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Field-theoretic simulations of colliding superconducting strings

[Takashi Hiramatsu]

Rikkyo University

with Daisuke Yamauchi (Kanagawa), Daniele Steer (APC), Marc Lilley (IAP)

Field-theoretic simulations of colliding superconducting strings

(and *gravitational waves from their network*)

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[colliding strings]

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[GWs from strings]

with Daisuke Yamauchi (Kanagawa)

Kibble mechanism

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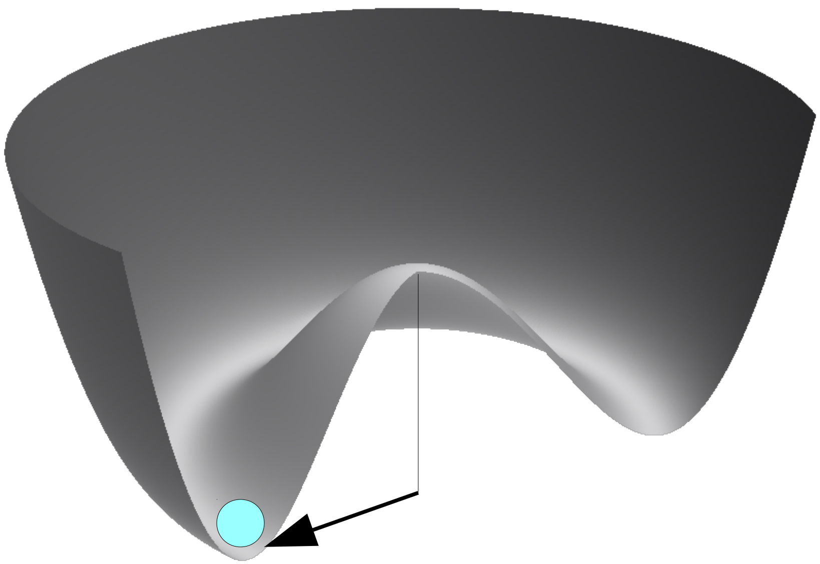
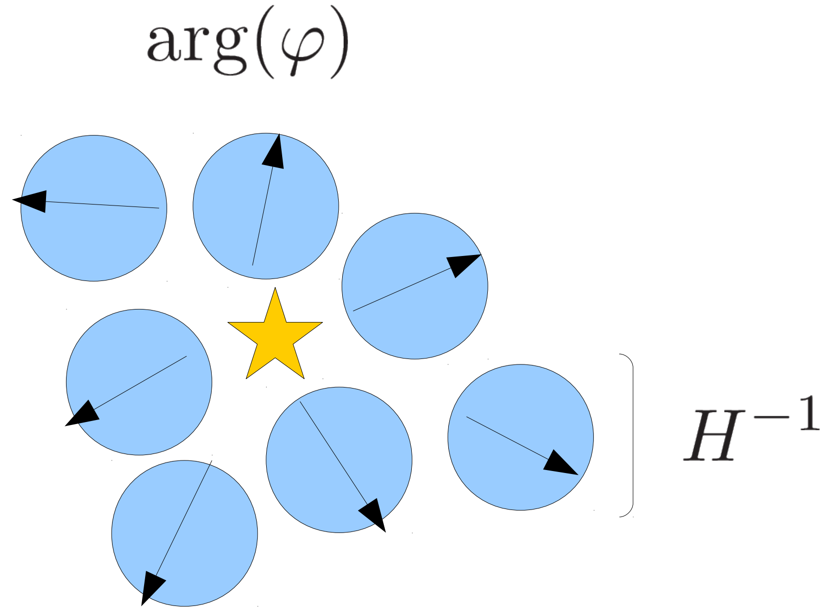
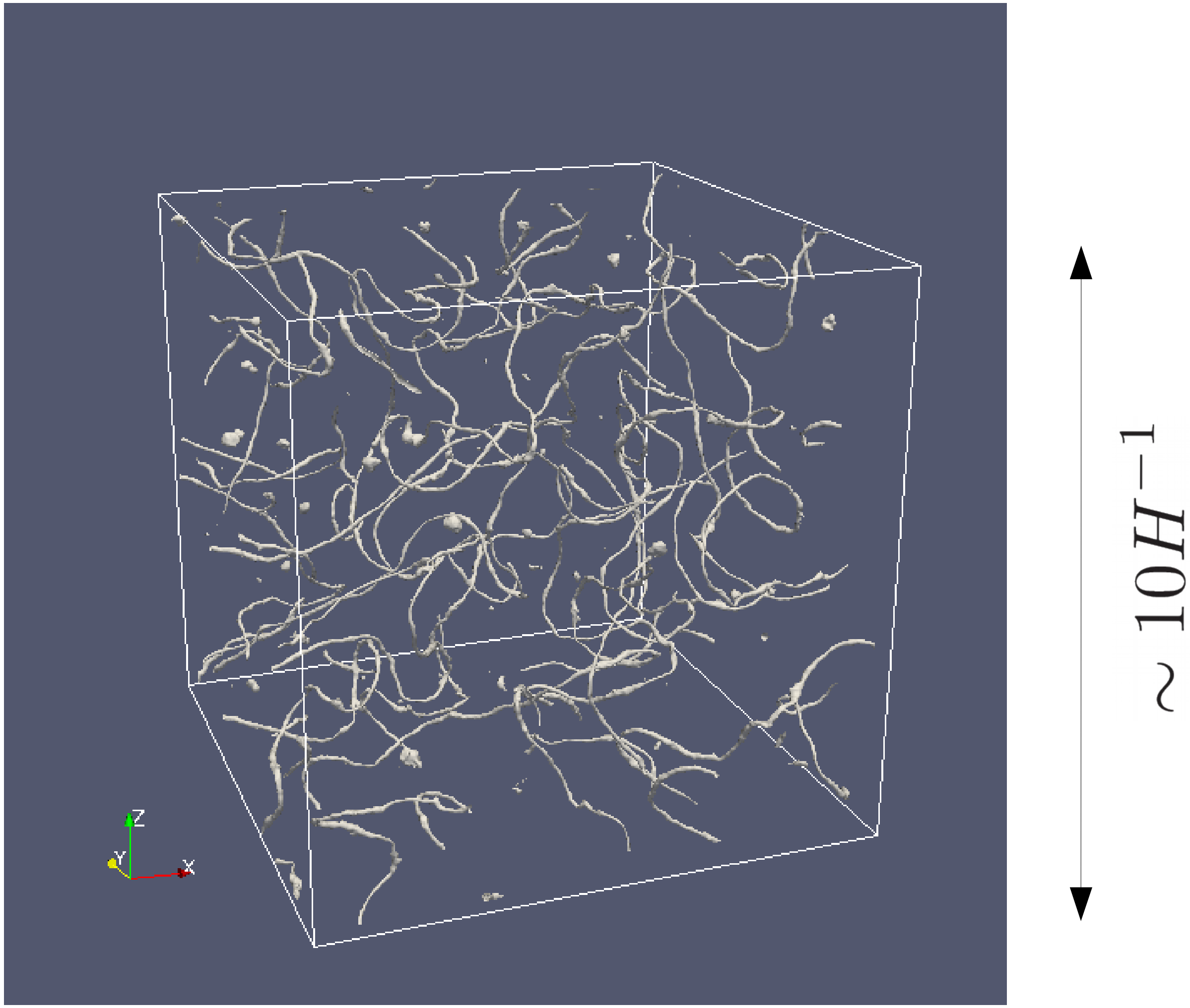




φ is aligned in a causal volume (=Hubble),
but is different in each volume in general.

If the above situation is realised,
a cosmic string appears at  .

String network



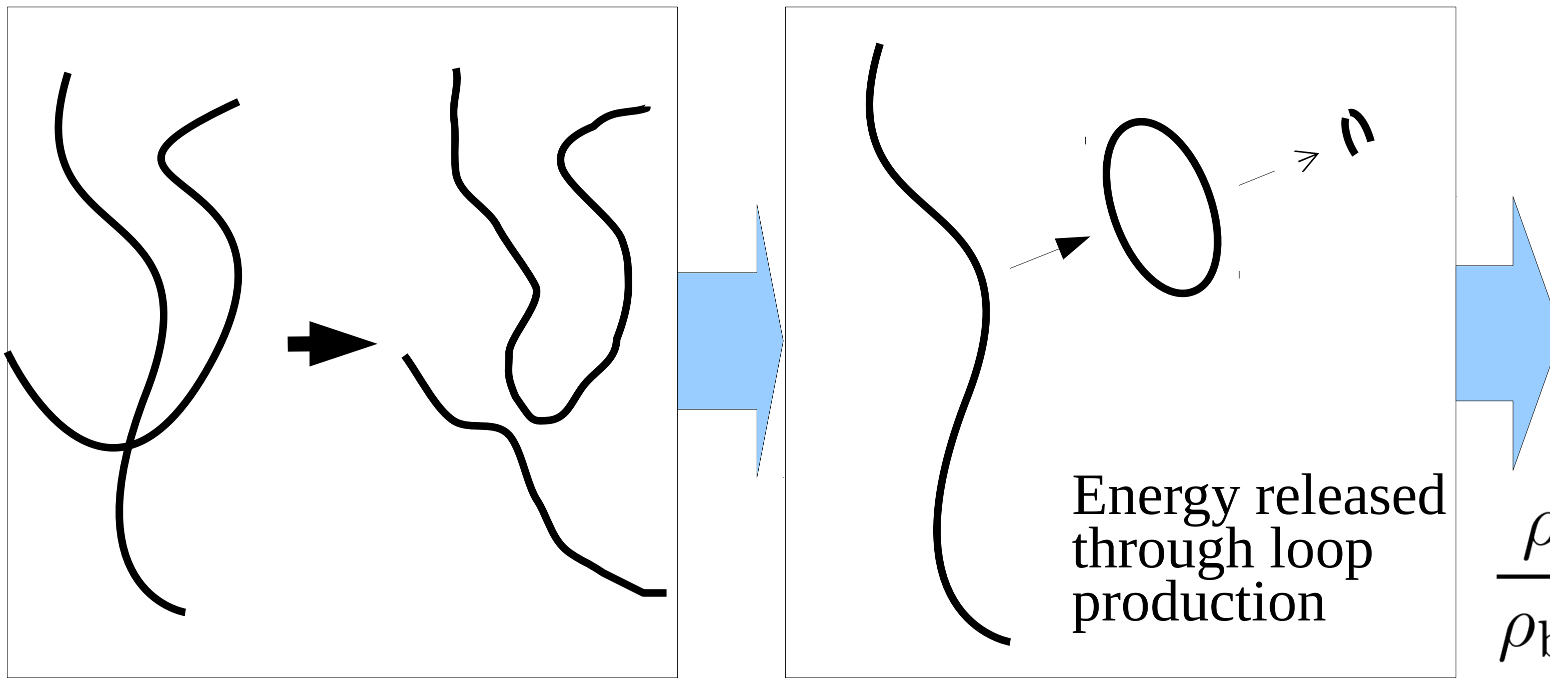
Interests : GUTs ? Superstrings ? Source of GWs ?

Non-trivial signals on CMB ? etc...

Decay rate of energy density

Naively, $\rho_{\rm str} \propto a^{-2}$ $\longrightarrow$ Eventually dominated, so highly suppressed.

But,



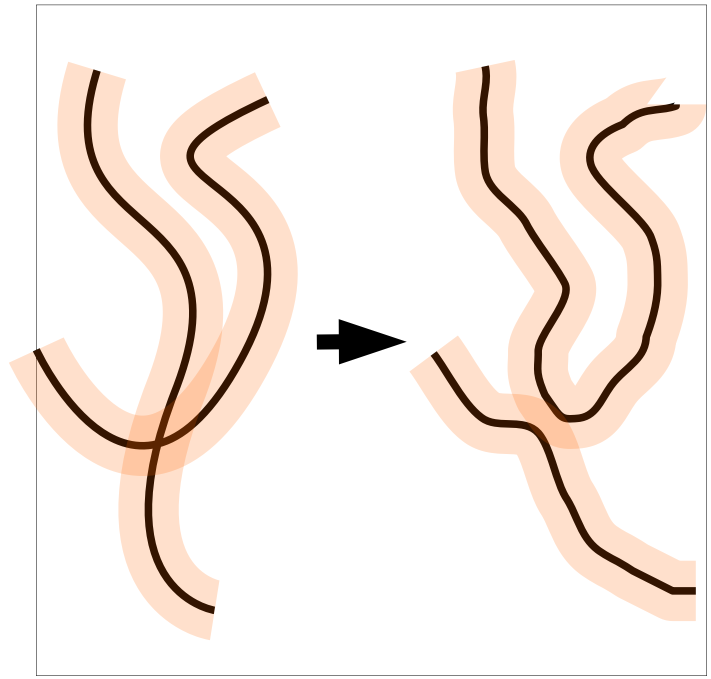
$\propto a^{-4}$ (RD)

$\propto a^{-3}$ (MD)

$\dfrac{\rho_{\rm str}}{\rho_{\rm back}}$ is always constant.

The fate of strings highly depends on the efficiency of reconnection process.

What we'd like to study



Reconnection process works even if strings couple with matter ?

Extend a past numerical study by Laguna and Matzner. Laguna and Matzner, PRD 41 (1990) 1751

Model : superconducting strings

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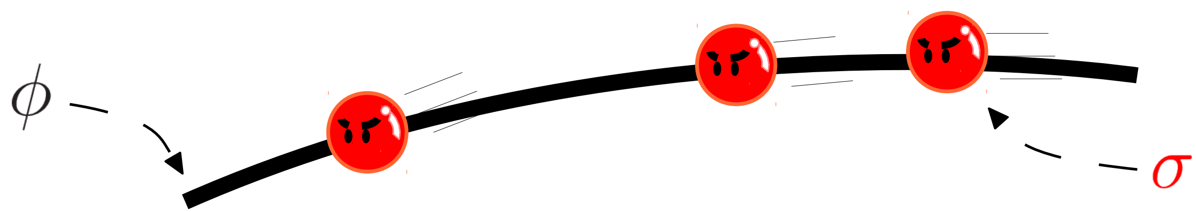
Model Lagrangian

Abelian-Higgs model ($U(1)$ gauge theory) + additional scalar field

$$S = -\int dx^4 \sqrt{-g} \left(\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \phi)^* (D^\mu \phi) + (\partial_\mu \sigma)^* (\partial^\mu \sigma) + V(\phi, \sigma) \right)$$

$$V(\phi,\sigma) = \frac{\lambda_\phi}{4}(|\phi|^2 - \eta^2)^2 + \boxed{\lambda_{\phi\sigma}(|\phi|^2 - \eta^2)|\sigma|^2} + \frac{\lambda_\sigma}{4}|\sigma|^4 + \frac{m_\sigma^2}{2}|\sigma|^2$$

$$D_\mu \equiv \partial_\mu - ieA_\mu \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$



Conserved current : $j_\mu = 2\mathrm{Im}(\sigma^* \partial_\mu \sigma)$

* Its realisability and observability in cosmological context is discussed by Witten.

Witten, NPB 249 (1985) 557

Preparation

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Ansatz and potential

$$V(\phi,\sigma)=\frac{\lambda_\phi}{4}(|\phi|^2-\eta^2)^2+\lambda_{\phi\sigma}(|\phi|^2-\eta^2)|\sigma|^2+\frac{\lambda_\sigma}{4}|\sigma|^4+\frac{m_\sigma^2}{2}|\sigma|^2$$

$$D_\mu=\partial_\mu-ieA_\mu$$

$$\phi(\mathbf{r})=\eta f(r)e^{in\theta}\qquad A_\theta(\mathbf{r})=\frac{n}{e}\alpha(r)\qquad \sigma(\mathbf{r})=\eta g(r)e^{i(kz-\omega t)}$$

Model parameters

winding number : $n=1$

gauge coupling : e

self-coupling of ϕ : $\beta_\varphi\equiv\frac{\lambda_\varphi}{2e^2}=1$

bear mass of σ : $\mu^2\equiv\frac{m_\sigma^2}{2e^2\eta^2}=0.01$

self-coupling of σ : $\beta_\sigma\equiv\frac{\lambda_\sigma}{2e^2}=1$

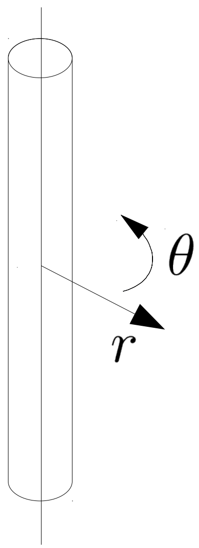
charge of σ : $\Omega^2\equiv\frac{\omega^2}{e^2\eta^2}=0$

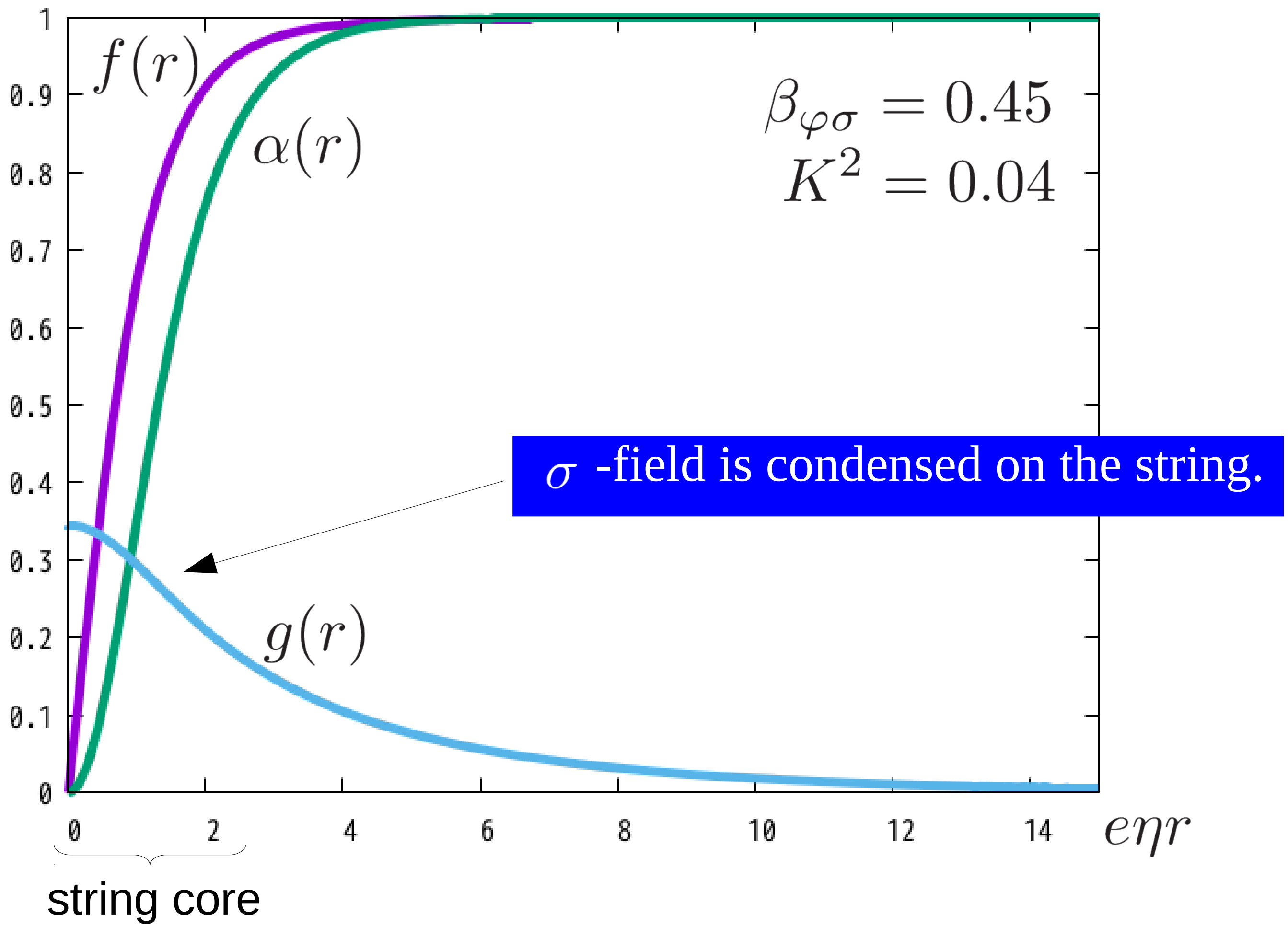
coupling between ϕ & σ : $\beta_{\varphi\sigma}\equiv\frac{\lambda_{\varphi\sigma}}{2e^2}$

current of σ : $K^2\equiv\frac{k^2}{e^2\eta^2}$

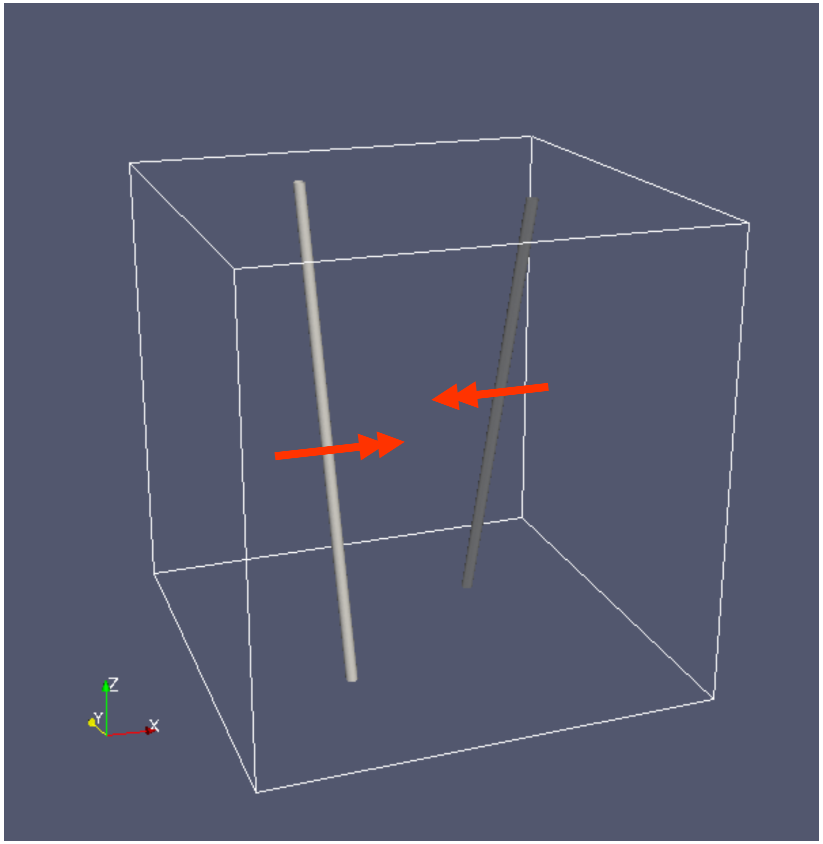
Find straight vortex solutions

$$\phi(\mathbf{r}) = \eta f(r) e^{i\theta} \qquad A_\theta(\mathbf{r}) = \frac{1}{e}\alpha(r) \qquad \sigma(\mathbf{r}) = \eta g(r) e^{ikz}$$





Setup of colliding simulations



Strategy

- Prepare 2 stable straight strings.
- Lorentz boost (velocity+rotation)

$$x^{\mu\prime} = \Lambda^{\mu}{}_{\nu} x^{\nu}$$

- Superposition

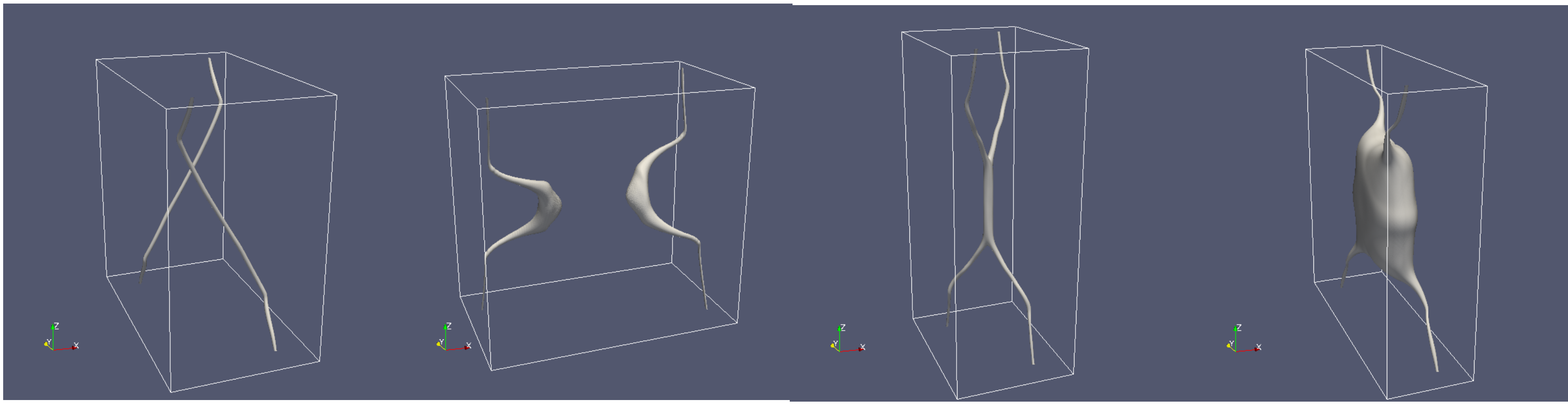
$$\phi = \frac{1}{\eta}\phi^{(1)}\phi^{(2)}$$
$$A_\mu = A_\mu^{(1)} + A_\mu^{(2)}$$
$$\sigma = \sigma^{(1)} + \sigma^{(2)}$$

Numerical methods

- Leap-Frog scheme
- 2nd-order finite difference
- Adaptive box size depending on velocity and angle, roughly $200^3 \sim 800^3$

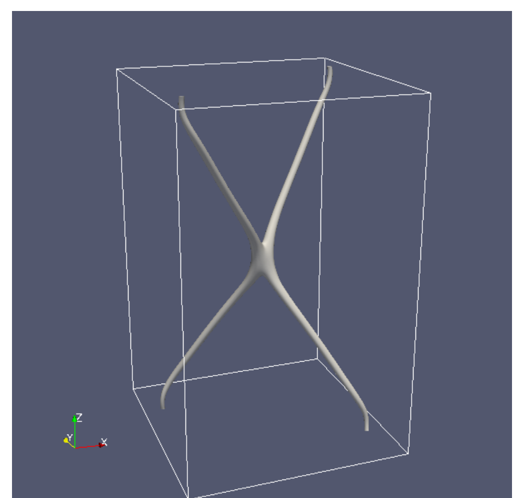
Result III : Phase diagram

Define 4 kinds of final states



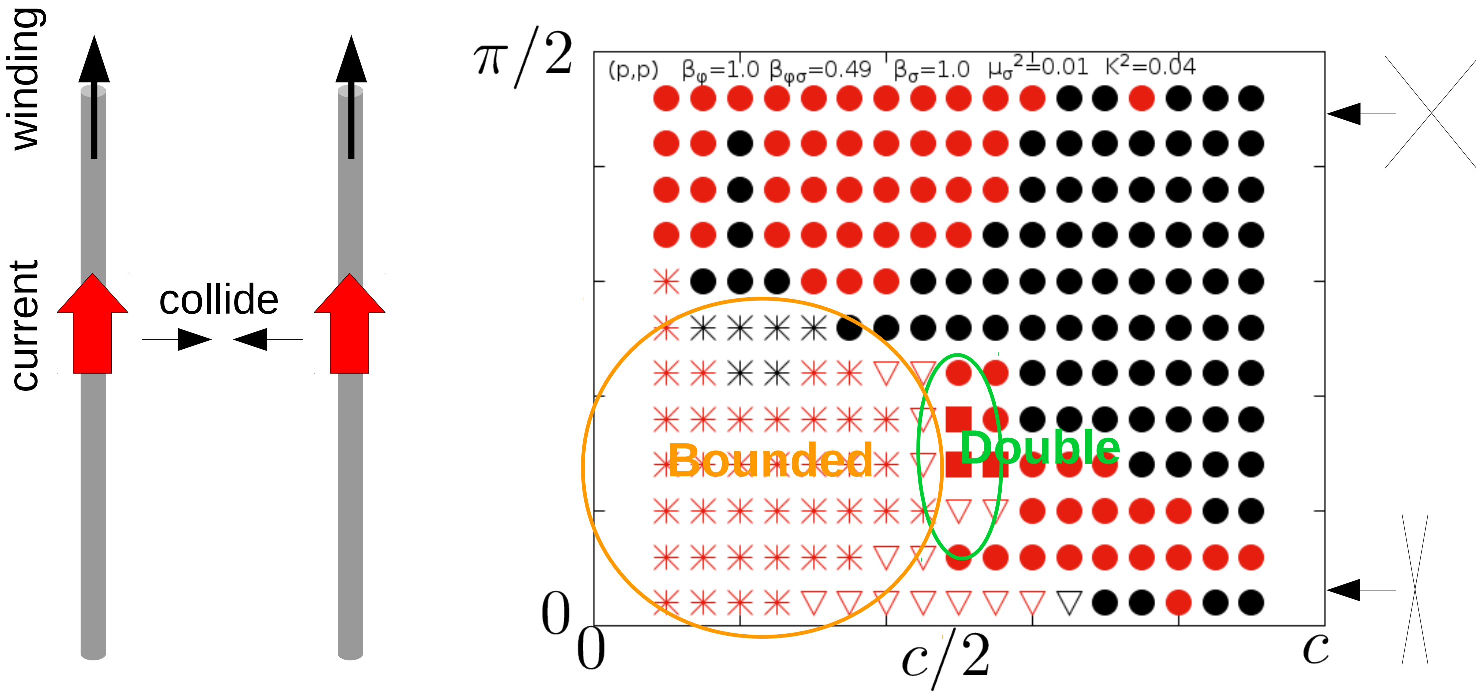
Reconnected — Doubly-reconnected (passing through) — Bounded — Expanding

current flow		no current flow
● (red)	reconnected	●
◎ (red)	doubly-reconnected	◎
+ (red)	bounded	+
× (red)	expanding	×
? (red)	indistinguishable	?



Indistinguishable

‘Phase diagram’ : fiducial setup



- Basically they reconnect with each other as usual
- Low-angle and low-velocity collisions leads to the bound state formation.
- The double reconnection takes place for intermediate-velocity collisions.
- The matter current tends to disappear for high-speed collisions.

→ Discussion on the bound states : Steer, Lilley, Yamauchi, TH, submitted to PRD, 1710.07475

‘Phase diagram’ : K^2 dependence

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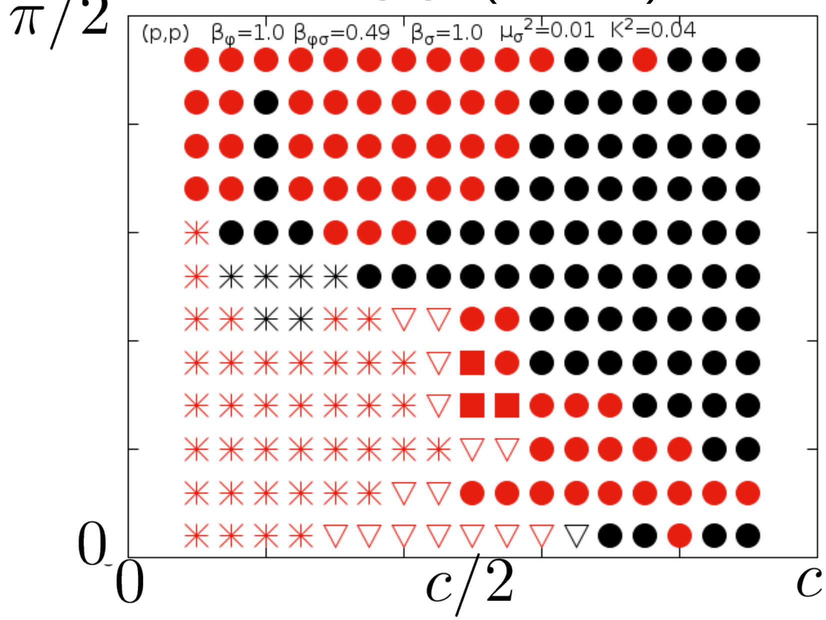
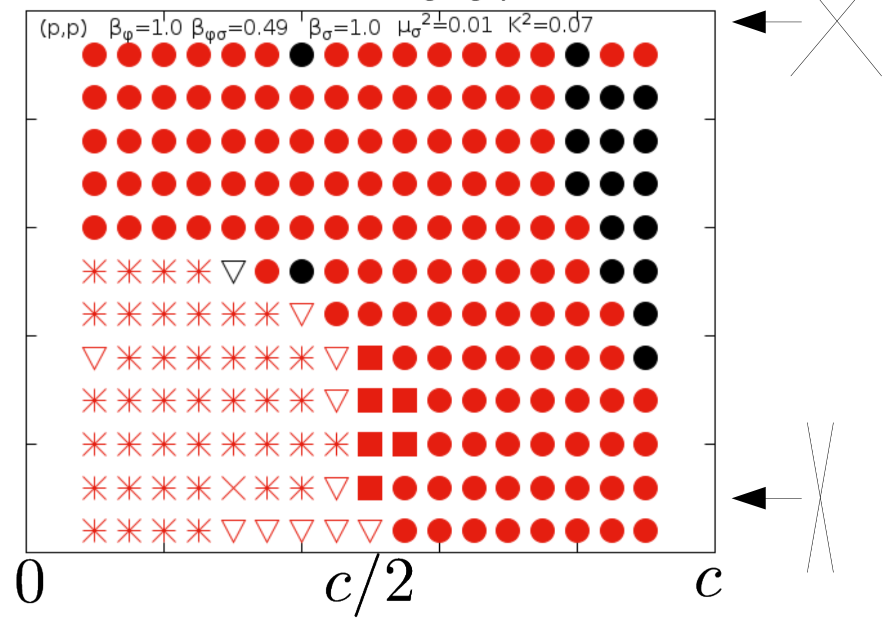
$K^2 = 0.04$ (fiducial)



$K^2 = 0.07$



- The current after collisions is easy to survive for large K^2
- The initial current strength K^2 does not affect the final configuration.

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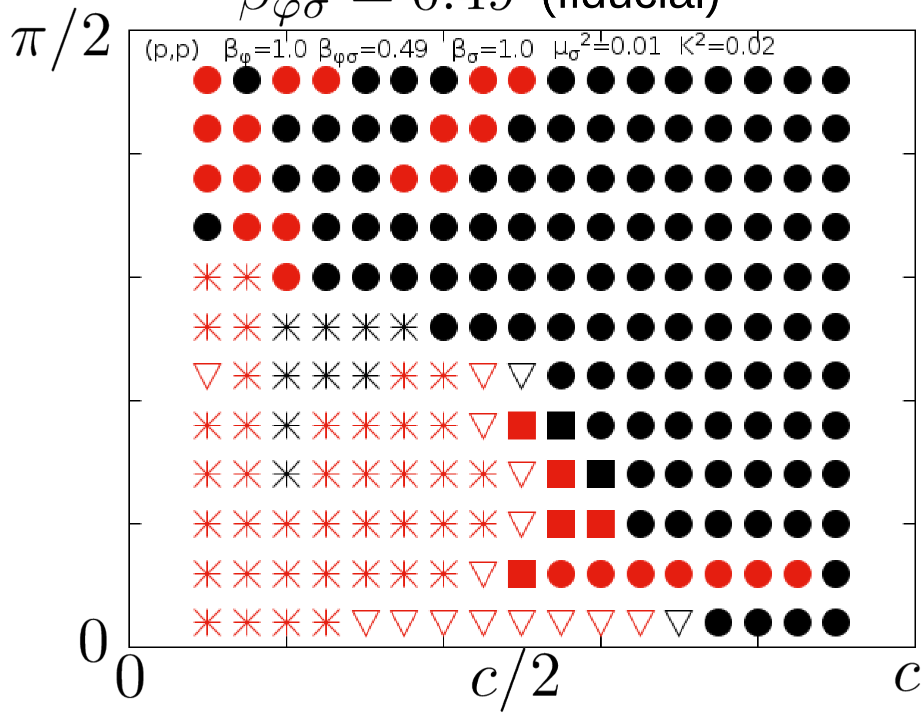
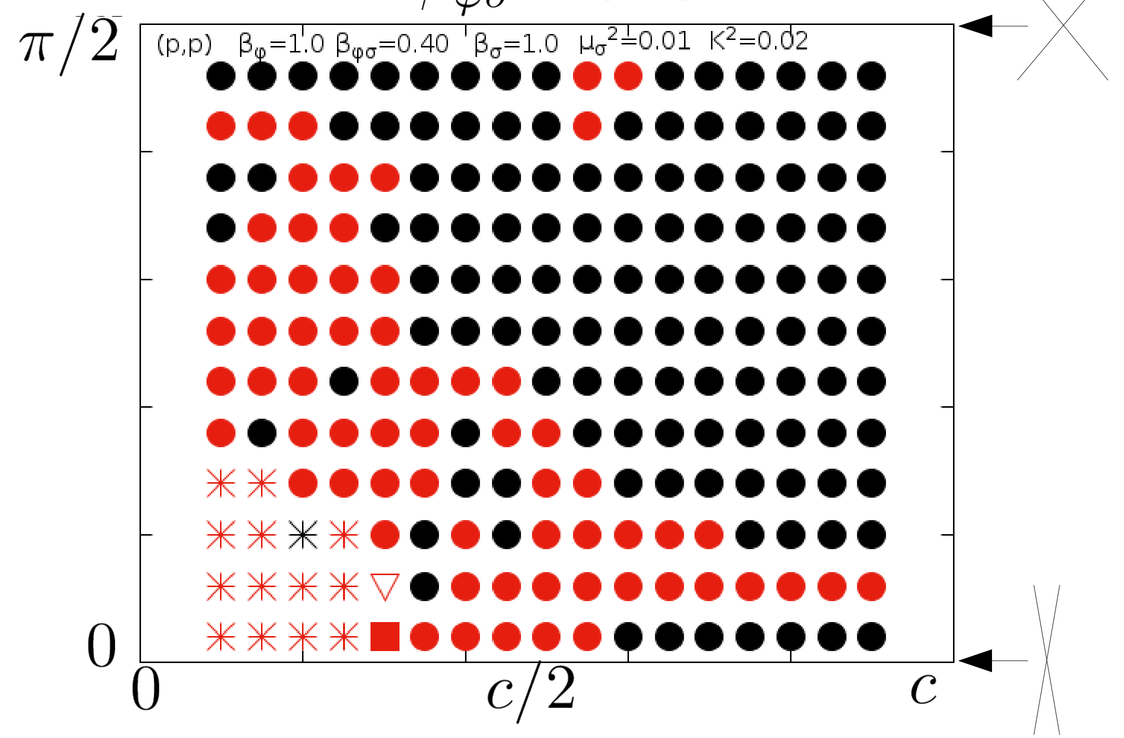
$\beta_{\varphi\sigma} = 0.49$ (fiducial)

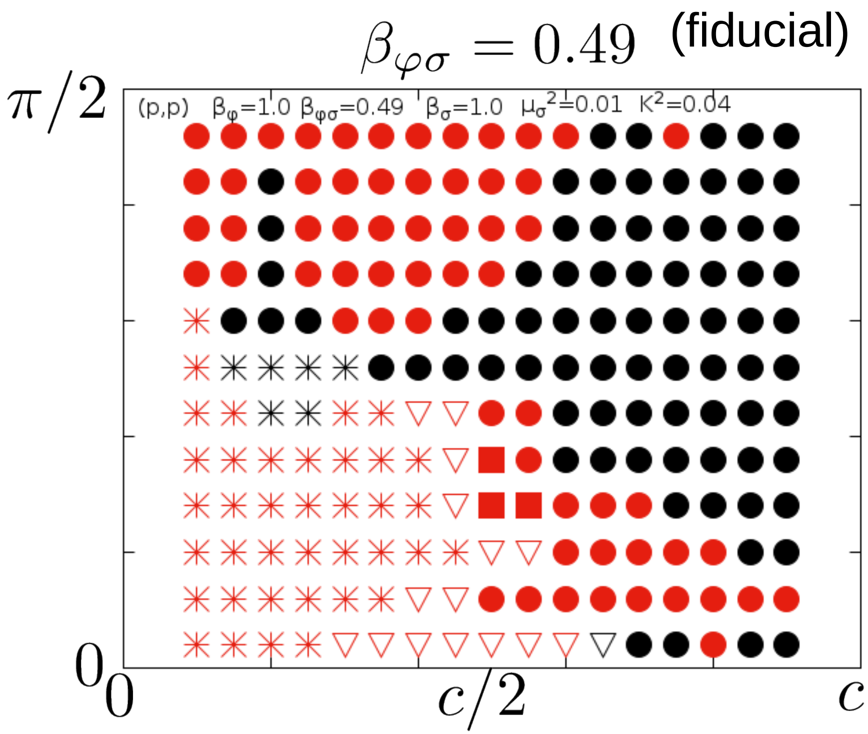


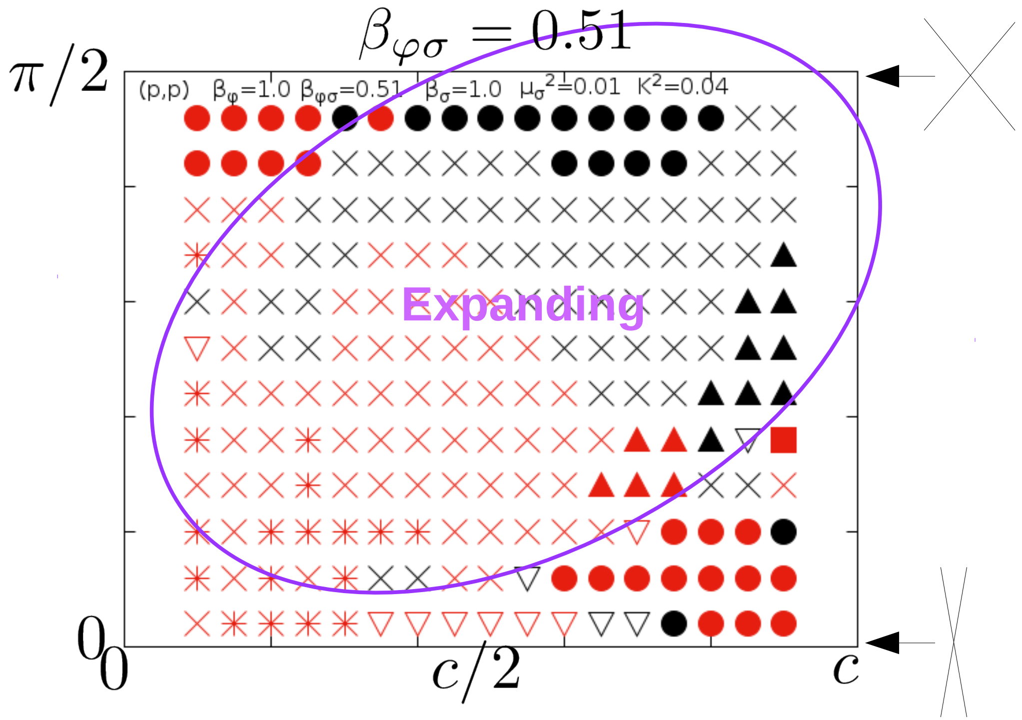
$\beta_{\varphi\sigma} = 0.40$



- The small $\beta_{\varphi\sigma}$ leads to less possibilities for forming bound states.
- $\beta_{\varphi\sigma}$ is not responsible for the final current strength.

‘Phase diagram’ : $\beta_{\varphi\sigma}$ dependence



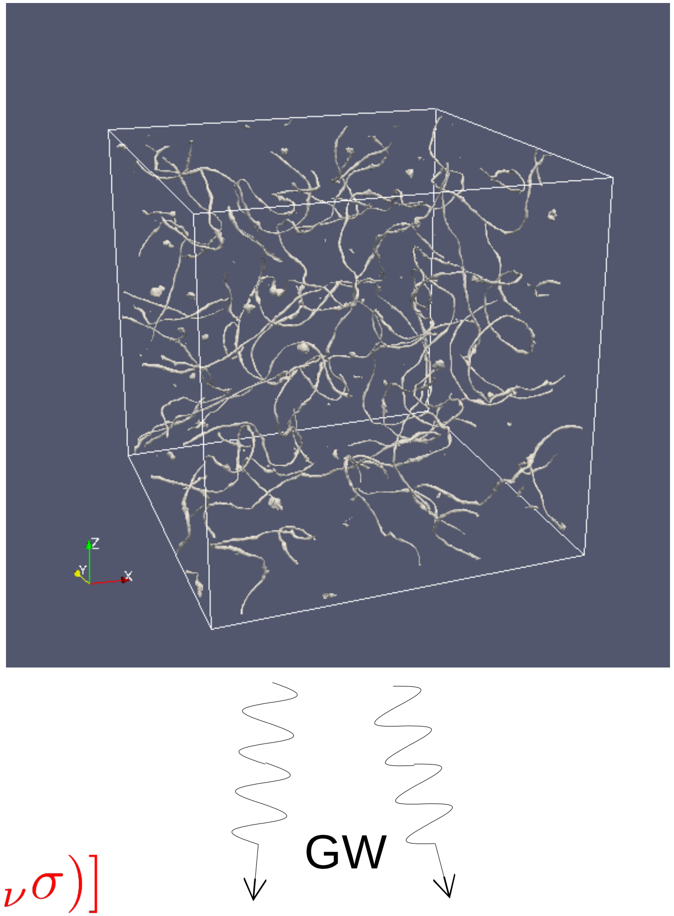


- The large $\beta_{\varphi\sigma}$ leads to the expanding bubble.
- Even so, high-speed collisions avoid the bubble nucleation.

$$
\begin{cases}
\dfrac{1}{\sqrt{-g}} D_\mu(\sqrt{-g} g^{\mu\nu} D_\nu \varphi) = V_{\varphi^*} \\
\dfrac{1}{\sqrt{-g}} \partial_\mu(\sqrt{-g} F^{\mu\nu}) = -2e_\varphi g^{\mu\nu} \mathrm{Im}(\varphi^* D_\mu \varphi) \\
\dfrac{1}{\sqrt{-g}} \partial_\mu(\sqrt{-g} \mathcal{F}^{\mu\nu}) = -2e_\sigma g^{\mu\nu} \mathrm{Im}(\sigma^* D_\mu \sigma) \\
\ddot{h}_{ij} + 2\mathcal{H}\dot{h}_{ij} - \triangle h_{ij} = \dfrac{2}{m_{\rm pl}^2}(T_{ij}^{\rm scalar} + T_{ij}^{\rm gauge})
\end{cases}
$$

$$T_{ij}^{\rm scalar} = 2\mathrm{Re}\left[(D_\mu\varphi)^*(D_\nu\varphi)\right] + 2\mathrm{Re}\left[(D_\mu\sigma)^*(D_\nu\sigma)\right]$$

$$T_{ij}^{\rm gauge} = F_{\mu\alpha}F_\nu{}^\alpha + \mathcal{F}_{\mu\alpha}\mathcal{F}_\nu{}^\alpha$$



GW spectrum [preliminary]

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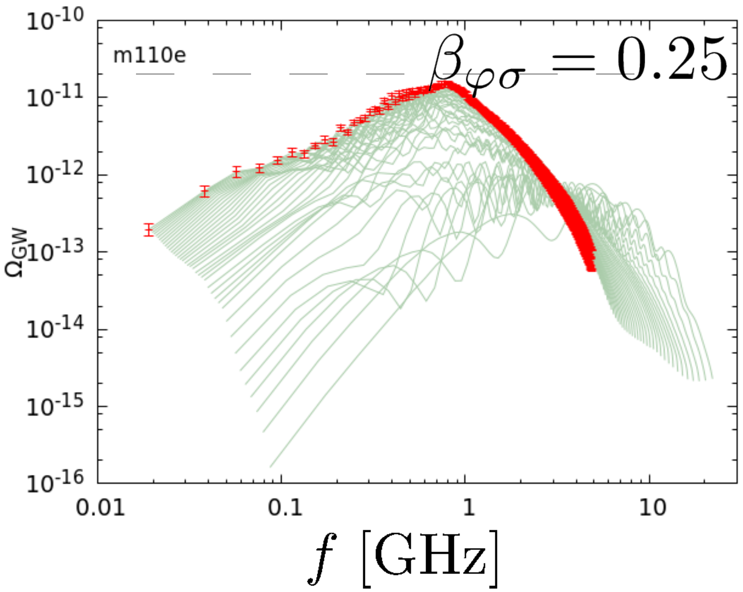
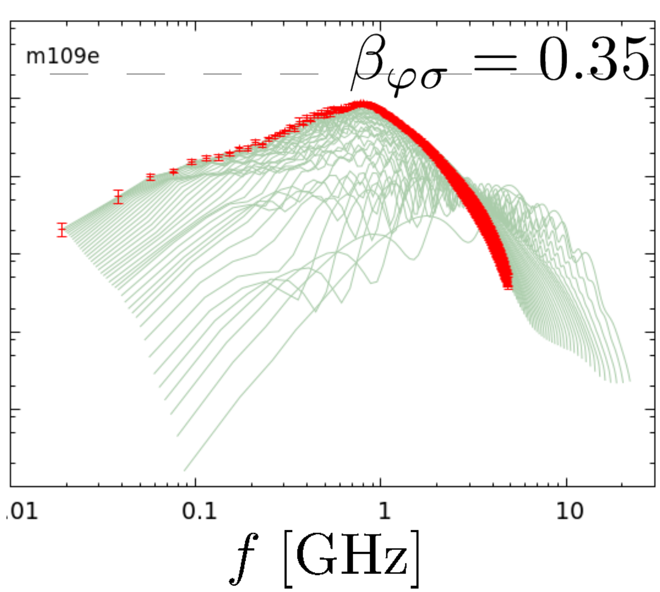
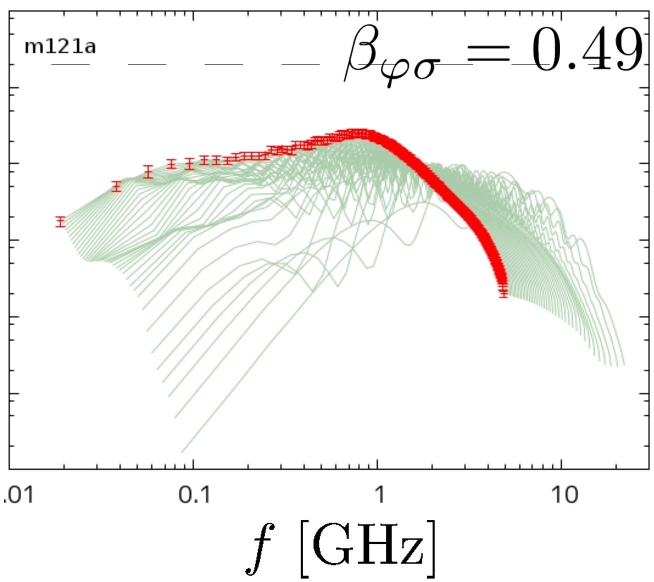
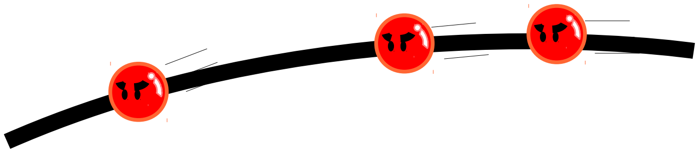
$$\beta_{\varphi\sigma} \equiv \lambda_{\varphi\sigma}/2e^2$$

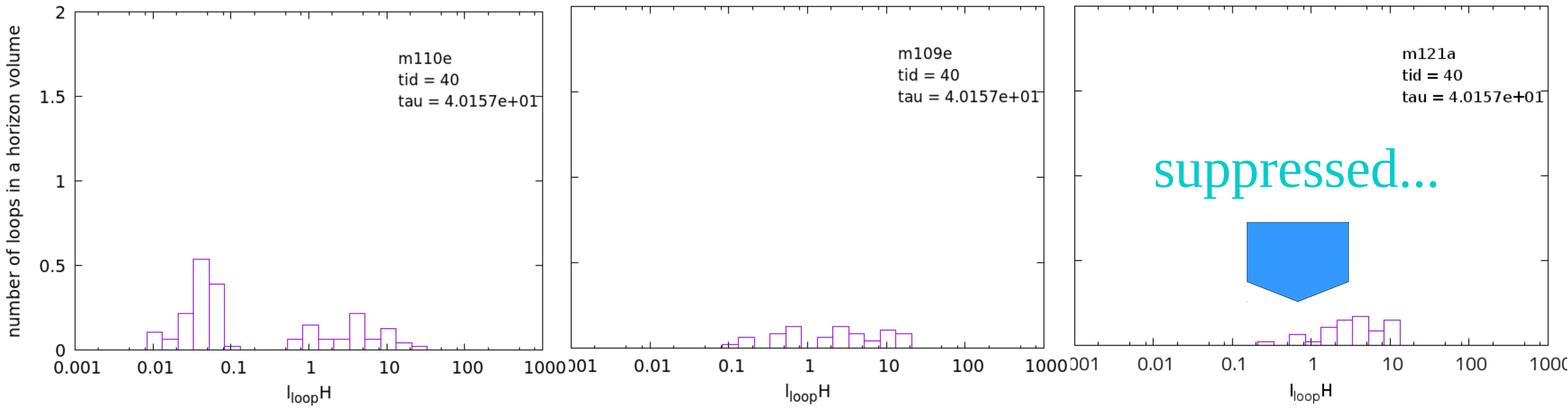


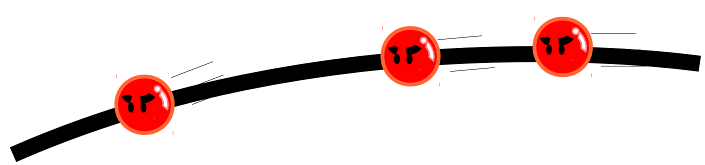






Loop distribution





Summary

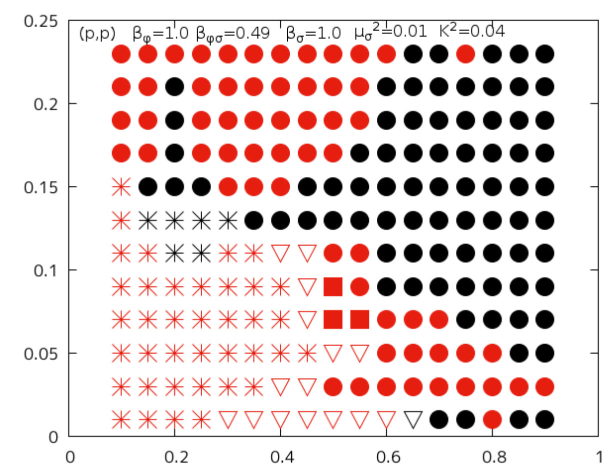
Q. Can strings reconnect in a usual way even if they couple to matter ?

Yes, but they have rich diversity of their final states

⟷ Always successful for critical AH strings

Main results:

- Stable pairs can form a bound state like Type-I AH strings.
- They can pass through each other by double-reconnection like Type-II AH strings.
- The final configuration depends on $\beta_{\varphi\sigma}$, not K^2.
- K^2 is responsible only for the final current strength.



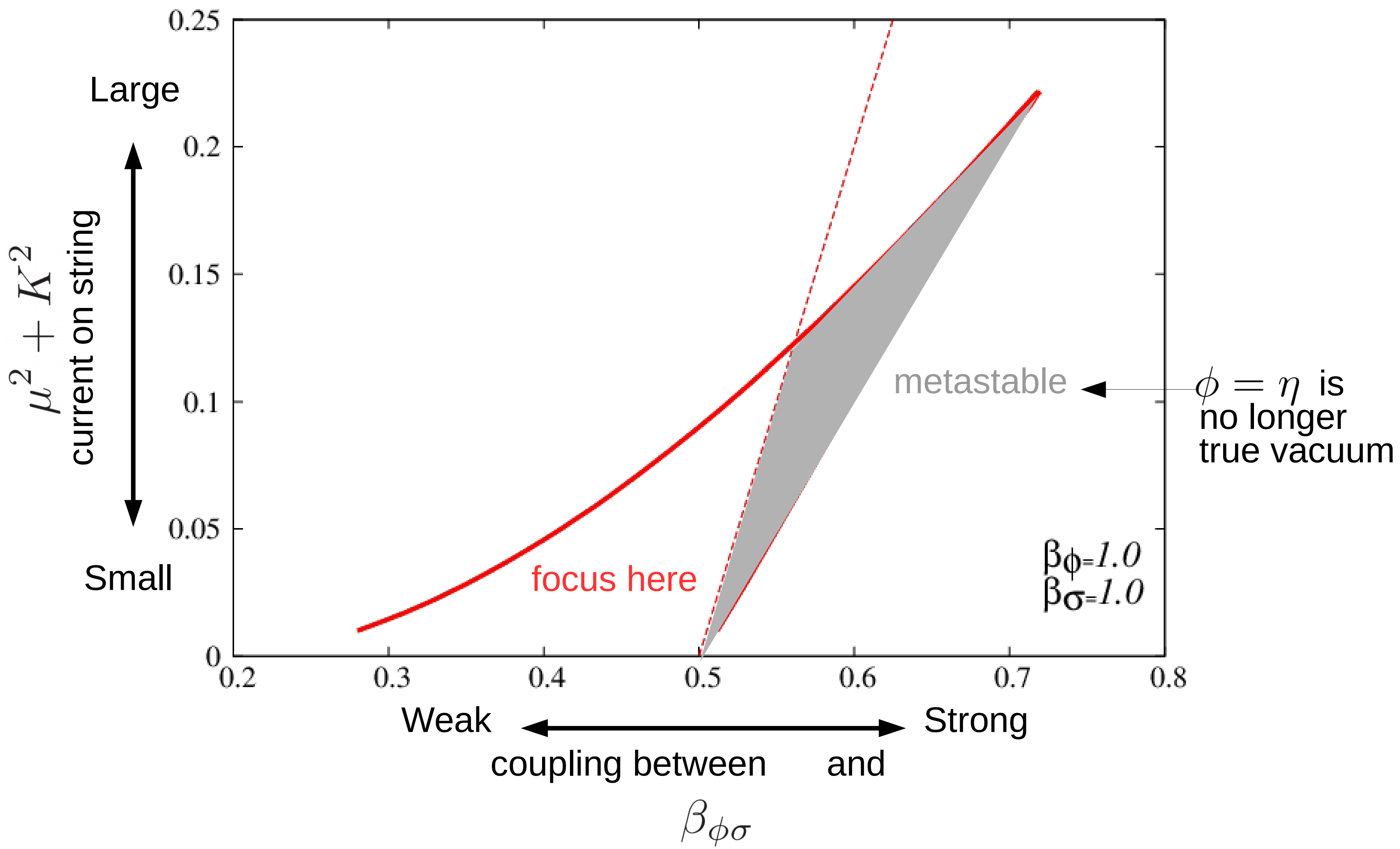
Preliminary results:

- GW signal is suppressed for superconducting cases
- strong coupling leads to the less population of loops ?

work in progress….

Result I : viable parameter region

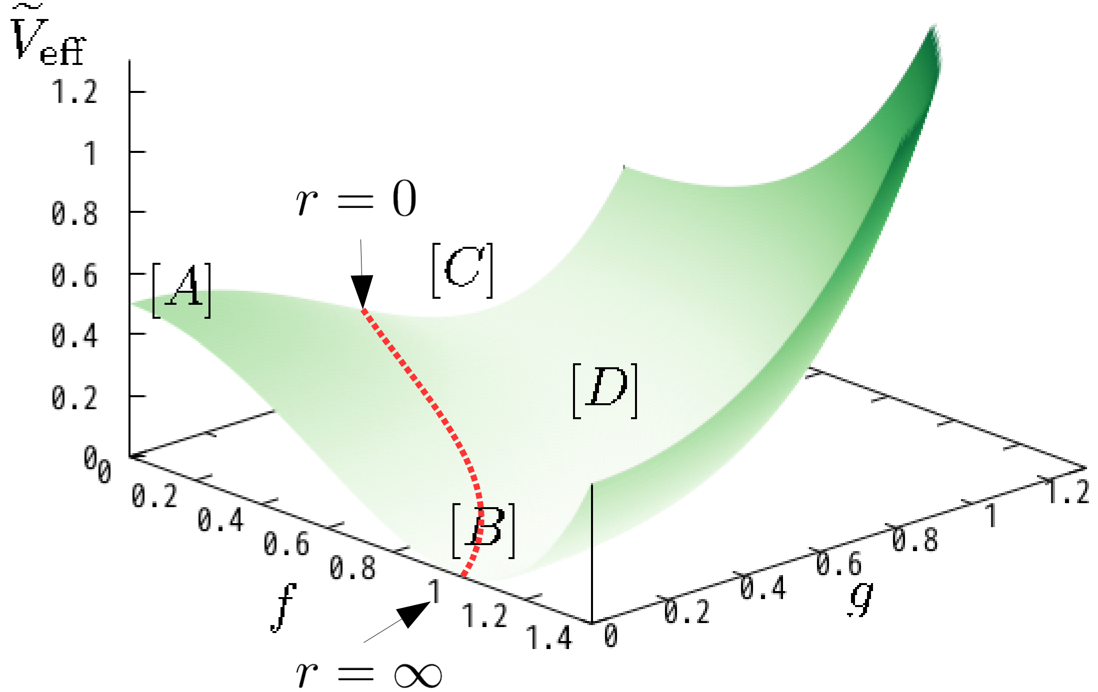
Superconducting string configuration is available only in the triangle region.



Effective potential

$$V(\phi,\sigma) = \frac{\lambda_\phi}{4}(|\phi|^2-\eta^2)^2 + \lambda_{\phi\sigma}(|\phi|^2-\eta^2)|\sigma|^2 + \frac{\lambda_\sigma}{4}|\sigma|^4 + \frac{m_\sigma^2}{2}|\sigma|^2$$

$$\widetilde{V}_{\rm eff}(\phi,\sigma) = \frac{\beta_\varphi}{2}(f^2-1)^2 + 2\beta_{\varphi\sigma}(f^2-1)g^2 + \frac{\beta_\sigma}{2}g^4 + \gamma g^2$$



Model : superconducting strings

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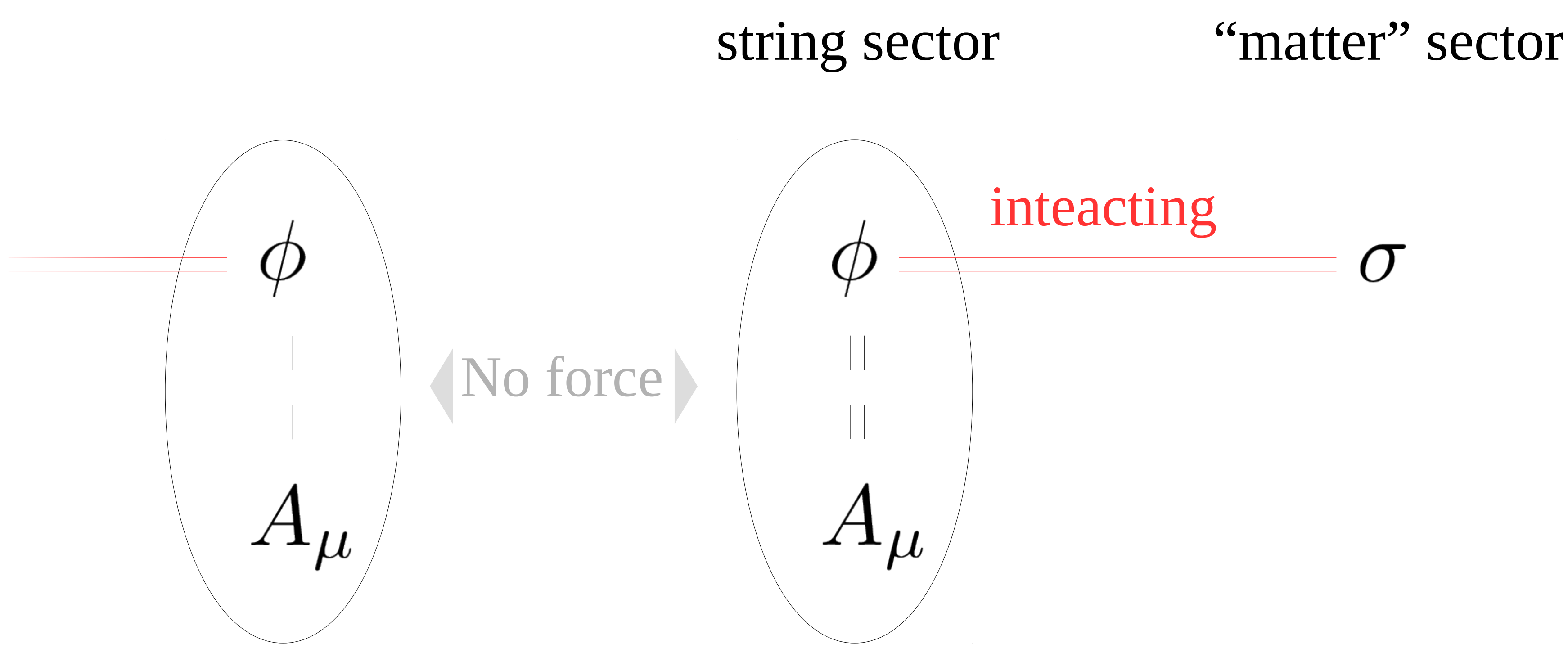


Motivations to study strings

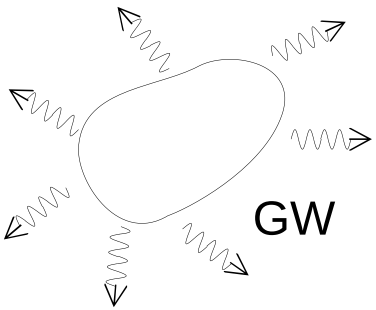
- ~~The seed of large-scale structure~~ → Inflation

- Massive strings can radiate GWs.

$$G\mu \approx \left(\frac{\eta}{M_{\rm pl}}\right)^2 = 10^{-6} \Rightarrow \mu \sim 10^{22}{\rm g/cm} \quad \mu H^{-1} \sim 10^{16} M_\odot$$

$G\mu \leq 1.3 \times 10^{-7}$ **Planck collaboration, A&A 594 (2016) A13**

$G\mu \leq 1.5 \times 10^{-8}$ **Janet et al., ApJ 653 (2006) 1571**



- Probe for very early Universe

- A clue of (grand) unified theories ?

- Superstrings ?

Jones, Stoica, Tye, JHEP 07 (2002) 051
Sarangi, Tye, Phys.Lett. B536 (2002) 185

Scaling solution

String networks *without any interactions* between strings have the attractor solution of their characteristic length (= mean separation of strings), which is growing as

$$\xi(t) \propto t \propto H^{-1}$$

$$\downarrow$$

The number of strings in the cosmological horizon scale is always constant.

$$n_{\rm str}(t) H^{-3} = {\rm const}$$

Confirmed in numerical simulations.

Vincent, Antunes, Hindmarsh, PRL 80 (1998) 2277
Moore, Shellard, PRD 65 (2001) 023503

String network : scaling solution

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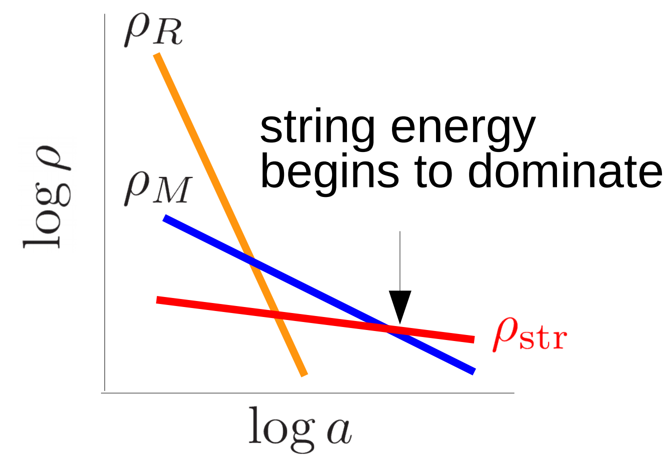
Energy density

Naively thinking ...

$$\rho_{\rm str} = \frac{\mu a L}{(aL)^3} \propto a^{-2}$$

→ eventually dominated by strings !

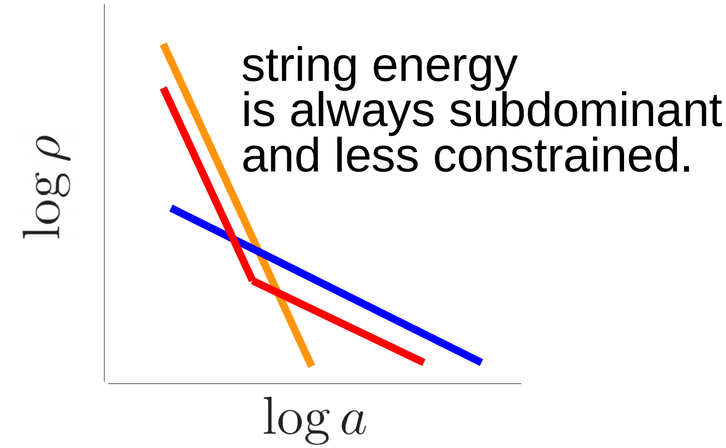
→ highly constrains amouts of strings



If strings follow scaling solutions,

$$\rho_{\rm str} = \frac{\mu \xi}{\xi^3} \propto H^2 \propto a^{-4} \quad \text{(if radiation dominent)}$$

$$\propto a^{-3} \quad \text{(if matter dominent)}$$



String can survive at the present time !