lorentz scalar (s=0)  
(1,2,1/2) of SU(3)XSU(2)XU(1)

# The Higgs potential

$$V(H) = \lambda \left( H^\dagger H - v^2/2 \right)^2$$

The most general renormalizable potential with a Tachyonic mass

Only two parameters

$$H = \begin{pmatrix} \phi^+ \\ \frac{v+h+i\phi^0}{\sqrt{2}} \end{pmatrix}$$

Lorentz scalar (s=0)  
(1,2,1/2) of SU(3)XSU(2)XU(1)

# Vacuum expectation value

$$m_W^2 = \frac{g^2 v^2}{4}, \quad m_Z^2 = \frac{(g'^2 + g^2)v^2}{4}$$

$$v = \left( \sqrt{2} G_F \right)^{-1/2} \approx 246 \text{ GeV}$$

known  
since 1970s

# The self coupling

the last piece of information within the SM

LHC (2012)

$$\lambda = \frac{m_h^2}{2v^2} \approx \frac{1}{8}$$

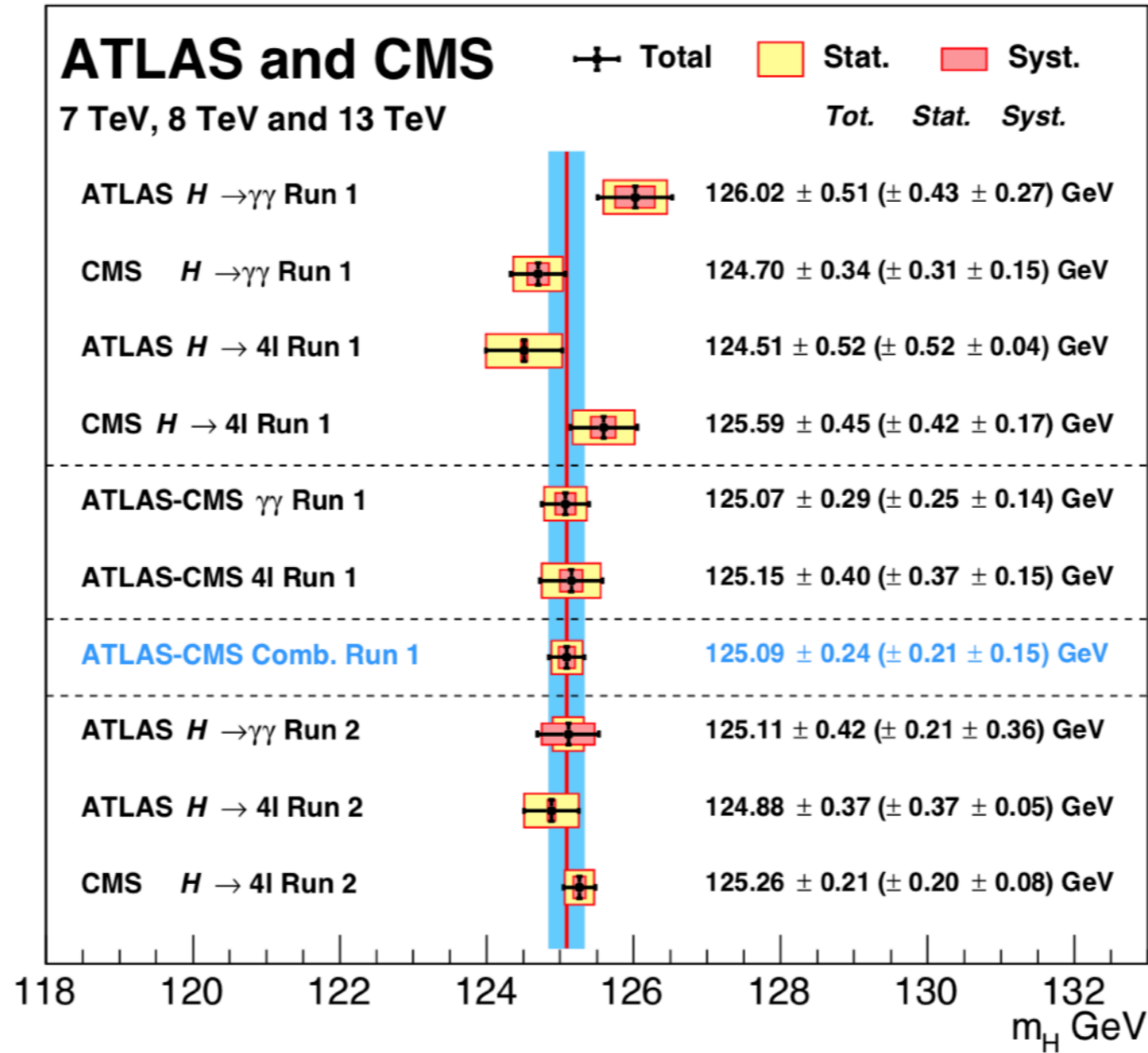
$$m_H = 125.03_{-0.27}^{+0.26}(\text{stat})_{-0.15}^{+0.13}(\text{syst})$$

CMS PAS HIG-14-009

$$m_H = 125.36 \pm 0.37(\text{stat.}) \pm 0.18(\text{syst.})\text{GeV}$$

ATLAS arXiv:1406.3827

# The Higgs mass



C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C, **40**, 100001 (2016) and 2017 update



# The SM Higgs potential

$$V(h) = \frac{1}{2}(125\text{GeV})^2 h^2 + \frac{1}{8}(246\text{GeV})h^3 + \frac{1}{32}h^4$$

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$$\lambda v^2 = \frac{m_h^2}{2}$$

measured

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measured

$$\lambda v$$

predicted

# The SM Higgs potential

$$V(h) = \frac{1}{2}(125\text{GeV})^2 h^2 + \frac{1}{8}(246\text{GeV}) h^3 + \frac{1}{32} h^4$$

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measured

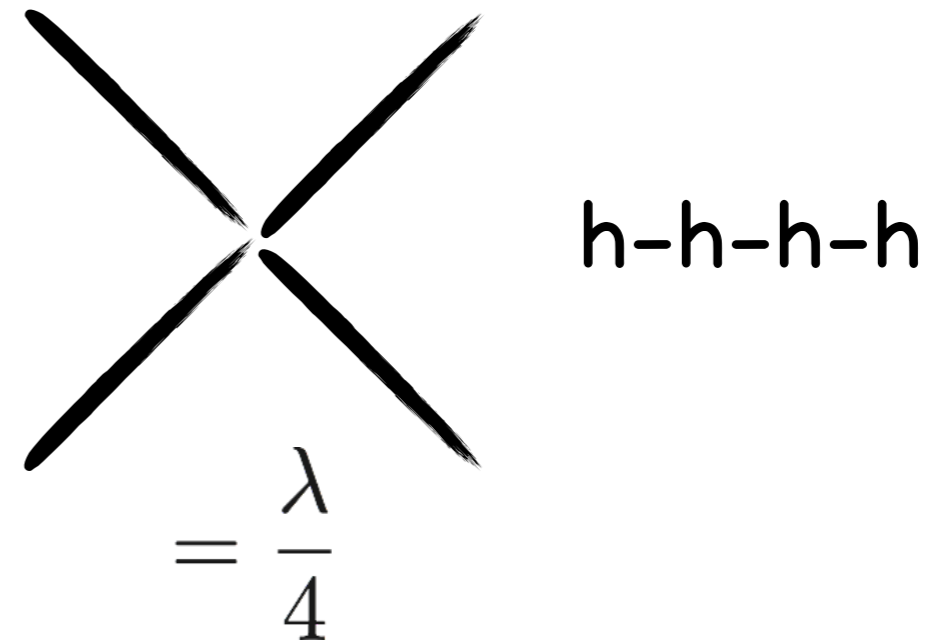
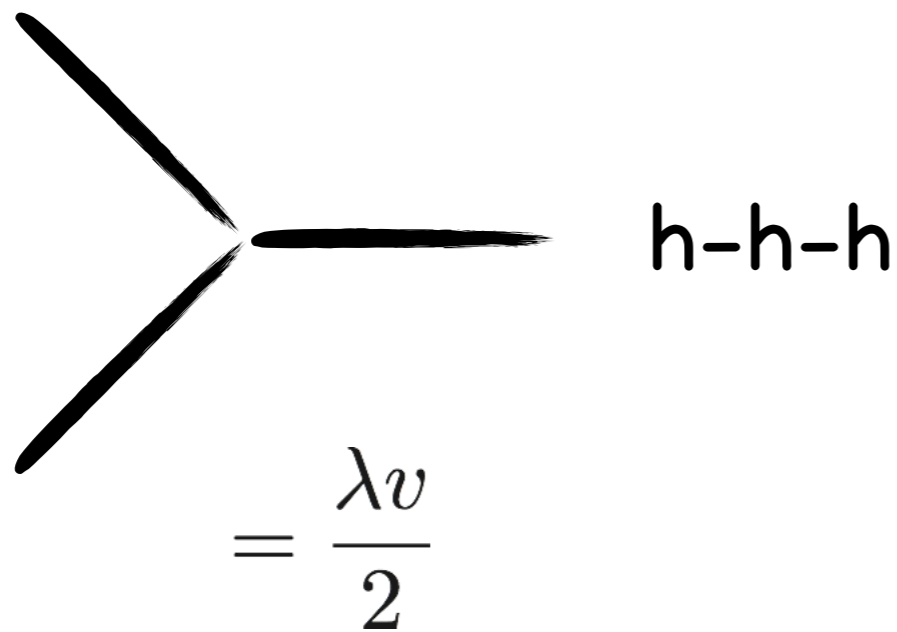
$$\lambda v$$

predicted

$$\frac{\lambda}{4}$$

predicted

# HIGGS SELF INTERACTIONS

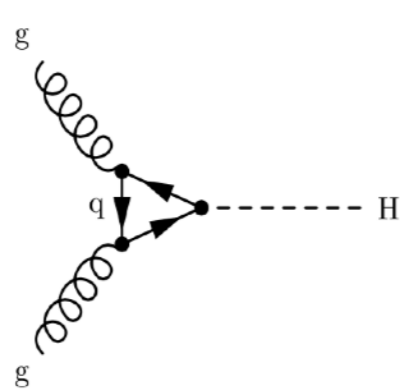


measured+predicted

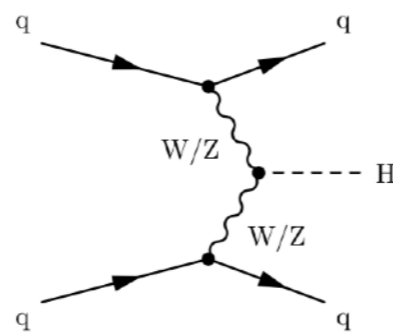
predicted

Theorists can make models predicting deviations (New physics).  
These are good topics for future colliders such as ILC or CEPC ...

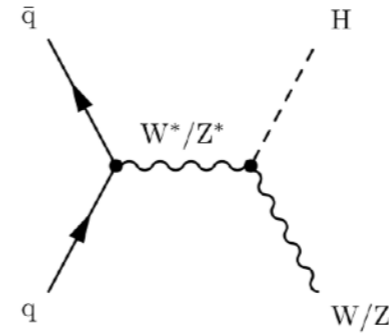
# Observed Higgs-other particle interactions



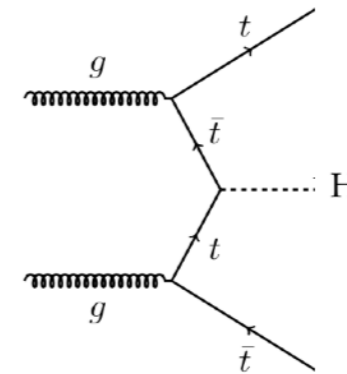
Gluon fusion  
ggH (19pb)



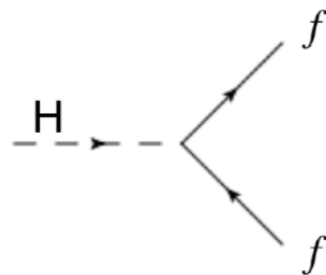
Vector Boson Fusion  
VBF (1.6pb)



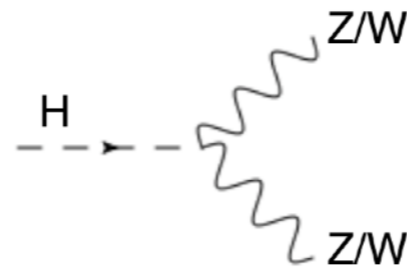
VH (1.1pb)



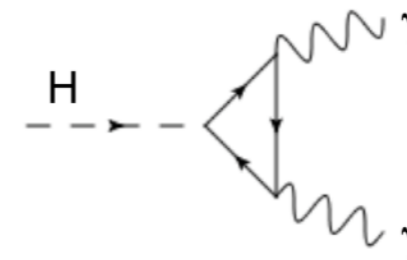
ttH (0.1pb)



bb (58%)  
 $\tau\tau$  (6.3%)



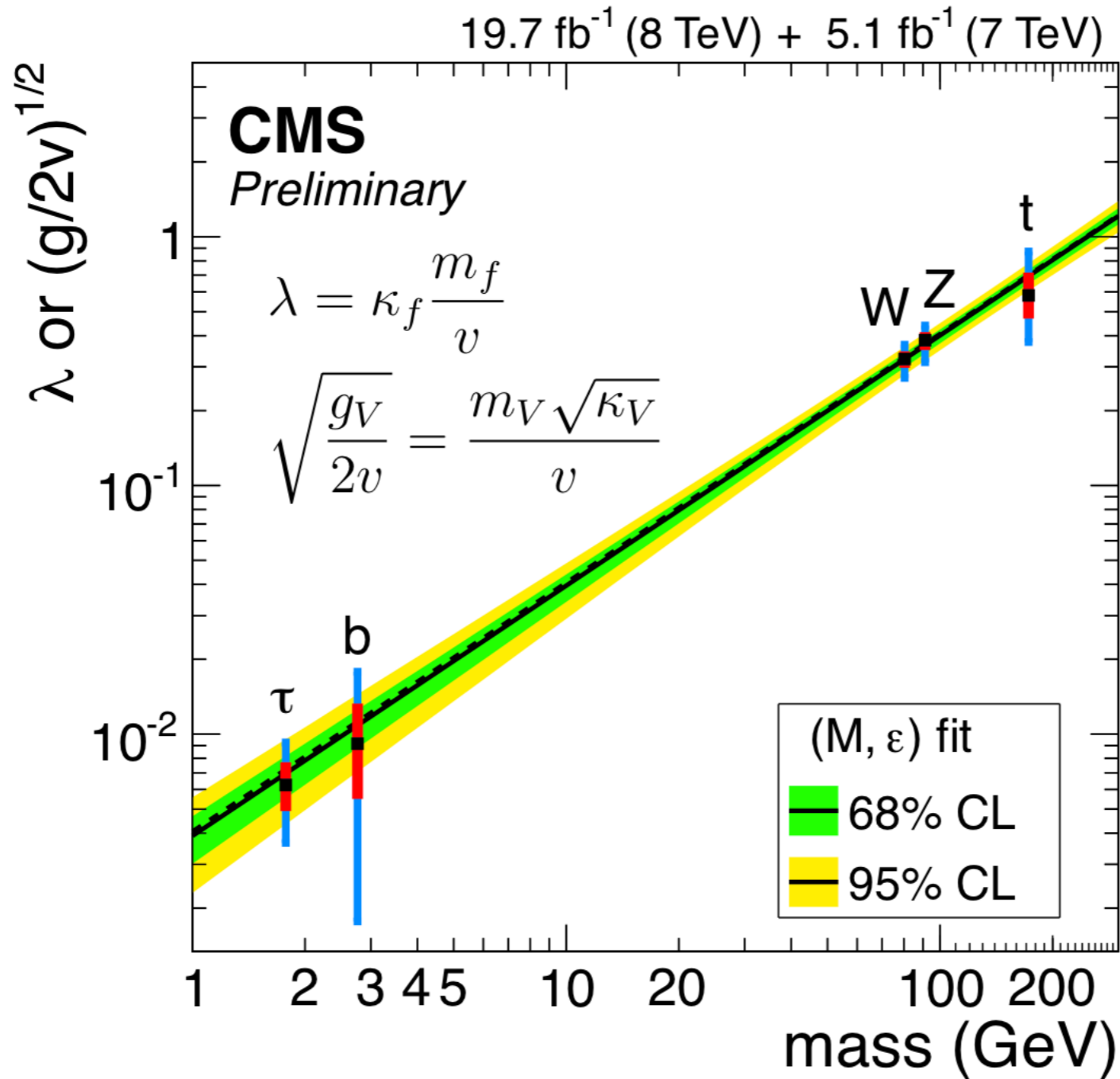
WW (21.5%)  
ZZ (2.6%)



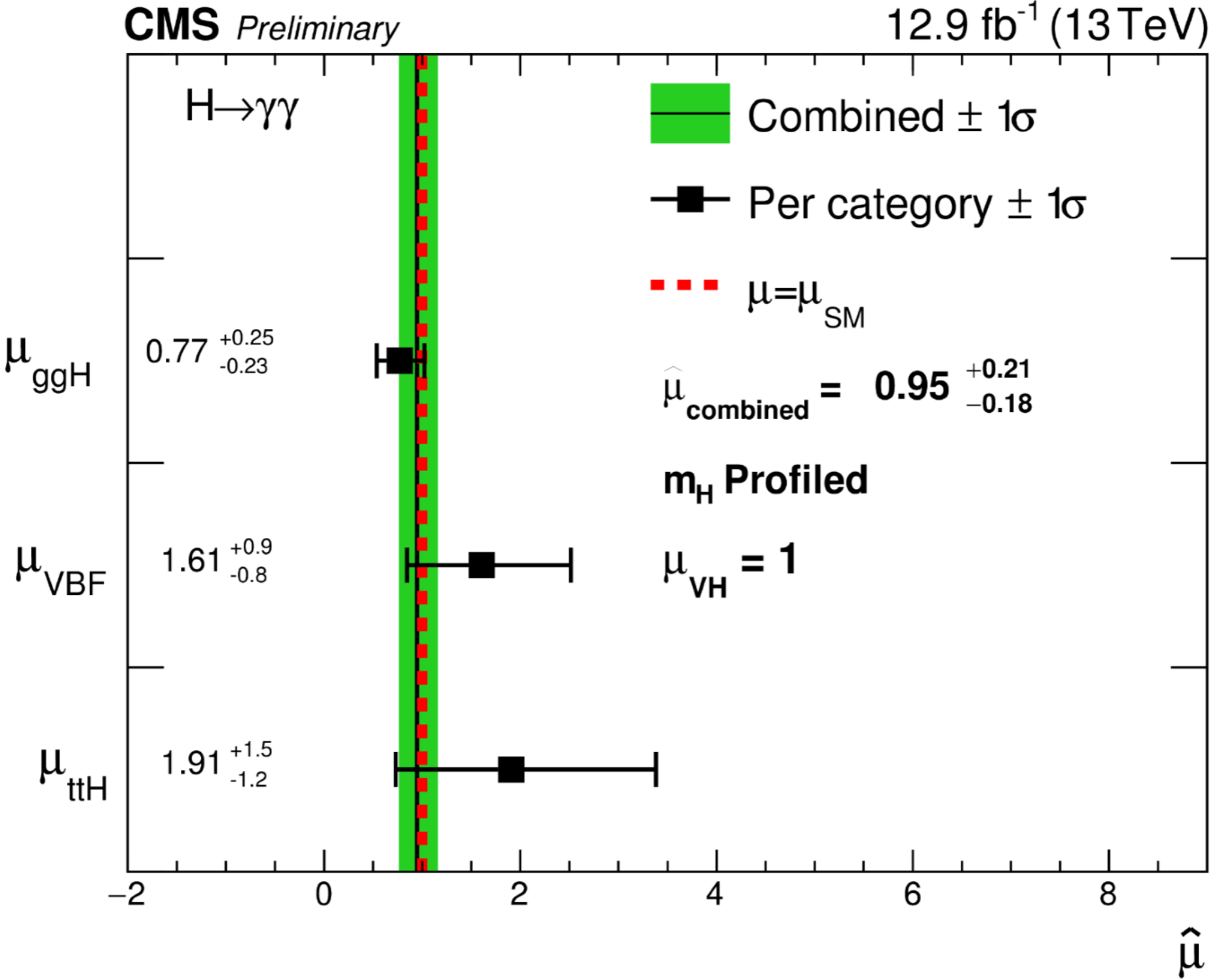
$\gamma\gamma$  (0.2%)

- Higgs couplings with gauge bosons and some fermions are observed through several processes (indirectly/directly)

# All are consistent with the SM



# consistent with the SM (LHC Run-2)





# The observed Decay pattern of the Higgs is consistent with the SM

$$\mu = \frac{N_{exp}}{N_{theory}}$$

$$\mu = 1.09 \pm 0.07 \text{ (stat)} \pm 0.04 \text{ (expt)} \pm 0.03 \text{ (th. bkg)} \pm 0.07 \text{ (th. sig)}$$

Confidently, we can tell that the observed Higgs  
is very very close to the SM Higgs!

# Beyond the collider energies?

- Let's be more ambitious and ask even more extrapolation of our knowledge further.
- **Q. What's role of the Higgs at very early era of universe, e.g., at the inflationary era?**
- To answer the question, we need to understand high energy & Large field behavior of the Higgs potential.

# High energy behavior

- In principle, we can ‘calculate’ everything  $E < \Lambda_{\text{SM}}$
- $\Lambda_{\text{SM}} > \text{TeV}$ , LHC
- $\Lambda_{\text{SM}} \sim M_{\text{P}}$ , in principle (Don’t forget renormalizability!)
- We may be able to see physics **near the Planck energy** assuming that the SM is good (which is not likely though).

# Higgs + Gravity

non-minimal coupling

$$S_{Higgs} = \int d^4x \left( \frac{M^2 + \xi |H|^2}{2} R + |D_\mu H|^2 - V(H) \right)$$

Grav. Mass

Kin. of Higgs  
(gauge-Higgs interaction)

Ricci curvature  
 $\sim 1/L^2$   
 $\sim M^2$

Pot. of Higgs

# Higgs + Gravity

$$V(H) = \lambda (|H|^2 - v^2/2)^2$$

$$S = \int d^4x \sqrt{g} \left[ \frac{M_P^2 + \xi |H|^2}{2} R + |DH|^2 - V(H) + \mathcal{L}_{\text{SM}} \right]$$

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- ‘**non-minimal coupling** term’ Dim=4 term (indeed minimally coupled in Einstein frame) is introduced.

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- This term is completely compatible with **all symmetries of the theory**, i.e. local covariance, Lorentz symmetry and also the SM gauge symmetry



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- There could be more terms  $\sim \mathbf{O(4+n)}/\Lambda^{n \geq 1}$  w/ strengths to be determined by future experiments or underlying theory you assume.

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- Below,  $\Lambda$ , those terms may be neglected.

# Non-minimally coupled gravity and chaotic inflation

without RG effect

[T. Futamase, K. Maeda, PRD(1989)]

$$S = \int d^4x \sqrt{-g} \left( \frac{M^2 - \xi \phi^2}{2} R + (\partial_\mu \phi)^2 - V(\phi) \right)$$

TABLE I. Chaotic inflationary scenario with a nonminimally coupled scalar. Inflation is realized for “Yes,” but not for “No.”

$\xi$	(non-minimal) (minimal)	$V = \frac{1}{4}\phi^4$	$V = \frac{1}{2}m^2\phi^2$
$\xi < 0$		Yes	No unless $ \xi  \lesssim 10^{-3}$
$\xi = 0$		Yes	Yes
$0 < \xi < \frac{1}{6}$		Yes if $\xi \lesssim 10^{-8}$	Yes if $\xi \lesssim 10^{-12}$
	(conformal)	Possible but unnatural if $10^{-8} < \xi \lesssim 10^{-3}$	Possible but unnatural if $10^{-8} < \xi$
		No if $\xi \gtrsim 10^{-3}$	No if $\xi \gtrsim 10^{-3}$
$\xi = \frac{1}{6}$		No	No
$\xi > \frac{1}{6}$		No	No

# Inflation with $V=K \cdot 2$

[SCP, S. Yamaguchi JCAP (2008)]

without RG  
effect

$$S = \int d^4x \sqrt{-g} \left( \frac{M^2 + K(\phi)}{2} R + (\partial_\mu \phi)^2 - V(\phi) \right)$$

General condition for flat potential at a large field limit:

$$\lim_{\phi \rightarrow \infty} \frac{V(\phi)}{K(\phi)^2} = \text{Const.} > 0$$

“attractor”!

(ex) monomial functions  $K(\phi) = \xi \phi^n, V(\phi) = \lambda \phi^{2n}$

**$n=2$  corresponds to ‘Higgs inflation’**

# Einstein frame

Bezrukov, Shaposhnikov (2008) for  $K(\phi)=\xi \phi^2$   
SCP, Yamaguchi (2008) for general  $K(\phi)$

$$S_J = \int d^4x \sqrt{g} \left[ \frac{M^2 + K(\phi)}{2} R + |DH|^2 + V(H) \right]$$



$$g_{\mu\nu} \rightarrow g_{\mu\nu}^E = e^{2\omega} g_{\mu\nu}$$

$$e^{2\omega} = \frac{M^2 + K}{M_P^2}$$

$$V_E = \frac{M_P^4}{(M^2 + K)^2} V_J$$

$$\frac{\partial H_E}{\partial H_J} = \sqrt{e^{-2\omega} + \frac{3}{2} e^{2\omega} K'^2}$$

$$S_E = \int d^4x \sqrt{g_E} \left[ \frac{M_P^2}{2} R_E + |DH_E|^2 - V(H_E) \right]$$

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for Higgs  
inflation:

$$V_J = \lambda H_J^4, K = \xi H_J^2$$

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for Higgs  
inflation:

$$V_J = \lambda H_J^4, K = \xi H_J^2 \rightarrow V_E \rightarrow \frac{\lambda}{\xi^2} M_P^4$$

# Einstein frame

Bezrukov, Shaposhnikov (2008) for  $K(\phi)=\xi \phi^2$   
SCP, Yamaguchi (2008) for general  $K(\phi)$

$$S_J = \int d^4x \sqrt{g} \left[ \frac{M^2 + K(\phi)}{2} R + |DH|^2 + V(H) \right]$$



$$g_{\mu\nu} \rightarrow g_{\mu\nu}^E = e^{2\omega} g_{\mu\nu}$$

$$V_E = \frac{M_P^4}{(M^2 + K)^2} V_J$$

$$e^{2\omega} = \frac{M^2 + K}{M_P^2}$$

$$\frac{\partial H_E}{\partial H_J} = \sqrt{e^{-2\omega} + \frac{3}{2} e^{2\omega} K'^2}$$

$$S_E = \int d^4x \sqrt{g_E} \left[ \frac{M_P^2}{2} R_E + |DH_E|^2 - V(H_E) \right]$$

flat potential

for Higgs  
inflation:

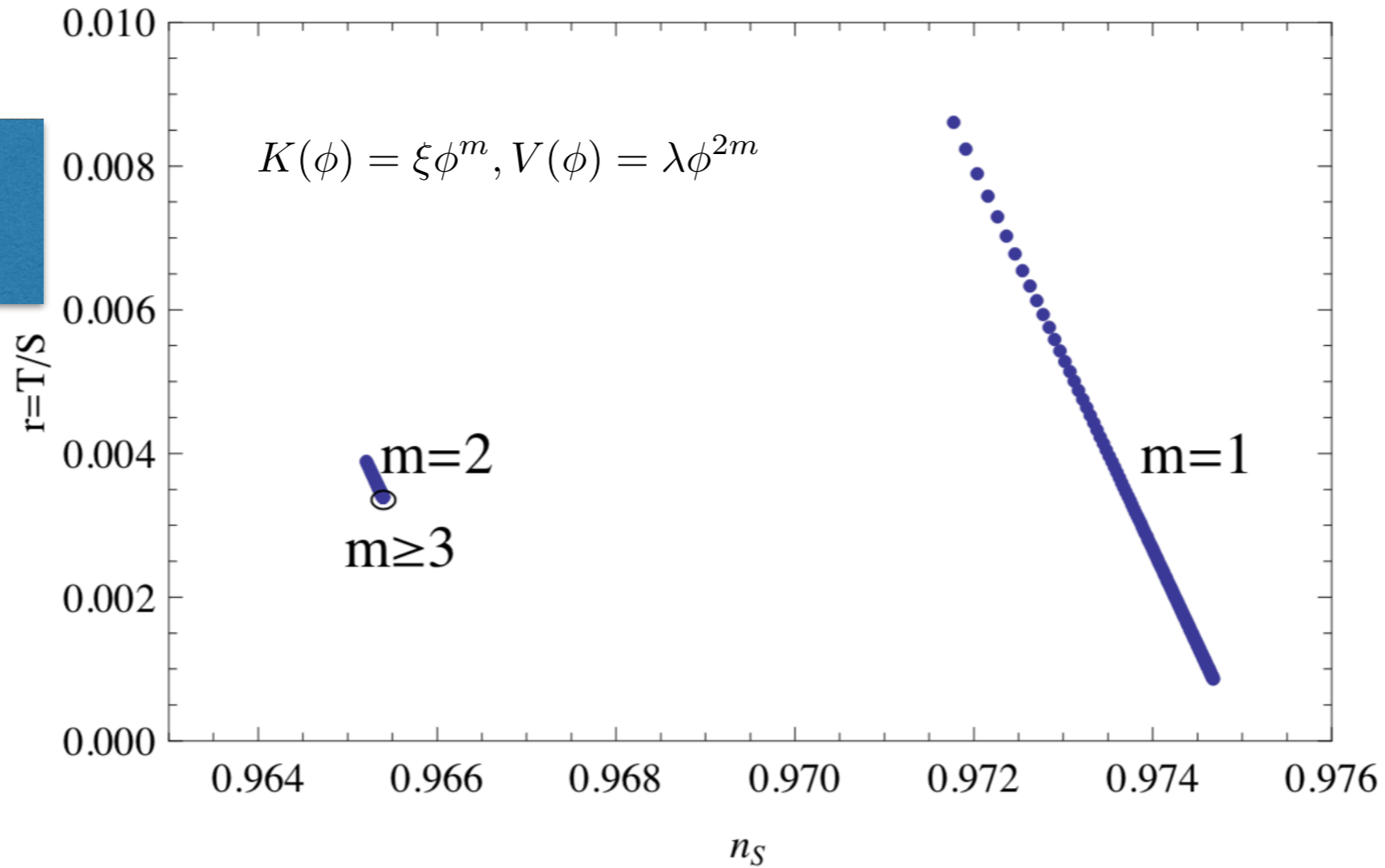
$$V_J = \lambda H_J^4, K = \xi H_J^2 \rightarrow$$

$$V_E \rightarrow \frac{\lambda}{\xi^2} M_P^4$$



[SCP, S. Yamaguchi JCAP (2008)]

without RG  
effect



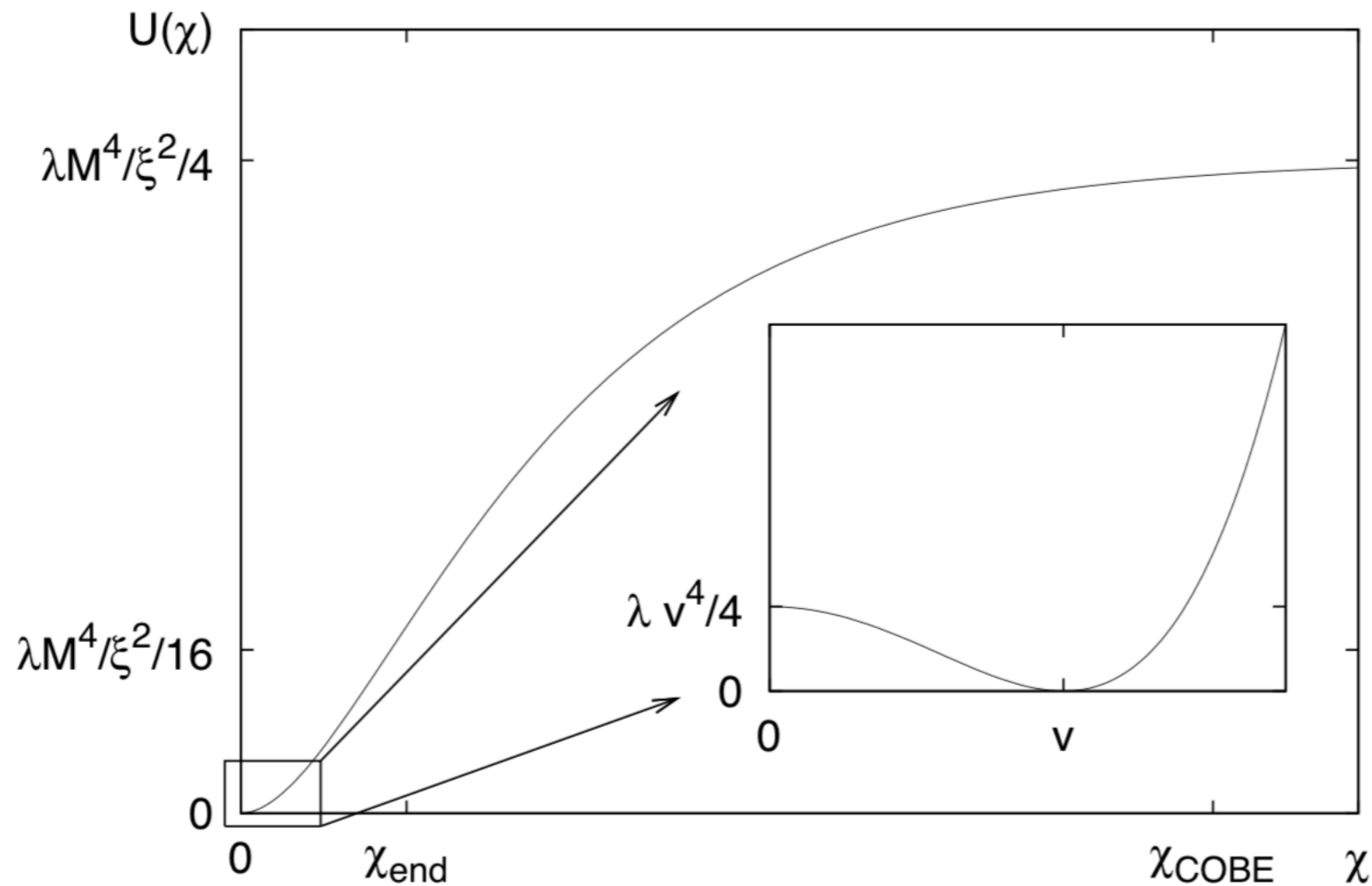
$$n_s = \begin{cases} 1 - \frac{3}{2a_0 N^{3/2}} - \frac{3}{2N}, & (m = 1) \\ 1 - \frac{9(1+1/(6a_0))}{2N^2} - \frac{2}{N}, & (m = 2) \\ 1 - \frac{9}{2N^2} - \frac{2}{N}, & (m \geq 3) \end{cases}, \quad r = \begin{cases} \frac{4}{a_0 N^{3/2}}, & (m = 1) \\ \frac{12(1+1/(6a_0))}{N^2}, & (m = 2) \\ \frac{12}{N^2}, & (m \geq 3) \end{cases}$$

# Higgs inflation

[Bezrukov, Shaposhnikov, PLB(2008)]

without RG  
effect

$$S = \int d^4x \sqrt{-g} \left( \frac{M^2}{2} R + \xi H^\dagger H R + \lambda (H^\dagger H - v^2/2)^2 + \dots \right)$$



# COBE normalization

without RG  
effect

$$\frac{U}{\epsilon} = (0.027 M_{\text{Pl}})^4 \quad \longrightarrow \quad \frac{\lambda}{\xi^2} \approx 10^{10}$$

or  $\lambda \sim 1, \xi \sim 10^5$

$$\mathcal{L}_G \ni \frac{M_P^2 + 10^5 \times \phi^2}{2} R + \dots$$

DOES **NOT** LOOK GOOD :-)

# WAIT!

Who told you this?  $\lambda \sim 1, \xi \sim 10^5$

Actually, when we take the **quantum** effect into account,  
the self coupling becomes small!

$$\lambda(100 \text{ GeV}) \approx 1/8 \rightarrow \lambda(\mu \gg 100 \text{ GeV}) \ll 1$$

# The RGE

$$(4\pi)^2 \frac{d\lambda}{d \ln \bar{\mu}^2} = -3y_t^4 + 6y_t^2\lambda + 12\lambda^2 + \frac{9}{16} \left( g_2^4 + \frac{2}{5}g_2^2g_1^2 + \frac{3}{25}g_1^4 \right) - \frac{9}{2}\lambda \left( g_2^2 + \frac{g_1^2}{5} \right) + \dots$$

**Top quark Yukawa coupling**

$$y_t = \frac{\sqrt{2}m_t}{v} = \frac{\sqrt{2} \cdot 172}{246} \approx 0.98$$

gauge & self interactions

# Top quark mass

## ***t*-Quark Mass (Direct Measurements)**

The following measurements extract a *t*-quark mass from the kinematics of  $t\bar{t}$  events. They are sensitive to the top quark mass used in the MC generator that is usually interpreted as the pole mass, but the theoretical uncertainty in this interpretation is hard to quantify. See the review “The Top Quark” and references therein for more information.

OUR AVERAGE of  $173.1 \pm 0.6$  GeV is an average of top mass measurements from LHC and Tevatron Runs. The latest Tevatron average,  $174.30 \pm 0.35 \pm 0.54$  GeV, was provided by the Tevatron Electroweak Working Group (TEVEWWG).

<u>VALUE (GeV)</u>	<u>DOCUMENT ID</u>	<u>TECN</u>	<u>COMMENT</u>
<b><math>173.1 \pm 0.6</math> OUR AVERAGE</b>	Error includes scale factor of 1.6. See the ideogram below.		
$172.84 \pm 0.34 \pm 0.61$	<sup>1</sup> AABOUD	16T ATLS	combination of ATLAS
$172.44 \pm 0.13 \pm 0.47$	<sup>2</sup> KHACHATRY...16AK	CMS	combination of CMS
$174.30 \pm 0.35 \pm 0.54$	<sup>3</sup> TEVEWWG	16 TEVA	Tevatron combination

• • • We do not use the following data for averages, fits, limits, etc. • • •

# Top quark mass

PDG (2017 update)

$m_t$ (GeV/ $c^2$ )	Source	$\int \mathcal{L} dt$	Ref.	Channel
$172.99 \pm 0.48 \pm 0.78$	ATLAS	4.6	[145]	$\ell$ +jets+ $\ell\ell$
$172.44 \pm 0.13 \pm 0.47$	CMS	19.7	[146]	$\ell$ +jets+ $\ell\ell$ +All jets
$172.35 \pm 0.16 \pm 0.48$	CMS	19.7	[146]	$\ell$ +jets
$172.22 \pm 0.18^{+0.89}_{-0.93}$	CMS	19.7	[152]	$\ell\ell$
$173.72 \pm 0.55 \pm 1.01$	ATLAS	20.2	[158]	All jets
$172.25 \pm 0.08 \pm 0.62$	CMS	35.9	[159]	$\ell$ +jets
$174.30 \pm 0.35 \pm 0.54$	CDF,DØ (I+II) $\leq 9.7$		[174]	publ. or prelim.
$173.34 \pm 0.27 \pm 0.71$	Tevatron+LHC $\leq 8.7 + \leq 4.9$		[2]	publ. or prelim.

(new)  
'smallish'

(old)  
'largish'

$m_t(\text{center}) = 173.1$

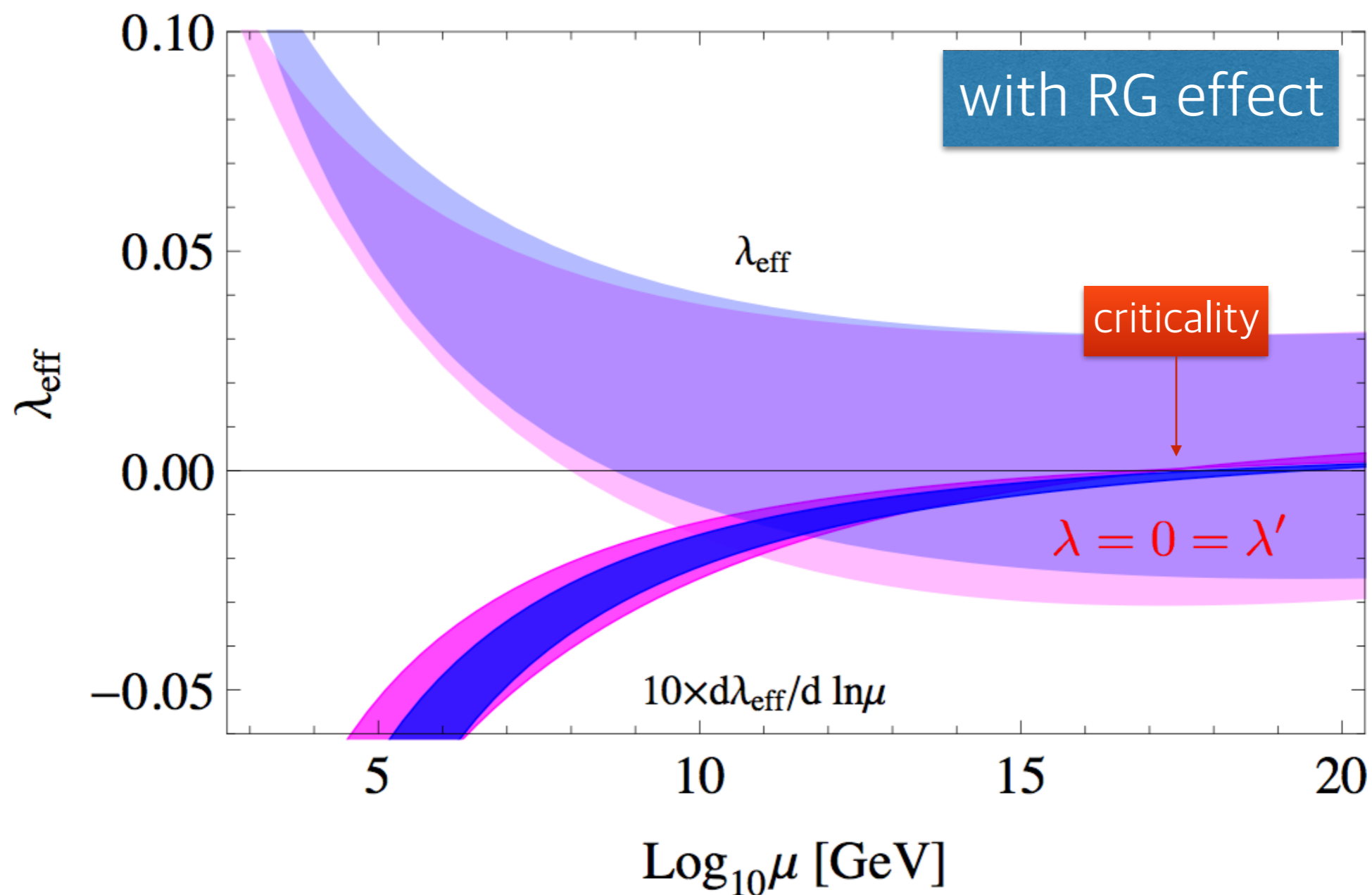
# Is Higgs vacuum unstable?

- Some of us say that “the Standard Model Higgs potential develops an instability at a scale of the order of  $10^{11}$  GeV”
- This statement is true only when you take ‘the current central values of the Higgs and top masses’
- **thus is premature since large uncertainties are involved**



- we should take the ‘measured top quark mass’ with **a lot of caution.**
- **Maybe our vacuum is stable after all** with a reasonable chance
- The ILC350 can provide more definite answer... (A hadron machine does not help much)

# 2-loop effective coupling

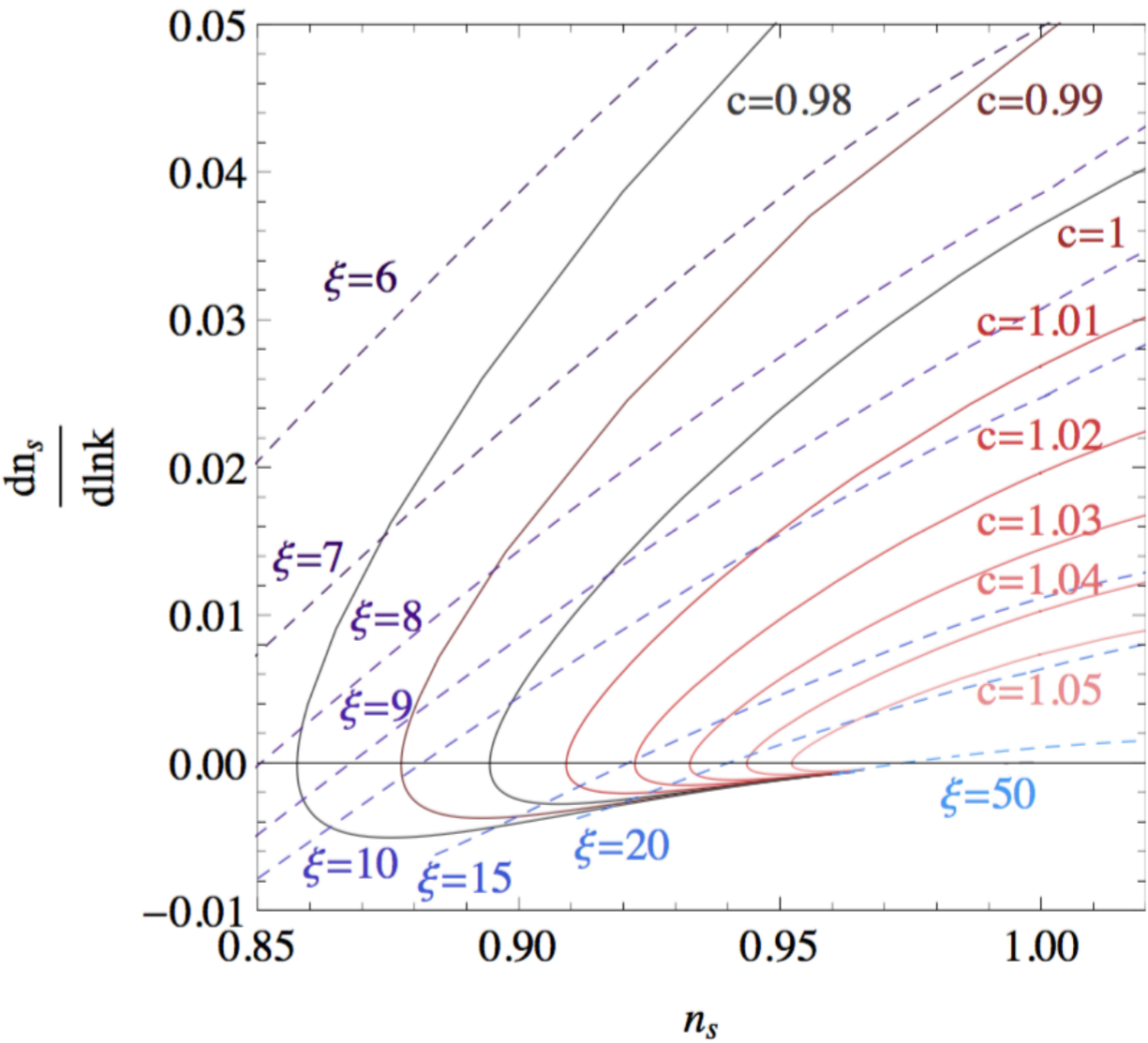
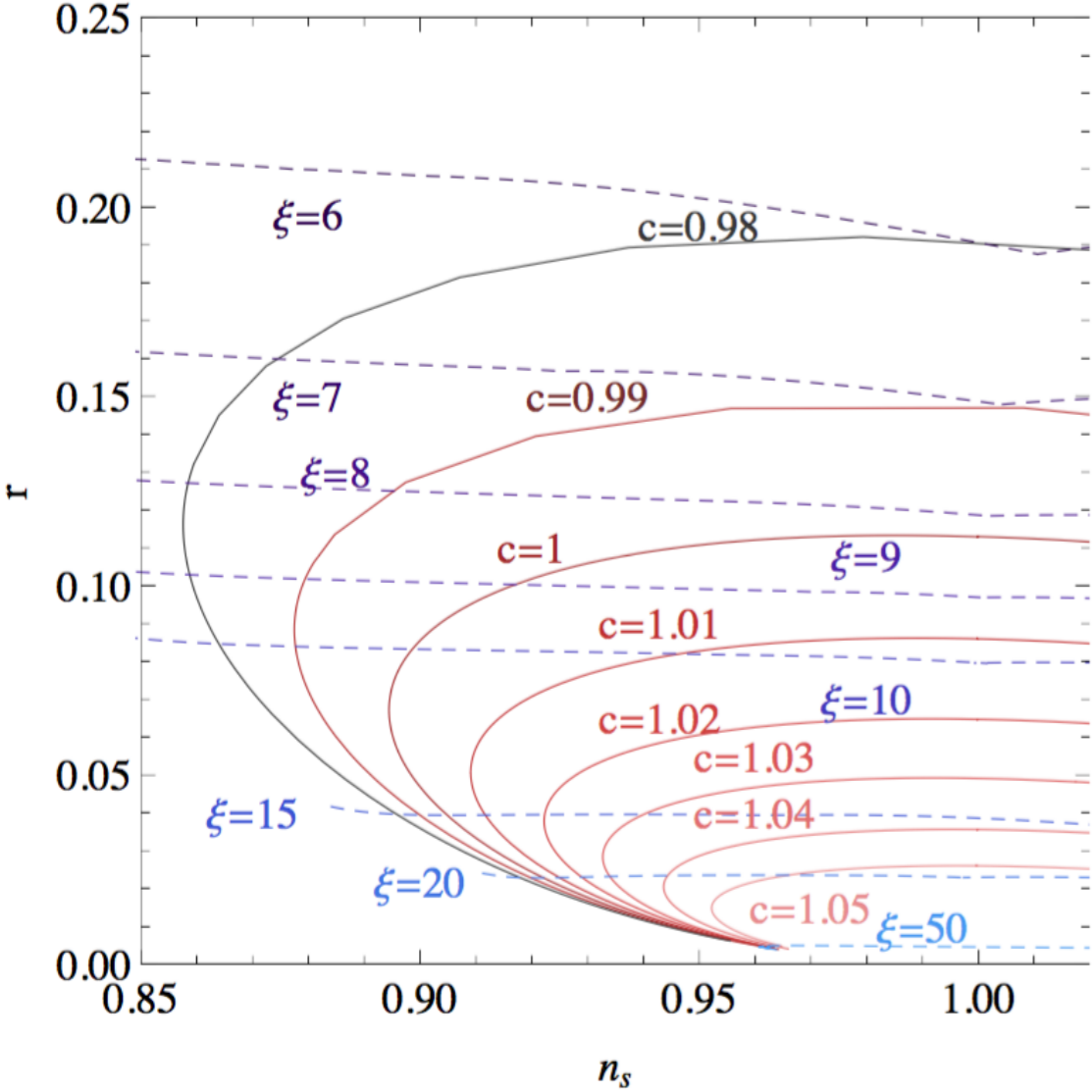


[Hamada, Kawai, Oda, SCP, PRL 2014]

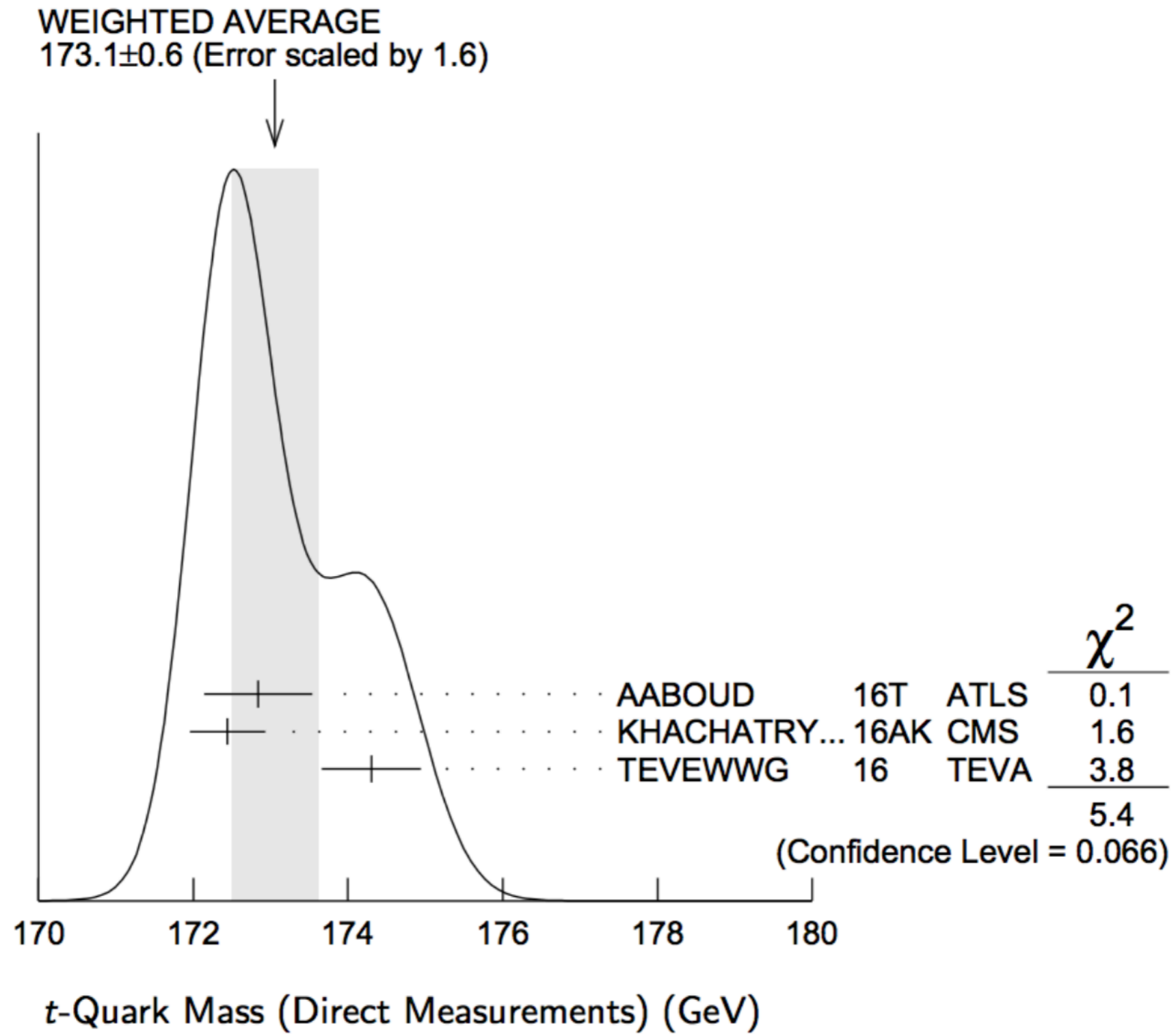
with RG effect

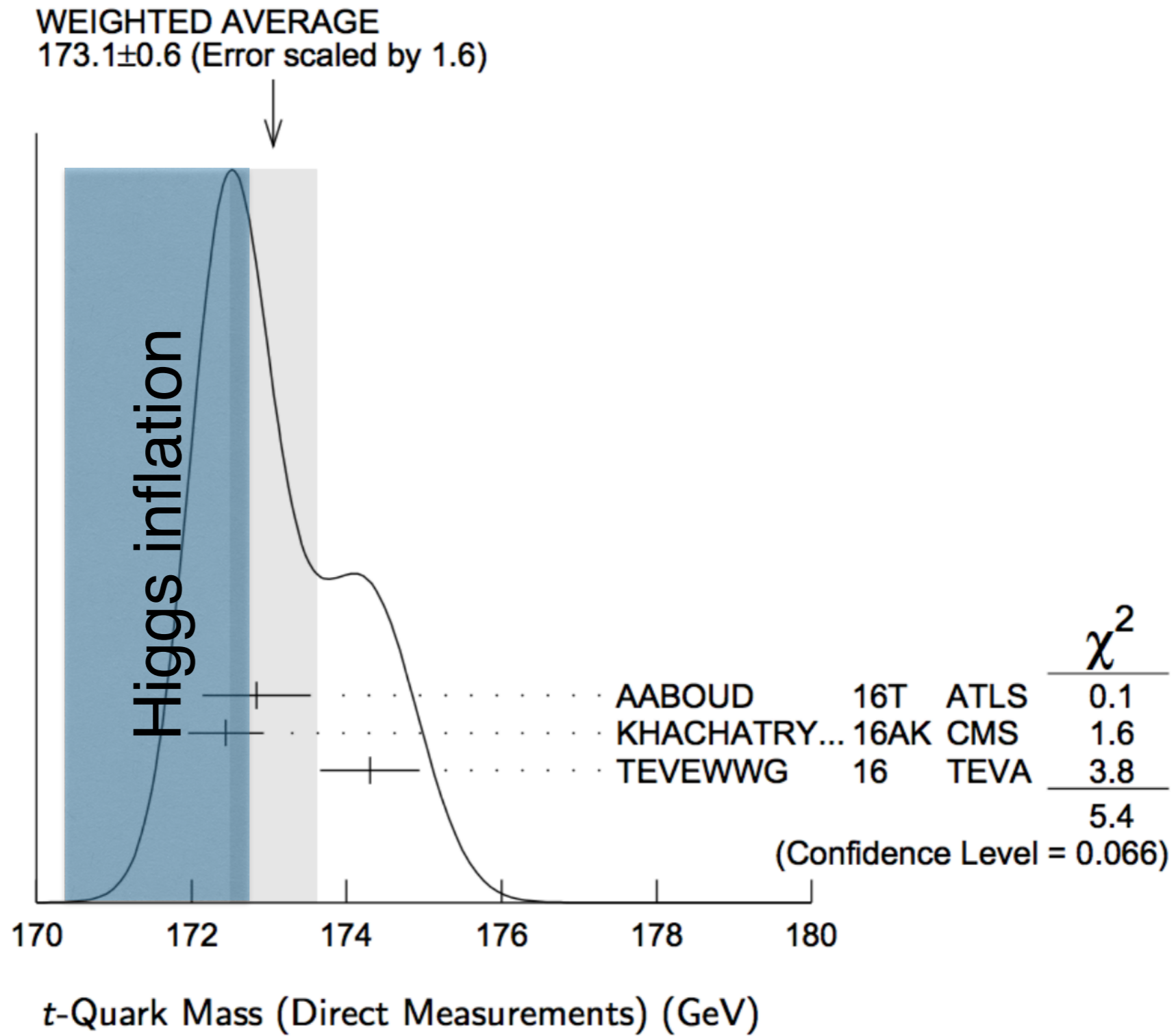
[Hamada, Kawai, Oda, SCP, PRL 2014]

[Hamada, Kawai, Oda, SCP, PRD2015]



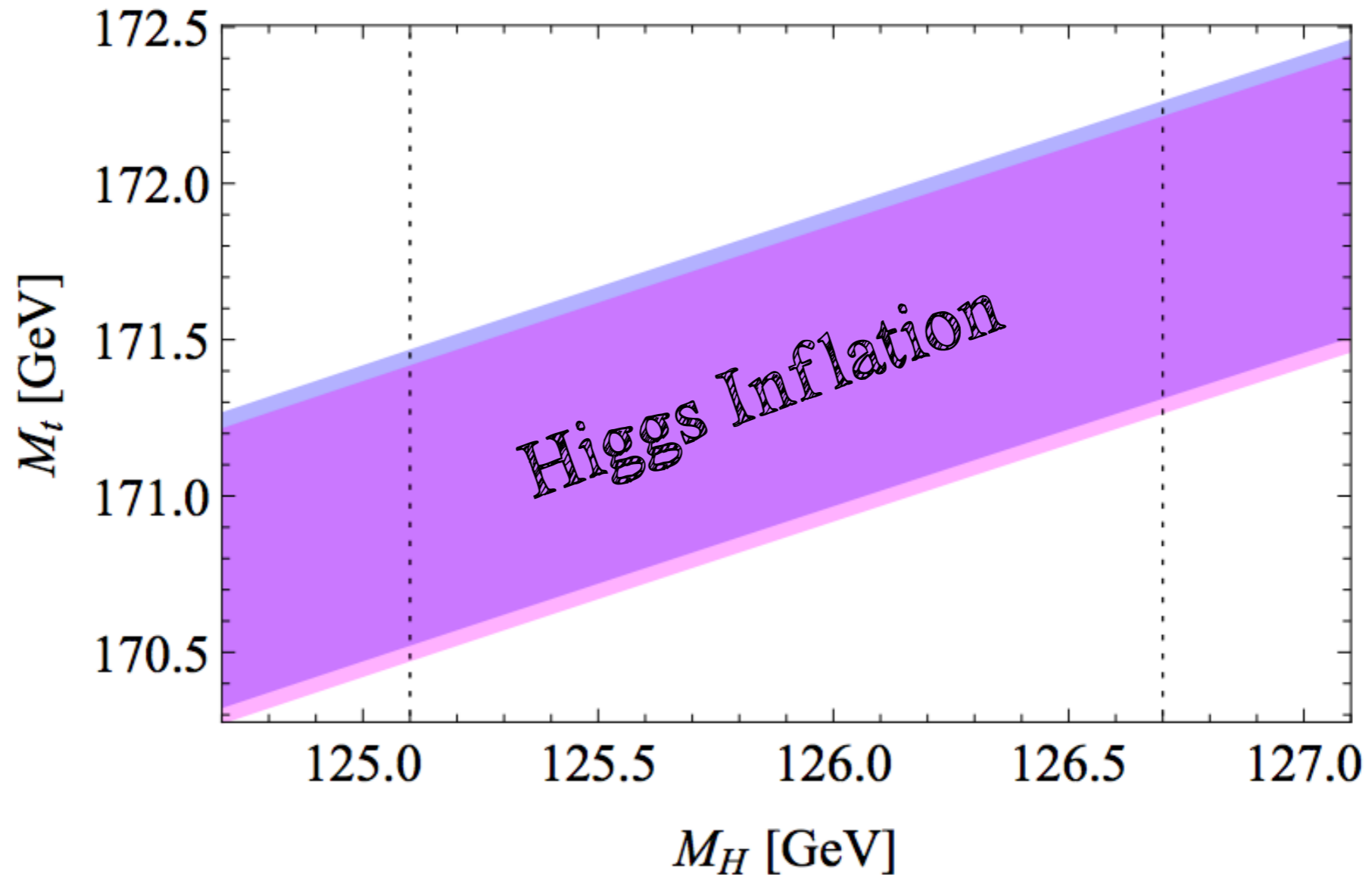
$\xi \sim O(1 - 10)$  This is nice!





$M_t$  &  $M_H$

from Higgs inflation



# Potential Synergy

Cosmological observation

e.g. primordial gravitational wave

=

Precision particle physics experiments

e.g. top quark mass

More discussions



# [topic #1] Cut off

We request  $E \ll \Lambda_{cut-off}$

$$E_{inf} \sim V_E^{1/4} = \frac{\lambda^{1/4}}{\sqrt{\xi}} M_P$$

Q. what's cut-off scale?

# background dependent cut-off scale

Cut-off depends on background  $g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}$   $\phi = \bar{\phi} + \delta\phi$

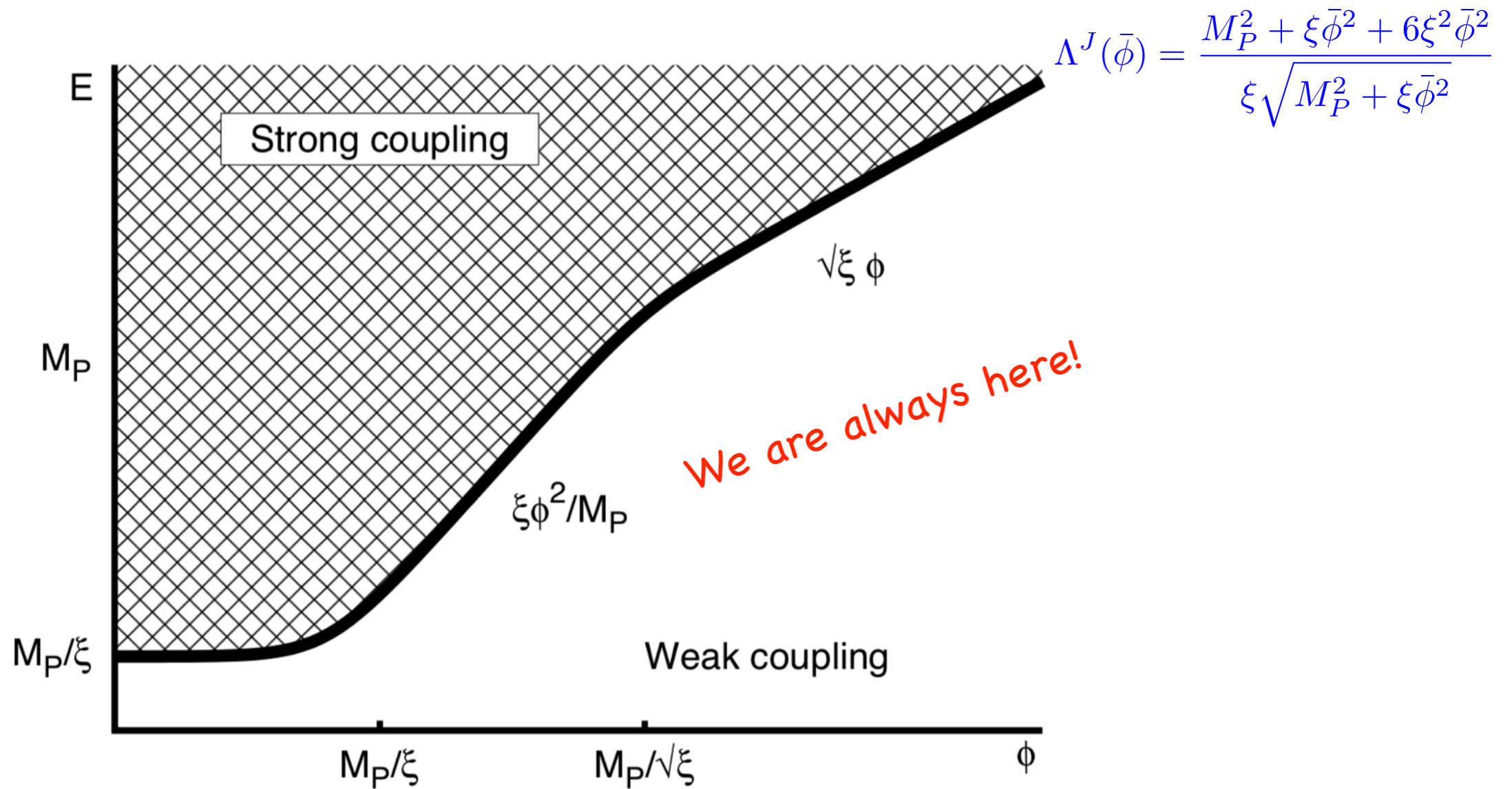
The normalized interaction term reads  $\mathcal{L}_{int} \ni \frac{1}{\Lambda^J} (\delta\hat{\phi})^2 \partial^2 \hat{h}$

thus the cutoff scale in Jordan frame:

$$\Lambda^J(\bar{\phi}) = \frac{M_P^2 + \xi\bar{\phi}^2 + 6\xi^2\bar{\phi}^2}{\xi\sqrt{M_P^2 + \xi\bar{\phi}^2}}$$

$\bar{\phi} \ll M_P/\xi$   $\frac{M_P}{\xi}$  [Burgess, Lee & Trott, 09']  
 [Barbon & Epinosa, 09']  
 $M_P/\xi \ll \bar{\phi} \ll M_P/\sqrt{\xi}$   $\frac{\xi\bar{\phi}^2}{M_P}$   
 $\bar{\phi} \gg M_P/\sqrt{\xi}$   $\sqrt{\xi}\bar{\phi}$

[Bezrukov, Magnin, Shaposhnikov & Sibiryakov, 10']



**Conclusion:**  
**Higgs inflation is consistent and stays perturbative in entire process!**

# [topic #2] non-minimal coupling

$$\frac{\xi}{2} H^\dagger H R$$

-same dimension of the Einstein term  
-allowed by all symmetries of the SM

*therefore* you are not allowed to drop this term without a reason

Dim 4 for Ricci

**Wait!**

$$R \sim (\partial_\rho h_{\mu\nu})^2 / M^2$$

Kinetic term of graviton

Thus, NM term  $\sim$

$$\phi^2 (\partial_\rho h_{\mu\nu})^2$$

Dim 6 for graviton dynamics

(sometimes, we should not forget *other Dim 6 terms, too*)

# General Lagrangian

$$S = \int d^4x \sqrt{-g} \left[ \frac{M_P^2}{2} A(\varphi) \mathcal{R} - \frac{1}{2} B(\varphi) g^{\mu\nu} (\partial_\mu \varphi \partial_\nu \varphi + A_\mu A_\nu \varphi^2) - V(\varphi) - C(\varphi) \bar{\psi} \gamma^\mu D_\mu \psi - \frac{y}{\sqrt{2}} D(\varphi) (\varphi \bar{\psi} \psi + \text{h.c.}) - \frac{E(\varphi)}{4g_A^2} F_{\mu\nu} F^{\mu\nu} \right],$$

$$A(\varphi) = 1 + a_2 \frac{\varphi^2}{M_P^2} + a_4 \frac{\varphi^4}{M_P^4} + \dots, \quad B(\varphi) = 1 + b_2 \frac{\varphi^2}{M_P^2} + b_4 \frac{\varphi^4}{M_P^4} + \dots,$$

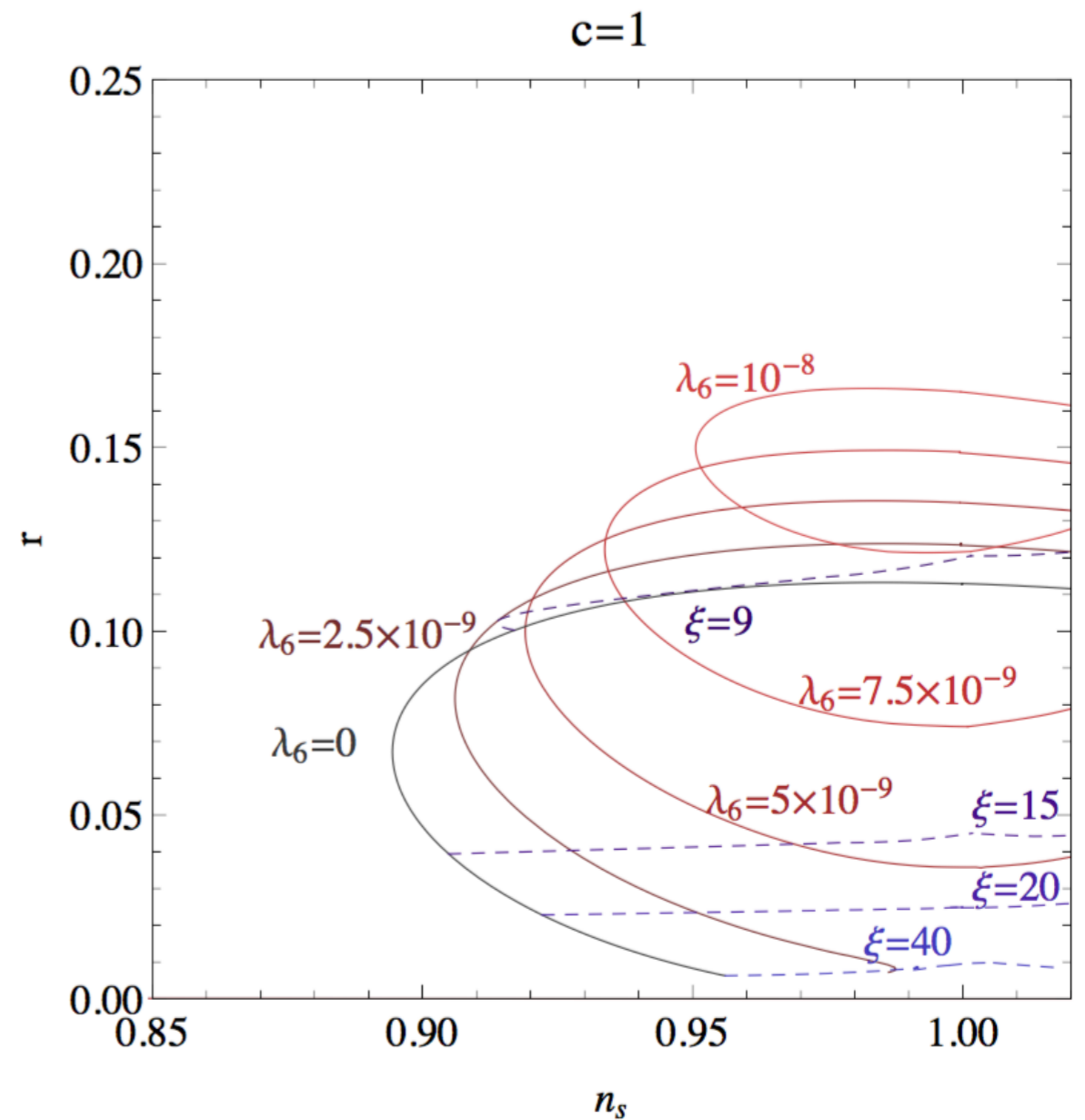
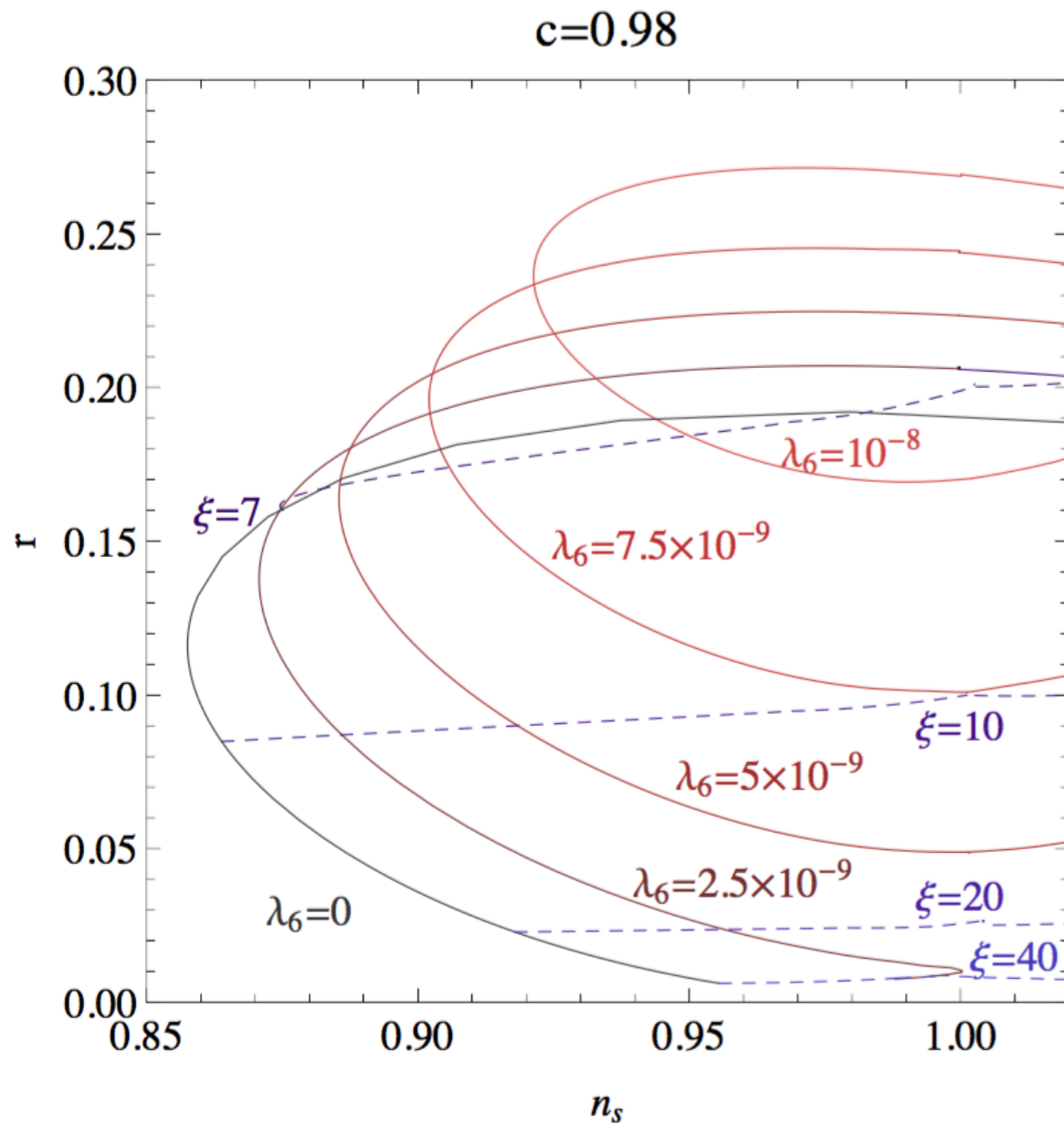
$$V(\varphi) = \frac{m^2}{2} \varphi^2 + \frac{\lambda}{4} \varphi^4 + \left( \lambda_6 \frac{\varphi^6}{M_P^2} + \lambda_8 \frac{\varphi^8}{M_P^4} + \dots \right).$$

[Hamada, Kawai, Oda, SCP, PRD2015]

“higher order terms”

$$V(\varphi) = \frac{m^2}{2}\varphi^2 + \frac{\lambda}{4}\varphi^4 + \left( \lambda_6 \frac{\varphi^6}{M_P^2} + \lambda_8 \frac{\varphi^8}{M_P^4} + \dots \right)$$

$$\mu_{\min} = c \frac{M_p}{\sqrt{\xi}}$$

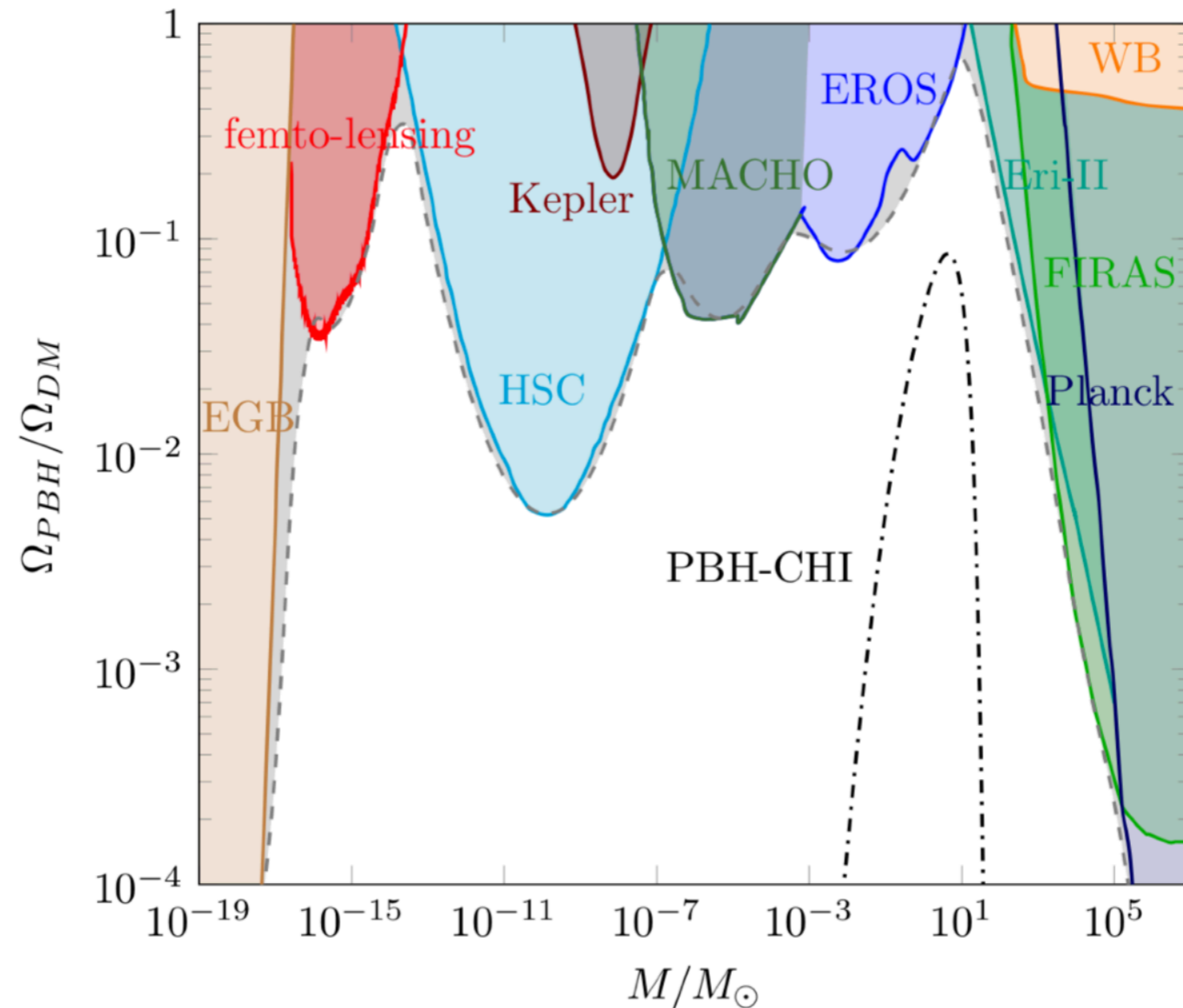


[Hamada, Kawai, Oda, SCP, PRD2015]

# [Topic #3]

## PBH in Higgs inflation

J. M. Ezquiaga, J. Garcia-Bellido and E. Ruiz Morales,  
"Primordial Black Hole production in Critical Higgs Inflation,"  
PLB (776), 345 (2018)



# Finally, take home messages

- Higgs inflation idea is an economic, effective and consistent idea of inflation fitting DATA well.
- Precision measurements of  $M_t$ ,  $M_H$ , self coupling are all very important.



# Higgs-f(R) duality

During the inflationary era, two theories effectively describe the same physics

$$S_{\text{Starobinsky}} \leftrightarrow S_{\text{Higgs}}$$

**We can enlarge the map in theory space**

# sketch of idea

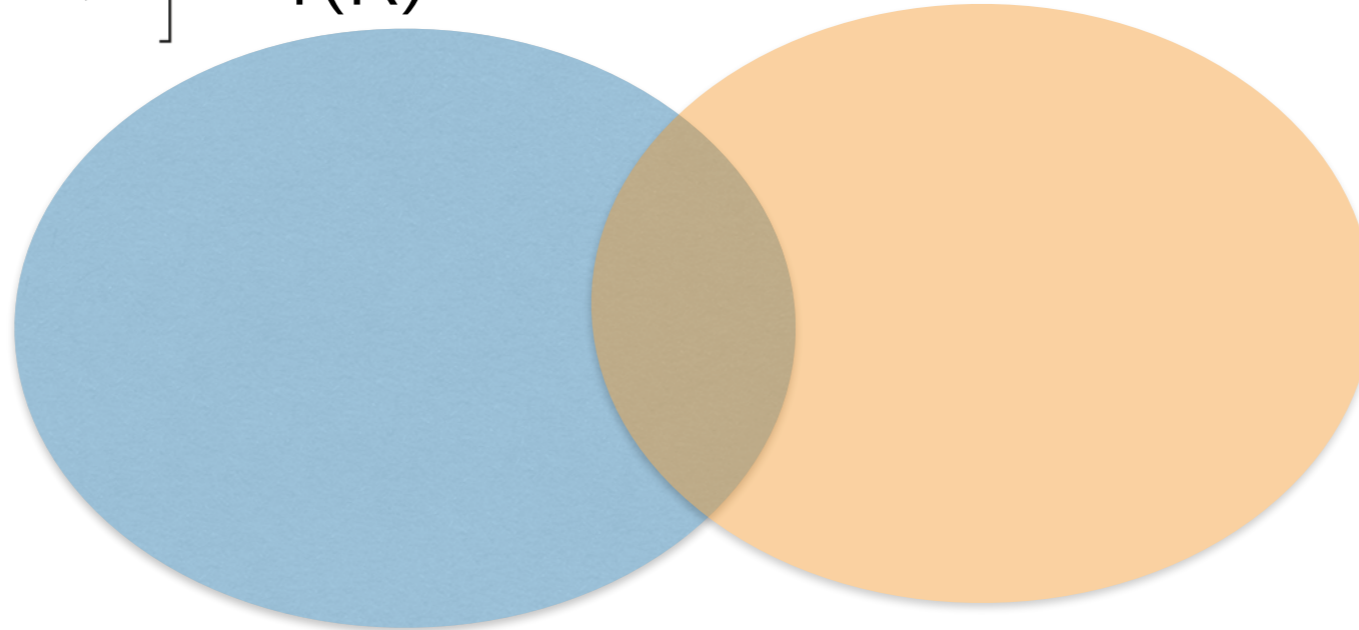
Starobinsky

$$S = \int d^4x \sqrt{-g} \left[ f(R) - \sum_i \xi_i(R) Q_i + \beta C^2 \right]$$

f(R)

Higgs

$$S = \int d^4x \sqrt{-g} \left[ \frac{M^2}{2} F(\Phi) R - \frac{1}{2} B(\Phi) g^{\mu\nu} (\partial_\mu \Phi \partial_\nu \Phi + A_\mu A_\nu \Phi^2) \right]$$



Matching  
@Einstein frame

[Gumrukcuoglu, Mukohyama, SCP (in preparation)]