Horava-Lifshitz cosmology revisited

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Implication of GW170817 on gravity theories

- $|c_{gw}^2 - c_\gamma^2| < 10^{-15}$
- Horndeski theoy (scalar-tensor theory with 2nd-order eom):
  Among 4 free functions, $G_4(\phi,X)$ & $G_5(\phi,X)$ are strongly constrained. Still $G_2(\phi,X)$ & $G_3(\phi,X)$ are free.
- Generalized Proca theory (vector-tensor theory):
  Among 6 (or more) free functions, $G_4(X)$ & $G_5(X)$ are strongly constrained. Still $G_2(X,F,Y,U)$, $G_3(X)$, $G_6(X)$, $g_5(X)$ are free.
- Horava-Lifshitz theory (renormalizable quantum gravity):
  The coefficient of $R^{(3)}$ is strongly constrained
  → IR fixed point with $c_{gw}^2 = c_\gamma^2$? How to speed up the RG flow?
- Ghost condensation (simplest Higgs phase of gravity):
  No additional constraint
- Massive gravity (simplest modification of GR):
  Upper bound on graviton mass ≈ 10^{-22}eV
  Much weaker than the requirement from acceleration

c.f. "All" gravity theories (including general relativity):
The cosmological constant is strongly constrained ≈ 10^{-120}.

Power counting

$$I \supset \int dt dx^3 \dot{\phi}^2$$

$$\int dt dx^3 \phi^n$$

$$\propto E^{-(1+3+ns)}$$

- **Scaling dim of ϕ**

  t → b t (E → b^{-1}E)

  x → b x

  ϕ → b^s ϕ

  1+3-2+2s = 0

  s = -1

- Renormalizability

  $$n \leq 4$$

- Gravity is highly non-linear and thus non-renormalizable

Abandon Lorentz symmetry?

$$I \supset \int dt dx^3 \dot{\phi}^2$$

- Anisotropic scaling
  $t \rightarrow b^z t$ ($E \rightarrow b^{-z} E$)
  $x \rightarrow b x$
  $\phi \rightarrow b^s \phi$
  $z+3-2z+2s = 0$
  $s = -(3-z)/2$
- $s = 0$ if $z = 3$

$$\int dt dx^3 \phi^n$$

$$\propto E^{-(z+3+ns)/z}$$

- For z = 3, any nonlinear interactions are renormalizable!
- **Gravity becomes renormalizable!?**

Horava-Lifshitz gravity

- HL gravity realizes **z=3 scaling @ UV** and thus is power-counting renormalizable
- **Renormalizability was recently proved** [Barvinsky, et al. 2016]
- Ostrogradsky ghost is absent and thus HL gravity is **likely to be unitary**
- Lorentz-invariance is broken @ UV
- **Lorentz-invariant IR fixed-point** is generic [Chadha & Nielsen 1983] (and may apply to GW as well; cf. $|c_{gw}^2 - c_\gamma^2| < 10^{-15}$ from GW170817) but running is slow (logarithmic)
- SUSY or/and strong dynamics can speed-up the RG running towards Lorentz-invariant IR fixed-point

Cosmological implications

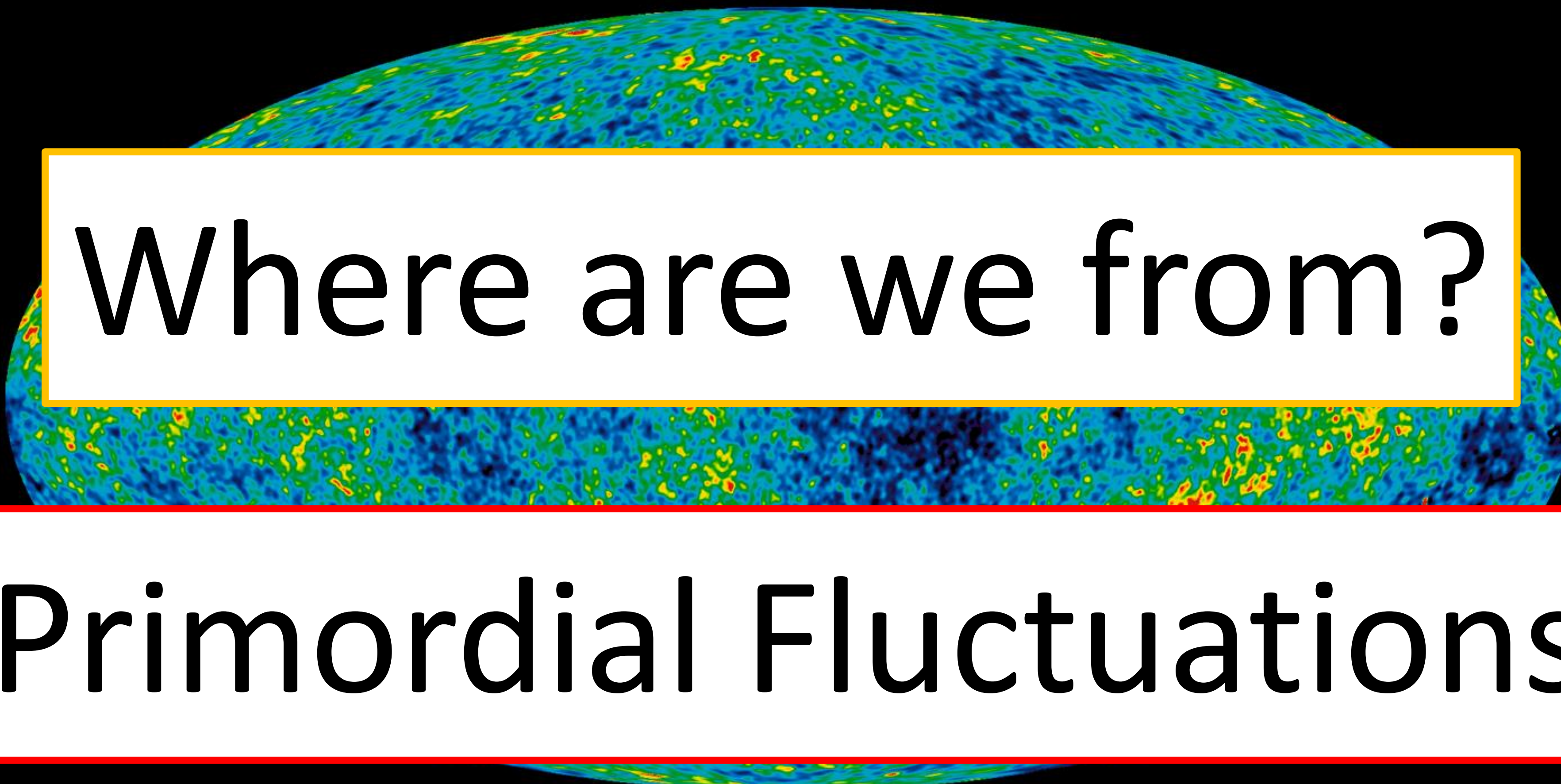
Horava-Lifshitz Cosmology: A Review, arXiv: 1007.5199

My talk

- The z=3 scaling solves the horizon problem and leads to (almost) scale-invariant cosmological perturbations without inflation (Mukohyama 2009).
- Higher curvature terms lead to regular bounce (Calcagni 2009, Brandenberger 2009).
- Higher curvature terms ($1/a^6$, $1/a^4$) might make the flatness problem milder (Kiritsis&Kofinas 2009).

Yota's talk

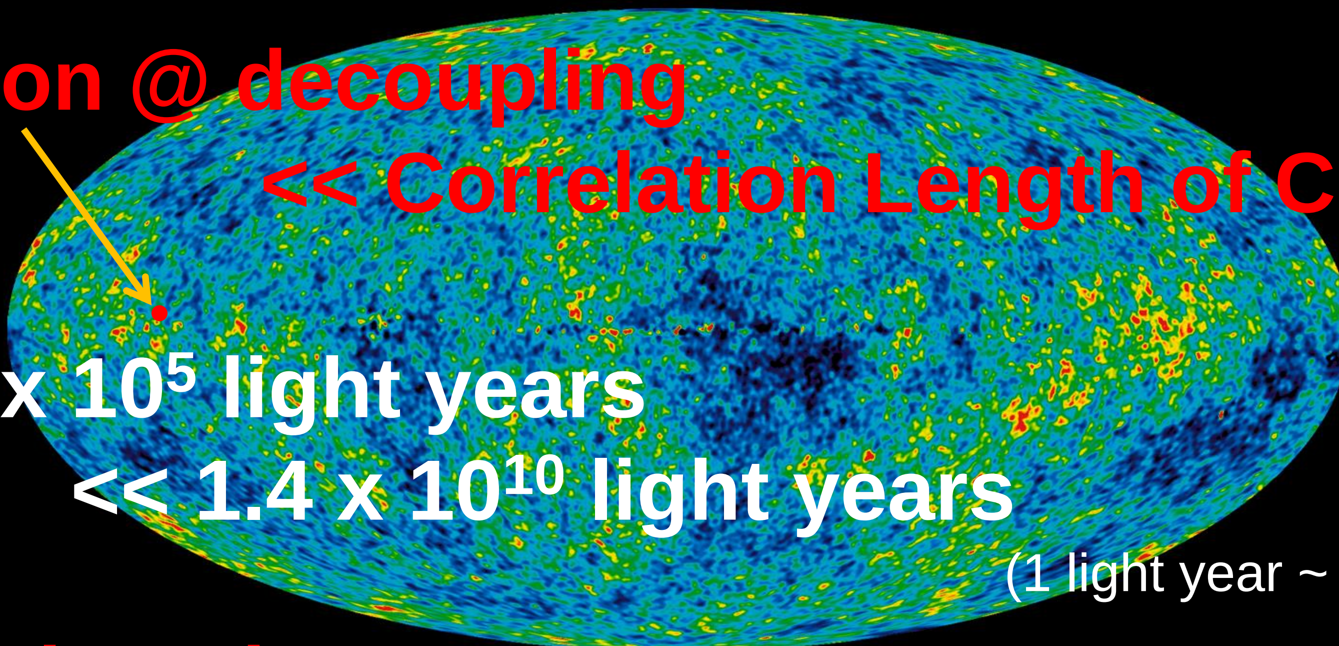
- The initial condition with z=3 scaling may actually solve the flatness problem. (Bramberger, Coates, Magueijo, Mukohyama, Namba and Watanabe 2017)
- Absence of local Hamiltonian constraint leads to DM as integration "constant" (Mukohyama 2009).



Where are we from?

Primordial Fluctuations

Horizon Problem
& Scale-Invariance



Horizon @ decoupling

<< Correlation Length of CMB

3.8 x 10^{5} light years

<< 1.4 x 10^{10} light years

(1 light year ~ 10^{18} cm)

Scale-invariant spectrum

Δ ~ constant

$$\langle \zeta_{\vec{k}} \zeta_{\vec{k}'} \rangle = (2\pi)^3 \delta^3(\vec{k} + \vec{k}') \frac{\Delta}{|\vec{k}|^3}$$

Usual story

- $\omega^2 >> H^2$: oscillate H = (da/dt) / a

  $\omega^2 << H^2$: freeze a : scale factor

  oscillation → freeze-out iff $d(H^2/\omega^2)/dt > 0$

  $\omega^2 = k^2/a^2$ leads to $d^2a/dt^2 > 0$

  Generation of super-horizon fluctuations requires accelerated expansion, i.e. inflation.

- Scaling law

  t → b t (E → b^{-1}E)

  x → b x

  $\phi \rightarrow b^{-1}\phi$

  ⇨ $\delta\phi \propto E \sim H$

  Scale-invariance requires almost const. H, i.e. inflation.

New story with z=3

Mukohyama 2009

- oscillation → freeze-out iff $d(H^2/\omega^2)/dt > 0$

  $\omega^2 = M^{-4}k^6/a^6$ leads to $d^2(a^3)/dt^2 > 0$

  OK for $a \sim t^p$ with $p > 1/3$

- Scaling law

  $t \to b^3 t$ ($E \to b^{-3} E$)

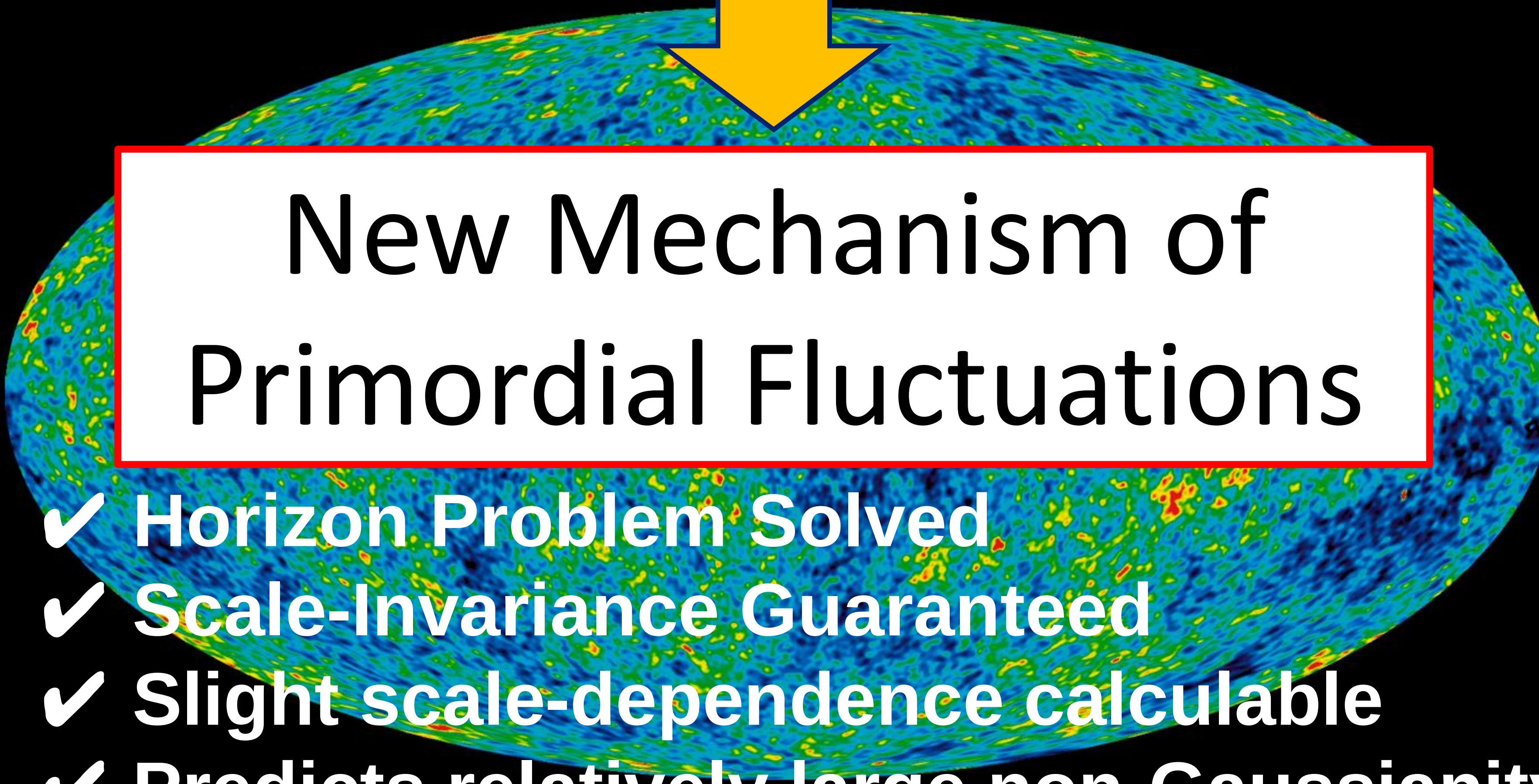
  $x \to b\, x$

  $\phi \to b^0 \phi$ ⇒ $\delta\phi \propto E^0 \sim H^0$

  Scale-invariant fluctuations!

- Tensor perturbation $P_h \sim M^2/M_{Pl}^2$

New Quantum Gravity



New Mechanism of Primordial Fluctuations

- ✔ **Horizon Problem Solved**
- ✔ **Scale-Invariance Guaranteed**
- ✔ **Slight scale-dependence calculable**
- ✔ **Predicts relatively large non-Gaussianity**

"Vainshtein screening" in projectable (N=N(t)) HL gravity

- Perturbative expansion breaks down in the $\lambda \rightarrow 1+0$ limit.
- Non-perturbative analysis shows continuity and GR is recovered in the $\lambda \rightarrow 1+0$ limit.

Screening scalar graviton

$$L = \left[f\left(\frac{\zeta_T}{\lambda - 1} \right) + g(\zeta_T, \lambda) \right] \frac{M_{Pl}^2 \dot{\zeta}_T^2}{\lambda - 1} - V(\zeta_T, D_i) \text{ + matter}$$

subleading

Independent of λ
No time derivative

Local in time, no time derivative
Non-local in space, each term has the same # of spatial derivatives in denominator and numerator

$\lambda \to 1$

$$L \sim \dot{\zeta}_c^2 \text{ + matter}$$

"Canonically normalized" scalar graviton decouples from the rest of the world.

Analogue of Vainshtein screening

“Vainshtein screening” in projectable (N=N(t)) HL gravity

- Perturbative expansion breaks down in the $\lambda \to 1+0$ limit.
- Non-perturbative analysis shows continuity and GR is recovered in the $\lambda \to 1+0$ limit.
  - ✓Spherically-sym, static, vacuum (Mukohyama 2010)
  - ✓Spherically-sym, dynamical, vacuum (Mukohyama 201?)
  - ✓Spherically-sym, static, with matter (Mukohyama 201?)
  - ✓General super-horizon perturbations with matter (Izumi-Mukohyama 2011; Gumrukcuoglu-Mukohyama-Wang 2011)
- “Vainshtein radius” can be pushed to infinity in the $\lambda \to 1+0$ limit.

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