Search for anomalous alignments of structures in PLANCK data using Tensor Minkowski Functionals

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Based on Chingangbam, Yogendran, Joby P. K. et al. 2017, and

Joby, Ganesan and Chingangbam, in preparation

Motivation

- **Scalar Minkowski Functionals (MF)** have been used to test Gaussianity of random fields.
- The trace of the rank two, translation invariant **Tensor Minkowski Functionals (TMFs)** yield the usual scalar MFs.
- Thus TMFs contain all the information that exists in the scalar MFs.
- Additionally, TMFs contain shape information.
- TMFs can be used for studying the statistical isotropy of fields.

Motivation

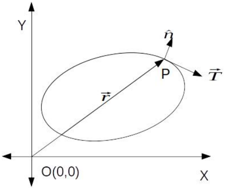
- Numerical work using stereographic projection has showed disagreement between PLANCK 2015 data and ΛCDM model at $4-\sigma$ for CMB E mode.

  (Vidhya and Chingangbam, 2017)

- Computation of TMF directly on the sphere is important before probing the causes for the disagreement.

TMFs in flat 2D space

Consider a closed curve in flat 2D space



where,

$\hat{n}$ is the unit normal to the curve.

$\vec{T}$ is tangent vector to the curve.

$\vec{r}$ is the position vector.

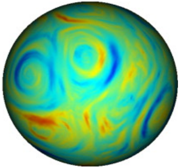
κ denotes the local curvature of the curve.

Translation invariant second rank Tensor Minkowski Functional:

$$W_2^{1,1} \equiv \frac{1}{4} \oint_C \vec{r} \otimes \hat{n}\, \kappa\, ds$$

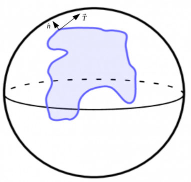
Closed curves on the sphere

For a field defined on the sphere:



- Red regions have field value greater than mean.
- Blue regions have field value less than the mean.
- Boundaries of iso colour regions form closed curves on the sphere.

TMF for closed curves on the sphere



- $\hat{n}$ is the unit normal to the curve at the given point on the curve.
- $\vec{T}$ is the tangent vector.
- Re-express $W_2^{1,1}$ in terms of $\vec{T}$ and generalize the definition to curved space as,

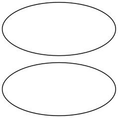
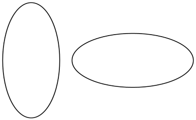
$$\mathcal{W}_1 \equiv \frac{1}{4} \oint_C \hat{T} \otimes \hat{T} \, ds$$

Measuring Statistical Isotropy

- Consider a random field, u, defined on the unit sphere.
- Choose a threshold, ν_t to obtain the excursion set.
- Find $\mathcal{W}_1$ for the boundaries of individual connected regions and holes.
- Obtain $\overline{\mathcal{W}}_1$ by averaging individual elements of $\mathcal{W}_1$ over all the structures.
- Let Λ_1 and Λ_2 denote the two eigenvalues of $\overline{\mathcal{W}}_1$, such that, $\Lambda_1 \leq \Lambda_2$.
- The alignment parameter is defined as,

$$\alpha \equiv \frac{\Lambda_1}{\Lambda_2}$$

Interpreting α

- α measures the net orientation of the structures.
- $\alpha = 1$ means that the field is statistically isotropic.
- For a circle, $\alpha = 1$, irrespective of the radius.
- For a pair of identical ellipses with their major axes aligned in the same direction, $0 < \alpha < 1$. 
- For a pair of identical ellipses with their major axes aligned perpendicularly to each other, $\alpha = 1$. 

Analytic computation of α on the sphere

For a pixelated map of a field u defined on the unit sphere, $\overline{\mathcal{W}}_1$ is given by,

$$\overline{\mathcal{W}}_1 = \frac{1}{4}\int_{S^2} da\,\delta(u-\nu_t)\,\frac{1}{|\nabla u|}\begin{pmatrix} u_{;2}^2 & u_{;1}u_{;2} \\ u_{;1}u_{;2} & u_{;1}^2 \end{pmatrix}$$

where,

- $u_{;i}$ is the i^{th} component of the covariant derivative of u,
- $|\nabla u| = (u_{;1}, u_{;2})$
- da is the area element,
- $\delta(u-\nu_t) = \frac{1}{\Delta\nu_t}$ when u lies between $\nu_t - \Delta\nu_t/2$ and $\nu_t - \Delta\nu_t/2$ and zero otherwise,
- $\kappa = \frac{2u_{;1}u_{;2} - u_{;1}^2 u_{;22} - u_{;2}^2 u_{;11}}{|\nabla u|^3}$

Chingangbam, Yogendran, Joby P. K. et al. (2017)

Method

- In this work we apply the analytic α expression to PLANCK 2015 CMB data.
- Study the behaviour of α upon masking the CMB field.
- We use the common mask UT78 for temperature which has a sky fraction of 77.6%.
- CMB FFP9 simulations provided by PLANCK which include instrumental effects such as the beam effect have been used in this analysis.

Effect of masking

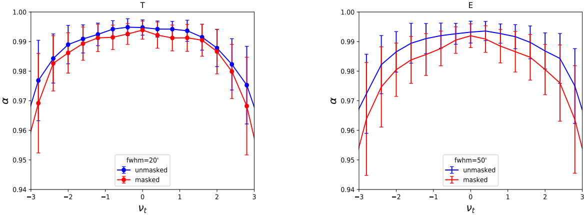


Figure: *Left panel*: α for T of CMB for unmasked and masked simulations. Average over 100 realizations. The error bars are the sample variance for the 100 realizations. *Right panel*: Same as left panel for E mode.

- The total number of structures (hotspots + coldspots) is smaller for threshold values further away from 0.
- Our ordering of Λ , along with fewer number of structures at larger thresholds, leads to smaller α values at these thresholds.
- Masking reduces the number of hotspots and coldspots and hence α for masked maps are found to be smaller than full sky maps.

Results: PLANCK T Low Frequency Bands

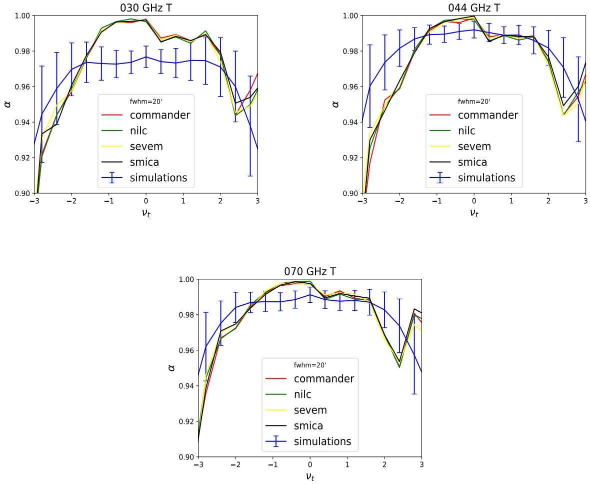


Figure: *Top*: Left panel shows a comparison of α for T of CMB obtained from simulations and from PLANCK 30GHz cleaned maps. Right panel displays the same for PLANCK 44GHz cleaned maps. *Bottom*: Same as top for PLANCK 70GHz cleaned maps.

Results: PLANCK T High Frequency Bands

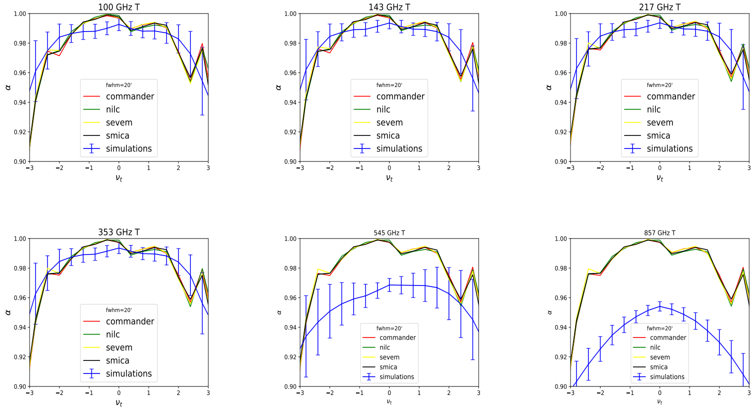


Figure: Top panel shows a comparison of α for T of CMB obtained from simulations and from PLANCK 100GHz, 143GHz and 217GHz cleaned maps. Bottom panel is the same as top for PLANCK 353GHz, 545GHz and 857GHz cleaned maps.

Quantifying the difference between Data and Simulations

- Compute α from PLANCK cleaned CMB maps.
- Compute the mean α, $\overline{\alpha}$, and the variance, σ_α, from 100 simulations.
- For each frequency band, compute the quantity, $\mathcal{D}_\alpha^X$, which is defined as follows,

$$\mathcal{D}_\alpha^X = \left| \frac{\alpha_{obs}^X - \overline{\alpha}^X}{\sigma_\alpha^X} \right|$$

  where, X can be T or E mode of CMB polarization.
- $\mathcal{D}_\alpha^X$ gives a measure of the deviation in statistical isotropy of the observed data from the theory.

$\mathcal{D}_\alpha$ for PLANCK frequency bands

$\mathcal{D}^T_\alpha(\nu_t = 0)$	30GHz	44GHz	70GHz	100GHz	143GHz	217GHz	353GHz	545GHz	857GHz
COM	3.5	1.4	1.4	1.0	0.8	0.9	1.0	6.5	13.5
NILC	3.3	1.2	1.8	1.4	1.3	1.3	1.4	6.9	14
SEVEM	3.5	1.5	1.4	1.0	0.9	1.0	1.0	6.6	13.6
SMICA	3.4	1.7	1.5	1.2	1.0	1.0	1.1	6.6	13.6

Table: $\mathcal{D}_\alpha(\nu_t = 0)$ for CMB Temperature for various PLANCK datasets.

Conclusions

- No significant alignment found in the PLANCK CMB Temperature data in the mid frequency range.
- Disagreement between ΛCDM model and PLANCK observations is seen at frequencies below 44 GHz and those above 545 GHz where foreground removal is expected to be less effective.
- $\mathcal{D}_\alpha(\nu_t = 0)$ values for CMB Temperature obtained analytically here are comparable to those from Vidhya and Chingangbam (2017).

Ongoing work:

- Comparison of analytic α computed from data and simulations, for E mode of CMB polarization.