Clockwork and its continuum limit

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CosPA 2017, Dec. 11, Kyoto

KC, S.H. Im, C.S. Shin, arXiv:1711.06228

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Institute for Basic Science

Outline

- Introduction

  Clockwork (CW) mechanism for

  * hierarchical couplings

  * exponentially enlarged field range

  * multiple number of nearly degenerate massive states at the threshold scale

- Continuum clockwork (CCW)

  Extra-dimensional realization of the (partial) CW mechanism

- Conclusion

Clockwork axions

Choi, Kim, Yun 14; Choi, Im 15; Kaplan, Rattazzi 15

Proposed to explain the hierarchical axion couplings (scales) required in the cosmological scenarios involving a rolling axion field:

$$f_{\rm eff} \sim \text{axion field excursion}$$

* Natural inflation: *Freese, Frieman, Olinto 90*

Instanton effect suggested by the weak gravity conjecture

$$V = \Lambda_{\rm inf}^4 \cos\left(\frac{\phi}{f_{\rm eff}}\right) + \Lambda_{\rm WG}^4 \cos\left(\frac{\phi}{f}\right) + \ldots$$

$$f_{\rm eff} \gtrsim \sqrt{N_e}\, M_{\rm Pl}, \quad f \lesssim \frac{M_{\rm Pl}}{S_{\rm ins}} \quad \rightarrow \quad \frac{f_{\rm eff}}{f} \gtrsim S_{\rm ins}\sqrt{N_e} \gtrsim 10^2$$

* Cosmological relaxation of the weak scale: *Graham, Kaplan, Rajendran 15*

Higgs-dependent barrier potential to stop the relaxion

$$V = \Lambda_{\rm Higgs}^4 \cos\left(\frac{\phi}{f_{\rm eff}}\right) + \mu^2|H|^2 \cos\left(\frac{\phi}{f}\right) + \ldots$$

$$\frac{\Lambda_{\rm Higgs}^4}{f_{\rm eff}} \sim \frac{\mu^2|H|^2}{f} \lesssim \frac{({\rm TeV})^4}{f} \quad \rightarrow \quad \frac{f_{\rm eff}}{f} \gtrsim \left(\frac{\Lambda_{\rm Higgs}}{\rm TeV}\right)^4$$

* ALP-driven magnetogenesis: *Anber, Sorbo 06*

ALP coupling for magnetogenesis

$$\mathcal{L}_{\rm ALP} = \Lambda_{\rm inf}^4 \cos\left(\frac{\phi}{f_{\rm eff}}\right) + \frac{\alpha_{\rm em}}{4\pi}\frac{\phi}{f}F^{\mu\nu}\tilde{F}_{\mu\nu}$$

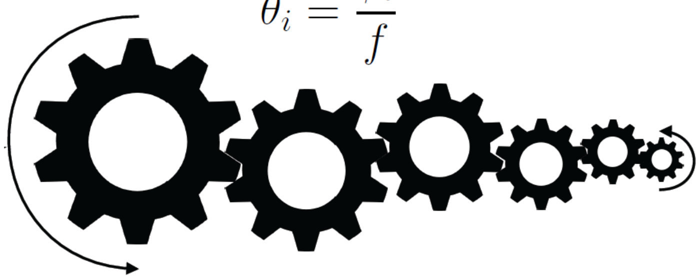
➔ $$\frac{f_{\rm eff}}{f} \gtrsim 10\frac{4\pi}{\alpha_{\rm em}} \sim 10^4$$

See the next talk by Tanmay Vachaspati
& the tomorrow parallel session talk by T. Sekiguchi

Clockwork axions

Choi, Kim, Yun 14; Choi, Im 15; Kaplan, Rattazzi 15

$$\theta_i = \frac{\phi_i}{f}$$



[Illustration from Giudice, McCullough]

$$q = \frac{\text{number of cogs of the } i\text{-th gear}}{\text{number of cogs of the } (i+1)\text{-th gear}} > 1$$

$$\mathcal{L} = -\frac{1}{2} f^2 \left[\sum_{i=0}^{N} (\partial_\mu \theta_i)^2 - 2 \sum_{i=0}^{N-1} m^2 \cos\left(\theta_{i+1} - q\theta_i\right) \right]$$

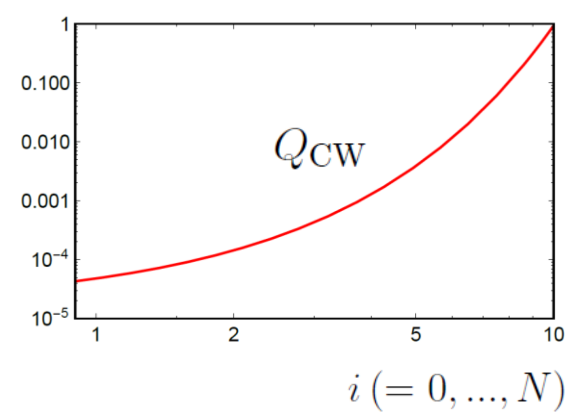
Symmetry breaking which results in an unbroken $U(1)_{CW}$ which is localized in the theory space, and has an exponentially enlarged (by q^N) range of the symmetry transformation:

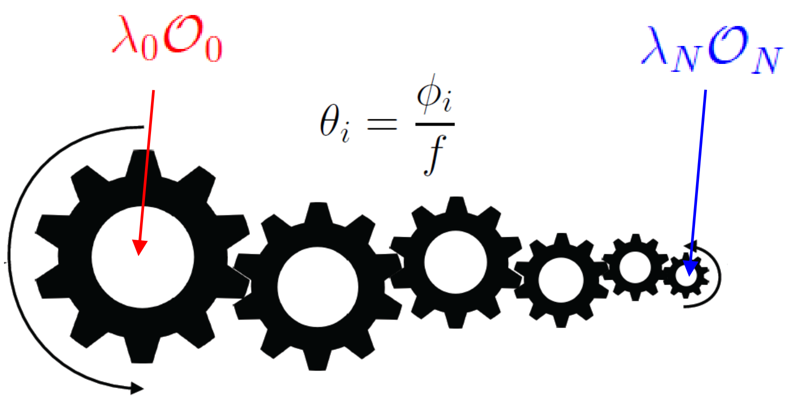
$$U(1)^{N+1} = \prod_i U(1)_i \quad \rightarrow \quad U(1)_{\rm CW}$$

$$\left[U(1)_i = e^{i\alpha_i Q_i} : \ \theta_i \rightarrow \theta_i + \alpha_i \right]$$

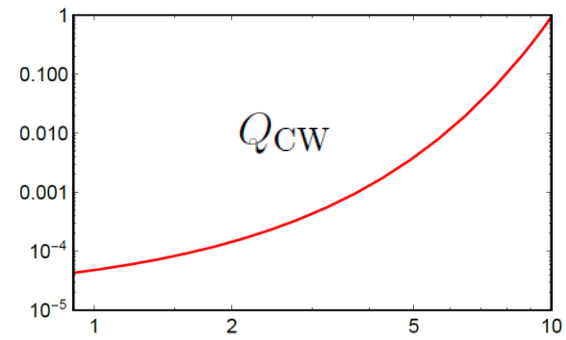
$$Q_{\rm CW} \propto \sum_{i=0}^{N} q^i Q_i$$

Localized distribution of unbroken symmetry (light axion) in the theory space





Localized (compact) $U(1)_{CW}$



* Physics associated with small symmetry transformation:

Localized (in theory space) light mode $\delta\theta_{\rm CW}$ (= axion, U(1) gauge boson, chiral fermion, ..) protected by $U(1)_{CW}$, which has hierarchical low energy couplings:

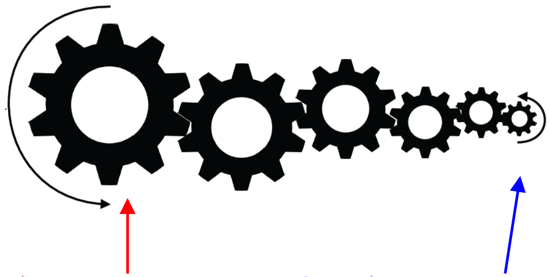
$$\mathcal{L}_{\rm UV} = \lambda_0\theta_0\mathcal{O}_0 + \lambda_N\theta_N\mathcal{O}_N \quad \rightarrow \quad \mathcal{L}_{\rm eff} = \delta\theta_{\rm CW}\left(\frac{\lambda_0}{q^N}\mathcal{O}_0 + \lambda_N\mathcal{O}_N\right)$$

* Physics associated with large symmetry transformation:

- Exponentially enlarged field range of axion (Nambu-Goldstone)
- Exponential hierarchy among the conserved (quantized) charge (Wigner-Weyl)

Light (canonically normalized) zero mode axion $a^{(0)}$ with

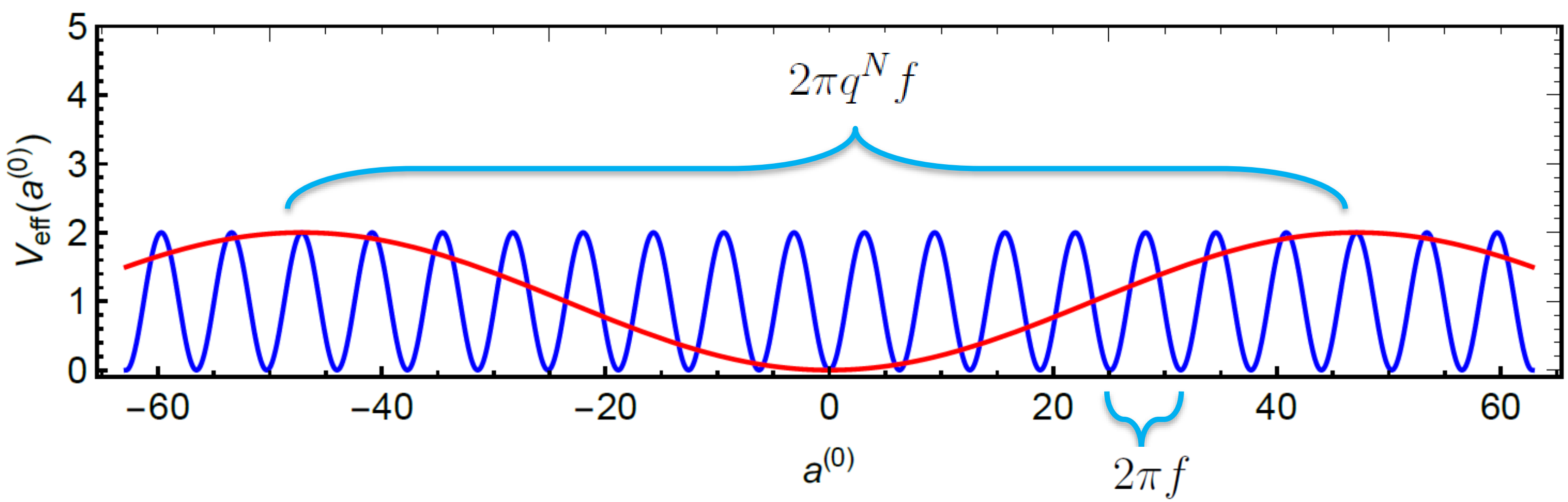
* exponentially enlarged field range (= axion periodicity)
* hierarchical couplings



$$\rightarrow \quad \Delta a^{(0)} \sim 2\pi q^N f \quad (2\pi f \equiv \Delta\phi_i)$$

$$\frac{1}{32\pi^2}\frac{\phi_0}{f}G_0^{\mu\nu}\tilde{G}_{0\mu\nu} + \frac{1}{32\pi^2}\frac{\phi_N}{f}G_N^{\mu\nu}\tilde{G}_{N\mu\nu} = \frac{a^{(0)}}{32\pi^2 f}\left(\frac{1}{q^N}G_0^{\mu\nu}\tilde{G}_{0\mu\nu} + G_N^{\mu\nu}\tilde{G}_{N\mu\nu}\right) + \ldots$$

Zero mode axion potential induced by two independent YM instantons

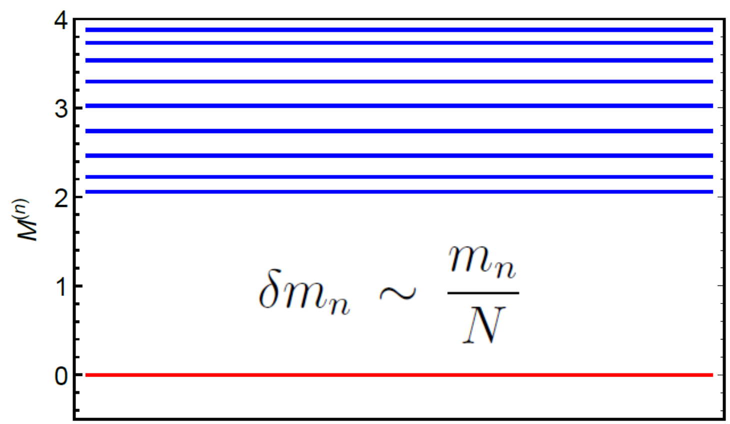


Multiple number of massive modes (N >> 1) with

* approximately degenerate masses
* unlocalized distribution in theory space

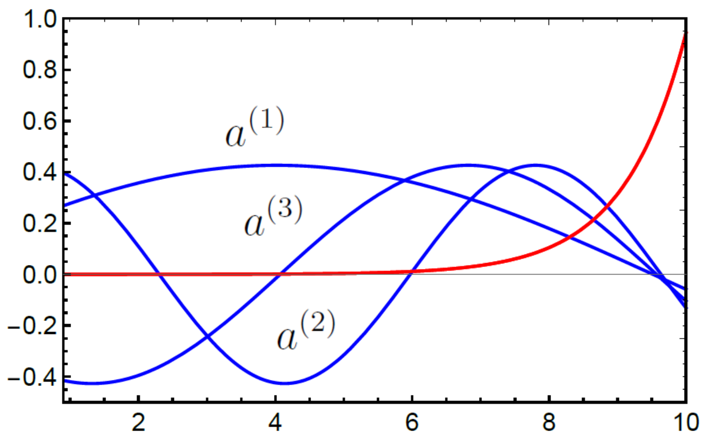
➔ Hierarchy between zero mode and massive mode couplings:

$$\mathcal{L}_{\rm UV} = \lambda_0 \theta_0 \mathcal{O}_0 \quad \rightarrow \quad \mathcal{L}_{\rm eff} = \lambda_0 \mathcal{O}_0 \left(\frac{\delta\theta_{\rm CW}}{q^N} + \delta\theta_{\rm massive} \right)$$



Mass spectrum

(*N = O(10)*, and there is no *O(10)* or bigger hierarchy in the UV parameters)



Distribution of mass eigenstates in the theory space

CW photons: Saraswat 16; Giudice, McCullough 16; Lee 17

$$\mathcal{L} = -\frac{1}{4g^2}\sum_{i=0}^{N} F_{\mu\nu i}F_i^{\mu\nu} - \frac{1}{2}m^2\sum_{i=0}^{N-1}\left[\partial_\mu\varphi_i - (A_{\mu i+1} - qA_{\mu i})\right]^2$$

Milli charged particle with exponentially small charge $Q_{\text{eff}} = Q/q^N$, which would avoid the naive WGC bounds $Q_{\text{eff}} > m/M_{\text{Planck}}$ and $\Lambda \lesssim Q_{\text{eff}} M_{\text{Planck}}$.

(Quantized gauge charges of the zero mode photon)

Other applications:

QCD axion, inflaton,

neutrino, flavor, WIMP,

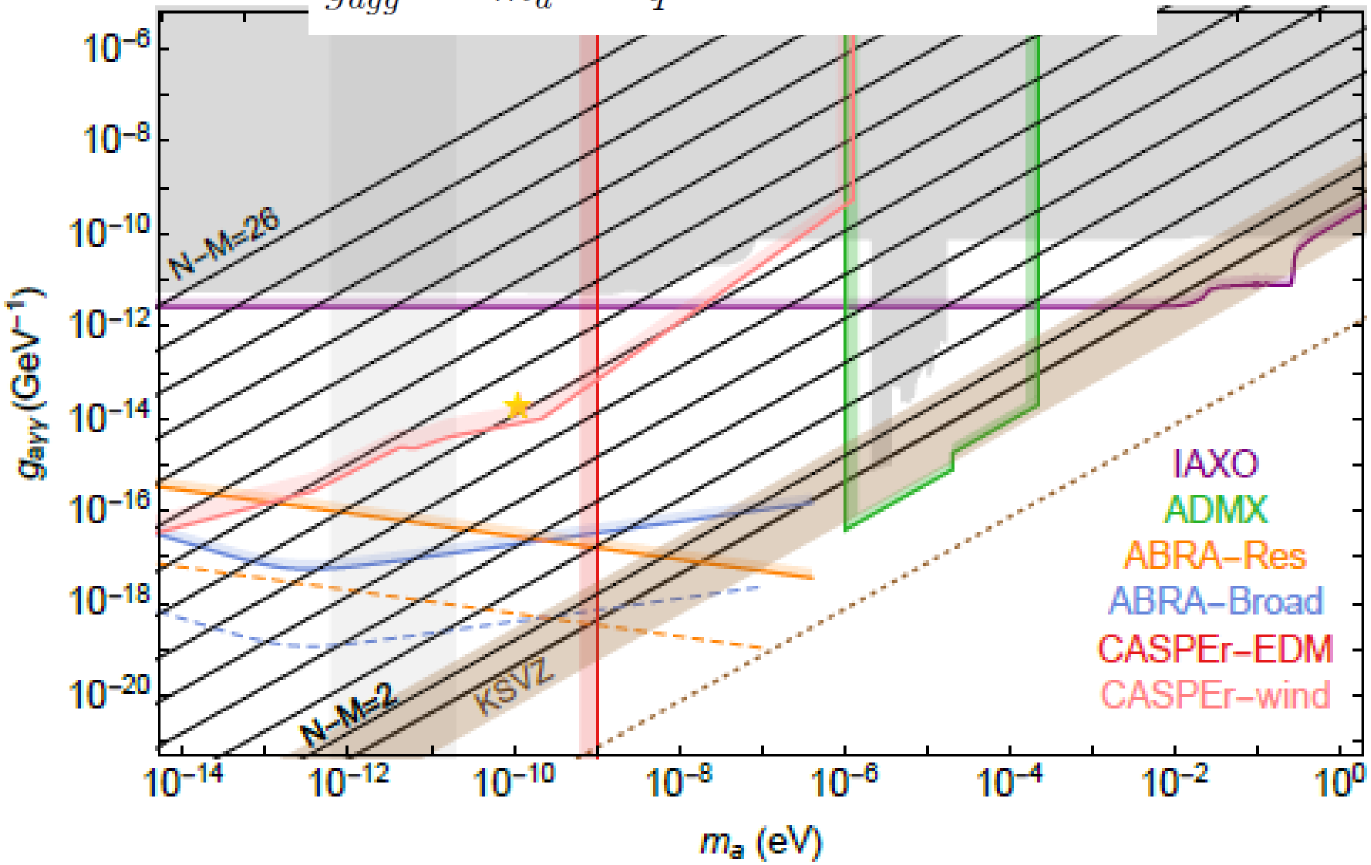
....

Higaki, Jeong, Kitajima, Takahashi 14,15;
Giudice, McCullough 16
Kehagias, Riotto 16
Farina, Pappadopulo, Rompineve, Tesi 16
Hambye, Teresi, Tytgat 16; Kim, McDonald 17;
Ahmed, Dillon 16
Coy, Frigerio, Ibe 17
Ben-Dayan 17
Im, Nilles, Trautner 17
Park, Shin 17
Gersdorff 17; Ibarra, Kushwaha, Vempati 17; Patel 17
Agrawal, Margues-Tavares, Xue, 17
Agrawal, Fan, Reece, Wang, 17; ...

Clockwork QCD axion:

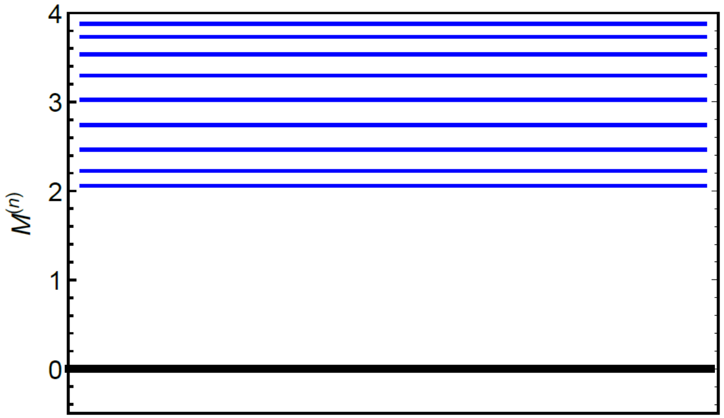
Higaki, Jeong, Kitajima, Takahashi 14,15
Farina, Pappadopulo, Rompineve, Tesi 16
Agrawal, Margues-Tavares, Xue, 17
Agrawal, Fan, Reece, Wang, 17

$$\frac{g_{a\gamma\gamma}}{g_{agg}} \propto \frac{g_{a\gamma}}{m_a} \propto \frac{q^{-M}}{q^{-N}} = q^{N-M} \quad (q = 3)$$



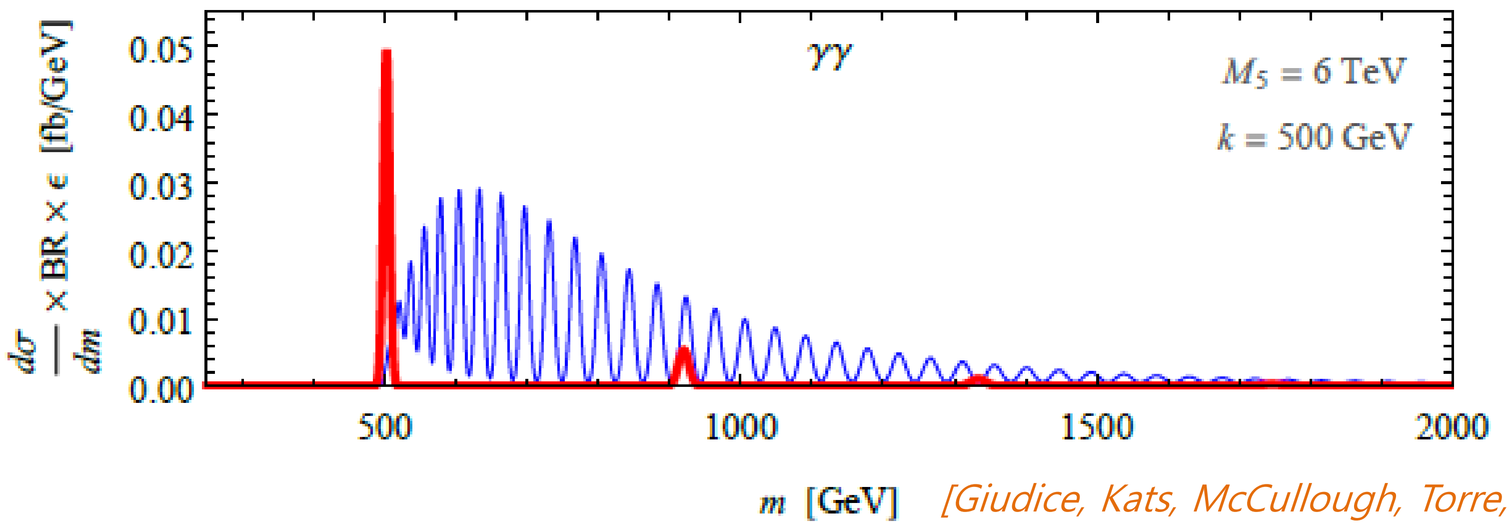
[Farina, Pappadopulo, Rimpineve, Tesi]

CW at Colliders:



Mass spectrum

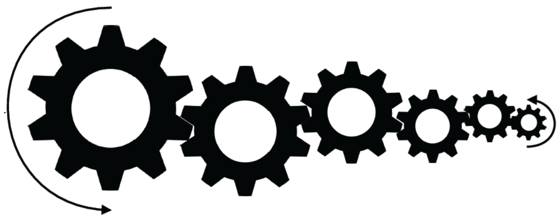
CW vs RS in di-photon invariant mass distribution



[Giudice, Kats, McCullough, Torre, Urbano]

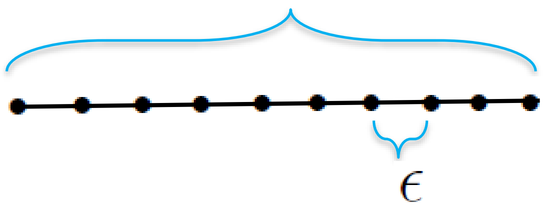
Continuum Clockwork (CCW)

Discrete theory space



$$\mathcal{L} = -\frac{1}{2} f^2 \left[\sum_{i=0}^{N} (\partial_\mu \theta_i)^2 - 2 \sum_{i=0}^{N-1} m^2 \cos\left(\theta_{i+1} - q\theta_i \right) \right]$$

Continuous extra spatial dimension



$$N = \frac{\pi R}{\epsilon}, \quad q = 1 + \mu\epsilon$$

Take $\epsilon \to 0$, while keeping R and μ finite:

$$q^N \quad \Rightarrow \quad e^{\mu\pi R}$$ (q is not a rational number anymore.)

$$\theta_{i+1} - q\theta_i = (\theta_{i+1} - \theta_i) - (q-1)\theta_i \quad \Rightarrow \quad \epsilon\left(\partial_y \theta(x,y) - \mu\theta(x,y) \right)$$

$$\sum \cos\left(\theta_{i+1} - q\theta_i \right) \quad \Rightarrow \quad \int dy \left(\partial_y \theta(x,y) - \mu\theta(x,y) \right)^2$$

In the continuum limit,

$$q^N \;\Rightarrow\; e^{\mu\pi R}$$

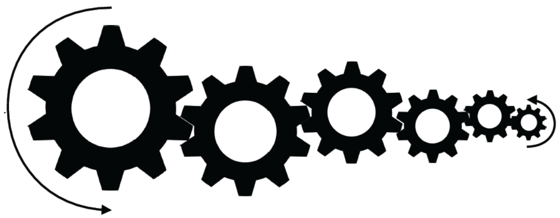
$$\sum \cos\left(\theta_{i+1} - q\theta_i\right) \;\Rightarrow\; \int dy \left(\partial_y\theta(x,y) - \mu\theta(x,y)\right)^2$$

* Large field (or large symmetry transformation) behavior of the original model is lost. (q is not a rational number anymore, and one can not get the original CW model by the dimensional deconstruction of CCW.)

* Unbroken CW symmetry becomes background-geometry-dependent, therefore the CW symmetry to protect light mode might not be a genuine symmetry, but broken for instance by 5D gravitational interactions:

$$Q_{\rm CW} \propto Q_0 + qQ_1 + \ldots + q^N Q_N \quad\Rightarrow\quad Q_{\rm CW}(R)$$

Two different approaches for CCW



$$\mathcal{L} = -\frac{1}{2}f^2\left[\sum_{i=0}^{N}(\partial_\mu\theta_i)^2 - 2\sum_{i=0}^{N-1} m^2\cos\Big(\theta_{i+1} - q\theta_i\Big)\right]$$

continuum limit

$$\Rightarrow \quad \frac{1}{2}f_5^3\int d^5x\left[\Big(\partial_\mu\theta\Big)^2 - \Big(\partial_y\theta - \mu\theta\Big)^2\right]$$

5D spacetime symmetry

$$\Rightarrow \quad ?$$

CCW1: [Giudice, McCullough 16]

$$\frac{1}{2}f_5^3\int d^5x\, e^{2\mu y}\Big((\partial_\mu\tilde{\theta})^2 - (\partial_y\tilde{\theta})^2\Big) = \frac{1}{2}\int d^5x\sqrt{-G}\,G^{MN}\partial_M\tilde{\theta}\partial_N\tilde{\theta}$$

$$\Big(\tilde{\theta} = e^{-\mu y}\theta, \quad ds^2 = G_{MN}dx^M dx^N = e^{4\mu y/3}(\eta_{\mu\nu}dx^\mu dx^\nu + dy^2)\Big)$$

(Redefined 5D axion in linear-dilaton background geometry)

CCW2: [Craig, Garcia, Sutherland 17]

$$\frac{1}{2}f_5^3\int d^5x\,\left((\partial_\mu\theta)^2 - (\partial_y\theta)^2 - \mu^2\theta^2 - \mu(\delta(y) - \delta(y-\pi R))\theta^2\right)$$

(Bulk and boundary masses tuned to be invariant under $\delta\theta = ce^{\mu y}$)

Discrete CW vs CCW1 vs CCW2

KC, Im, Shin, arXiv:1711.06228

Key features of the discrete CW:

1) Unbroken $U(1)_{\rm CW}$ to protect zero mode, which is respected by gravity

2) Hierarchical couplings of zero mode

3) Exponentially enlarged range of $U(1)_{\rm CW}$ (zero mode axion)

4) Approximately degenerate multiple massive modes

5) Hierarchy between zero mode coupling and massive mode coupling

CCW1 can realize only 1), 4) & 5), while CCW2 can realize 2), 4) & 5)

CCW can reproduce only partial features of the discrete CW model. Yet, CCW is interesting enough as it can be combined with the extra-dimensional solution of the hierarchy problem, which would result in interesting phenomenological consequences.

For detailed discussions, see
Giudice, McCullough 16; Craig, Garcia, Sutherland 17;
KC, Im, Shin 17; Giudice, Kats, McCullough, Torre, Urbano 17

Continuum CW axion vs Discrete CW axion

KC, Im, Shin, arXiv:1711.06228

$$\frac{1}{2}f^2\sum_{i=0}^{N}\left(\partial_\mu\theta_i(x)\right)^2+m^2f^2\sum_{i=0}^{N-1}\cos\left(\theta_{i+1}-q\theta_i\right)$$

$$\Rightarrow \quad \frac{1}{2}f_5^3\int d^5x\left[\left(\partial_\mu\theta\right)^2-\left(\partial_y\theta-\mu\theta\right)^2\right]$$

To address the axion field range, we need to start with a periodic 5D axion:

$$\Rightarrow \quad \frac{1}{2}f_5^3\int d^5x\left[\left(\partial_\mu\theta\right)^2-\left(\partial_y\theta-\mu\sin\theta\right)^2\right]$$

$$=\frac{1}{2}f_5^3\int d^5x\left[\partial_M\theta\partial^M\theta+\frac{1}{2}\mu^2\cos 2\theta+2\mu\cos\theta\Big(\delta(y)-\delta(y-\pi R)\Big)\right]$$

CCW2 for periodic 5D axion

With periodic 5D axion, we can study the behavior of the theory over the full range of the zero mode axion, and compare it with the case of discrete CW axions.

Canonically normalized zero mode axion:

$$\frac{a^{(0)}(x)}{f_4} = \left(\frac{e^{2\pi\mu R} - 1}{2\pi\mu R}\right)^{1/2} \int^{\varphi^{(0)}} d\xi \left(1 + \frac{\xi^2}{4}\right)^{-1/2} \left(1 + \frac{\xi^2}{4} e^{2\mu\pi R}\right)^{-1/2}$$

$$\left(\tan\frac{\theta(x,y)}{2} = \frac{1}{2} e^{\mu y} \varphi^{(0)}(x), \quad f_4^2 = \int_0^{\pi R} dy\, f_5^3 = f_5^3 \pi R\right)$$

Field range of the zero mode axion:

$$\Delta a^{(0)} \simeq 2\pi f_4 \times \sqrt{\frac{\pi\mu R}{2}} \qquad \left(\mu R \gtrsim 1\right)$$

Zero mode axion coupling to the boundary YM instantons:

$$\int d^5x\,\theta(x,y)\left(\delta(y)\frac{\kappa_0}{32\pi^2}G_0^{\mu\nu}\tilde{G}_{0\mu\nu}+\delta(y-\pi R)\frac{\kappa_\pi}{32\pi^2}G_\pi^{\mu\nu}\tilde{G}_{\pi\mu\nu}\right)$$

$$\left(\kappa_{0,\pi}=\text{integers of order unity}\right)$$

$$\delta\theta(x,0)=e^{-\mu\pi R}\frac{\delta a^{(0)}}{f_4}+\text{massive fluctuations}$$

$$\delta\theta(x,\pi R)=\frac{\delta a^{(0)}}{f_4}+\text{massive fluctuations}$$

$$\Rightarrow\qquad \frac{e^{-\mu\pi R}\kappa_0}{32\pi^2}\frac{\delta a^{(0)}}{f_4}G_0^{\mu\nu}\tilde{G}_{0\mu\nu}+\frac{\kappa_\pi}{32\pi^2}\frac{\delta a^{(0)}}{f_4}G_\pi^{\mu\nu}\tilde{G}_{\pi\mu\nu}$$

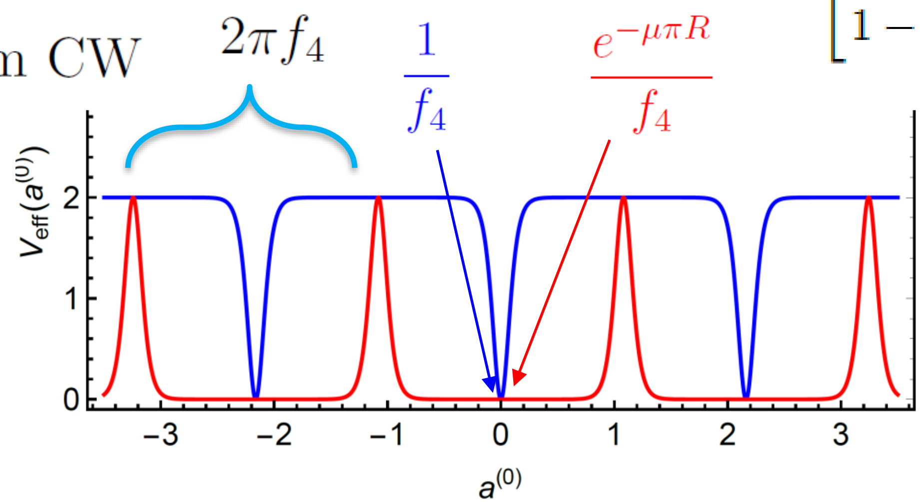
There is no significant enhancement of the axion field range, although there is exponential hierarchy in the couplings of the zero mode axion fluctuation.

Zero mode axion potential (over the full field range) induced by the boundary YM instantons $G_0\tilde{G}_0$ and $G_\pi\tilde{G}_\pi$:

$$\left[\frac{e^{-2m(\pi R-y_b)}\mathrm{sn}^2(\frac{a}{2if}|e^{-2m\pi R})}{1-e^{-2m(\pi R-y_b)}\mathrm{sn}^2(\frac{a}{2if}|e^{-2m\pi R})}\right]$$

$(y_b = 0, \quad y_b = \pi R)$

Continuum CW

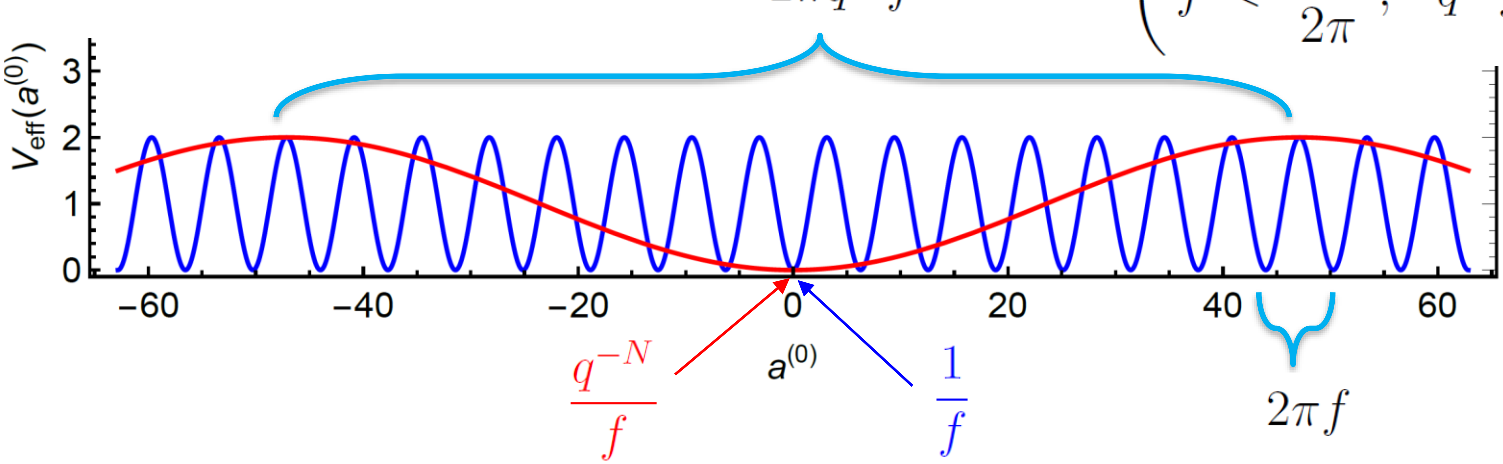


$$\left(\frac{f_4}{M_{\rm Pl}} \sim \left(\frac{f_5}{M_5}\right)^{3/2}\right)$$

Discrete CW

$2\pi q^N f$

$$\left(f < \frac{M_{\rm Pl}}{2\pi}, \quad q^N f \gg M_{\rm Pl}\right)$$



CW symmetry to protect the zero mode

* Discrete CW axion

$$\mathrm{U(1)_{CW}}: \quad \delta\theta_i(x) = c q^i \quad (|c| \ll 1)$$

Localized in theory space, and compatible with all spacetime symmetries and other gauge symmetries

* Continuum CW axion (CCW2)

$$\mathrm{U(1)_{CW}}: \quad \delta(\tan(\theta(y,x)/2)) = c e^{\mu y} \quad (|c| \ll 1)$$

Background-geometry-dependent, so not respected by 5D gravitational interactions

Conclusion

- Clockwork (CW) is a mechanism to generate a localized symmetry in low energy effective theory, which can result in

  * Exponential hierarchy of the couplings of light particle protected by localized symmetry

  * Exponentially enlarged field range

  * Approximate degenerate multiple number of massive states

- Continuum CW is an extra-dimensional realization of the clockwork mechanism, but has certain limitations due to the 5D locality and spacetime symmetries, and therefore can reproduce only partial features of the discrete CW model.

- Yet the continuum CW is interesting as it can connect the CW mechanism with the known extra-dimensional solution of the hierarchy problem, which can have many interesting phenomenological implications.