Cosmic Birefringence and anisotropies in the CMB polarization

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December 14, 2017, CosPA 2017

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Cosmic Birefringence

Electrodynamic Lagrangian with the Chern-Simons term:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\beta}{2M}\phi F_{\mu\nu}F^{\mu\nu}$$

Left-circular polarized light and right-circular polarized light travel with different velocities. So a plane polarized wave would be rotated by an angle $\Delta\psi$ which is given by:

$$\Delta\psi = \frac{\beta}{2M}\Delta\phi$$

[*Harari and Sikivie, 1992; Carroll 1998*]

This would cause a rotation of the CMB polarization field. The Stokes parameters Q and U of CMB polarization rotate as:

$$[Q \pm iU]'(\hat{n}) = e^{\mp i2\Delta\psi}[Q \pm iU](\hat{n})$$

[*Li and Zhang, 2008*]

Motivation and context

Cosmic Birefringence would produce EB and TB correlations in the CMB. E-mode transforms as $(-1)^l$ and B-mode transforms as $(-1)^{l+1}$. These terms violate parity.

The rotation angle $\Delta\psi$ may not necessarily be isotropic over the sky. If the rotation angle is position dependent in the sky, it will cause an apparent violation of statistical isotropy of the CMB polarization.

$$\langle a^X_{lm} a^{X'*}_{l'm'} \rangle \neq C^{XX'}_l \delta_{ll'} \delta_{mm'}$$

Position Dependent Rotation

In general the rotation can be a position dependent rotation $\Delta\psi = \Delta\psi(\hat{n})$ and we retain upto first order.

We now can write $\Delta\psi(\hat{n}) = \sum_{LM} \psi_{LM} Y_{LM}(\hat{n})$.

We are interested in finding:

$$C_L^{\psi\psi} = \frac{1}{L(L+1)} \sum_M \psi_{LM} \psi_{LM}^*$$

A position dependent rotation would cross-correlate different multipoles of the CMB polarization:

$$\langle a_{lm}^X a_{l'm'}^{X'*} \rangle \neq C_l^{XX'} \delta_{ll'} \delta_{mm'}$$

In this sense there will be an induced anisotropy in CMB polarization.

The Polarization Correlations i

In general:

$$\langle X_{l_1 m_1} (X'_{l_2 m_2})^* \rangle = \sum_{LM} D^{LM,XX'}_{l_1 l_2} \xi^{LM}_{l_1 m_1 l_2 m_2}$$

where, from calculation one gets:

$$D^{LM,XX'}_{ll'} = 2\psi_{LM} Z^{XX'}_{ll'} H^{L}_{ll'}$$

with:

XX'	BE	EB	EE	BT	TB	ET	TE
$Z^{XX'}_{l_1 l_2}$	$-\tilde{C}^{EE}_{l_2}$	$-\tilde{C}^{EE}_{l_1}$	$-i\left(\tilde{C}^{EE}_{l_2} - \tilde{C}^{EE}_{l_1}\right)$	$-\tilde{C}^{TE}_{l_2}$	$-\tilde{C}^{TE}_{l_1}$	$-i\tilde{C}^{TE}_{l_2}$	$-\tilde{C}^{TE}_{l_1}$

The Polarization Correlations ii

with

$$H^L_{l_1 l_2} = \begin{pmatrix} l_1 & l_2 & L \\ 2 & -2 & 0 \end{pmatrix}$$

$$\xi^{LM}_{l_1 m_1 l_2 m_2} = \sqrt{\frac{(2l_1+1)(2l_2+1)(2L+1)}{4\pi}} \begin{pmatrix} l_1 & l_2 & L \\ -m_1 & m_2 & M \end{pmatrix}$$

Using the orthogonality properties of the Wigner 3j symbols we write the minimum-variance estimator for $D^{LM,XX'}_{l_1 l_2} \xi^{LM}_{l_1 m_1 l_2 m_2}$ as:

$$\hat{D}^{LM,XX',map}_{l_1 l_2} \xi^{LM}_{l_1 m_1 l_2 m_2} = (G^L_{l_1 l_2})^{-1} \sum_{mm'} X^{\mathrm{map}}_{l_1 m_1} X'^{\mathrm{map}*}_{l_2 m_2} \xi^{LM}_{l_1 m_1 l_2 m_2}$$

and $G^L_{l_1 l_2} = \frac{(2l_1+1)(2l_2+1)}{4\pi}$

The Minimum Variance Estimator for ψ_{LM}

Following *Gluscevic, V. et al. 2009* we construct:

$$\hat{\psi}_{LM} = N_L^{-1} \sum_{l_2 \geq l_1} G^L_{l_1 l_2} \sum_{AA'} F^{L,A'}_{l_1 l_2} \hat{D}^{LM,A,map}_{l_1 l_2} \left[(\mathcal{C}^{l_1 l_2})^{-1} \right]_{AA'}$$

where:

$$N_L = \sum_{l_2 \geq l_1} G^L_{l_1 l_2} \sum_{AA'} F^{L,A}_{l_1 l_2} F^{L,A'}_{l_1 l_2} \left[(\mathcal{C}^{l_1 l_2})^{-1} \right]_{AA'}$$

The $F^{L,A}_{l_1 l_2} = 2 Z^A_{l_1 l_2} H^L_{l_1 l_2} W_{l_1} W_{l_2}$ uses the theoretical unrotated C^A_ls which are calculated from CAMB and $\mathcal{C}^{l_1 l_2}_{AA'}$ is the covariance of $\hat{D}^{LM,A,map}_{l_1 l_2}$ in AA' space.
Example: $\mathcal{C}^{l_1 l_2}_{EB,EB} = C^{BB,map}_{l_2} C^{EE,map}_{l_1}$, $\mathcal{C}^{l_1 l_2}_{BE,BE} = C^{BB,map}_{l_1} C^{EE,map}_{l_2}$.

For our EB and TB estimators $l_1 + l_2 + L = even$

Gaussian Bias

We are probing the trispectrum $\langle EBEB \rangle$ or $\langle TBTB \rangle$. There would be contributions from the disconnected terms which will bias the result and would require correcting for.

$$EBEB \qquad TBTB$$

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$$EBEB \qquad TBTB$$

For TB or EB correlations only the middle term makes significant contributions. The first term couples to the monopole only.

Gaussian Bias Correction

For Gaussian CMB fluctuations with noise the Gaussian bias from the disconnected term is given by:

$$C^{\psi\psi}_{L,\text{Gauss}} = \langle \hat{\alpha}_{LM} \hat{\alpha}^{*}_{LM} \rangle$$

Since we take two different maps for $\langle \hat{\alpha}^{I}_{LM} \hat{\alpha}^{II*}_{LM} \rangle$ correlations. The EB Gaussian bias:

$$\langle \hat{\alpha}^{I}_{LM} \hat{\alpha}^{II*}_{LM} \rangle_{\text{Gauss}} = N^{-1}_{L,I} N^{-1}_{L,II} \sum_{l_1,l_2} G^{L}_{l_1,l_2} \left[\frac{(F^{L,BE}_{l_1 l_2})^2 C^{BB,I\times II}_{l_1} C^{EE,I\times II}_{l_2}}{C^{BB,I}_{l_1} C^{EE,I}_{l_2} C^{BB,II}_{l_1} C^{EE,II}_{l_1}} + \frac{F^{L,BE}_{l_1 l_2} F^{L,EB}_{l_1 l_2} C^{BE,I\times II}_{l_1} C^{EB,I\times II}_{l_2}}{C^{BB,I}_{l_1} C^{EE,I}_{l_2} C^{BB,II}_{l_2} C^{EE,II}_{l_1}} \right]$$

The first term contributes most to the bias. For TB correlations, $E \leftrightarrow T$ in the above expression.

Data and Simulations

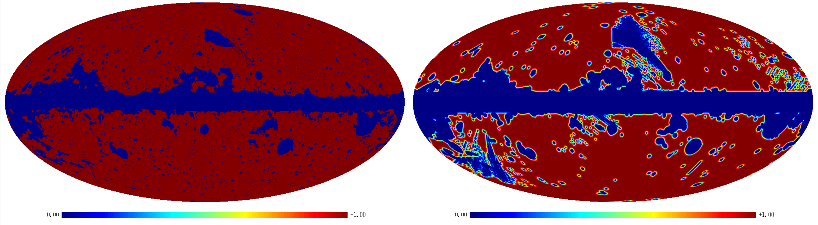
Data: We have used 2015 Planck Commander and SMICA temperature and polarization half mission data.

Mask:We used union masks UT78 for temperature and UPB77 polarization mask both with half mission missing pixels. We apodize the polarization maps with a FWHM of 2°.

Downgrading: We performed our analysis at $N_{\mathrm{side}} = 256$ by using

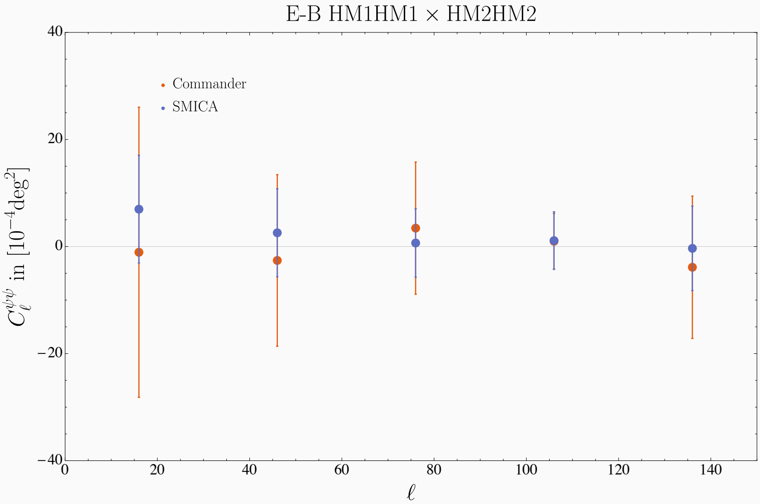
$$a_{lm}^{\mathrm{OUT}} = \frac{B_l^{\mathrm{OUT}} P_l^{\mathrm{OUT}}}{B_l^{\mathrm{IN}} P_l^{\mathrm{IN}}} a_{lm}^{\mathrm{IN}}$$

Masks

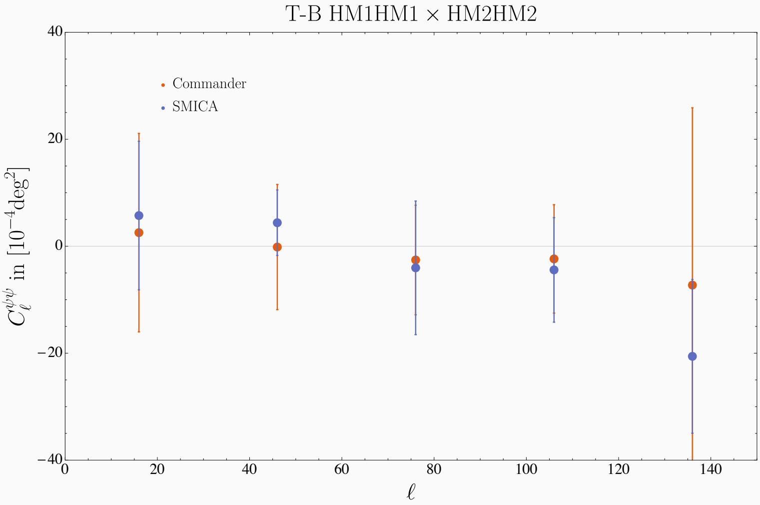


(a) UT78 $\bigcup$ HMIS (b) UPB77 $\bigcup$ HMIS

Reconstructed Rotation Power Spectrum i



Reconstructed Rotation Power Spectrum ii



Summary

Compared to the previous results from WMAP the estimates of $C_l^{\psi\psi}$ have improved by a factor of $10^3 \sim 10^4$.

These are the most stringent limit on parity violating EB and TB correlations in the CMB data.

We hope to complete the MC simulations to estimate other uncertainties in the final result soon.