KK Towers in the Early Universe:
Phase Transitions, Relic Abundances, and Applications to Axion Cosmology

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[arXiv:1612.08950]
[arXiv:1509.00470]

collaborators on this work:
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Scalars in the Early Universe

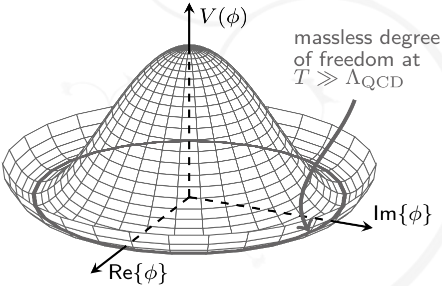
Impact of Mass-Generating Phase Transitions

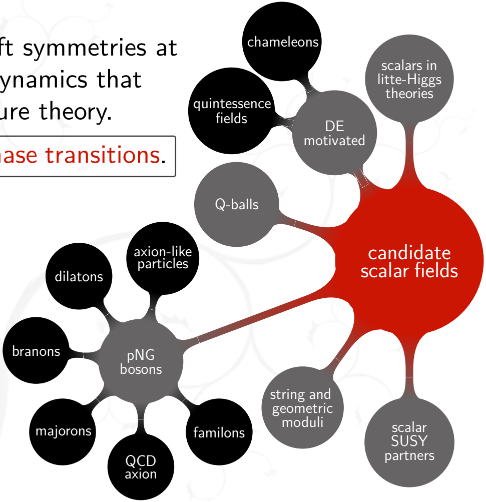
- Additional scalar fields commonly appear in extensions of the SM, and tend to play an important role in early-universe cosmology.

- These fields are often light due to shift symmetries at high scales, but are broken by some dynamics that enters in the effective lower-temperature theory.

i.e., they undergo mass-generating phase transitions.

example: QCD Axion





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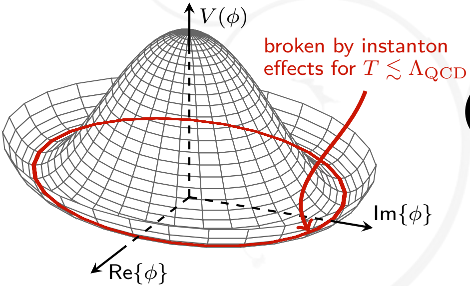
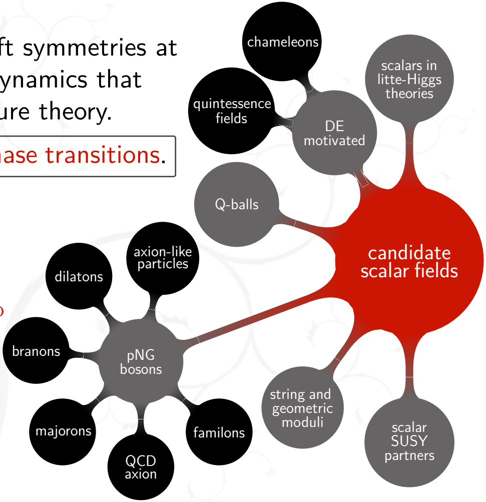




Scalars in the Early Universe

Impact of Mass-Generating Phase Transitions

- All of this can be important for model building: the energy density ρ carried by these scalar(s) at late times (used to compute abundances, overclosure bounds, *etc.*) is generally **sensitive to the timescale Δ_G over which such a phase transition unfolds**.

- With ***multiple fields*** $\{\phi_\lambda\}$, such transitions can generate off-diagonal elements in the mass matrix $\mathcal{M}^2$, and thus **mixing is also generated** amongst the fields in a dynamical, time-dependent way.

$$V_{\text{eff}}(\phi_0, \phi_1, \ldots) \supset \frac{1}{2}\sum_{k,\ell} \phi_k \overbrace{\mathcal{M}^2_{k\ell}(t)}^{\text{mass matrix}} \phi_\ell$$

$$\mathcal{M}^2(t) = \underbrace{\begin{bmatrix} M_0^2 & 0 & \cdots & 0 \\ 0 & M_1^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & M_{N-1}^2 \end{bmatrix}}_{\text{constant masses } M_i^2} + \underbrace{\begin{bmatrix} m_{00}^2 & m_{01}^2 & \cdots & m_{0,N-1}^2 \\ m_{01}^2 & m_{11}^2 & \cdots & m_{1,N-1}^2 \\ \vdots & \vdots & \ddots & \vdots \\ m_{0,N-1}^2 & m_{1,N-1}^2 & \cdots & m_{N-1,N-1}^2 \end{bmatrix}}_{m_{ij}^2(t) \text{ generated during phase transition}}$$

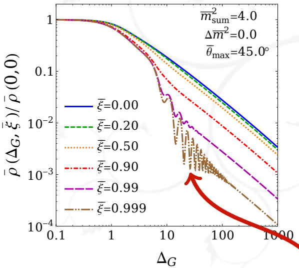
A Two-Field Toy Model

A *Very* Brief Review

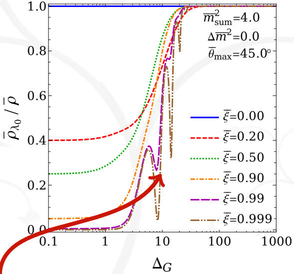
- This has been found to have a surprising influence, even in the context of a simple but **generic two-component toy model** [arXiv:1509.00470]:

$$\mathcal{M}^2(t) \;=\; \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & M^2 \end{bmatrix}}_{\text{constant term}} + \underbrace{\begin{bmatrix} \overline{m}_{00}^2 & \overline{m}_{01}^2 \\ \overline{m}_{01}^2 & \overline{m}_{11}^2 \end{bmatrix}}_{\text{generated terms}} \underbrace{h^2(t)}_{\text{time-dependence}}$$

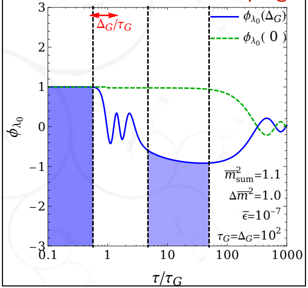
both enhancements and large suppressions to late-time energy density



distribution amongst fields extremely sensitive to phase transition timescale



additional dynamical over/underdamping transitions: "reoverdamping"



sequence of parametric resonances appear as mixing is saturated

$\Delta_G \equiv$ phase transition timescale
$\overline{\xi} \equiv$ mixing $\in [0, 1)$

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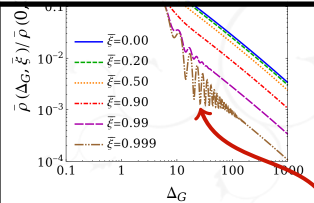
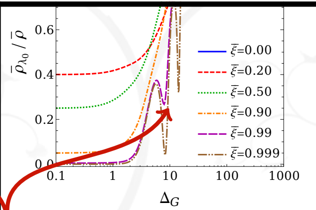
distribution amongst fields

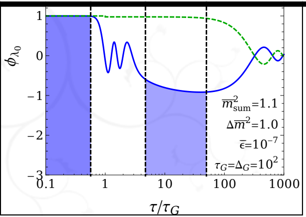
additional dynamical

This is under the minimal assumption of only ***two* components**.

$\Rightarrow$ what happens in models with **larger collections** of fields, such as those furnished by models with extra dimensions?







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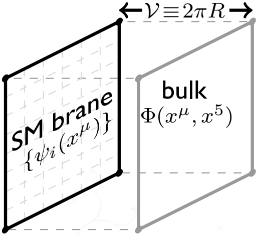
Mass Generation in a KK Tower

The Framework

- Consider a spacetime geometry $\mathcal{M} \times S^1/\mathbb{Z}_2$, *i.e.* an extra dimension compactified on a line segment, with a bulk scalar $\Phi(x^\mu, x^5)$:

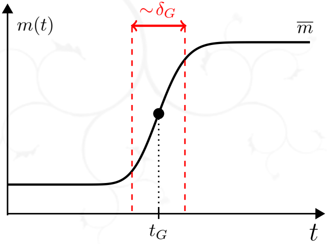


$$\mathcal{S} = \int d^4x dx^5 \left[\underbrace{\frac{1}{2}\partial_M \Phi^* \partial^M \Phi}_{\Phi \text{ shift symmetry forbids bulk mass}} + \delta(x^5) \mathcal{L}_{\text{brane}}(\psi_i, \Phi) \right]$$

interactions with fields on brane can lead to an *effective* 4D mass $m(t)$:

$$\frac{1}{\mathcal{V}} \mathcal{L}_{\text{brane}}(\Phi) = -\frac{1}{2} m^2(t) \, |\Phi|^2 + \ldots$$

and we parameterize our ignorance:



- The 4D mass matrix then mixes the fields:

$$\mathcal{M}^2 = m^2(t) \begin{bmatrix} 1 & \sqrt{2} & \sqrt{2} & \cdots \\ \sqrt{2} & 2 + \frac{M_c^2}{m^2(t)} & 2 & \cdots \\ \sqrt{2} & 2 & 2 + \frac{4M_c^2}{m^2(t)} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

$$M_c \equiv 1/R \geq 4.49 \cdot 10^{-12} \text{GeV}$$

- $\overline{m} \gg M_c$ indicates highly mixed ensemble

Evolving the System

In a flat FRW cosmology the KK modes $\{\phi_k\}$ evolve as

$$\ddot{\phi}_k + 3H(t)\dot{\phi}_k + \sum_{\ell=0}^{\infty} \mathcal{M}^2_{k\ell}(t)\phi_\ell \ = \ 0 \ ,$$

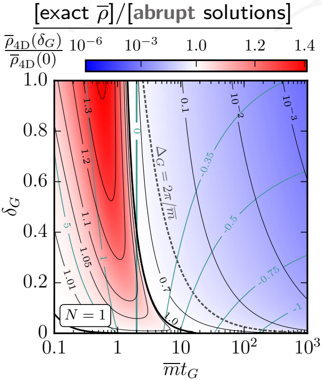
which in general cannot be solved analytically due to the time-dependence in $\mathcal{M}^2_{k\ell}$ near the phase transition.

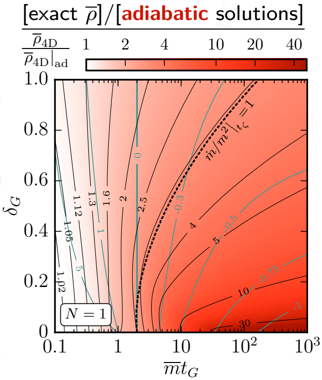
$\Rightarrow$ perform numerics on *truncated* tower of N modes, and recover features through $N \to \infty$ limiting behavior.

Survey of Four-Dimensional ($N = 1$) Limit

Standard Approximations

- Two approximations are commonly use in the literature to compute late-time abundances in single-field models that undergo such phase transitions:
  - **abrupt** approximation $\overline{\rho}_{\rm 4D}$ (where $\delta_G \to 0$)
  - **adiabatic** approximation $\overline{\rho}_{\rm 4D}|_{\rm ad}$ (where $\dot{m}/m^2 \ll 1$)





- Even for $N = 1$, there are regions of parameter space that are **inaccesible to the standard approximations**, particularly in the $m \gg 1/t_G$ regime.

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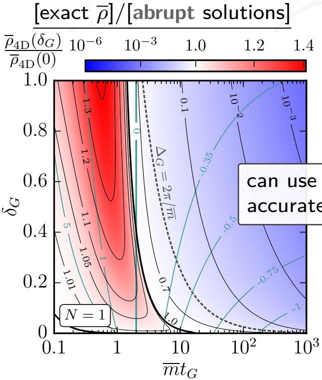
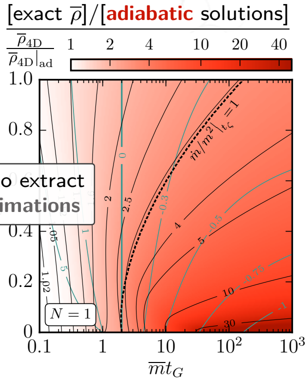


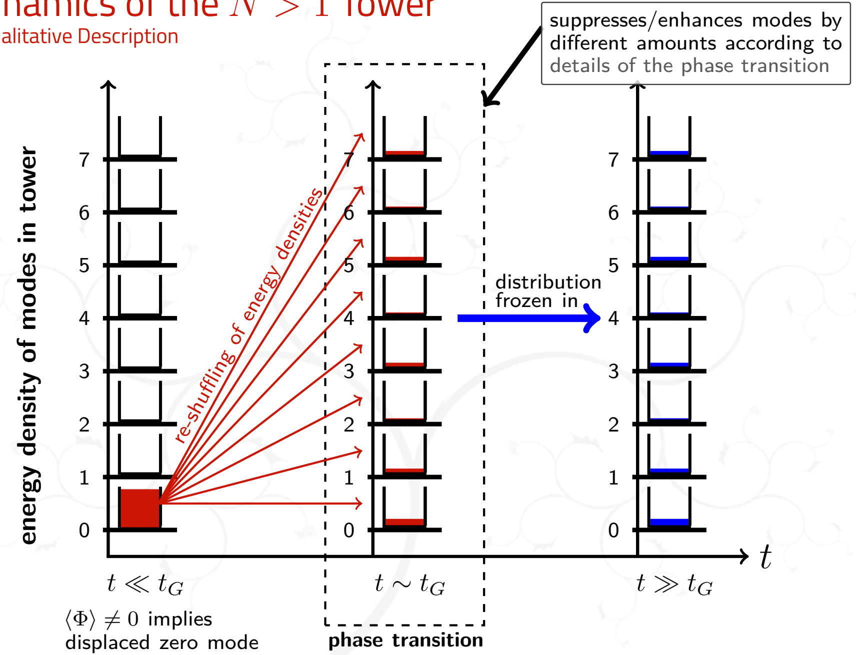


can use numerical results to extract accurate **analytical approximations**

- Even for $N = 1$, there are regions of parameter space that are **inaccesible to the standard approximations**, particularly in the $m \gg 1/t_G$ regime.

Dynamics of the $N > 1$ Tower

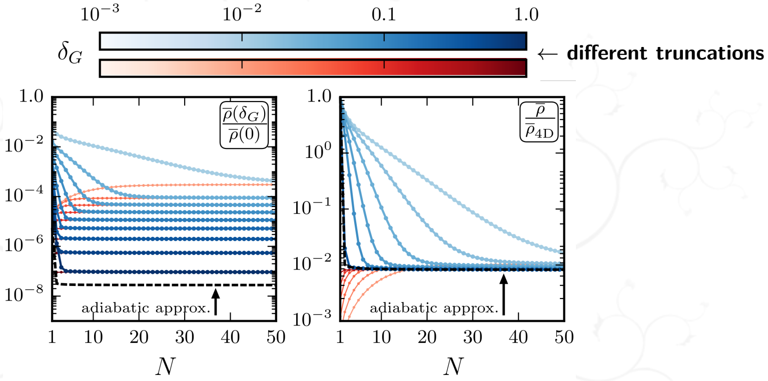
A Qualitative Description



Approaching Asymptotia: $N \to \infty$

Behavior of the Solutions

- It is instructive to examine the $N \to \infty$ asymptotic behavior of various late-time quantities while varying δ_G (and taking $\overline{m} = 100 M_c$):



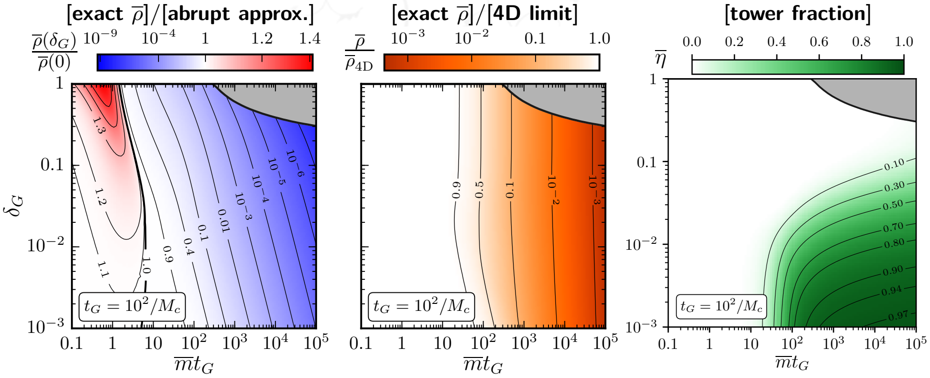
[exact $\overline{\rho}$]/[abrupt approx.] **[exact $\overline{\rho}$]/[4D limit]**

> the phase transition suppresses modes that exceed $\delta_G t_G \gtrsim \sqrt{2\pi}/\lambda$, *i.e.* it **accelerates the $N \to \infty$ convergence** — often leaving only a few modes that appreciably contribute to the total ρ.

The KK Tower Limit: Extracting $N \to \infty$ Limit

Suppressions, Tower Fractions, and Distributions

- Equipped with a method to efficiently compute asymptotia for large N, we now have the ability to compute results effectively for the **full KK tower**.



$$\eta \equiv 1 - \max_\lambda \left\{ \frac{\overline{\rho}_\lambda}{\overline{\rho}} \right\}$$

fraction of abundance in subdominant modes

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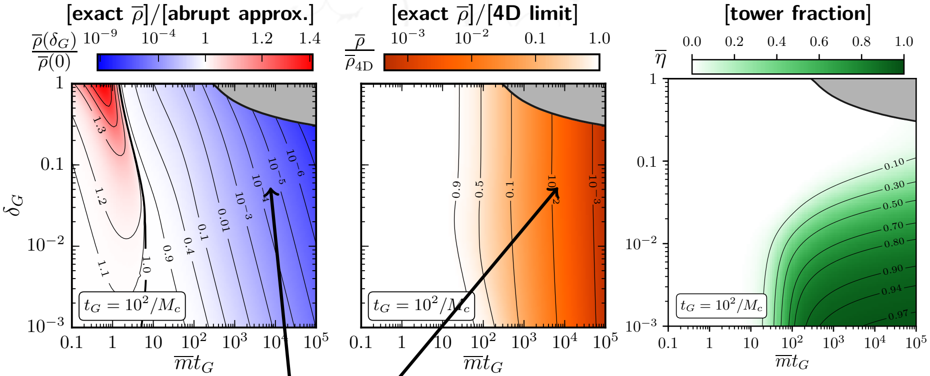

again can extract general **analytical approximations** in different regions

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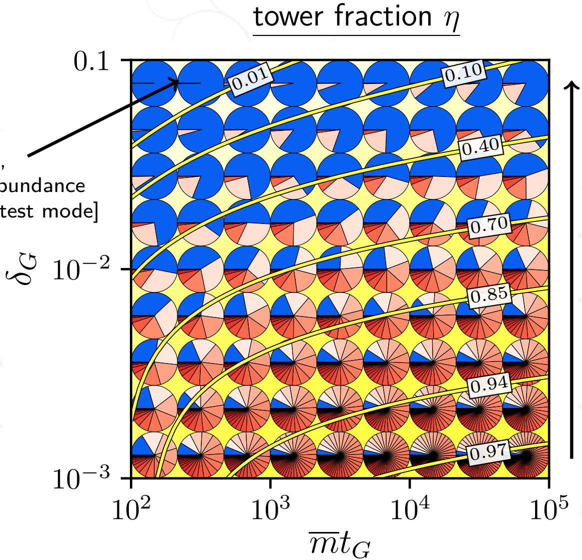

slices give $\overline{\rho}_\lambda/\overline{\rho}$, the fractional abundance [**blue slice**≡lightest mode]

heavier modes **increasingly suppressed** with larger δ_G (as $\delta_G t_G \gtrsim \sqrt{2\pi}/\lambda$)

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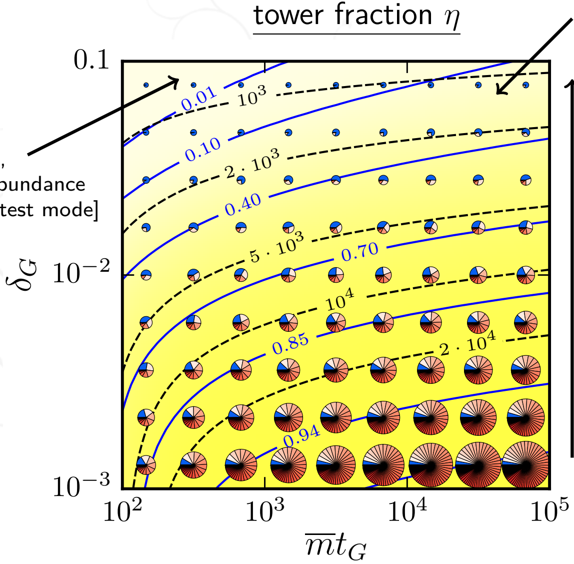
area of pies $\sim$ total abundance of tower

slices give $\overline{\rho}_\lambda/\overline{\rho}$, the fractional abundance [**blue slice**$\equiv$lightest mode]



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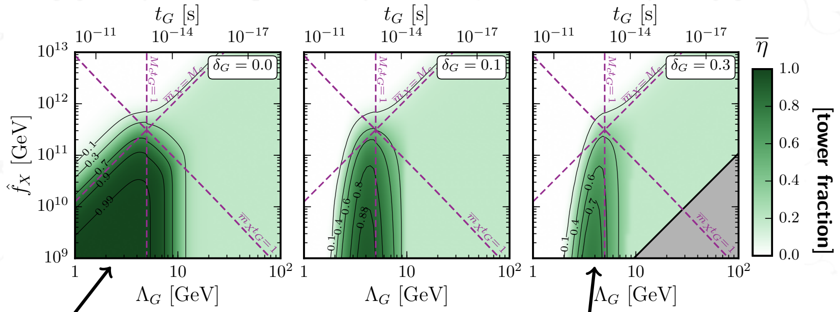
Example: Axion in the Bulk

- At this point we can drop the generality of Φ and apply our machinery to a specific model: for example a **bulk axion-like particle (ALP)**.
- Our $\{t_G, \overline{m}_X, M_c\}$ parameter space is mapped onto $\{\Lambda_G, \hat{f}_X, M_c\}$

associated confinement scale

effective 4D decay constant

$$t_G = \sqrt{\frac{45 g_*(T_{\rm RH})}{2\pi^2}} \frac{T_{\rm RH}^2 M_P}{g_*(\Lambda_G)\Lambda_G^4} \qquad \overline{m}_X^2 = \frac{\mathcal{C}^2 g^2}{32\pi^2} \frac{\Lambda_G^4}{\hat{f}_X^2}$$



maximum tower fraction in $1 \lesssim M_c t_G \lesssim \overline{m}_X t_G$

transition suppresses heavier modes, confining maximum to $1 \sim M_c t_G \lesssim \overline{m}_X t_G$

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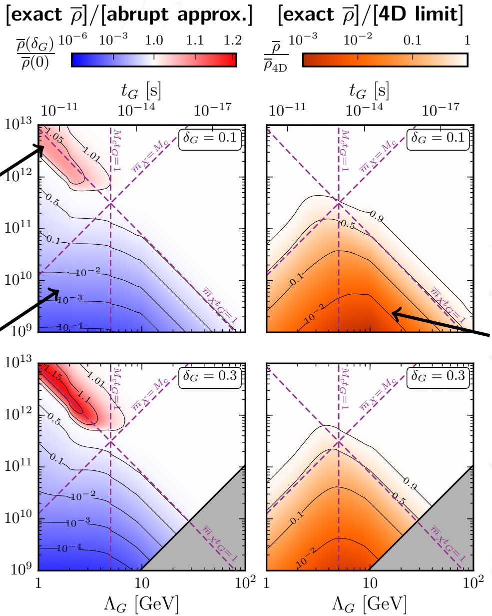

enhancements and large **suppressions** relative to abrupt approximation

presence of extra dimension produces **significant additional suppression** of $\overline{\rho}$ for $\overline{m} \gtrsim M_c \cup \overline{m} t_G \gtrsim 1$

The Take-Away Message

- Models of non-minimal scalar sectors that undergo mass-generating phase transitions in general are **very sensitive to phase transitions details**.
  - both the **total energy density** and its **distribution** across individual modes in the ensemble show this — both in a simple but generic two-field model, and in model with a bulk scalar.
  - we derived a variety of asymptotic scaling behaviors and **analytic expressions** for the energy densities of the tower as functions of relevant model parameters
  - applied the general machinery of our framework to the **example of a bulk axion**, allowing us to determine where the standard approximations succeed/fail — and may suggest the weakening of overclosure bounds in certain regions
- **There are many possible future directions**:
  - we assumed a single flat extra dimension, but what phenomena arise with a **warped geometry** and/or multiple extra spatial dimensions?
  - we operated under assumption that the fields $\rho_\phi \ll \rho_{\rm crit}$ during the mass-generation epoch, but what is the effect of the **backreaction on H** away from this regime [*i.e.*, where scalars play role during inflation/(p)reheating]?

THANKS FOR YOUR ATTENTION!