Gravitational waves, solitons, and causality in modified gravity

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December 14, 2017

General ideas of causality

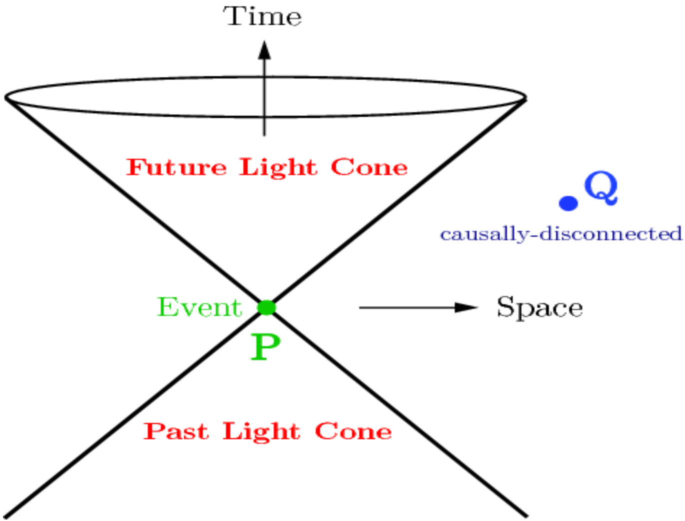
Causality as a hand wave

Two events are causally connected if one may have influenced the other

– Maxwell electrodynamics $\implies$ two electromagnetic events are causally connected if light can travel between them (null).

– One can associate an (electromagnetic) causal domain with every spacetime event; draw light cones around the point.

Simple demonstration of Maxwellian causality



Gravitational causality

Newtonian

Gravity is a long-range force, and all events are (gravitationally-) causally connected

Einsteinian

Energy influences spacetime structure, and gravitational information travels at some finite speed, determined from the field equations;

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi T_{\mu\nu}.$$

Gravitational waves are the physical medium which transports this information.

Gravitational waves?

– Gravitational waves are non-linear objects; they arise as solutions to the Einstein equations. But often treated as linear.

– *linear* gravitational waves predicted in 1916;

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}; ||\mathbf{h}|| \ll 1,$$

$$\Box_{\boldsymbol{\eta}} \bar{h}_{\mu\nu} + \mathcal{O}(\bar{h}^2) = 8\pi T_{\mu\nu}.$$

Linear waves travel at the speed of light.

– As it turns out, within general relativity, linear and non-linear gravitational waves propagate at speed c. Proven by Rosen (amongst others); highly non-trivial to even see the wave structure in

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi T_{\mu\nu}$$

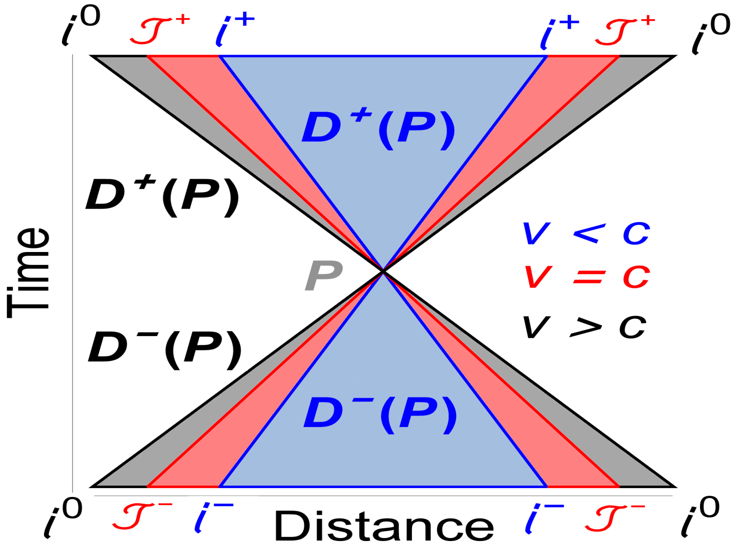
So what if gravity is non-GR?

– Since gravitational waves are sensitive to the field equations, the nature of gravitational waves in a non-Einstein theory of gravity are likely to be different to their GR counterparts. Waveforms, amplitudes, ...

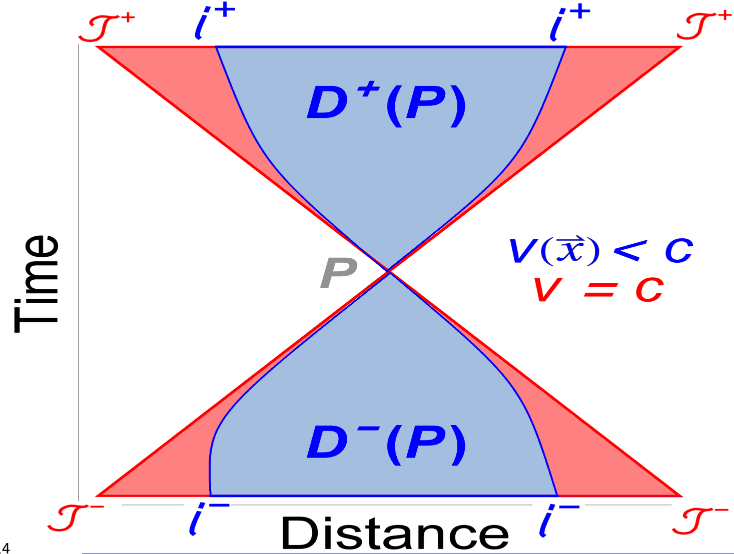
– The nature of gravitational causality is fundamentally altered. Often only linear theory treated – but may not extend to non-linear.

– To study causality in a modified theory of gravity (scalar-tensor, higher-order curvature, $\cdots$) we can search for exact gravitational wave/soliton solutions and analyse their properties (AGS and Melatos 17, PRD).

Simple kinds of modified causality



Slightly more complicated causality



$f(R)$ gravity

$f(R)$ gravity – "non-linear" generalisation of Einstein-Hilbert

$$f'(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} + [g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu]\,f'(R) = \kappa T_{\mu\nu}$$

– One example we are interested in is the $f(R)$ class of theories, which are motivated for a variety of reasons (Dark matter/energy, 'natural' correction, derivable from string theory considerations)

– $f(R) = R$ returns general relativity; what about theories with quadratic or higher-order corrections?

Linear vs. non-linear?

– For analytic theories of the form

$$f(R) = \sum_{i=0,1,2,\cdots} a_i R^i$$

.

– Field equations linearised about a Minkowski or de-Sitter background only care about a_0, a_1, and a_2. Higher-order pieces do not appear in the linearised equations.

– A result of Berry and Gair (PRD, 2011) states that the above theory predicts the existence of linear scalar modes with group velocity ($a_2 \neq 0$)

$$c_{\mathrm{g}} = c\frac{\sqrt{\omega^2 + a_1 \left(3a_2\right)^{-1}}}{\omega}, \tag{1}$$

$\implies$ tachyonic for $a_2 > 0$.

An example exact solution (AGS & Melatos, '17)

Of the Einstein-Rosen (1937) form, Jordan-Ehlers-Kompaneets line element in Weyl coordinates (t, ρ, ϕ, z)

$$ds^2 = e^{-2\psi} \left[e^{2\gamma} \left(-dt^2 + d\rho^2\right) + \rho^2 d\phi^2\right] + e^{2\psi} dz^2, \qquad (2)$$

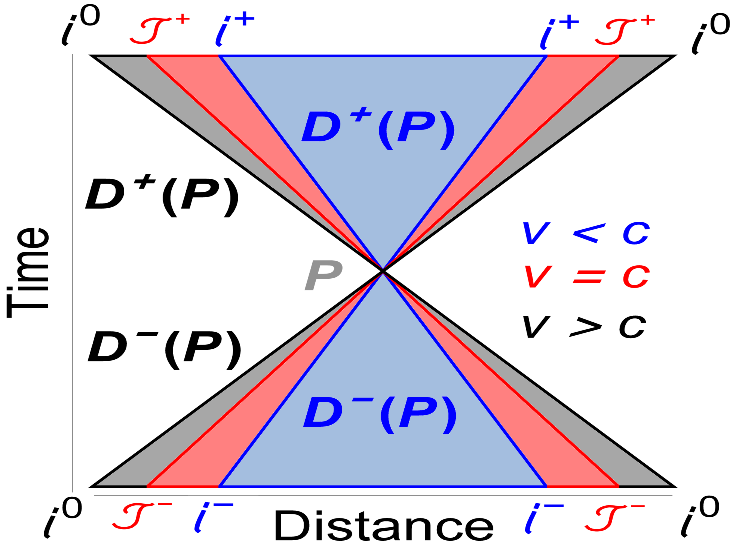
where ψ and γ are functions of t and ρ.

– With the choices $\psi = 0$ and

$$\gamma = \frac{1}{2} \ln \left\{ A \left(1 - v^2\right) \operatorname{csch} \left[\delta + \omega \left(t - v\rho\right)\right]^2 \right\},$$

we find that the Einstein-Rosen metric is a solution, for any $v \neq 1$, to an $f(R)$ theory with polynomial f for *any* values of a_1 or a_2.
$\Longrightarrow$ Linear theory does not capture essential properties of GWs even for small amplitudes A!

All except the red (GR) surfaces!



Astrophysical implications

If tachyonic GWs exist, they may be able to escape the (null defined) event horizon of a black hole. Beyond horizon information during ringdown?

Sub/Super-luminal gravitational interactions will couple to the Friedmann equations and modify cosmological dynamics – annihilation and decays of superluminal particles into ordinary ones will release very large amounts of kinetic energy from the rest masses ($E \sim mv^2, v \gg c$) which could generate a fast expansion of the Universe.

Summary

– Causality in modified gravity is sensitive to properties of non-linear GWs; many different ways causality can be altered.

– A linear analysis may fail to capture essential features.

– Through data analysis of BH-BH mergers we can test GR; if non-Kerr features then we can study non-linear GWs to rule out classes of theories based on tachyonic GWs (maybe?).

– Future things to think about: connection with initial data and other mathematical considerations.