Primordial gravitational waves and early universes

Asuka Ito (Kobe Univ.)

with Jiro Soda (Kobe Univ.)

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1. Observations of cosmological magnetic fields and magnetogenesis in the early universe

   Inflation vs cyclic universe

2. Primordial gravitational waves from magnetic field production in a cyclic universe

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Contents

1. **Observations of cosmological magnetic fields and magnetogenesis in the early universe**

   Inflation vs cyclic universe

2. Primordial gravitational waves from magnetic field production in a cyclic universe

3. Conclusion

Cosmological magnetic fields

Several observations imply there exist magnetic fields on galaxy and galaxy cluster scales

$$B_{galaxy} \sim 10^{-6} \text{ Gs} \quad , \qquad B_{cluster} \sim 10^{-5} \text{ Gs}$$

In particular, Gamma ray burst observation infer existence of magnetic fields in void

$$B_{void} \gtrsim 10^{-16} \text{ Gs}$$

They imply that the seed of magnetic fields must be produced in the early universe

How can we produce?

Magnetogenesis

Ratra suggested a mechanism of magnetogenesis during inflation (1992)

$$\boxed{S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}(\partial_\mu\phi)^2 - V(\phi) - \frac{1}{4}f(\phi)F_{\mu\nu}F^{\mu\nu}\right]}$$

This coupling prevent dilution of the gauge field due to expansion

$$f(\phi) = \left(\frac{a}{a_{end}}\right)^n$$ **(a: scale factor, n: positive constant)**

However, this mechanism suffers from
the strong coupling problem during inflation, namely, $f(\phi) < 1$

On the other hand…

Magnetogenesis

In cyclic universe models,
there is no strong coupling problem during the contracting phase

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}(\partial_\mu\phi)^2 - V(\phi) - \frac{1}{4}f(\phi)F_{\mu\nu}F^{\mu\nu}\right]$$

$$f(\phi) = \left(\frac{a}{a_{end}}\right)^n \longrightarrow f(\phi) > 1$$ **during contracting phase**

Magnetogenesis in $f(\phi)FF$

Inflation	**vs**	**cyclic universe**
difficult		**succesfull**
		(F. A. Memviela (2013))

ekpyrotic scenario

As a explicit cyclic universe model, we consider an ekpyrotic scenario.

In 4-dim, it is described by a scalar field rolling on an effective potential.

$$\boxed{S = \int d^4x \sqrt{-g} \left[-\frac{1}{2} (\partial_\mu \phi)^2 - V(\phi) \right]}$$

where ϕ **represents the size of a extra dimension.**

$$\left(\begin{array}{lll} \dot{\phi} > 0 & \longrightarrow & \text{expanding} \\ \dot{\phi} < 0 & \longrightarrow & \text{contracting} \end{array} \right.$$

- **In the ekpyrotic scenario,**
  quantum fluctuations are produced during a contracting phase

PGWs in cyclic universe

Quantum fluctuations of GWs in the contracting phase is blue spectrum $P_h(k) \propto k^2$

This is common to other cyclic universe models!

We can not observe PGWs in cyclic universe models

It is believed that detection of PGWs kills cyclic universe

However, I will show that this is not true
if magnetic field production occurs in cyclic universe!

Magnetogenesis in ekpyrotic

$$\boxed{S=\int d^4x\sqrt{-g}\left[-\frac{1}{2}(\partial_\mu\phi)^2-V(\phi)\right]}$$

In the contracting phase for quantum fluctuations,
the potential is given by $\boxed{V(\phi)=V_0 e^{\lambda\frac{\phi}{M_{pl}}}}$

(V_0 **and** λ **are negative constants)**

We also consider a exponential type gauge kinetic function

$$\boxed{S=\int d^4x\sqrt{-g}\left[-\frac{1}{2}(\partial_\mu\phi)^2-V_0 e^{\lambda\frac{\phi}{M_{pl}}}-\frac{1}{4}e^{2\rho\frac{\phi}{M_{pl}}}F_{\mu\nu}F^{\mu\nu}\right]}$$

expected from dimensional reduction

Magnetogenesis in ekpyrotic

$$\boxed{S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}(\partial_\mu\phi)^2 - V_0 e^{\lambda\frac{\phi}{M_{pl}}} - \frac{1}{4}e^{2\rho\frac{\phi}{M_{pl}}}F_{\mu\nu}F^{\mu\nu}\right]}$$

We treat the electromagnetic field as a perturbation.
Then the background solution is given by

$$\boxed{a(\tau) = a_{end}\left(\frac{-\tau}{\tau_{end}}\right)^{\frac{2}{\lambda^2-2}} \quad , \quad \frac{\phi}{M_{pl}} = \phi_0 - \frac{2\lambda}{\lambda^2-2}\ln\left(\frac{-\tau}{M_{pl}}\right)}$$

Solving the perturbative equation,
we get magnetic fields at the end of the contracting phase as

$$\boxed{B_k(\tau) \propto k^{\frac{1}{2}\frac{\lambda^2-4\rho\lambda-2}{\lambda^2-2}} H_{end}^2}$$

H_{end} **: Hubble at the end of the contracting phase**

Magnetogenesis in ekpyrotic

$$\boxed{B_k(\tau) \propto k^{\frac{1}{2}\frac{\lambda^2-4\rho\lambda-2}{\lambda^2-2}} H_{end}^2}$$

We assume scale invariant magnetic fields which might be favored by observations

$$\left(\frac{1}{2}\frac{\lambda^2-4\rho\lambda-2}{\lambda^2-2} = -\frac{3}{2} \right)$$

$$\boxed{B_k(\tau) = \frac{3\sqrt{2}}{8k^{3/2}}(\lambda^2-2)^2 H_{end}^2}$$

ex. $H_{end} \sim 10^{-5} M_{pl},\ |\lambda| \sim 17$ → 10^{-12} Gs **(at present)**

$\therefore$ **This mechanism could explain observed magnetic fields!**

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PGWs from magnetic field production

$$S_{EM} = \int d^4x\sqrt{-g}\left[-\frac{1}{4}e^{2\rho\frac{\phi}{M_{pl}}}F_{\mu\nu}F^{\mu\nu}\right]$$

3d order perturbation

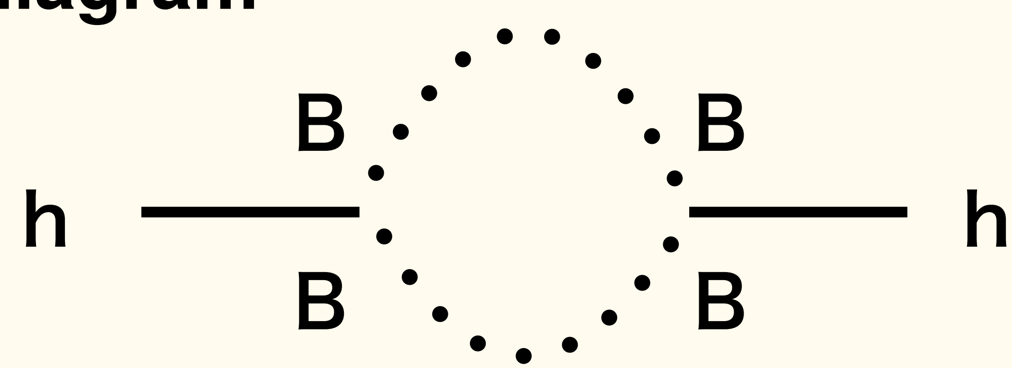
$$\left(\vec{B} = \frac{e^{\rho\frac{\phi}{M_{pl}}}}{a^2}\left(\nabla\times\vec{A}\right)\right)$$

$$S_{GW} = \int d^4x\sqrt{-g}\left[\frac{1}{2}B_iB_jh^{ij}\right]$$

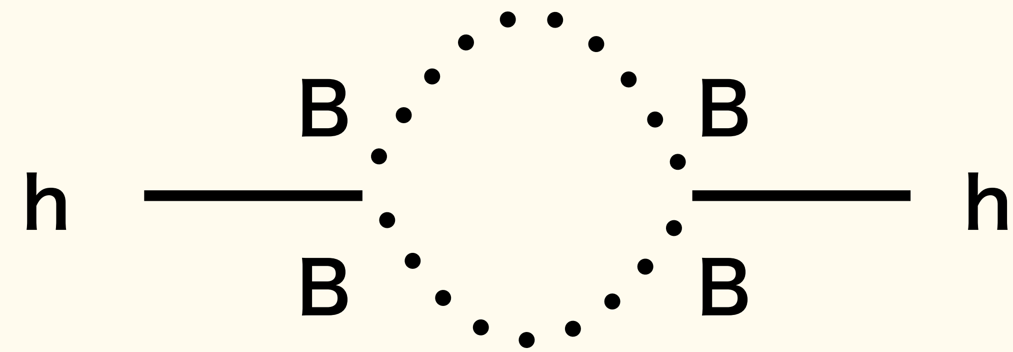
B_i **: physical magnetic field**

h_{ij} **: gravitational wave**

Magnetic field production induces primordial gravitational waves through a following diagram



PGWs from magnetic field production



Using in-in formalism,

we can get the power spectrum of GW come from above diagram

$$\boxed{P_s(k) \simeq \frac{27}{16\pi^4}\lambda^8 \left(\frac{H_{end}}{M_{pl}}\right)^4 \ln\left(\frac{k}{k_{in}}\right)}$$

nearly scale invariant (slightly blue)

To produce observed magnetic fields, we set

$$H_{end} \sim 10^{-5} M_{pl}, \ |\lambda| \sim 17 \quad \longrightarrow \quad \underline{P_s(k) \simeq 10^{-11}}$$

It is comparable to PGWs from inflation $P_{inf}(k) \simeq \left(\frac{H_{inf}}{\pi M_{pl}}\right)^2$

Conclusion

- As to the magnetic field production in $f(\phi)FF$ mechanism, cyclic universe is more favored than inflation

- Magnetic field production in cyclic universe models can induce abundant PGWs

  → Detection of PGWs can not kill cyclic universe

- To distinguish cyclic universe and inflation we need to see

  ex. (
  - spectrum of PGWs using various observations
  - non-gaussianity
  - if PGWs are quantum origin or not

  (
  - standard clock in non-gaussianity of scalar perturbation

  (X.Chen, M.H.Namjoo, Y.Wang (2016)))

Conclusion

- As to the magnetic field production,
  cyclic universe is more favored than inflation

- Magnetic field production in cyclic universe models
  can induce abundant PGWs

  → Detection of PGWs does not deny cyclic universe

- To distinguish cyclic universe and inflation we need to see

  ex.
  - spectrum of PGWs on various scales
  - PGWs are quantum origin or not
  - (standard clock in non-gaussianity of scalar perturbation
    (X.Chen, M.H.Namjoo, Y.Wang (2016)))

Magnetic field production in ekpyrotic

$$\boxed{S_{EM} = \int d^4x\sqrt{-g}\left[-\frac{1}{4}e^{2\rho\frac{\phi}{M_{pl}}}F_{\mu\nu}F^{\mu\nu}\right]}$$

($F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ **)**

coulomb gauge

$$\boxed{S_{EM} = \frac{1}{2}\int d^4x e^{2\rho\frac{\phi}{M_{pl}}}\left[(A_i')^2 - (\partial_i A_j)^2\right]}$$

Fourier transformation

$$\left(A_i = \int \frac{d^3k}{(2\pi)^{3/2}} e^{-\rho\frac{\phi}{M_{pl}}} u_{\boldsymbol{k}} e^{-i\boldsymbol{kx}}\epsilon_i\right)$$

$$\boxed{u_{\boldsymbol{k}}'' + \left(k^2 - \frac{(e^{\rho\frac{\phi}{M_{pl}}})''}{e^{\rho\frac{\phi}{M_{pl}}}}\right)u_{\boldsymbol{k}} = 0}$$

An ekpyrotic universe

As a explicit bouncing universe model,
we consider an ekpyrotic universe.

It is described by a scalar field rolling on an effective potential.