Special Relativity from Soft Gravitons

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with McCullen Sandora, PRD 96 084048 (1704.05071)

Can the laws of special relativity
be violated in principle? Are they exact?

Special Relativity in QED

QED   Spin 1 Photons Coupled to Spin 1/2 Electrons

Special Relativity in QED

QED $$\mathcal{L} = -\frac{1}{4} F_{\mu\alpha}\eta^{\mu\nu}\eta^{\alpha\beta}F_{\nu\beta} + \bar{\psi}(i\gamma^{\mu}D_{\mu} - m)\psi$$

Special Relativity in QED

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$$\{\gamma^{\mu}, \gamma^{\nu}\} = 2\,\eta^{\mu\nu}$$

$$\eta^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -c^2 & 0 & 0 \\ 0 & 0 & -c^2 & 0 \\ 0 & 0 & 0 & -c^2 \end{pmatrix}$$

$$E_q^2 = c^2|\mathbf{q}|^2 \qquad E^2 = c^2|\mathbf{p}|^2 + m^2c^4$$

Violate Special Relativity in QED

QED $$\mathcal{L} = -\frac{1}{4} F_{\mu\alpha} \eta^{\mu\nu} \eta^{\alpha\beta} F_{\nu\beta} + \bar{\psi}(i\gamma_e^\mu D_\mu - m)\psi$$

$$\{\gamma_e^\mu, \gamma_e^\nu\} = 2\,\eta_e^{\mu\nu}$$

$$\eta^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -c^2 & 0 & 0 \\ 0 & 0 & -c^2 & 0 \\ 0 & 0 & 0 & -c^2 \end{pmatrix} \qquad \eta_e^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -c_e^2 & 0 & 0 \\ 0 & 0 & -c_e^2 & 0 \\ 0 & 0 & 0 & -c_e^2 \end{pmatrix}$$

$$E_q^2 = c^2|\mathbf{q}|^2 \qquad E^2 = c_e^2|\mathbf{p}|^2 + m^2 c_e^4$$

Violate Special Relativity in QED

$c_e \neq c$ Cherenkov radiation in vacuum

(Related processes constrain faster than light neutrinos)
(Cohen, Glashow)

Violate Special Relativity in Standard Model

Colladay and Kostelecky 1998
Coleman and Glashow 1998

46 Lorentz violating couplings (CPT even)

Violate Special Relativity in Standard Model

- Local
- Causal
- Unitary
- Renormalizable (EFT)
- No vacuum instability
- No gauge anomalies
- Same # degrees of freedom
- Obeys laws of thermodynamics
- (Gauge invariant)
- …

More Violations of Special Relativity

$$\mathcal{L} = -\frac{1}{4} F_{\mu\alpha} \eta^{\mu\nu} \eta^{\alpha\beta} F_{\nu\beta} + \sum_n \bar{\psi}_n (i\gamma_n^\mu D_\mu - m_n)\psi_n$$

$$\eta^{\mu\nu} \to \eta^{\mu\nu}(-\Delta) \qquad \gamma_n^i D_i \to f_n(\gamma_n^i D_i)$$

$$E_q^2 = K_1(\mathbf{q}) \qquad E_n^2 = \tilde{K}_n(\mathbf{p}_n)$$

$$\downarrow \qquad \downarrow$$

$$K_1(\mathbf{q}) = c^2|\mathbf{q}|^2 + d|\mathbf{q}|^4 + \ldots \qquad \tilde{K}_n(\mathbf{p}) = c_n^2|\mathbf{p}|^2 + m_n^2 c_n^4 + d_n|\mathbf{p}|^4 + \ldots$$

In this talk

Principles: Locality (and Rot. + Trans. Invariance)

In this talk

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Apply to Spin 2

In this talk

Principles: Locality (and Rot. + Trans. Invariance)

Apply to Spin 2

↓

Special Relativity

$$E^2 = p^2c^2 + m^2c^4$$

Particle Spin

Rotation invariance organizes particles by spin

Spin $$s = 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \ldots$$

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"Massless" $$h = \pm s$$

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“Massless” Spin 1 and Spin 2

State: $\langle \mathbf{x} | \psi \rangle = \epsilon(\mathbf{q})\, e^{i\mathbf{q}\cdot\mathbf{x}}$

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Spin 1 $\epsilon_i, \quad \epsilon_i\, q_i = 0$ (Transverse)

"Massless" Spin 1 and Spin 2

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Spin 1 $\epsilon_i\,, \quad \epsilon_i\, q_i = 0$ (Transverse)

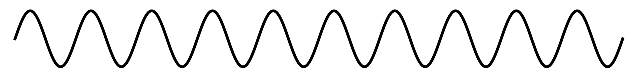
Spin 2 $\epsilon_{ij}\,, \quad \epsilon_{ij}\, q_i = 0, \quad \epsilon_{ii} = 0$ (TT)

Manifest rotation invariance

(no need for gauge invariance for manifest symmetry)

Propagator for Spin 1 and Spin 2

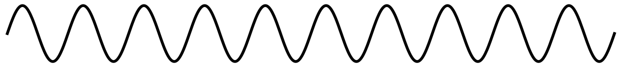
Spin 1

ϵ_i  ϵ_j

$$G_{ij} = \frac{\delta_{ij} - \frac{q_i q_j}{|\mathbf{q}|^2}}{E^2 - K_1(\mathbf{q})}$$

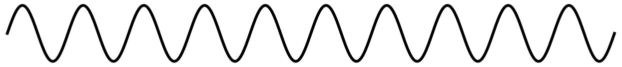
Propagator for Spin 1 and Spin 2

Spin 1

ϵ_i  ϵ_j

$$G_{ij} = \frac{\delta_{ij} - \frac{q_i q_j}{|\mathbf{q}|^2}}{E^2 - K_1(\mathbf{q})}$$

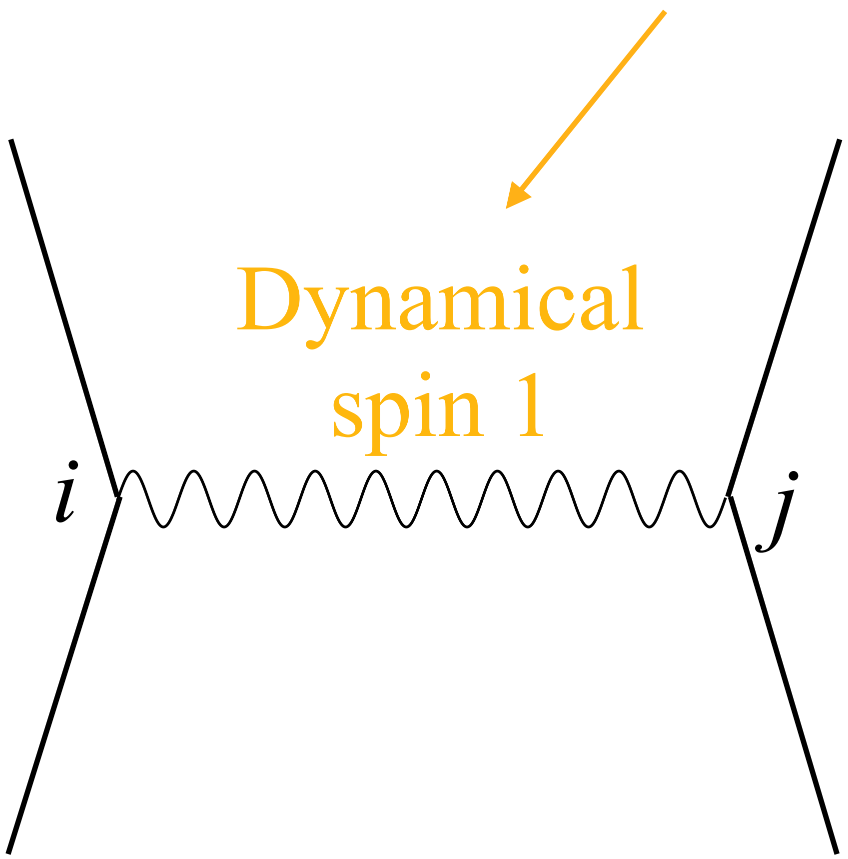
Spin 2

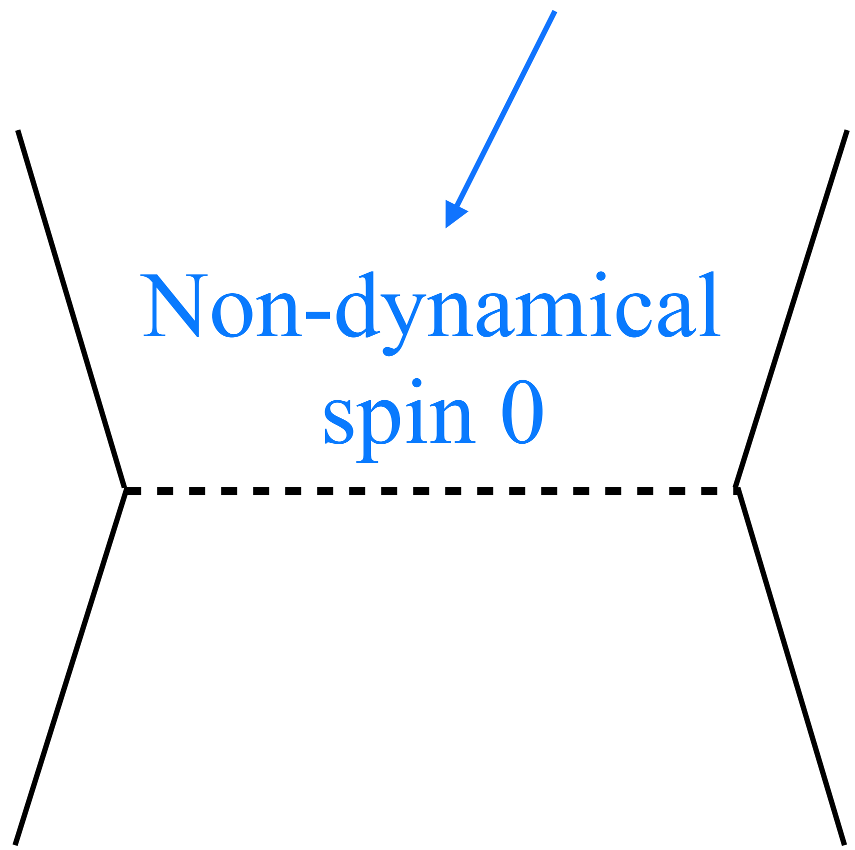
ϵ_{ij}  ϵ_{kl}

$$G_{ijkl} = \frac{\left(\delta_{ij} - \frac{q_i q_j}{|\mathbf{q}|^2}\right)\left(\delta_{kl} - \frac{q_k q_k}{|\mathbf{q}|^2}\right) - (j \leftrightarrow k, l)}{E^2 - K_2(\mathbf{q})}$$

Action from Tree-Exchange

$$\Delta S = \int \frac{d^4 q}{(2\pi)^4} \left[\tilde{j}^i(q) \frac{\delta_{ij} - \frac{q_i q_j}{|\mathbf{q}|^2}}{\omega^2 - K_1(\mathbf{q})} \tilde{j}^{j*}(q) + \frac{\tilde{\rho}(q)\tilde{\rho}^*(q)}{L_1(\mathbf{q})} \right]$$





Magnetic+Electric
(Non-Local)

Electric [Coulombic]
(Non-Local)

Locality

Principle: No instantaneous action at a distance

Locality

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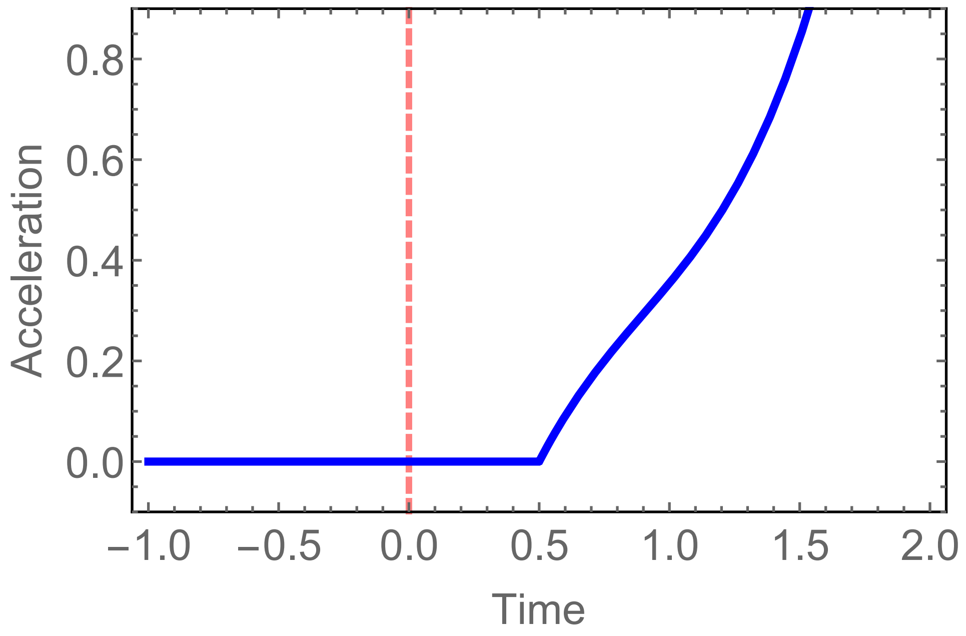
Turn on sources at t=0

$$\frac{d^p}{dt^p}\mathbf{x}_n(t=0)=0 \quad (p \geq 2)$$



Locality

$$\Delta S = \int \frac{d^4 q}{(2\pi)^4} \left[\tilde{J}^i(q) \frac{\delta_{ij} - \frac{q_i q_j}{|\mathbf{q}|^2}}{\omega^2 - K_1(\mathbf{q})} \tilde{J}^{j*}(q) + \frac{\tilde{\rho}(q)\tilde{\rho}^*(q)}{L_1(\mathbf{q})} \right]$$

Constitutive relation $\quad q_i \, \tilde{J}^i(q) = M_1(\mathbf{q}) \, \omega \, \tilde{\rho}(q)$

Locality

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$$\frac{d^p}{dt^p} \mathbf{x}_n(t=0) = 0 \quad (p \geq 2) \qquad \Longrightarrow$$

$$\Delta S = \int \frac{d^4q}{(2\pi)^4} \left[\sum_p \frac{\mathrm{Poly}_p(|\mathbf{q}|^2)}{\omega^p} \right]$$

Locality

$$\Delta S = \int \frac{d^4q}{(2\pi)^4} \left[\tilde{J}^i(q) \frac{\delta_{ij} - \frac{q_i q_j}{|\mathbf{q}|^2}}{\omega^2 - K_1(\mathbf{q})} \tilde{J}^{j*}(q) + \frac{\tilde{\rho}(q)\tilde{\rho}^*(q)}{L_1(\mathbf{q})} \right]$$

Constitutive relation $\quad q_i \, \tilde{J}^i(q) = M_1(\mathbf{q}) \, \omega \, \tilde{\rho}(q)$

$$K_1(\mathbf{q}) = c^2|\mathbf{q}|^2 + |\mathbf{q}|^4 P_a(|\mathbf{q}|^2)$$

$$L_1(\mathbf{q})^{-1} = \frac{1}{|\mathbf{q}|^2} + P_b(|\mathbf{q}|^2)$$

$$M_1(\mathbf{q})^2 = 1 + |\mathbf{q}|^2 P_c(|\mathbf{q}|^2)$$

(locality)

Local Lagrangian for Photon Fields

Field: $(\epsilon_i(\mathbf{q})\,\hat{a}_{\mathbf{q}} + \epsilon_i^*(\mathbf{q})\,\hat{a}^\dagger_{-\mathbf{q}})/\sqrt{2E_q} \to A_i(\mathbf{x})$

$$\mathcal{L} = \frac{1}{2}|\dot{\mathbf{A}}|^2 - \frac{1}{2}\nabla \times \mathbf{A} \cdot \frac{K_1(-\Delta)}{-\Delta}\nabla \times \mathbf{A} + \mathbf{A}\cdot\mathbf{J} - \phi\,\rho + \frac{1}{2}\phi\,L_1(-\Delta)\phi + \dot{\mathbf{A}}\cdot\mathcal{M}_1(-\Delta)\nabla\phi + \lambda(\nabla\cdot\mathbf{A})^2$$

Not gauge invariant

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Not gauge invariant

$$K_1 = -c^2\Delta + \mathcal{O}(\Delta^2), \quad L_1 = -\Delta + \mathcal{O}(\Delta^2), \quad \mathcal{M}_1 = 1 + \mathcal{O}(\Delta)$$

Soft Gauge Invariance for Spin 1

$$\overset{\mathbf{E}\downarrow}{}\qquad\qquad\overset{\mathbf{B}\downarrow}{}$$

$$\mathcal{L} = \frac{1}{2}|\dot{\mathbf{A}} + \nabla\phi|^2 - \frac{c^2}{2}|\nabla \times \mathbf{A}|^2 + \mathbf{A}\cdot\mathbf{J} - \phi\,\rho + \mathcal{O}(\Delta^2)$$

4-component vector: $A_\mu \equiv (-\phi, \mathbf{A})$

Locality $\Longrightarrow$ $A_\mu \equiv A_\mu + \partial_\mu \alpha$ (slowly varying α)

$$\epsilon_\mu(q) \to \epsilon_\mu(q) + q_\mu\,\tilde{\alpha} \quad (\text{soft } \tilde{\alpha})$$

Soft Gauge Invariance for Spin 2

Soft Gauge Invariance for Spin 2

Appendix:– Here we extend the locality analysis to the spin 2 case. The most general interaction from tree-level graviton exchange is

$$\Delta S = \int \frac{d^4 q}{(2\pi)^4}\Big[\tilde{\tau}_{ij}\frac{r_{ik}r_{jl}-\frac{1}{2}r_{ij}r_{kl}}{\omega^2 - K_2(\mathbf{q})}\tilde{\tau}^*_{kl} + \frac{2\,\tilde{\pi}_i\,\tilde{\pi}^*_i}{N_2(\mathbf{q})} - \frac{\tilde{\sigma}\,\tilde{\tau}^*_{ii}}{R_2(\mathbf{q})} - \frac{1}{2}\frac{\tilde{\sigma}\,\tilde{\sigma}^*}{L_2(\mathbf{q})} - \frac{1}{2}\frac{\omega^2\tilde{\sigma}\,\tilde{\sigma}^*}{|\mathbf{q}|^2 L_2'(\mathbf{q})}\Big], \quad (34)$$

where $r_{ij} \equiv \delta_{ij} - \frac{q_i q_j}{|\mathbf{q}|^2}$. For non-localities to cancel, we need the constitutive relations: $q_i\,\tilde{\tau}_{ij} = M_2(\mathbf{q})\,\omega\,\tilde{\pi}_i$ and $q_i\,\tilde{\pi}_i = M_2'(\mathbf{q})\,\omega\,\tilde{\sigma}$. Imposing locality we find that these functions must be related to polynomials as:

$$K_2(\mathbf{q}) = m_2^2\,\delta_{P_2 P_2',0} + c_g^2|\mathbf{q}|^2 + |\mathbf{q}|^4 P_d(|\mathbf{q}|^2), \quad (35)$$

$$N_2(\mathbf{q})^{-1} = \frac{P_2}{|\mathbf{q}|^2} + P_e(|\mathbf{q}|^2), \quad (36)$$

$$R_2(\mathbf{q})^{-1} = \frac{\sqrt{P_2 P_2'}}{|\mathbf{q}|^2} + P_f(|\mathbf{q}|^2), \quad (37)$$

$$L_2(\mathbf{q})^{-1} = \frac{c_g^2 P_2 P_2'}{|\mathbf{q}|^2} + P_g(|\mathbf{q}|^2), \quad (38)$$

$$L_2'(\mathbf{q})^{-1} = \frac{P_2 P_2'}{|\mathbf{q}|^2} + P_h(|\mathbf{q}|^2), \quad (39)$$

$$M_2(\mathbf{q})^2 = P_2 + |\mathbf{q}|^2 P_i(|\mathbf{q}|^2), \quad (40)$$

$$M_2'(\mathbf{q})^2 = P_2' + |\mathbf{q}|^2 P_j(|\mathbf{q}|^2). \quad (41)$$

So a long range force requires $c_g^2 P_2 P_2' \neq 0$. Hence the graviton must be massless, and we can set $P_2 = P_2' = 1$, and we need $c_g \neq 0$. By Fourier transforming to the local field representation $(\epsilon_{ij}(\mathbf{q})\,\hat{a}_{\mathbf{q}} + \epsilon^*_{ij}(\mathbf{q})\,\hat{a}^\dagger_{-\mathbf{q}})/\sqrt{2E_q} \to h_{ij}(\mathbf{x})$, we can construct the following Lagrangian:

$$\mathcal{L} = \frac{1}{2}|\dot{\mathbf{h}}|^2 - \frac{1}{2}\nabla\times\mathbf{h}\cdot\mathcal{K}_2(-\Delta)\nabla\times\mathbf{h} + h_{ij}\,\tau^{ij} + 2\,\psi_i\,\pi_i + \phi\,\sigma + \frac{1}{2}\nabla\psi\cdot\mathcal{N}_2(-\Delta)\nabla\psi + \nabla\psi\cdot\mathcal{M}_2(-\Delta)\dot{\mathbf{h}} + \phi\,\mathcal{R}_2(-\Delta)\nabla\nabla\cdot\mathbf{h} + \lambda(\nabla\cdot\mathbf{h})^2. \quad (42)$$

For ease of notation, we have defined the dot product between two matrices as $A\cdot B \equiv A_{ij}B_{ij} - A_{ii}B_{jj}$. The functions here are related to the above as $\mathcal{K}_2(-\Delta) \equiv K_2(-\Delta)/(-\Delta)$, $\mathcal{N}_2(-\Delta) \equiv N_2(-\Delta)/(-\Delta)$, $\mathcal{M}_2(-\Delta) \equiv M_2'(-\Delta)R_2(-\Delta)/(-\Delta)$, and $\mathcal{R}_2(-\Delta) \equiv R_2(-\Delta)/(-\Delta)$. Demanding that the spin 2 exchange arises from an action places further consistency conditions on the functions $L_2^{-1} = K_2/R_2^2$, $L_2'^{-1} = 4M_2'^2/N_2 - 3|\mathbf{q}|^2/R_2^2$, and $M_2 = M_2' R_2/N_2$. Note that in our convention, the gravitational couplings are included in the sources τ^{ij}, π^i, σ.

We then find that to leading order in a derivative expansion, the first, second, sixth and seventh terms in (42) assemble into $\mathcal{L}_2 = \frac{1}{2}|\dot{\mathbf{h}} + \nabla\psi|^2 - \frac{c_g^2}{2}|\nabla\times\mathbf{h}|^2$, which is invariant under the gauge transformation $h_{ij} \to h_{ij} + \nabla_{(i}\alpha_{j)}$, $\psi_i \to \psi_i - \dot{\alpha}_i$. Likewise, the seventh and eighth terms in (42) are invariant under the gauge transformation $\psi_i \to \psi_i - \nabla_i\alpha_0$, $\phi \to \phi + \dot{\alpha}_0$ (up to a total derivative). A 4×4 matrix $h_{\mu\nu}$ can then be assembled as $h_{0i} = -\psi_i$, $h_{00} = \phi$, and we obtain soft gauge invariance, as reported in Eq. (12).

Soft Gauge Invariance for Spin 2

$$G_{ijkl} = \frac{\left(\delta_{ij} - \frac{q_i q_j}{|\mathbf{q}|^2}\right)\left(\delta_{kl} - \frac{q_k q_k}{|\mathbf{q}|^2}\right) - (j \leftrightarrow k, l)}{E^2 - K_2(\mathbf{q})}$$

2 types of non-localities

Add non-dynamical h_{00} (Newton) h_{0i}

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2 types of non-localities

Add non-dynamical h_{00} (Newton) h_{0i}

Find $V(r) \sim \frac{1}{r} + \dots$ $K_2(\mathbf{q}) = c_g^2 |\mathbf{q}|^2 + \dots$

Locality $\Longrightarrow h_{\mu\nu} \equiv h_{\mu\nu} + \partial_\mu \alpha_\nu + \partial_\nu \alpha_\mu$ (slowly varying α_μ)

$$\epsilon_{\mu\nu}(q) \to \epsilon_{\mu\nu}(q) + q_\mu \tilde{\alpha}_\nu + q_\nu \tilde{\alpha}_\mu \quad (\text{soft } \tilde{\alpha}_\mu)$$

Soft Gauge Invariance for Spin 1 and 2

Soft gauge invariance is required to ensure that the non-dynamical fields are mixed with the propagating degrees of freedom, such that long range forces inherit the finite speed of propagation of the spin 1 or spin 2 particles

(Soft) Gauge Invariant Spin 2 Theories

Difficult. Simple attempts fail… Leading to many questions:

— What restrictions are placed on $E_n^2 = \tilde{K}_n(\mathbf{p}_n)$?

— What if we include fermions and gauge/vector bosons?

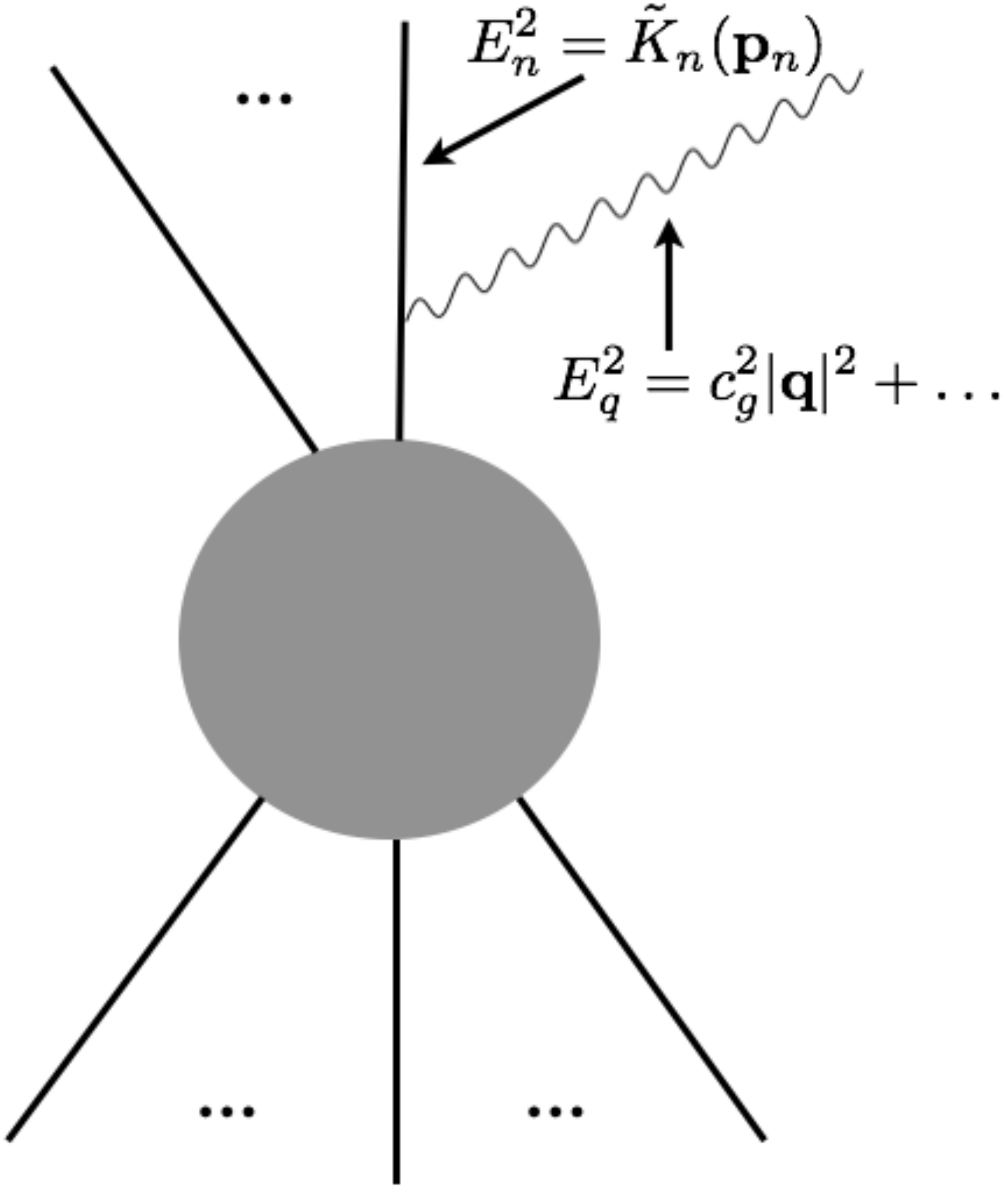
— What constraints apply to $E_q^2 = \tilde{K}_q(\mathbf{q})$?

— Is the equivalence principle still required for consistency?

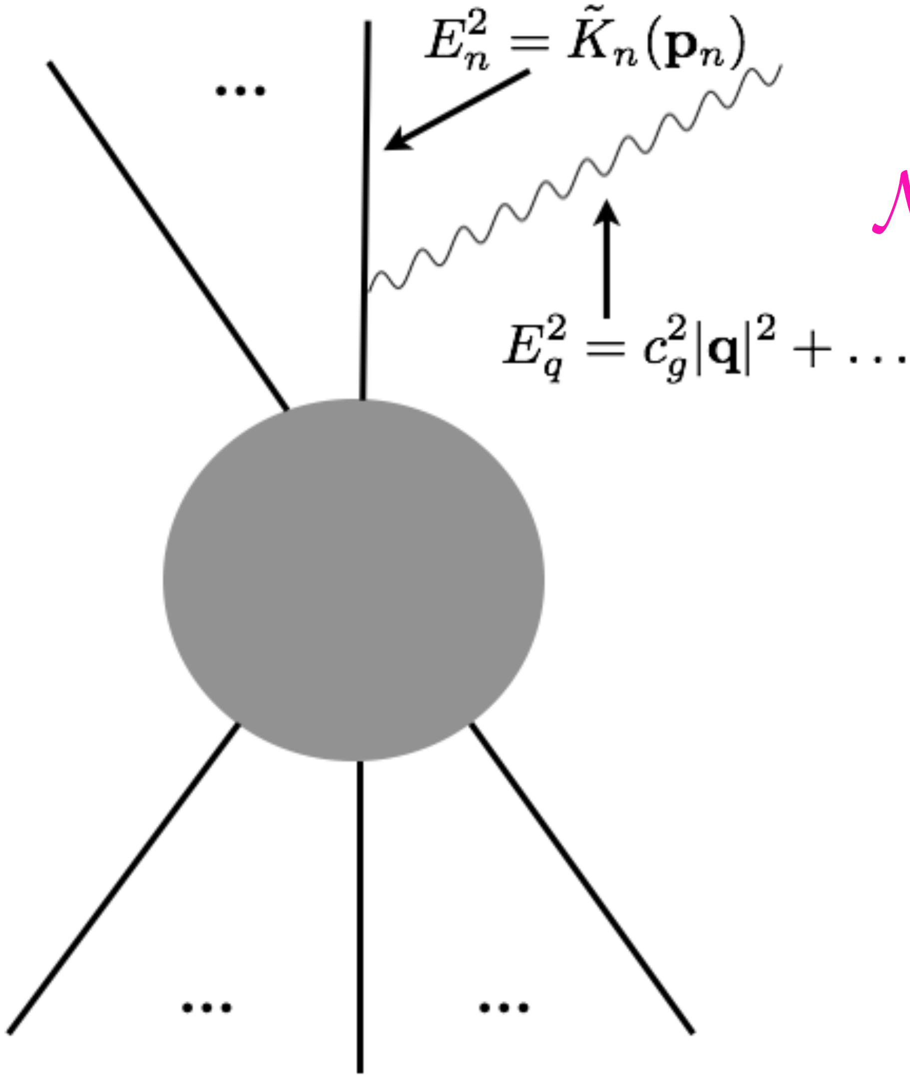
— What objects can we couple to?

Need systematic analysis

Soft Graviton Scattering

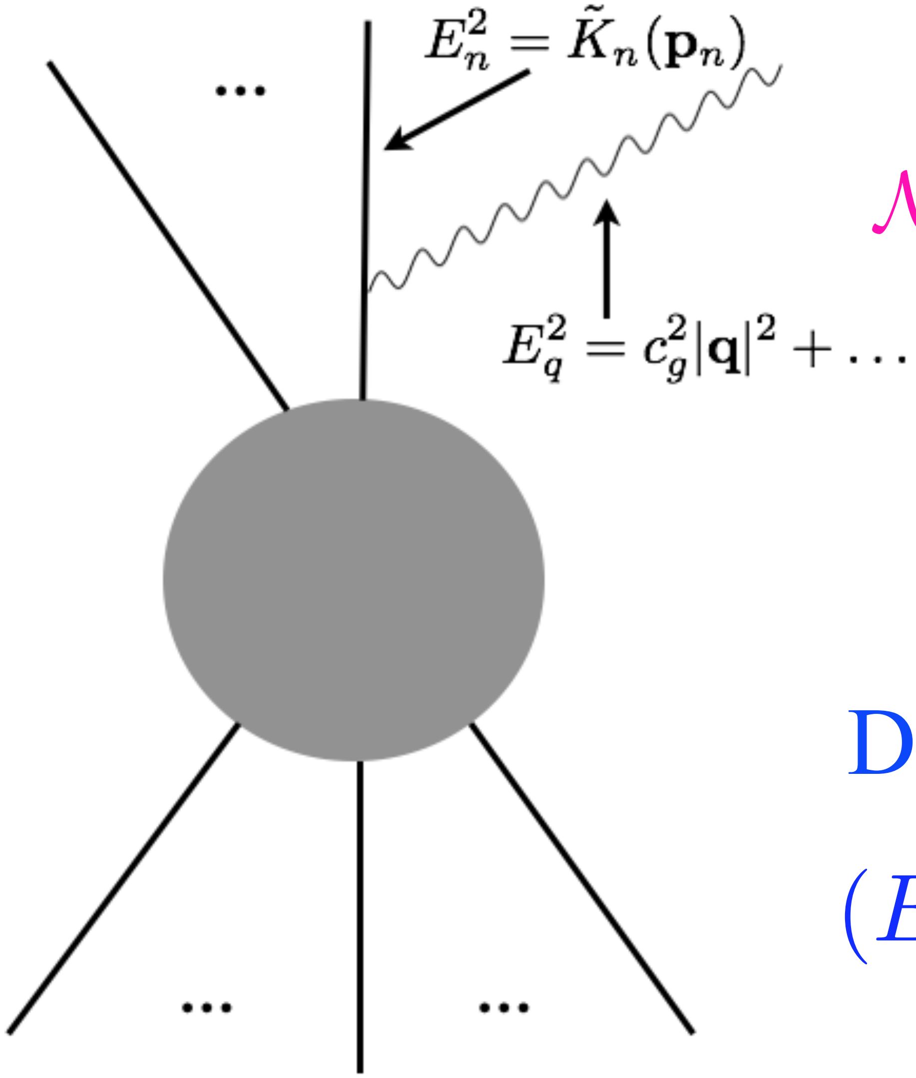


Soft Graviton Scattering



$$\mathcal{M}_{N+1} = \mathcal{M}_N \times \left[\sum_i \frac{\epsilon_{\mu\nu}(q)\, \mathcal{T}_i^{\mu\nu}(p_i)}{(E_i - E_q)^2 - \tilde{K}_i(\mathbf{p}_i - \mathbf{q})} + \sum_f \frac{\epsilon_{\mu\nu}(q)\, \mathcal{T}_f^{\mu\nu}(p_f)}{(E_f + E_q)^2 - \tilde{K}_f(\mathbf{p}_f + \mathbf{q})} \right]$$

Soft Graviton Scattering



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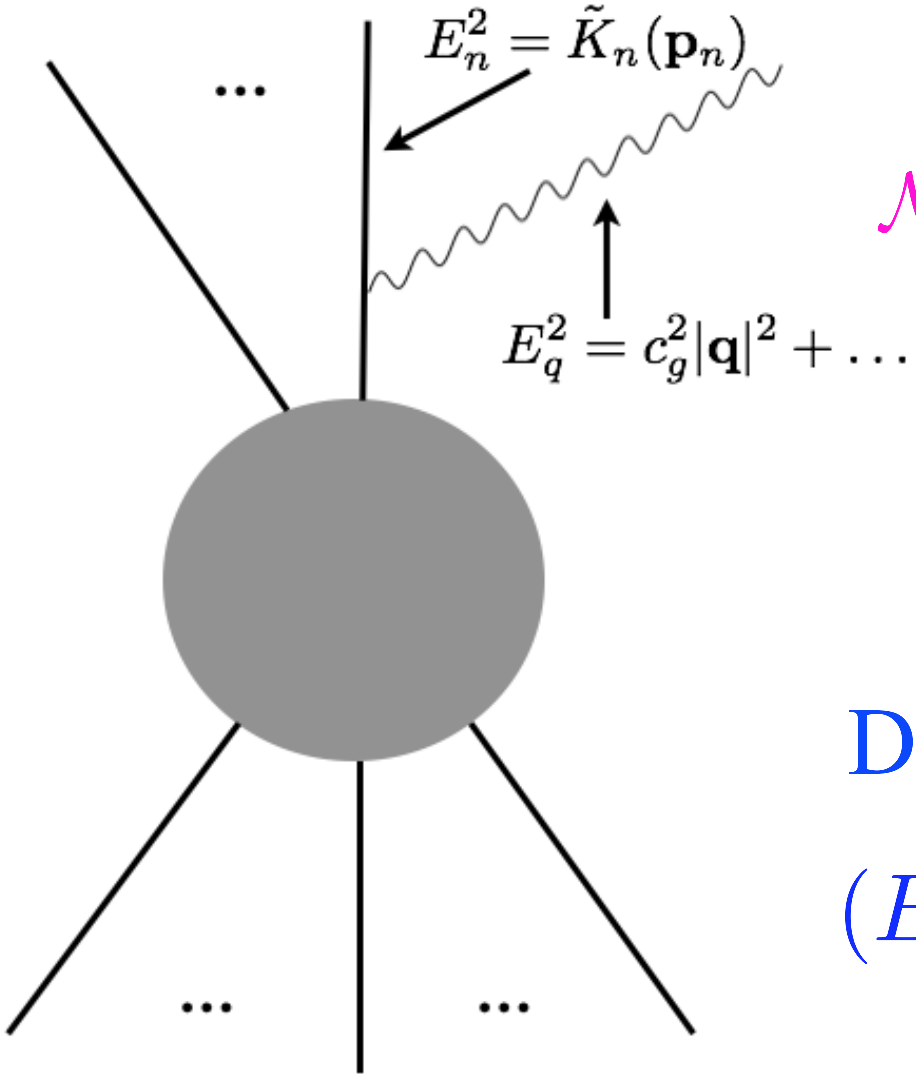
Denominator:

$$(E_n \mp E_q)^2 - \tilde{K}_n(\mathbf{p}_n \mp \mathbf{q}) = \mp 2\, q_\alpha\, \zeta_n^\alpha(\mathbf{p}_n)$$

$$q_\alpha \equiv (E_q, -\mathbf{q}) \qquad \zeta_n^\alpha(p_n) \equiv \left(E_n, \frac{1}{2} \frac{\partial \tilde{K}_n}{\partial \mathbf{p}} \Big|_{\mathbf{p}_n} \right)$$

Soft Graviton Scattering



$$\mathcal{M}_{N+1} = \mathcal{M}_N \times \left[-\sum_i \frac{\epsilon_{\mu\nu}(q)\, \mathcal{T}_i^{\mu\nu}(p_i)}{2\, q_\alpha\, \zeta_i^\alpha(\mathbf{p}_i)} + \sum_f \frac{\epsilon_{\mu\nu}(q)\, \mathcal{T}_f^{\mu\nu}(p_f)}{2\, q_\alpha\, \zeta_f^\alpha(\mathbf{p}_f)} \right]$$

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Unitarity and Locality Constraint

$$\epsilon_{\mu\nu}(q) \to \epsilon_{\mu\nu}(q) + q_\mu \, \tilde{\alpha}_\nu + q_\nu \, \tilde{\alpha}_\mu \quad (\text{soft } \tilde{\alpha}_\mu)$$

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Cancel graviton momenta

$$\mathcal{T}_n^{\mu\nu}(p_n) \propto \zeta_n^\mu(p_n)$$

Unitarity and Locality Constraint

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Cancel graviton momenta

$$\mathcal{T}_n^{\mu\nu}(p_n) = g_n(E_n)\, \zeta_n^\mu(p_n)\, \zeta_n^\nu(p_n)$$

Unitarity and Locality Constraint

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Cancel graviton momenta

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$$\sum_i g_i(E_i)\, \zeta_i^\nu(p_i) = \sum_f g_f(E_f)\, \zeta_f^\nu(p_f)$$

Conservation Laws

$\underline{\nu = 0}$

$$\sum_i g_i(E_i)\, E_i = \sum_f g_f(E_f)\, E_f$$

(Grav. Coupling)

$$g_n(E_n) = \kappa$$

Conservation Laws

$\underline{\nu = 0}$

$$\sum_i g_i(E_i)\, E_i = \sum_f g_f(E_f)\, E_f$$

(Grav. Coupling)

$$\boxed{g_n(E_n) = \kappa + \frac{Q_n}{E_n}}$$

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$$\sum_i g_i(E_i) \frac{\partial \tilde{K}_i}{\partial \mathbf{p}}\Big|_{\mathbf{p}_i} = \sum_f g_f(E_f) \frac{\partial \tilde{K}_f}{\partial \mathbf{p}}\Big|_{\mathbf{p}_f}$$

$$\boxed{g_n(E_n) \frac{\partial \tilde{K}_n}{\partial \mathbf{p}}\Big|_{\mathbf{p}_n} = a\, \mathbf{p}_n}$$

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$$\boxed{g_n(E_n) \frac{\partial \tilde{K}_n}{\partial \mathbf{p}}\Big|_{\mathbf{p}_n} = a\, \mathbf{p}_n}$$

(Disp. Relation)

$$\boxed{\kappa\, E_n^2 + 2\, Q_n\, E_n = \frac{a}{2} |\mathbf{p}_n|^2 + b_n}$$

Dispersion Relation

$$\kappa\, E_n^2 + 2\, Q_n\, E_n = \kappa\, c_g^2 |\mathbf{p}_n|^2 + b_n$$

Complete the square: $$E_n \to E_n - \frac{Q_n}{\kappa},$$

Dispersion Relation

$$\kappa\, E_n^2 + 2\, Q_n\, E_n = \kappa\, c_g^2 |\mathbf{p}_n|^2 + b_n$$

Complete the square: $$E_n \to E_n - \frac{Q_n}{\kappa},$$

$$\boxed{E_n^2 = |\mathbf{p}_n|^2\, c_g^2 + m_n^2\, c_g^4}$$

Special Relativistic Dispersion Relation

Some Consequences

- Lorentz violating Standard Model

- Lorentz violating approaches to quantum gravity

- Doubly/deformed special relativity

- Lorentz violating alternatives to inflation

Brings into doubt the viability of these models

Conclusions

- Locality requires soft gauge invariance
- For spin ≤ 1, special relativity is easily violated in principle
- For spin 2, special relativity is difficult to violate:
- — The relativistic energy-momentum relation must be exact
- — The leading interactions must be Lorentz invariant
- It remains to be proven if all higher order interactions are too