Generalized spatially covariant gravity

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Based on:

- ***XG,** Phys.Rev. D **90** (2014) 081501(R), [arXiv: 1406.0822]*
- ***XG**, Phys.Rev. D **90** (2014) 104033, [arXiv: 1409.6708]*
- ***XG** and **Zhi-bang Yao**, work in progress*

k-essence, Horndeski and beyond

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1915 • GR

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Introducing second derivatives?

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2011 Generalized galileon

*[Deffayet, **XG**, Steer & Zahariade*
***Phys.Rev. D** 84 (2011) 064039]*

In *D*=4:

$$\begin{aligned}\mathcal{L} = {} & G_0(\phi, X) + G_1(\phi, X)\Box\phi \\ & + G_2(\phi, X)\, R + \frac{\partial G_2}{\partial X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] \\ & + G_3(\phi, X)\, G^{\mu\nu}\nabla_\mu\nabla_\nu\phi \\ & - \frac{1}{6}\frac{\partial G_3}{\partial X}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]\end{aligned}$$

with $X \equiv -\frac{1}{2}(\partial\phi)^2$

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Generalized galileon/Horndeski theory:

- The most general theory for a scalar field coupled to gravity, of which the Lagrangian/EoMs involve up to the second derivatives of the scalar field and the metric.
- Propagates 1 scalar + 2 tensor dofs.

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*Even **beyond galileon/Horndeski** theory?*

- Higher derivatives of the scalar field and the metric.
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We need some alternative approach.

Spatially covariant gravity

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2 tensor + 1 scalar

Spatially covariant gravity

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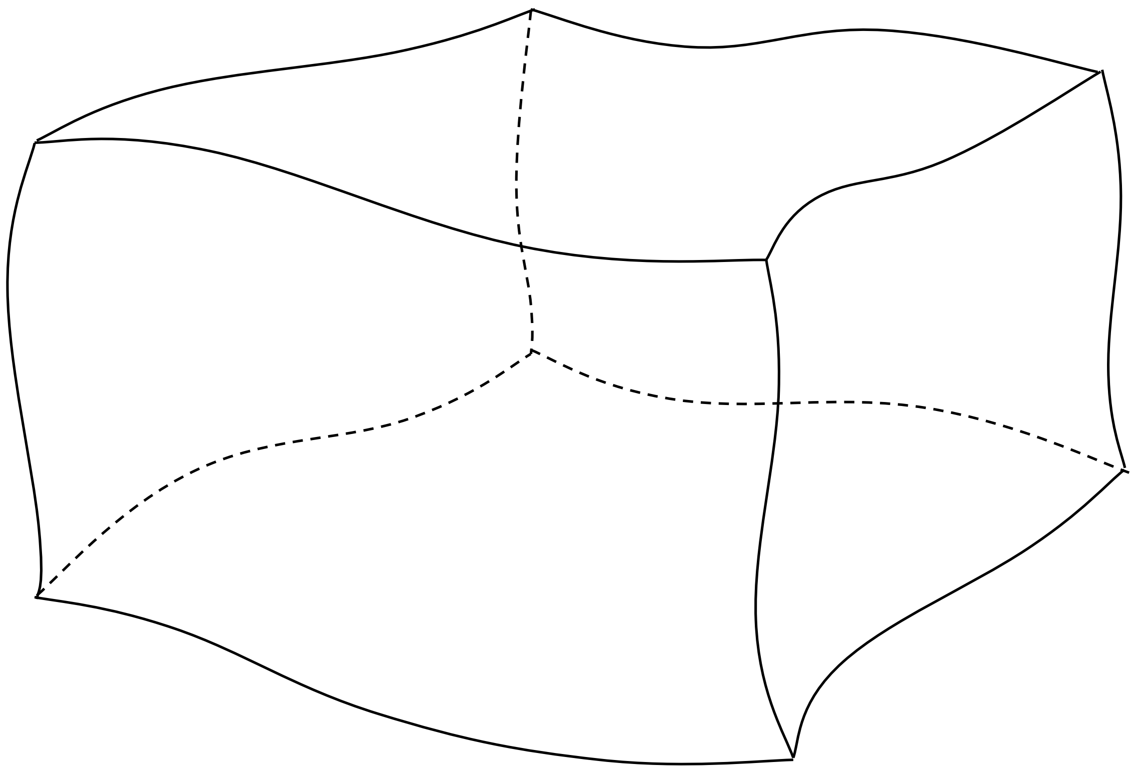
Breaking U(1) →	Massive vector (2 vector + 1 scalar)
Breaking whole spacetime diff →	Massive gravity (2 tensor + 2 vector + 1 scalar)
Breaking time diff, respecting only spatial symmetries →	2 tensor + 1 scalar

Foliation of spacetime



Spacetime covariant

4-D quantities

$(g_{\mu\nu}, R_{\mu\nu\rho\sigma}, \nabla_\mu)$

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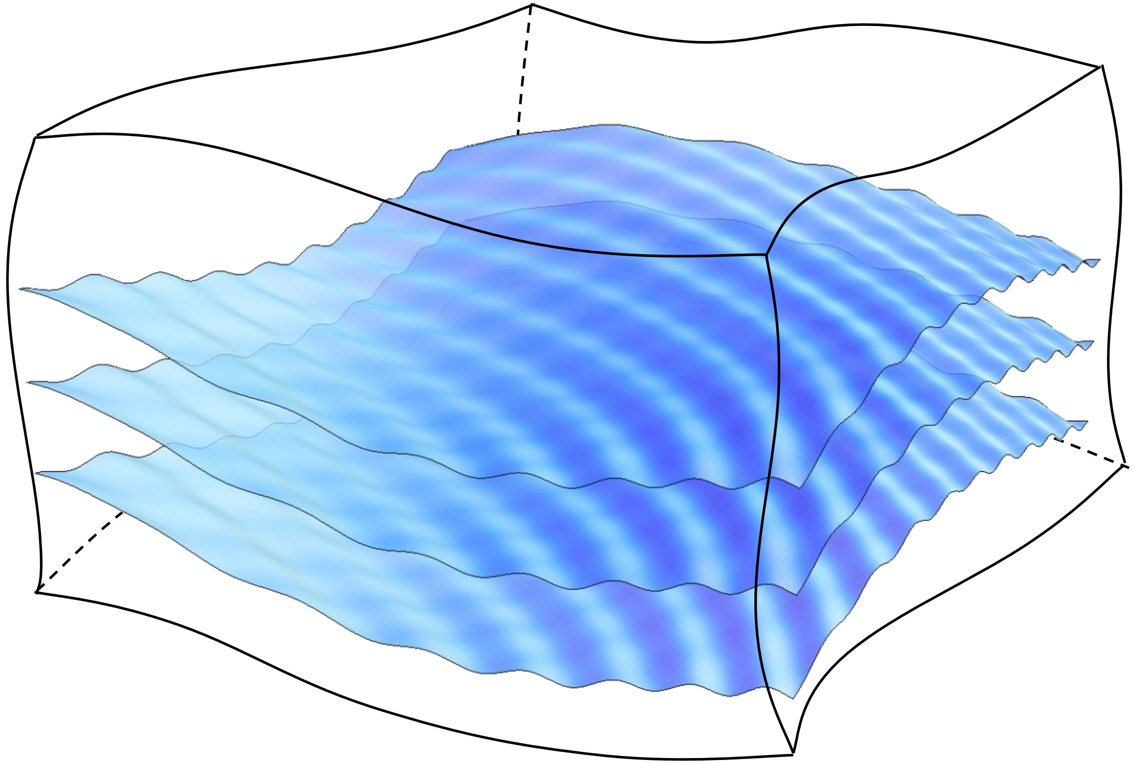

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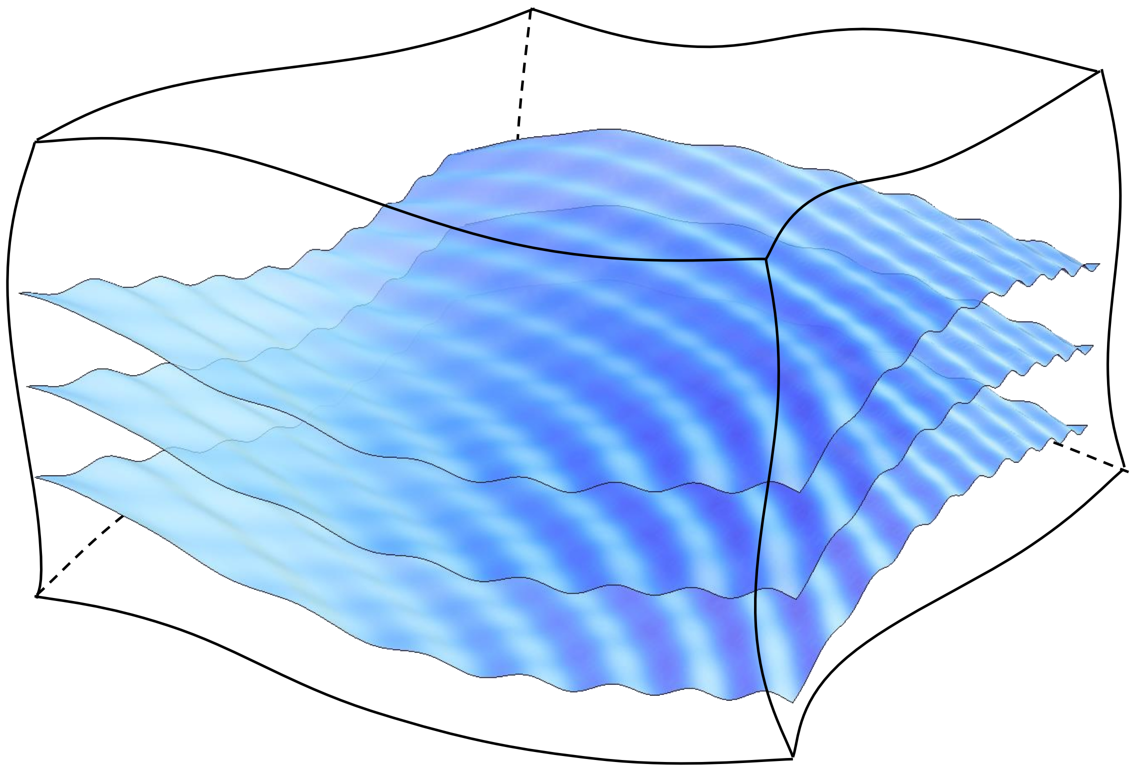

Spacetime covariant

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→

Spatially covariant

3-D quantities

$(t, N, h_{ij}, R_{ij}, \nabla_i) \quad K_{ij}$

Examples of spatially covariant theories

2004 Ghost condensation

[Arkani-Hamed, Cheng, Luty & Mukohyama]

2007 Effective field theory of inflation

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The first explicit example of scalar-tensor theories beyond Horndeski.

Two equivalent languages

Spacetime covariant
Scalar-tensor theories

$$\mathcal{L}\left(g_{\mu\nu}, R_{\mu\nu\rho\sigma}, \phi, \nabla_{\mu}\right)$$

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Spatially covariant
gravity theories

$$\mathcal{L}\left(t, N, h_{ij}, R_{ij}, K_{ij}, \nabla_{i}\right)$$

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Gauge recovering: $t \to \phi\left(t, \vec{x}\right)$
(Stueckelberg trick)

Two equivalent languages

Gauge fixing: $\phi(t, \vec{x}) \to t$
(unitary gauge)

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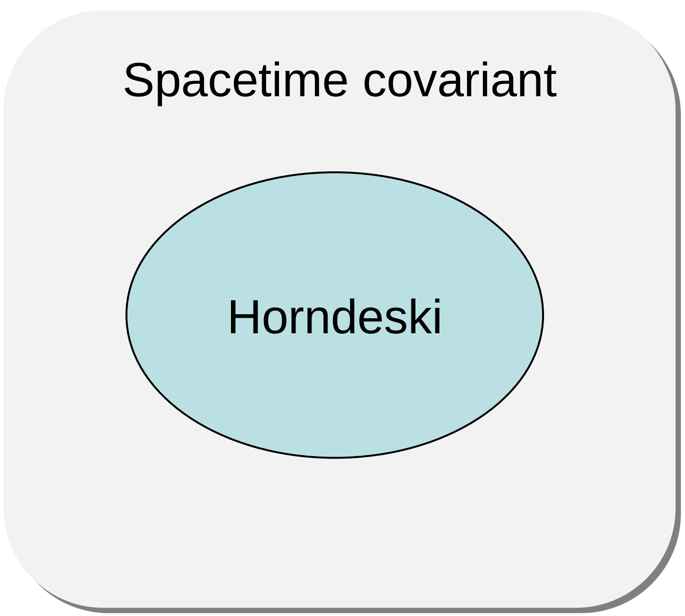

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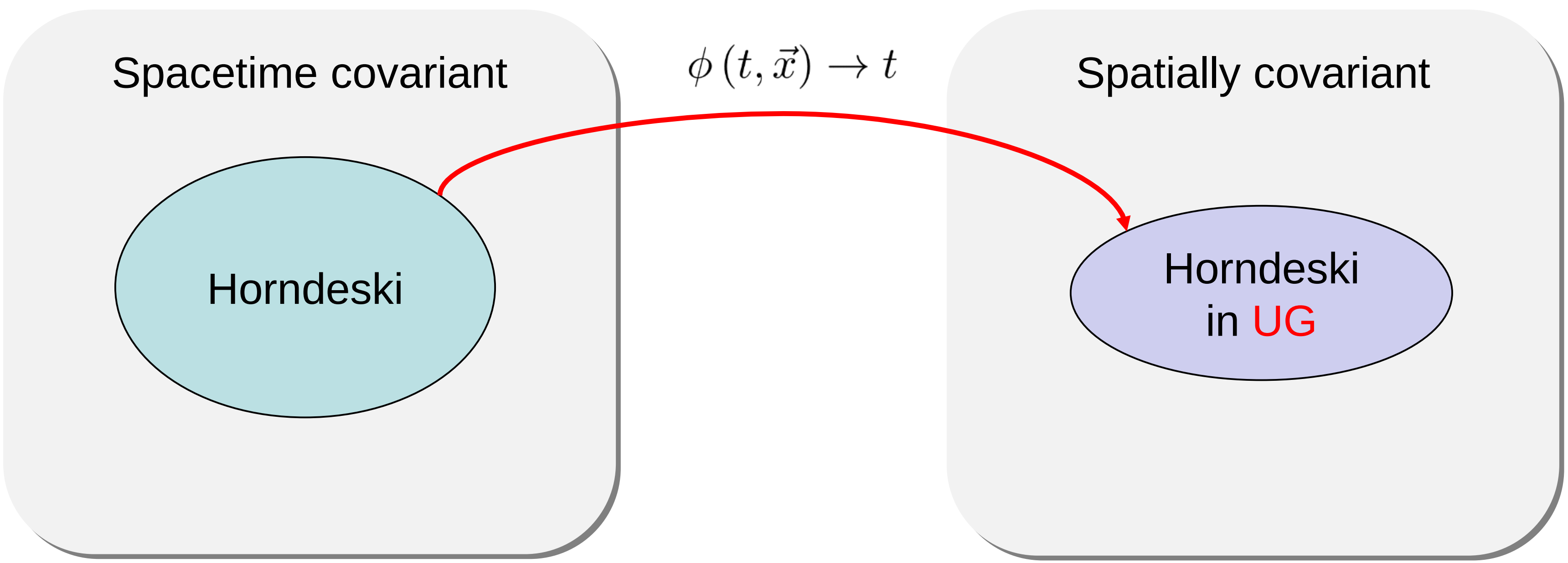

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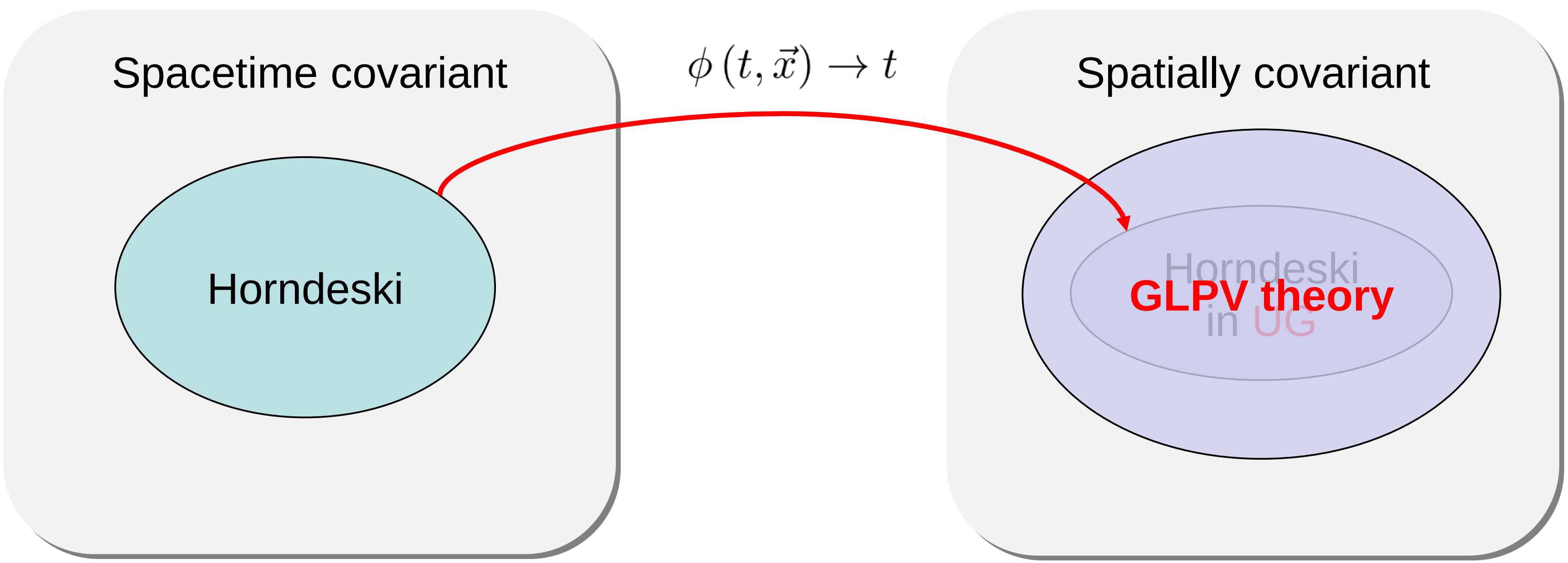

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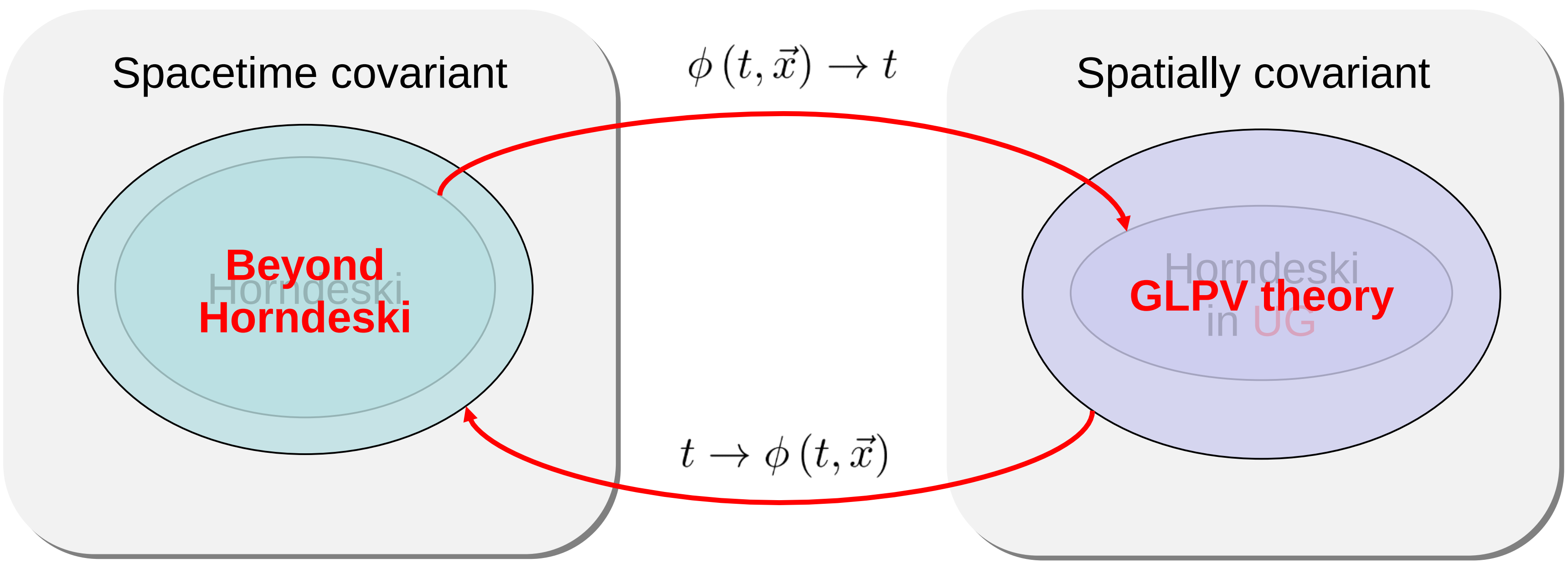

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2014 — **Spatially covariant gravity**
*[**X. Gao**, **Phys.Rev. D** 90 (2014) 081501 (**Rapid Communication**)]*

$$S=\int \mathrm{d}t\mathrm{d}^3x\, N\sqrt{h}\,\mathcal{L}(t,N,h_{ij},R_{ij},K_{ij},\mathrm{D}_i)$$

2 tensor + 1 scalar DoFs with higher derivative EoMs.
[X.Gao, Phys.Rev. D90 (2014) 104033]

Evolution of the theories

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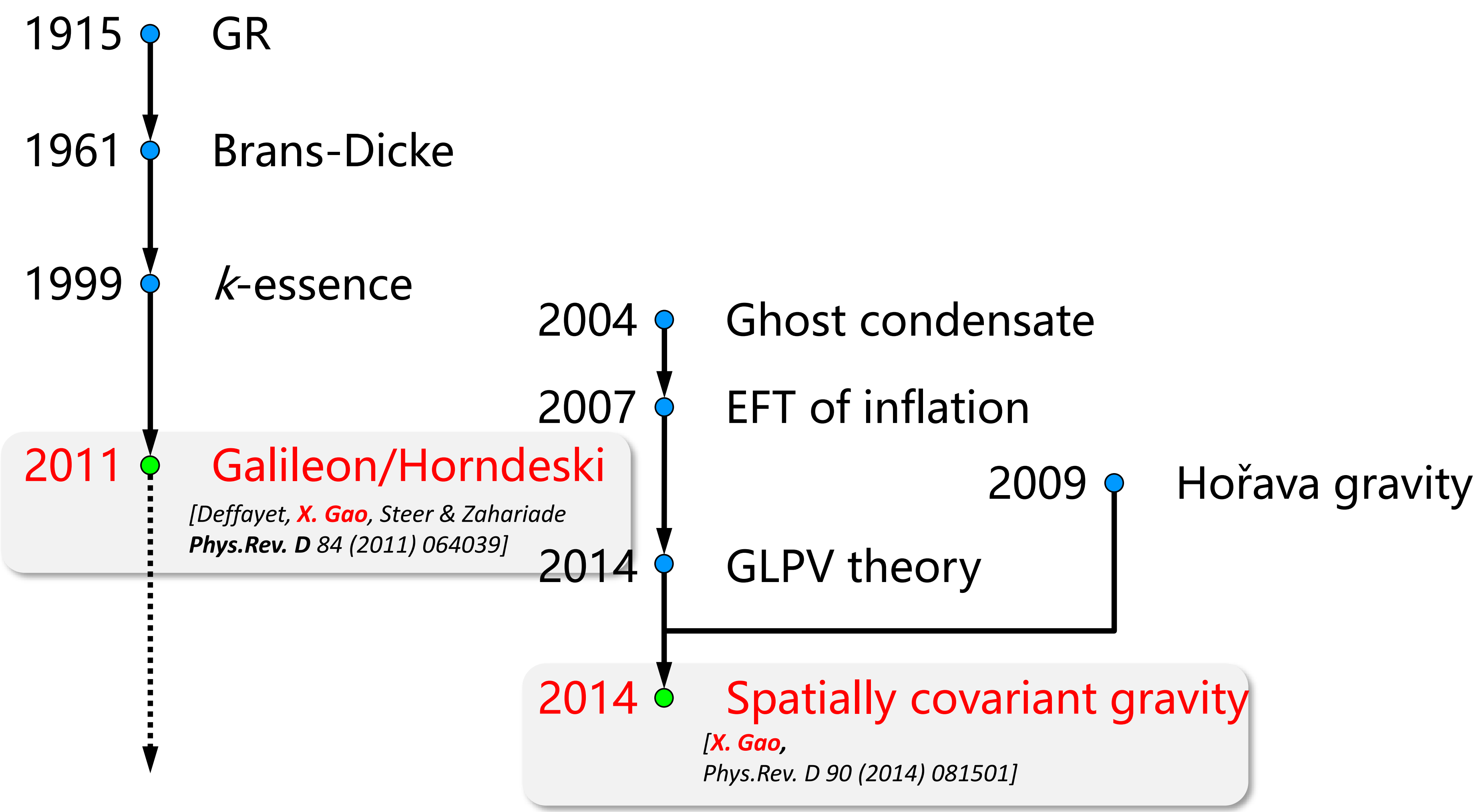

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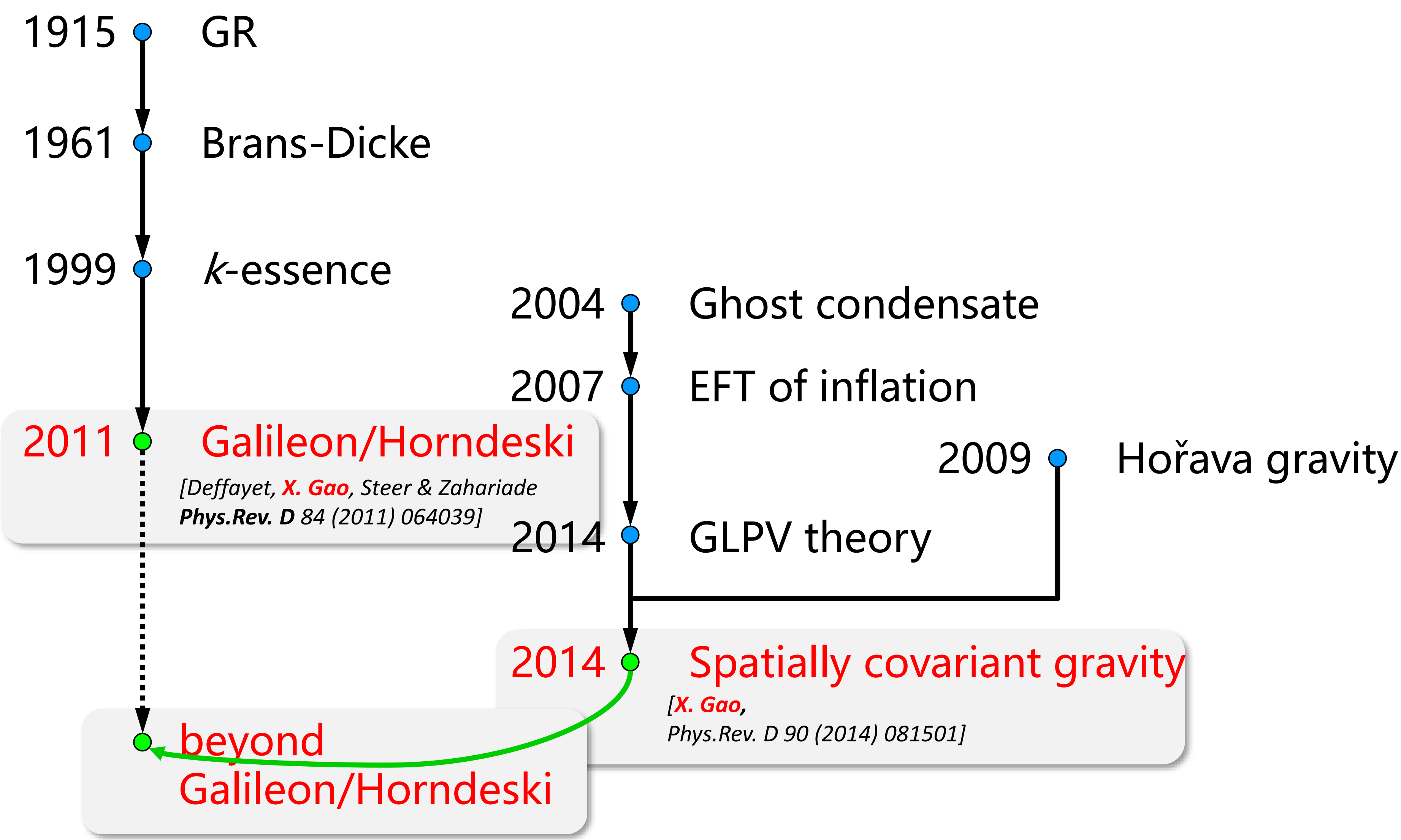

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Generalized Spatially covariant gravity

*[**XG**, and Zhi-bang Yao, work in progress]*

$$S=\int \mathrm{d}t\mathrm{d}^3x\, N\sqrt{h}\mathcal{L}\left(t,N,h_{ij},R_{ij},K_{ij},\pounds_{\boldsymbol{n}}N,\mathrm{D}_i\right)$$

Structure of the theory is constrained!

$$\Phi\equiv \pi_N+2\mathcal{A}\,h_{ij}\pi^{ij}-\sqrt{h}\mathcal{F}\approx 0$$

[G. Domènech, S. Mukohyama, R. Namba, A. Naruko, R. Saitou, and Y. Watanabe, Phys. Rev. D92 (2015), no. 8 084027]

Message from this talk

1. Beyond Horndeski theories can be further generalized.

2. A class of spatially covariant gravity theories is proposed.

Thank you for your attention!