Revisiting the stochastic background of primordial GWs

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Brief Overview

We evaluate the amplitude of primordial gravitational waves which originated from quantum fluctuations in the early universe. This issue has earlier been discussed in the decelerating universe (gr-qc/0002035v1). The evaluation in the accelerating universe has been done without considering the possible quantum effect on these gravitational waves (astro-ph/0501329). Here we place the primordial gravitational waves in the squeezed vacuum state and study the stochastic background of these waves in the accelerating universe (1511.07120v2[gr-qc]). We observe that the squeezing effect depends on each evolutionary stage of the universe and observe dramatic effects on the amplitude due to the squeezing effect. The resulting spectrum is compared with the sensitivity curves of various detectors.

Primordial GWs - Intro

- generated by strong variable gravitational field of early universe through mechanism of parametric amplification of zero point quantum fluctuations.
- initial vacuum state $\rightarrow$ multi-particle quantum state.
- Amplitude and power spectrum of stochastic background of primordial GWs depend on the behavior of the different expansion stage of the universe throughout the span of cosmic time.

Expansion history of the universe

The different evolutionary stages of the universe can be characterized in terms of the scale factor as follows

$$
\begin{aligned}
a(\eta) &= l_0|\eta|^{1+\beta}, \quad -\infty < \eta \leq \eta_1, \\
a(\eta) &= l_0 a_z(\eta-\eta_p)^{1+\beta_s}, \quad \eta_1 \leq \eta \leq \eta_s, \\
a(\eta) &= l_0 a_e(\eta-\eta_e), \quad \eta_s \leq \eta \leq \eta_2, \\
a(\eta) &= l_0 a_m(\eta-\eta_m)^2, \quad \eta_2 \leq \eta \leq \eta_E, \\
a(\eta) &= l_H|\eta-\eta_a|^{-1}, \quad \eta_E \leq \eta \leq \eta_H.
\end{aligned}
$$

The constant l_0 can be expressed as,

$$
l_0 = l_H b \zeta_E^{-(2+\beta)} \zeta_2^{\frac{\beta-1}{2}} \zeta_s^{\beta} \zeta_1^{\frac{\beta-\beta_s}{1+\beta_s}}, \tag{1}
$$

where $b = |1+\beta|^{-(1+\beta)}$, $\zeta_E \equiv \frac{a(\eta_H)}{a(\eta_E)}$, $\zeta_2 \equiv \frac{a(\eta_E)}{a(\eta_2)}$, $\zeta_s \equiv \frac{a(\eta_2)}{a(\eta_s)}$, $\zeta_1 \equiv \frac{a(\eta_s)}{a(\eta_1)}$.

Primordial GWs

The perturbed metric of a flat FLRW universe can be written as

$$dS^2 = a^2(\eta)[-d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j]. \tag{2}$$

The gravitational wave field $h_{ij}(\mathbf{x}, \eta)$,

$$h_{ij}(x,\eta) = \frac{C}{(2\pi)^{\frac{3}{2}}} \int_{-\infty}^{+\infty} \frac{d^3k}{\sqrt{2k}} \sum_{p=1}^{2} [h_k^{(p)}(\eta) c_k^{(p)} + h_k^{(p)*}(\eta) c_k^{(p)\dagger}] \varepsilon_{ij}^{(p)*}(k) e^{-ik.x}, \tag{3}$$

where $C = \sqrt{16\pi} l_{pl}$, $l_{pl} = \sqrt{G}$. $k = (\delta_{ij} k^i k^j)^{\frac{1}{2}}$ and is related to wavelength λ by $\lambda = \frac{2\pi a}{k}$.

Primordial GWs

The dynamical evolution equation of the primordial gravitational waves in the flat FLRW universe can be written as,

$$h''_k(\eta) + 2\frac{a'}{a}h'_k(\eta) + k^2 h_k(\eta) = 0. \tag{4}$$

The gravitational wave mode $h_k(\eta)$ can be rescaled in terms of mode function as,

$$h_k(\eta)a(\eta) = \mu_k(\eta), \tag{5}$$

where the mode functions can have the following form

$$\mu_k(\eta) = u_k(\eta) + v^*_k(\eta), \tag{6}$$

which then satisfies the equation of motion

$$\mu_k'' + \left(k^2 - \frac{a''}{a}\right)\mu_k = 0, \tag{7}$$

- $k > a'/a$: h_k decreases adiabatically with time.
- $k < a'/a$: h_k does not decrease adiabatically, the initial vacuum state evolves into a squeezed vacuum state.

Squeezing effect

The functions $u_k(\eta)$ and $v_k(\eta)$ in Eq.(6) can be represented in terms of three real functions: the squeezing parameter r_k, squeezing angle ϕ_k and the rotation angle θ_k as,

$$\begin{aligned} u_k &= e^{i\theta_k}\cosh r_k, \\ v_k &= e^{-i(\theta_k - 2\phi_k)}\sinh r_k. \end{aligned} \tag{8}$$

The equations of motion for these two complex functions lead to the equations governing the three real functions mentioned above:

$$\begin{aligned} r_k' &= \frac{a'}{a}\cos 2\phi_k, \\ \phi_k' &= -k - \frac{a'}{a}\sin 2\phi_k \coth 2r_k, \\ \theta_k' &= -k - \frac{a'}{a}\sin 2\phi_k \tanh r_k. \end{aligned} \tag{9}$$

Squeezing angle and squeezing parameter

In the adiabatic regime, the wavelength is shorter than the Hubble radius, therefore k is dominant. Thus the squeezing angle can be given by,

$$\phi_k = -k(\eta + \eta_k). \tag{10}$$

In the long wavelength regime, k can be neglected, and the squeezing angle becomes,

$$\phi_k \propto \tan^{-1}\left(\frac{1}{a^2(\eta)}\right). \tag{11}$$

Thus the squeezing angle also varies for each frequency interval.

Squeezing angle and squeezing parameter

The frequency dependent squeezing parameter r_k grows as,

$$r_k \approx \ln \frac{a_{**}(k)}{a_*(k)}, \tag{12}$$

where a_* is the value of $a(\eta)$ at η_*, the time beginning of the range, i.e., the higher frequency end of the range and a_{**} denotes $a(\eta)$ at η_{**}, the end of time range, i.e., the lower frequency end of range.

Squeezing angle and squeezing parameter

The squeezing parameter corresponding to each frequency range,

$$
\begin{aligned}
r_k &= \ln\left(\frac{k}{k_1}\right)^{\beta-\beta_s}, \quad k_s \le k \le k_1, \\
r_k &= \ln\left[\left(\frac{k}{k_s}\right)^{\beta}\left(\frac{k_s}{k_1}\right)^{\beta-\beta_s}\right], \quad k_2 \le k \le k_s, \\
r_k &= \ln\left[\left(\frac{k}{k_2}\right)^{\beta-1}\left(\frac{k_2}{k_1}\right)^{\beta}\left(\frac{k_s}{k_1}\right)^{-\beta_s}\right], \quad k_H \le k \le k_2, \\
r_k &= \ln\left[\left(\frac{k}{k_H}\right)^{\beta+1}\left(\frac{k_H}{k_2}\right)^{\beta-1}\left(\frac{k_2}{k_1}\right)^{\beta}\left(\frac{k_s}{k_1}\right)^{-\beta_s}\right], \quad k_E \le k \le k_H, \\
r_k &= \ln\left[\left(\frac{k}{k_H}\right)^{\beta}\left(\frac{k_E}{k_H}\right)\left(\frac{k_H}{k_2}\right)^{\beta-1}\left(\frac{k_2}{k_1}\right)^{\beta}\left(\frac{k_s}{k_1}\right)^{-\beta_s}\right], \quad k \le k_E.
\end{aligned}
$$

Power Spectrum

The power spectrum of the gravitational waves,

$$\langle 0|h_{ij}(\mathbf{x},\eta)h^{ij}(\mathbf{x},\eta)|0\rangle = \frac{C^2}{2\pi^2}\int_0^\infty k^2|h_k(\eta)|^2\frac{dk}{k}, \tag{13}$$

where

$$h^2(k,\eta) = \frac{C^2}{2\pi^2}k^2|h_k(\eta)|^2, \tag{14}$$

gives the mean-square value of the gravitational waves with interval k.

Power Spectrum

Using $C = \sqrt{16\pi} l_{pl}$, we get the power spectrum as

$$|h(k,\eta)| = \frac{4 l_{pl}}{\sqrt{\pi}} k |h_k(\eta)|. \quad (15)$$

The amplitude of the primordial gravitational waves for the wave interval $k_E \leq k \leq k_1$ becomes

$$h(k,\eta_H) = 8\sqrt{\pi}\left(\frac{l_{pl}}{l_H}\right)\left(\frac{k}{k_H}\right)\left(1 + 2\sinh^2 r_k + \sinh 2r_k \cos 2\phi_k\right)^{1/2}. \quad (16)$$

Spectral energy density

- The wave number k varies proportionally as the frequency ν.
- The fractional energy density of gravitational waves ($\rho_{gw}/\rho_c < 1$) can be defined in terms of spectral energy density $\Omega_{gw}(\nu)$ as:

$$\frac{\rho_{gw}}{\rho_c} = \int \Omega_{gw}(\nu) \frac{d\nu}{\nu}, \tag{17}$$

where spectral energy density is written as

$$\Omega_{gw}(\nu) = \frac{\pi^2}{3} h^2(\nu) \left(\frac{\nu}{\nu_H} \right)^2. \tag{18}$$

- The frequency ranges are taken as $\nu_E = 1.5 \times 10^{-18}$ Hz, $\nu_H = 2 \times 10^{-18}$ Hz, $\nu_2 = 117 \times 10^{-18}$ Hz, $\nu_s = 10^8$ Hz and $\nu_1 = 3 \times 10^{10}$ Hz.

Allowed models on β

The wavelength of the gravitational wave at the time when it enters the long wavelength regime must be greater than the Planck length l_{pl}. Thus,

$$b\frac{l_{pl}}{l_0}\left(\frac{\nu}{\nu_H}\right)^{2+\beta} < 1. \tag{19}$$

Thus at the highest frequency $\nu = \nu_1$, we get

$$\left(\frac{\nu_1}{\nu_H}\right)^{2+\beta} < 1.63 \times 10^6, \tag{20}$$

which provides the upper bound on β which is -1.77. The allowed values of β_s with respect to $\beta = -1.8, -1.9, -2.0$ are respectively $\beta_s = 0.598, -0.552, -1.689$. The case $\beta = -2$ denotes the exact de Sitter expansion.

The consistency of these models in the absence ($r_k = 0$) and presence ($r_k \neq 0$) of the squeezing effect can be examined.

For $\beta = -1.8$,

$$\begin{aligned} \frac{\rho_{gw}}{\rho_c} &= 9.29 \times 10^{-4}, \quad r_k = 0, \\ \frac{\rho_{gw}}{\rho_c} &= 6.83 \times 10^{-5}, \quad r_k \neq 0. \end{aligned} \tag{21}$$

For $\beta = -1.9$,

$$\begin{aligned} \frac{\rho_{gw}}{\rho_c} &= 2.34 \times 10^{-6}, \quad r_k = 0, \\ \frac{\rho_{gw}}{\rho_c} &= 1.79 \times 10^{-7}, \quad r_k \neq 0. \end{aligned} \tag{22}$$

For $\beta = -2$,

$$\begin{aligned} \frac{\rho_{gw}}{\rho_c} &= 8.97 \times 10^{-7}, \quad r_k = 0, \\ \frac{\rho_{gw}}{\rho_c} &= 9.97 \times 10^{-8}, \quad r_k \neq 0. \end{aligned} \tag{23}$$

Amplitude

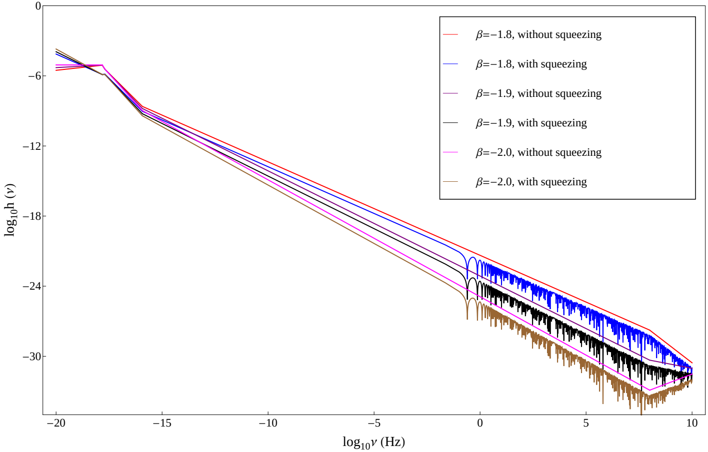

Figure: Amplitude of stochastic background of GWs as for $\beta = -1.8$, $\beta = -1.9$, $\beta = -2.0$ in presence and absence of squeezing effect.

β = -1.8

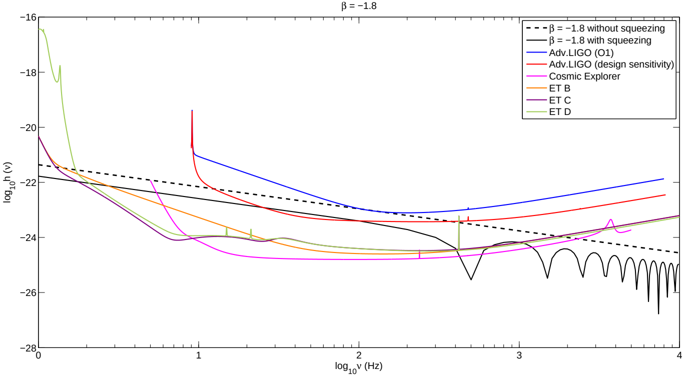

Figure: Amplitude of GWs for $\beta = -1.8$ compared with sensitivity curves of Advanced LIGO (during O1 run), Advanced LIGO (design sensitivity), Cosmic Explorer and Einstein Telescope.

β = -1.8

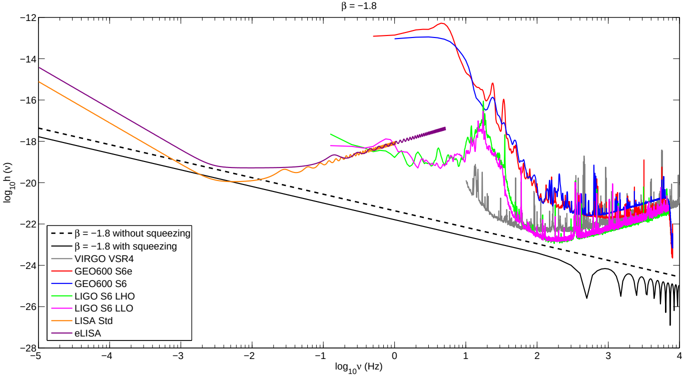

Figure: Amplitude of Gws for $\beta = -1.8$ compared with sensitivity curves of VIRGO, GEO-600, LIGO S6, LISA and eLISA.

$\beta = $ -1.9

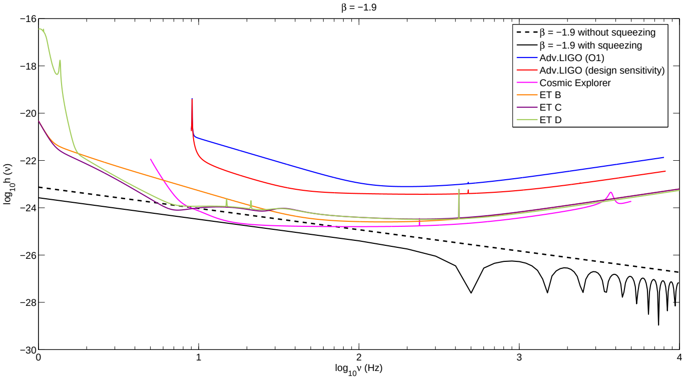

Figure: Amplitude of GWs for $\beta = -1.9$ compared with sensitivity curves of Advanced LIGO (during O1 run), Advanced LIGO (design sensitivity), Cosmic Explorer and Einstein Telescope.

β = -1.9

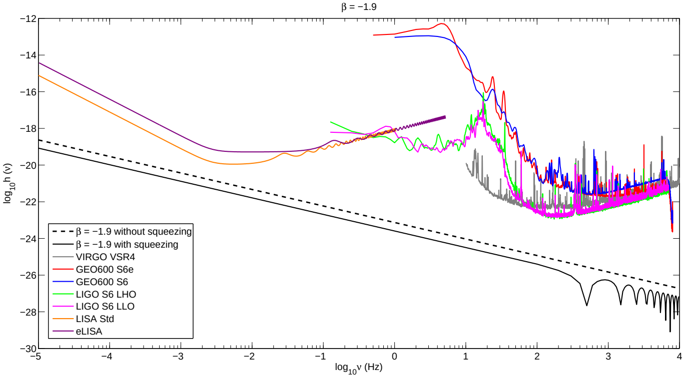

Figure: Amplitude of GWs for $\beta = -1.9$ compared with sensitivity curves of VIRGO, GEO-600, LIGO S6, LISA and eLISA.

β = -2.0

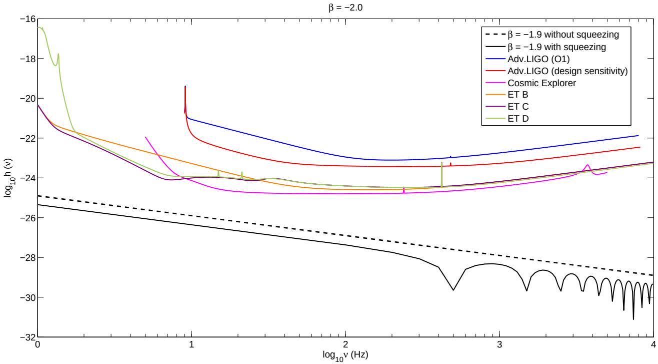

Figure: Amplitude of GWs for $\beta = -2.0$ compared with sensitivity curves of Advanced LIGO (during O1 run), Advanced LIGO (design sensitivity), Cosmic Explorer and Einstein Telescope.

$\beta = $ -2.0

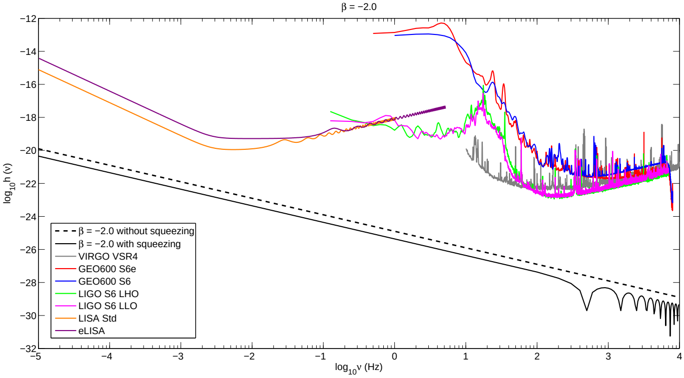

Figure: Amplitude of GWs for $\beta = -2.0$ compared with sensitivity curves of VIRGO, GEO-600, LIGO S6, LISA and eLISA.

Observations and discussions

- For the model $\beta = -1.8$ both in the absence and presence of squeezing effect, the field lies within the sensitivity range of Einstein Telescope, Advanced LIGO(O1 run and design sensitivity), Cosmic Explorer and LISA.
- The field for the model $\beta = -1.9$ in the absence of squeezing effect lies within the sensitivity range of Einstein Telescope.
- The gravitational wave field for $\beta = -2.0$ both in the absence and presence of squeezing effect lies below the sensitivity curves of all the mentioned detectors.

Observations and discussions

- We observe strong oscillating features towards higher frequency.
- As the wave enters the long wavelength regime (lower frequency), the oscillation decreases.
- There is decrease in the amplitude for each model due to the squeezing effect as compared to models in the absence of squeezing effect.
- However, while the amplitude for the models without squeezing effect starts decreasing when the wave enters the long wavelength regime, the amplitude for the models in the presence of squeezing effect keeps on increasing. This is due to the fact that the variance of the gravitational wave mode's phase is strongly squeezed while its amplitude is strongly increased, a feature which continues upto the very end of the amplifying regime