Stable cosmology in chameleon bigravity

CosPA 27, 17.12.14

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Based on

Stable cosmology in chameleon bigravity

arXiv 1711.04655, with
A. De Felice, S. Mukohyama, and Yota Watanabe

Outline

1. Describe the theory
2. Our goal: *realistic background cosmology*
3. Construct the tools
   i. The action
   ii. Scaling solutions
   iii. Stability
4. Numerics and results
5. Conclusion

Chameleon bigravity

De Felice, Uzan, Mukohyama, 1702.04490

- A theory of 2 gravitons and 1 scalar field
- Chameleon → Environment-dependent graviton mass Khoury and Weltman, astro-ph/0309300
- This extends **massive bigravity** and addresses:
  Hassan and Rosen, 1109.3515
  a. Higuchi bound $m_T^2 \gtrsim H^2$
  b. No fine-tuning needed to pass solar system tests…
  c. … and to have an interesting phenomenology

De Felice, Gumrukcuoglu, Mukohyama, Tanahashi, Tanaka, 1404.0008

Goal of the work

- Show that the theory can accommodate a “realistic” background cosmology !

Does everything work as planned?

- Higuchi bound
- Stability
- Modes

The action

Chameleon bigravity side

$$s^{\mu}{}_{\nu} \equiv \left(\sqrt{g^{-1}f}\right)^{\mu}{}_{\nu}$$

$$\boxed{\begin{aligned} S_{\mathrm{EH}} &= \frac{M_g^2}{2}\int R[g]\sqrt{-g}d^4x + \frac{\kappa M_g^2}{2}\int R[f]\sqrt{-f}d^4x\,, \\ S_m &= M_g^2 m^2 \int \sum_{i=0}^{4} \beta_i(\phi) e_i[s]\sqrt{-g}d^4x\,, \\ S_\phi &= -\frac{1}{2}\int g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\sqrt{-g}d^4x \end{aligned}}$$

Matter side

$$\boxed{S_{\mathrm{mat}} = \int \mathcal{L}_{\mathrm{mat}}(\psi, \tilde{g}_{\mu\nu})d^4x}$$

$$\tilde{g}_{\mu\nu} \equiv A(\phi)^2 g_{\mu\nu}$$

Scaling solutions

Exponential couplings $A(\phi) = e^{\beta\phi/M_g}\,, \quad \beta_i(\phi) = -c_i e^{-\lambda\phi/M_g}$

- Exact **radiation** dominated and **Λ**-dominated solutions

$$ds_g^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j\,, \quad ds_f^2 = \xi^2(t)\left[-c^2(t)dt^2 + a^2(t)\delta_{ij}dx^i dx^j\right]$$

$$\boxed{\begin{array}{c} \dfrac{d\varphi}{dN_e} = \dfrac{n}{\lambda}, \quad \dfrac{1}{h}\dfrac{dh}{dN_e} = -\dfrac{2}{n}, \\ \xi = const\,, \quad c = const \end{array}} \qquad \varphi \equiv \phi/M_g\,, \quad h \equiv \frac{H}{m}$$

$$N_e = \ln\left(a(t)/a_i\right)$$

- **Dust**-dominated, under condition

$$\beta\left(\lambda^2 - \frac{3c}{c+\kappa\xi^2}\right) = 0$$

- When $\beta \ll 1$ yields an **approximate scaling solution.**

n = 4 (rad.)
n = 3 (dust)
(n = 0) (Λ)

Scaling solutions

Scaling solutions under homogeneous perturbations

$$\begin{cases} \ln h = \ln h_0 - \frac{n}{2} N_e + \epsilon h^{(1)} \,, \\ \varphi = \frac{n N_e}{\lambda}\left(1 + \epsilon \varphi^{(1)}\right) , \\ \xi = \bar{\xi} + \epsilon \xi^{(1)} \,, \\ c = c^{(0)} + \epsilon c^{(1)} \,, \end{cases}$$

yields

$$\varphi^{(1)\prime\prime} + \left(1 + \frac{2}{N_e}\right) \varphi^{(1)\prime} + \mathcal{A}_r \varphi^{(1)} = 0 \,, \qquad {}^{\prime} \equiv \frac{d}{dN_e}$$

$$\varphi^{(1)\prime\prime} + \left(\frac{3}{2} + \frac{2}{N_e}\right) \varphi^{(1)\prime} + \mathcal{A}_m \varphi^{(1)} = 0$$

$$\mathcal{A}_i > 0$$

Inhomogeneous perturbations

ADM splitting

$$ds_g^2 = -\mathcal{N}^2 dt^2 + \gamma_{ij}(\mathcal{N}^i dt + dx^i)(\mathcal{N}^j dt + dx^j)\,, \quad ds_f^2 = -\tilde{\mathcal{N}}^2 dt^2 + \tilde{\gamma}_{ij}(\tilde{\mathcal{N}}^i dt + dx^i)(\tilde{\mathcal{N}}^j dt + dx^j)$$

Perturbations

$$\phi = \bar{\phi} + \delta\phi\,, \quad \psi_\alpha = \bar{\psi}_\alpha + \delta\psi_\alpha$$

$$\mathcal{N} = N(1+\Phi)\,, \quad \mathcal{N}_i = N_i + \delta N_i\,, \quad \gamma_{ij} = a^2\delta_{ij} + \delta\gamma_{ij}\,,$$
$$\tilde{\mathcal{N}} = \tilde{N}(1+\tilde{\Phi})\,, \quad \tilde{\mathcal{N}}_i = \tilde{N}_i + \delta\tilde{N}_i\,, \quad \tilde{\gamma}_{ij} = \tilde{a}^2\delta_{ij} + \delta\tilde{\gamma}_{ij}$$

tensor vector scalar

Decomposition in SO(3) representations

$$\delta N_i = Na(\partial_i B + B_i)\,, \quad \delta\gamma_{ij} = a^2\left[2\delta_{ij}\Psi + \left(\partial_i\partial_j - \frac{\delta_{ij}}{3}\Delta\right)E + \partial_{(i}E_{j)} + h_{ij}\right]\,,$$

$$\delta\tilde{N}_i = \tilde{N}\tilde{a}(\partial_i\tilde{B} + \tilde{B}_i)\,, \quad \delta\tilde{\gamma}_{ij} = \tilde{a}^2\left[2\delta_{ij}\tilde{\Psi} + \left(\partial_i\partial_j - \frac{\delta_{ij}}{3}\Delta\right)\tilde{E} + \partial_{(i}\tilde{E}_{j)} + \tilde{h}_{ij}\right]$$

Inhomogeneous perturbations

2x2 tensor modes

- $c_{T1} = 1,\ c_{T2} = c$
- 2 massive modes

$$m_T^2 = m^2 \Gamma \frac{c + \kappa\xi^2}{\kappa\xi}$$

& 2 massless modes

Non trivial no-ghost condition:

$$c > 0$$

1x2 vector modes

- $c_V = m^2 \Gamma \frac{c+1}{2\xi J}$
- 2 massive modes

$$m_V^2 = m_T^2$$

Non trivial no-ghost condition:

$$J > 0$$

Non trivial no-gradient instability condition:

$$\Gamma > 0$$

2 scalar modes

- massive modes
- non trivial sound speeds

+ matter modes

Non trivial no-ghost condition (large expression)

Non trivial no-gradient instability condition (large expression)

Equations for numerics

Set of equations to integrate

$$\begin{cases} h' = h'(h, \xi, \varphi, \varphi')\,, \\ \varphi'' = \varphi''(h, \xi, \varphi, \varphi')\,, \\ \xi' = \xi'(h, \xi, \varphi, \varphi')\,, \end{cases}$$

Start near a radiation scaling solution

$$h_i' \approx -2h_i\,, \quad \varphi_i' \approx \varphi_{\rm sc}' = \frac{4}{\lambda}\,, \quad \varphi_i'' \approx 0\,, \quad \xi_i' \approx 0$$

Parameters for numerics

We chose some parameters so that $J > 0$ is always satisfied

$$c_3 c_1 - c_2^2 = \mathfrak{A}\,, \quad c_1 + 2c_2 + c_3 = \mathfrak{B}$$

Finally we chose the example parameters

$$c_{\rm in} = \frac{101}{100}\,, \quad c_{\rm V,in}^2 = 1\,, \quad \mathfrak{A} = 1\,, \quad \mathfrak{B} = 1\,,$$

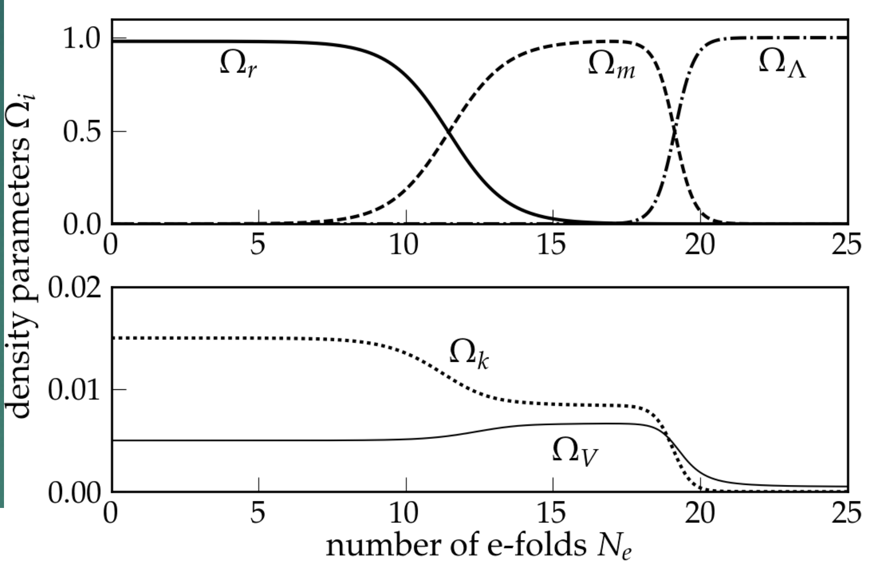
$$\Omega_{\Lambda i}^{\rm EF} = 1 \times 10^{-30}\,, \quad \Omega_{di}^{\rm EF} = 1 \times 10^{-5}\,, \quad \Omega_{ki}^{\rm EF} = \frac{3}{200}\,, \quad \Omega_{Vi}^{\rm EF} = \frac{1}{200}\,,$$

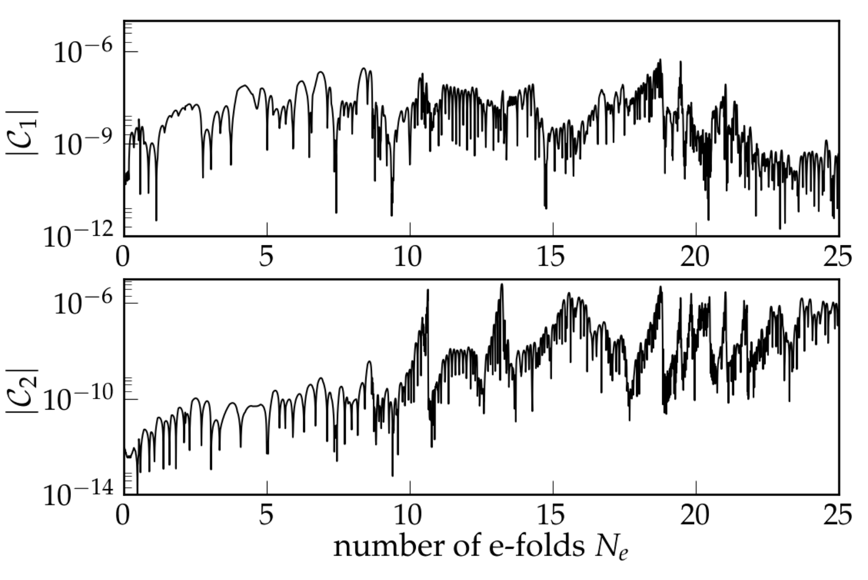
$$\beta = 1 \times 10^{-2}\,, \quad \lambda = \frac{40}{3}$$

Numerical results

Evolution as planned !

- Radiation – dust – Λ domination
- Scaling solutions stable
- Small numerical errors



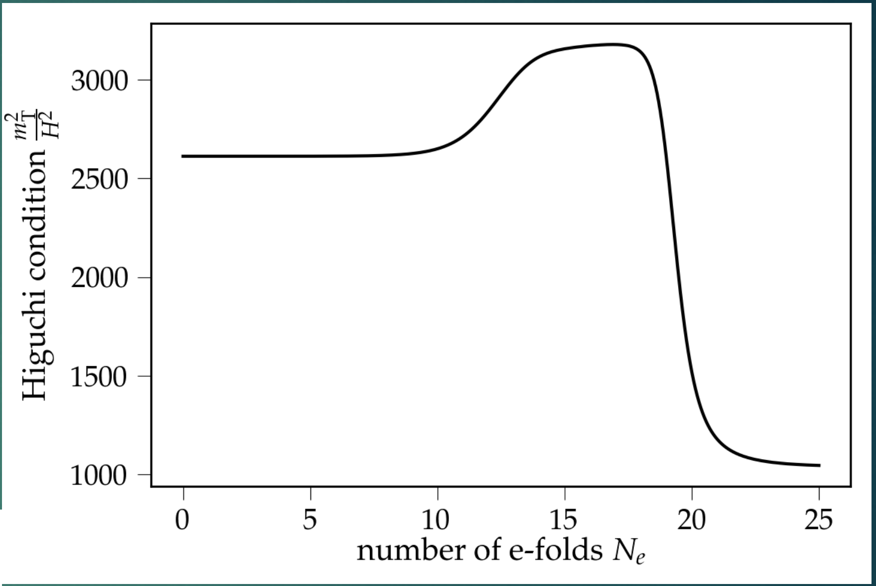


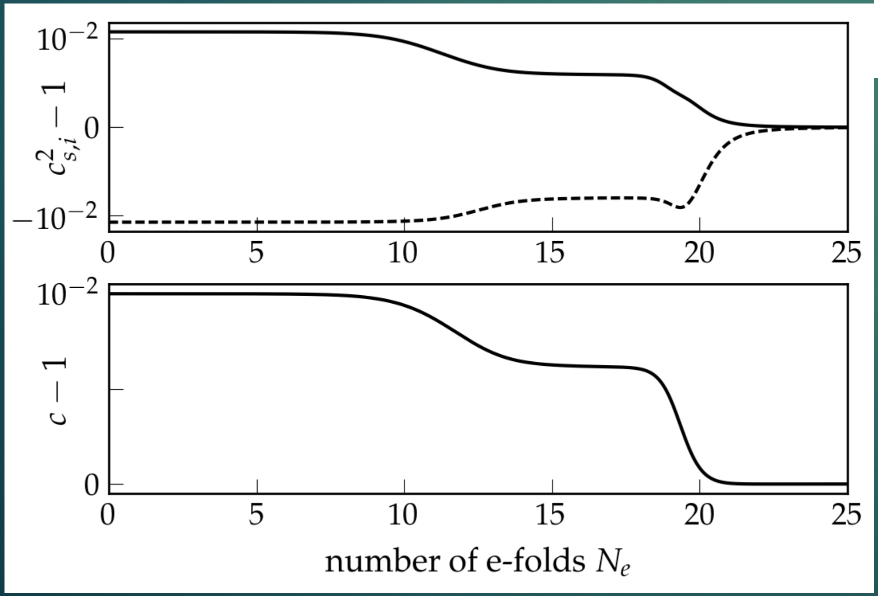
What about the Higuchi bound and the sound-speeds?...

Numerical results

Again just as planned !

- $m_T^2 \gg H^2$ at all times
- Positive sound-speeds, close to 1
- No-ghost conditions satisfied





Promising!

Proof of existence for a stable cosmology in chameleon bigravity !

Summary

i. **Chameleon bigravity** solves the fine-tuning problems of bigravity and extends its reach

ii. Scaling solutions were described

iii. Stability conditions under homogeneous and inhomogeneous perturbations were found

iv. The model propagates 2x2 tensor, 1x2 vector, 2 scalar + matter modes

v. Numerical integration and example background cosmology achieved

Future outlook

A promising model, with avenues for further study !

E.g. constraints from:

i. More precise background cosmology
ii. Evolution of perturbations
iii. Solar-system tests
iv. Modification of wave-forms of GW due to graviton oscillation

Thank you for your attention !

((c) marvel)

GRAVITON BROTHERS