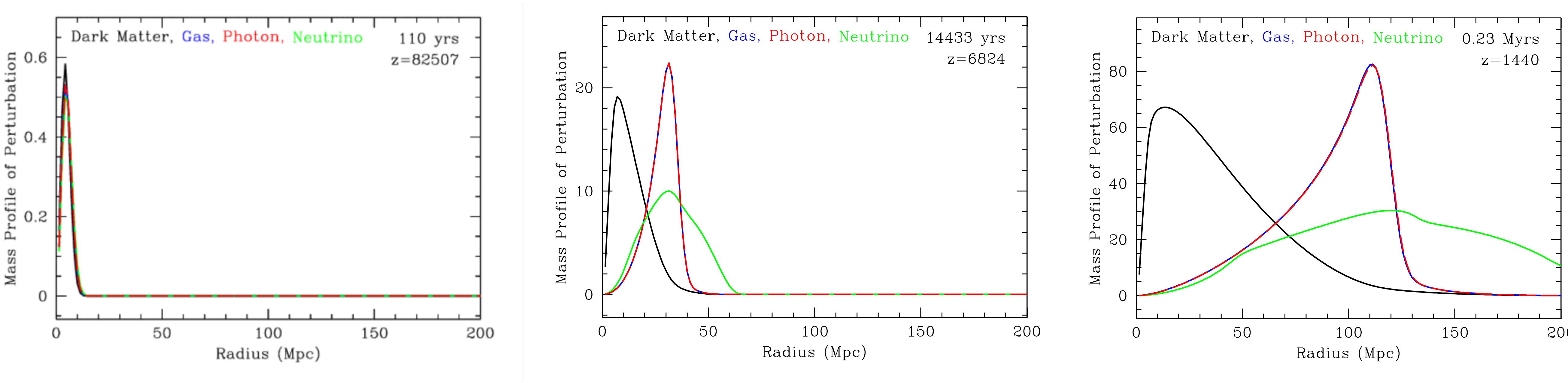
Infrared Effects at the BAO Scale

Gabriele Trevisan

CosPA 2017

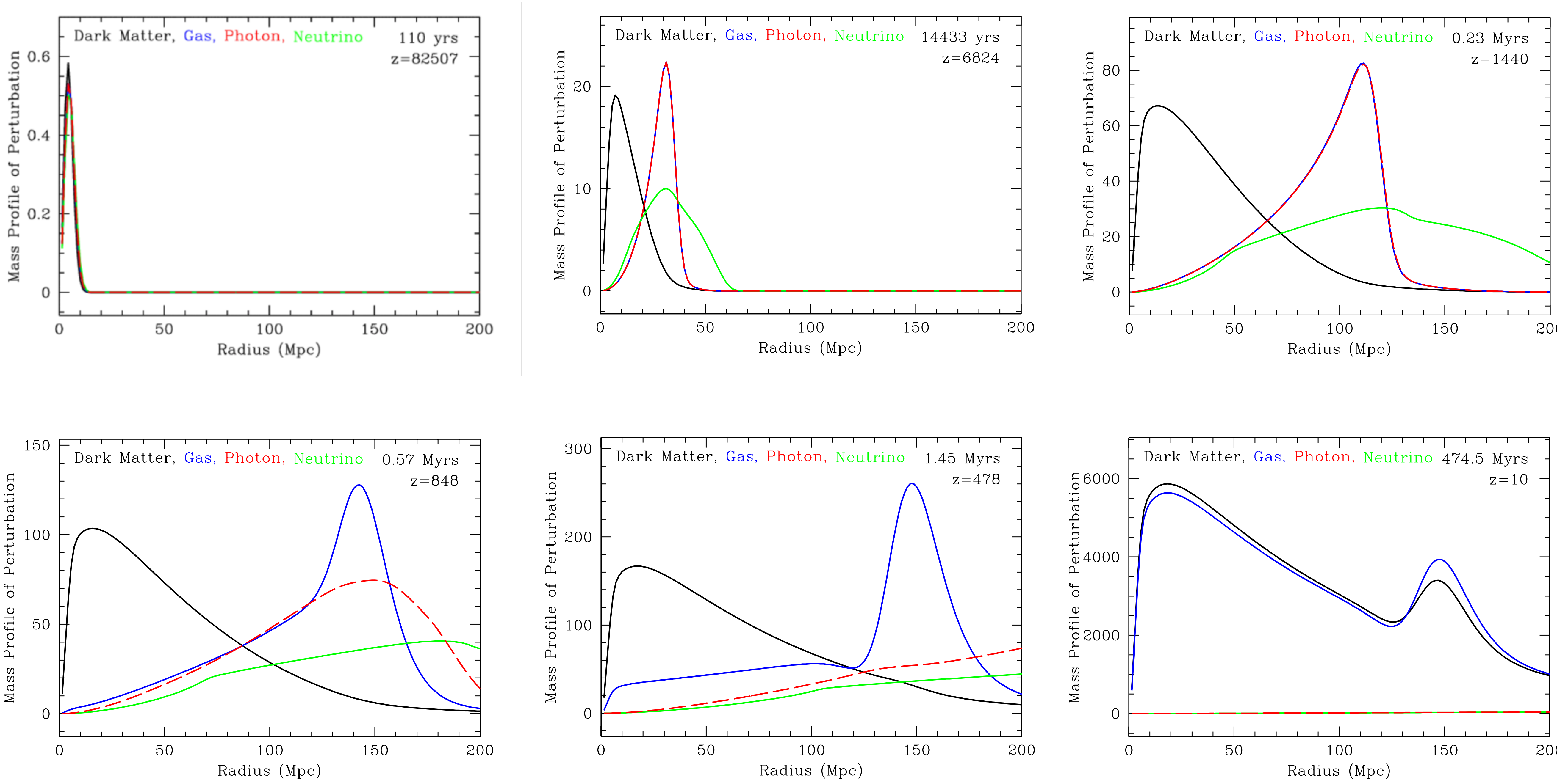
Senatore, Trevisan: arXiv 1710.02178, Scoccimarro, Trevisan: arXiv xxxx.xxxx

BAO in pictures



Eisenstein et al. 0604361

BAO in pictures

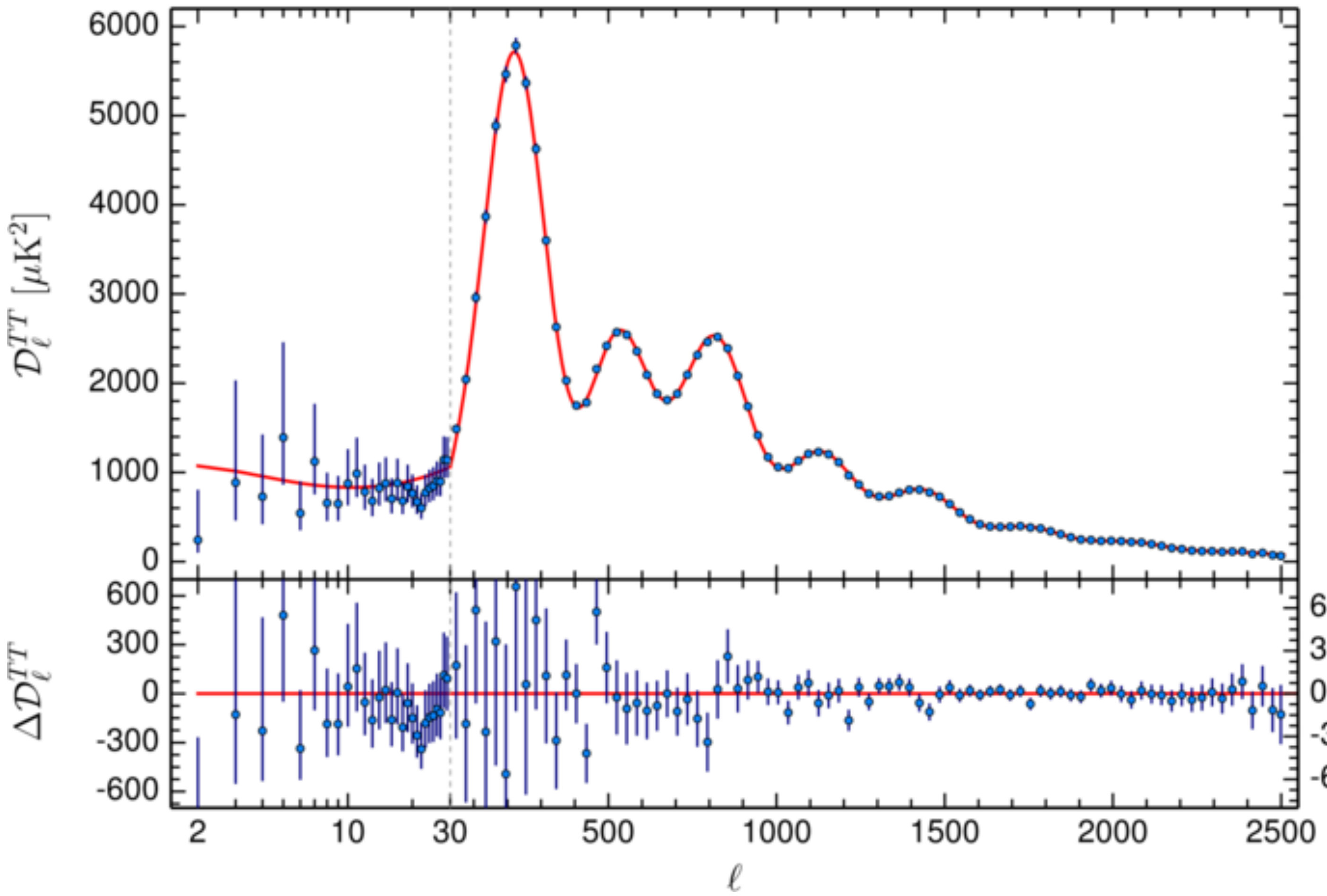


Eisenstein et al. 0604361


R = 150 Mpc

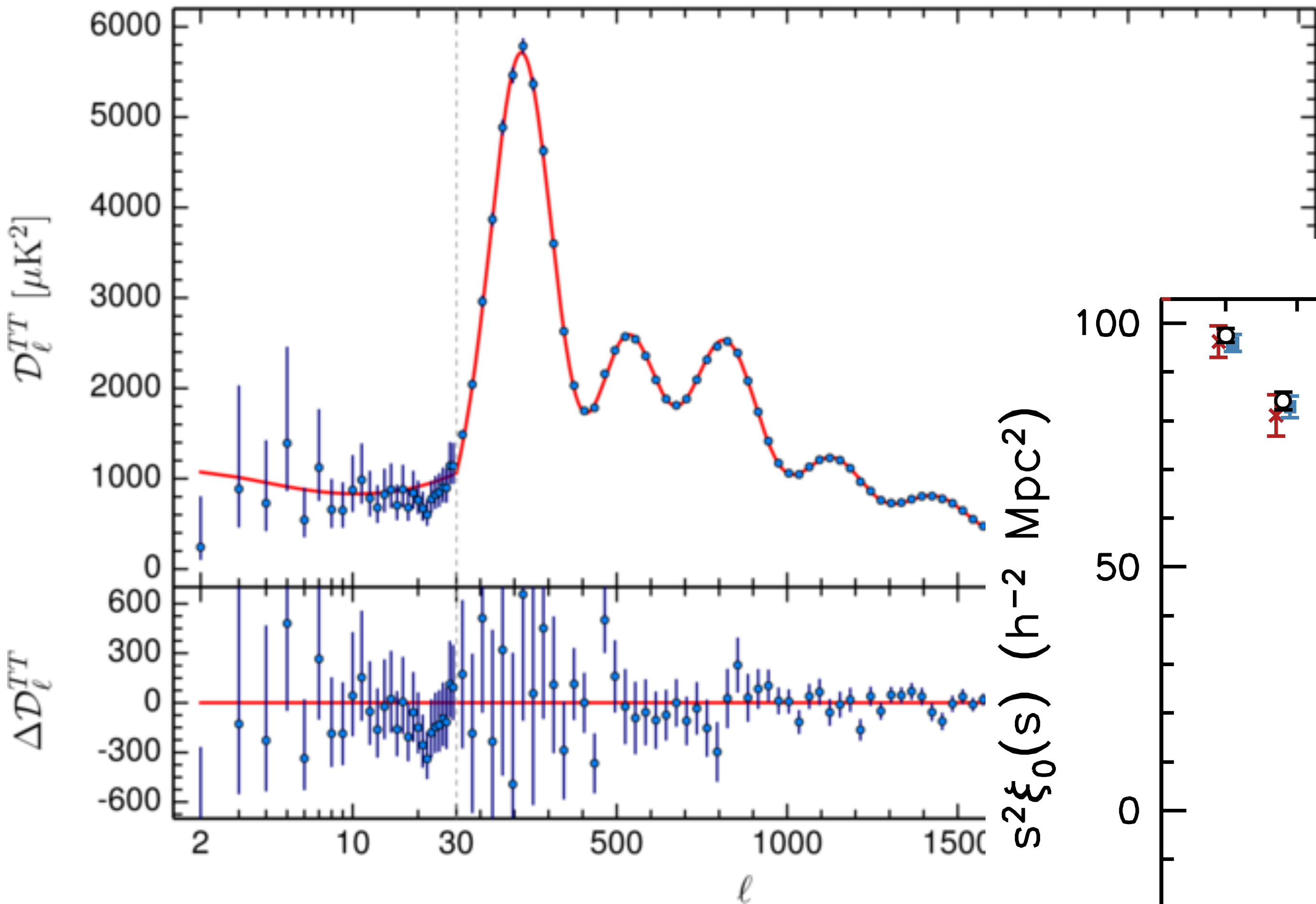
BAO in the sky

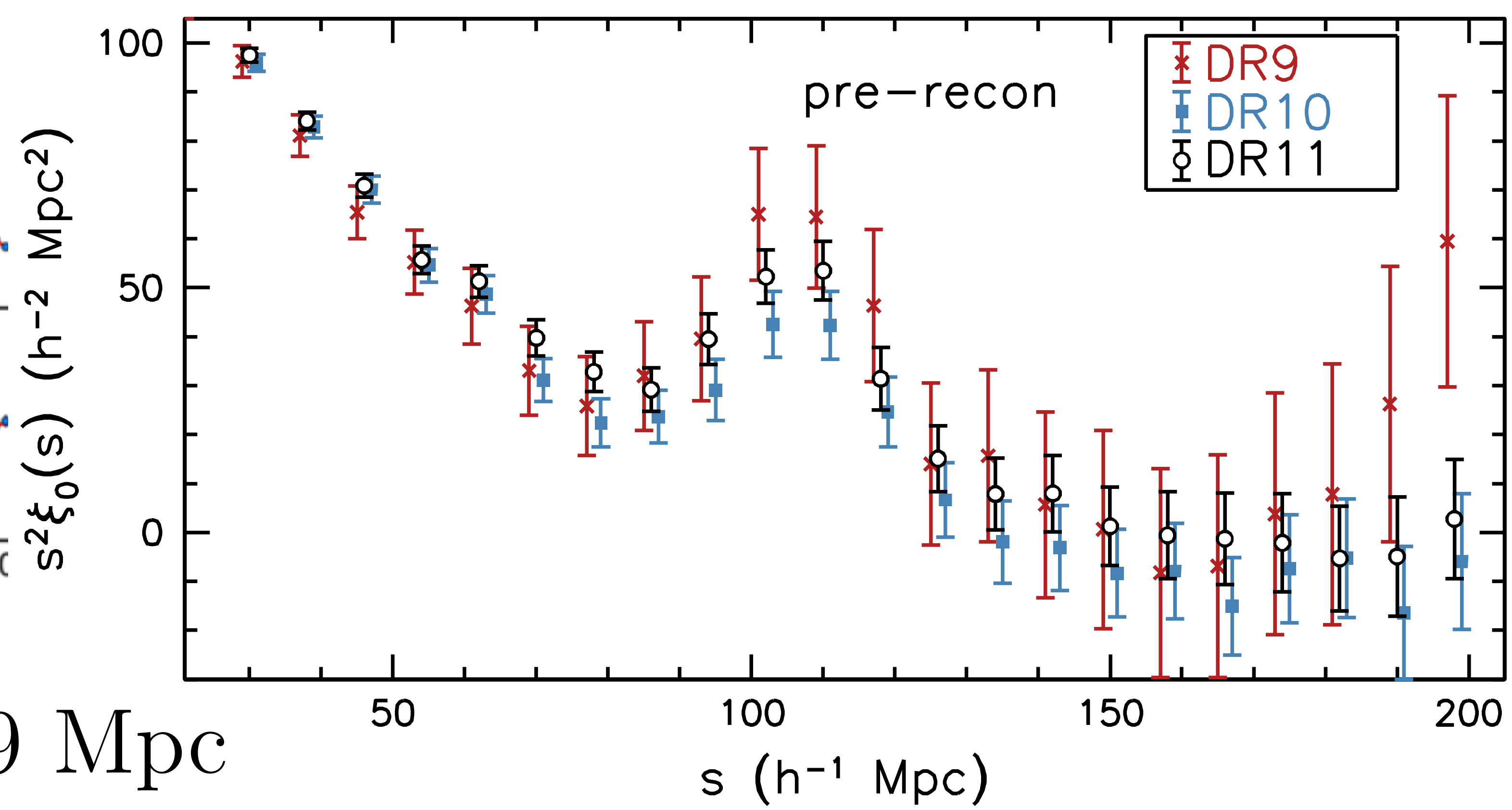
Planck 2015



$$r_{drag} = 147.33 \pm 0.49 \text{ Mpc}$$

BAO in the sky





$$r_{drag} = 147.33 \pm 0.49 \text{ Mpc}$$

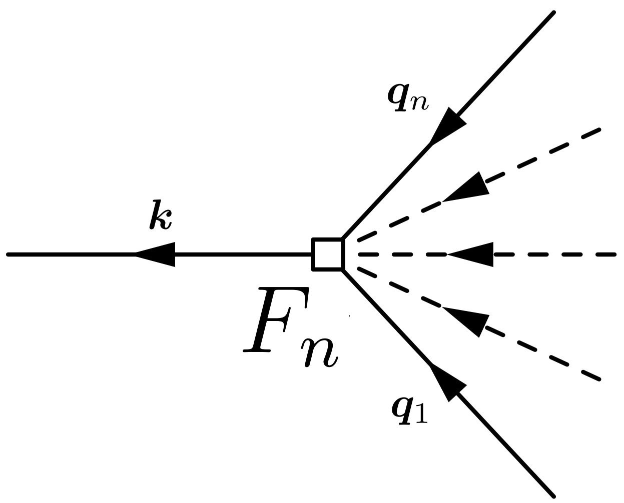
$$D_V(z = 0.57) = 2056 \pm 20 \text{ Mpc}$$

Standard Perturbation Theory (SPT)

Fluid Equations:

$$\partial_\tau \delta + \vec{\nabla} \cdot [(1+\delta)\vec{v}] = 0,$$
$$\partial_\tau \vec{v} + \mathcal{H}\,\vec{v} + \vec{v} \cdot \vec{\nabla}\vec{v} + \vec{\nabla}\phi = 0$$
$$\triangle \phi = \frac{3}{2}\mathcal{H}^2 \Omega_{\rm m} \delta$$

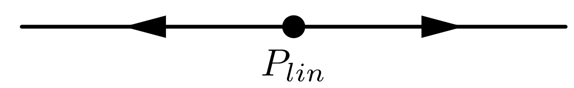
Perturbative solution: $\delta_n(\vec{k})$

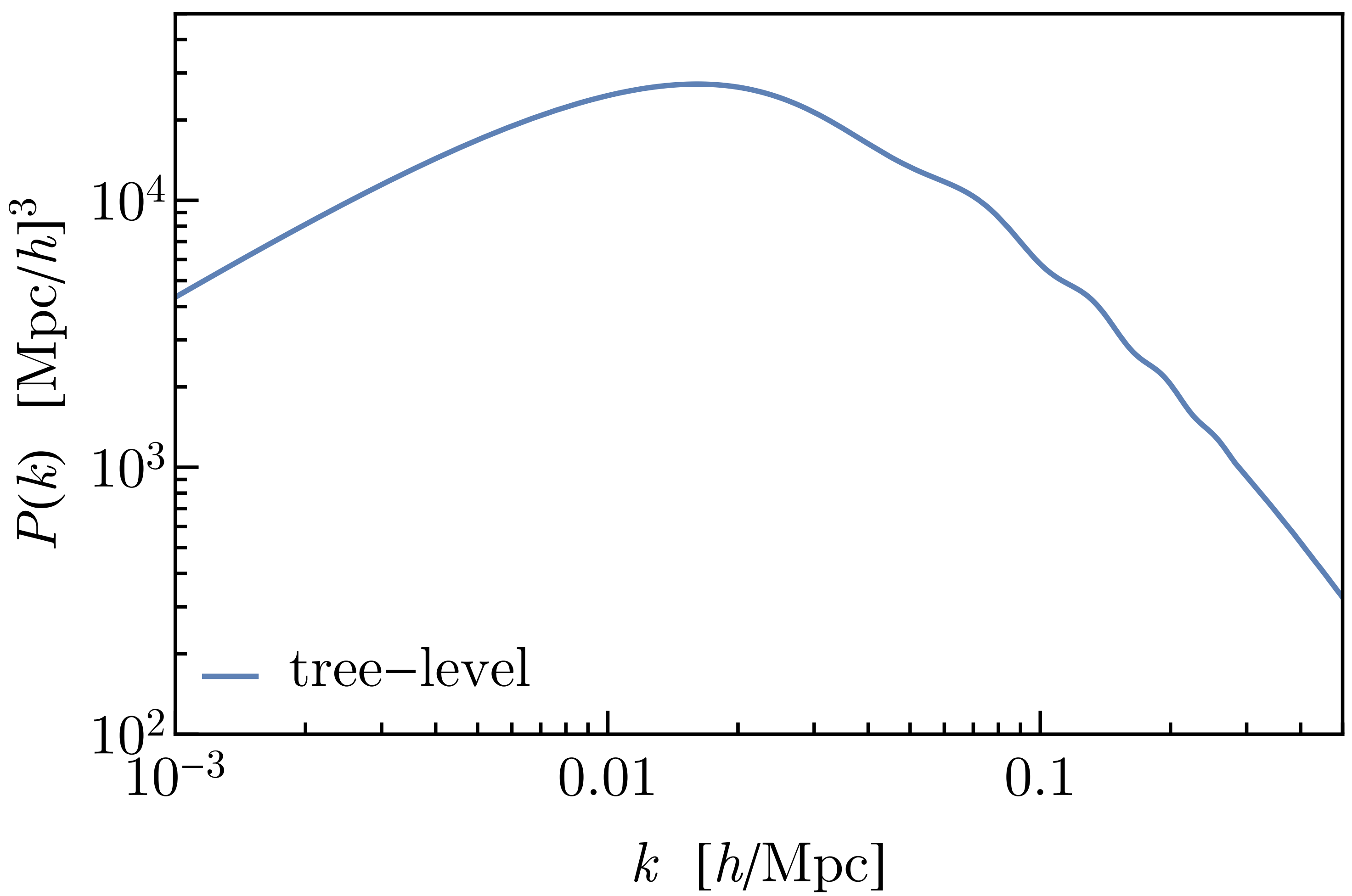


$\delta^n_{\rm linear}(\vec{q})$

SPT 2-Point Function

$$P(k)|_{tree-level} =$$



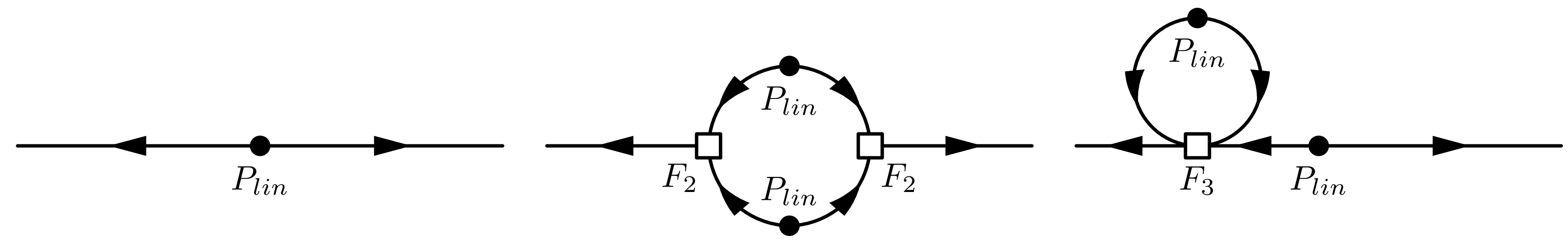


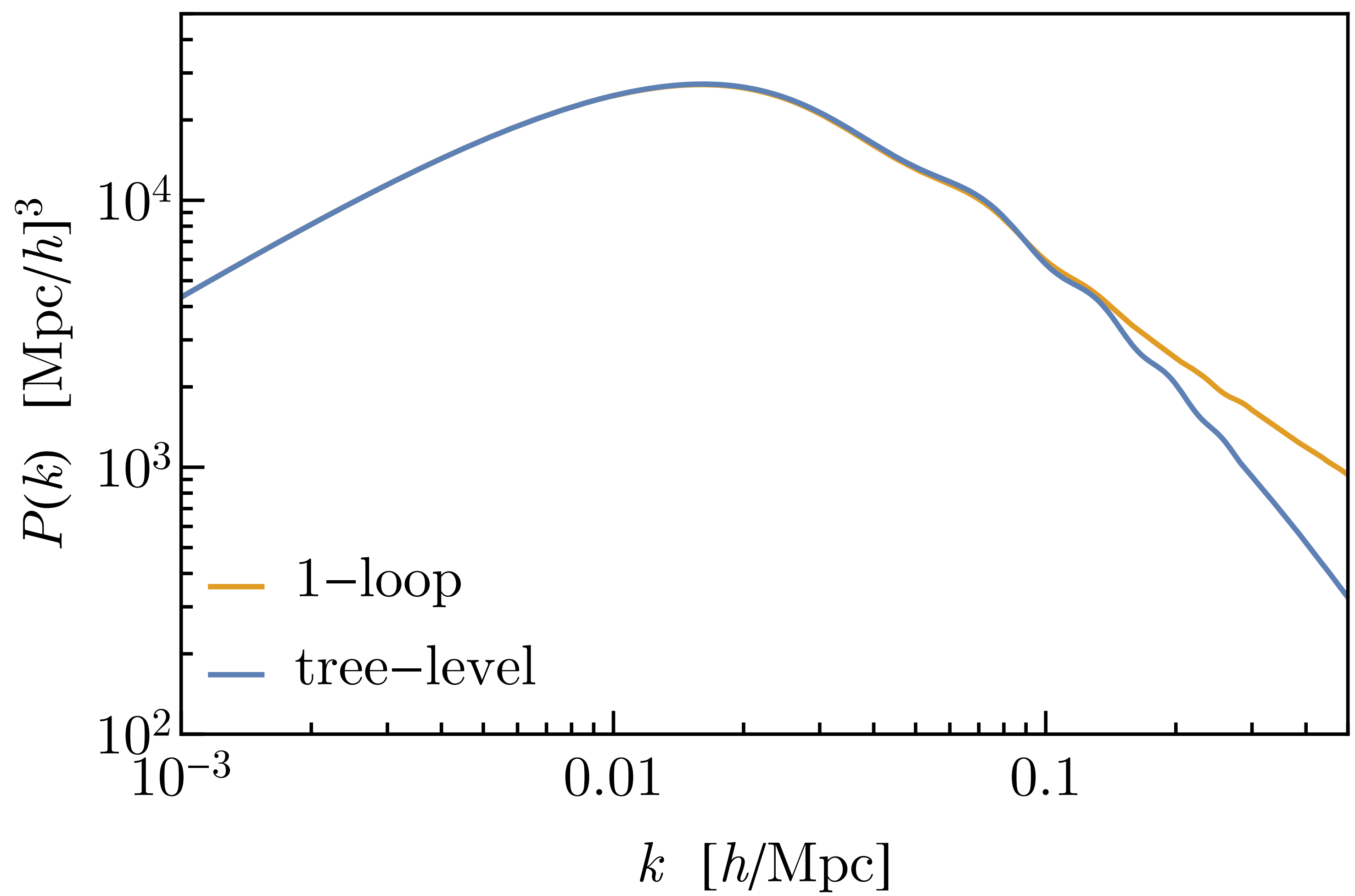
Where is the BAO in Fourier space?

The BAO signal is ~5% oscillation with freq. 1/150 Mpc

SPT 2-Point Function

$$P(k)|_{1-loop} =$$

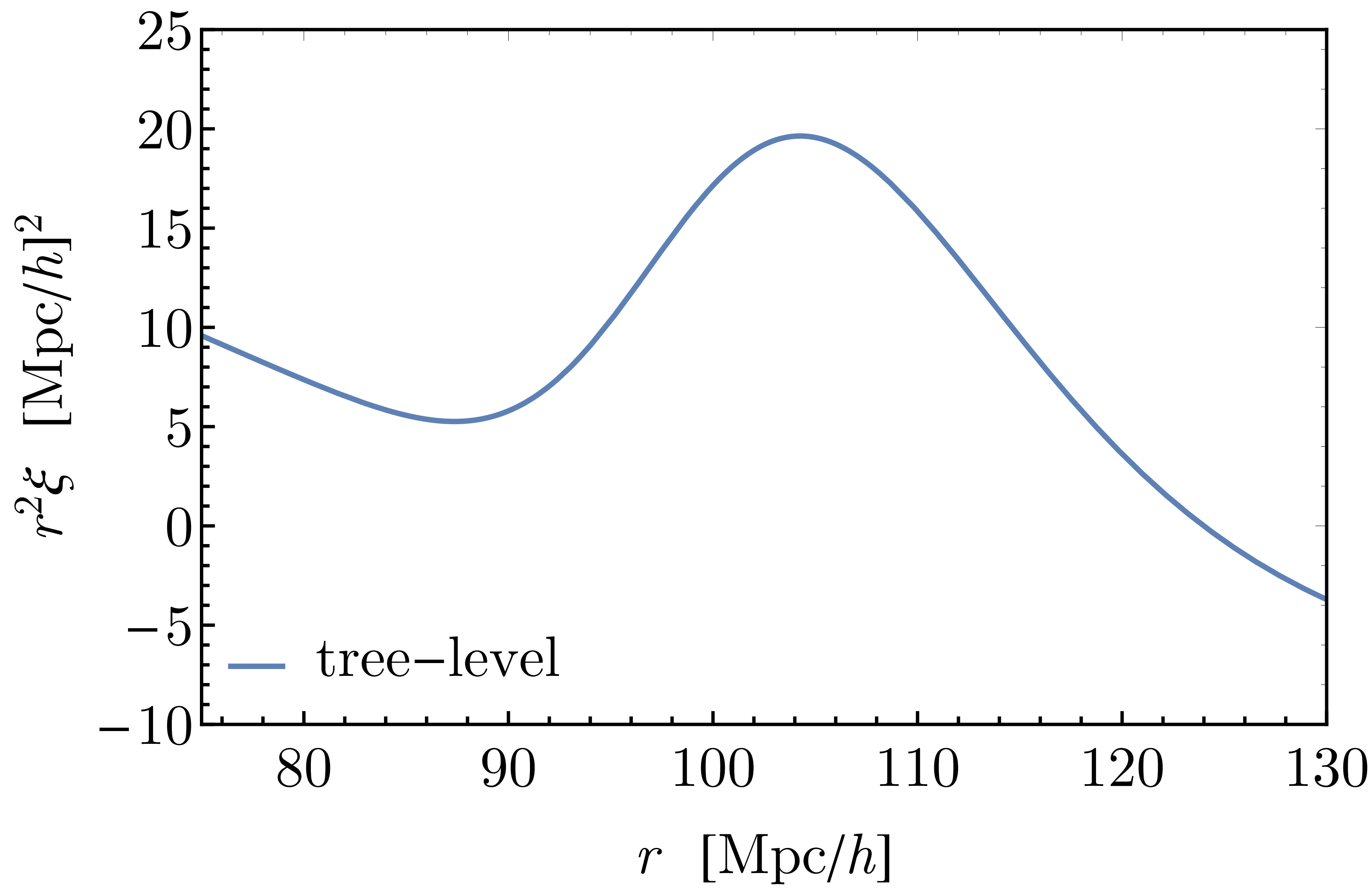




Perturbation Theory should recover the BAO,
so the more loops the better, right?

SPT 2-Point Function

SPT completely fails around the BAO scale



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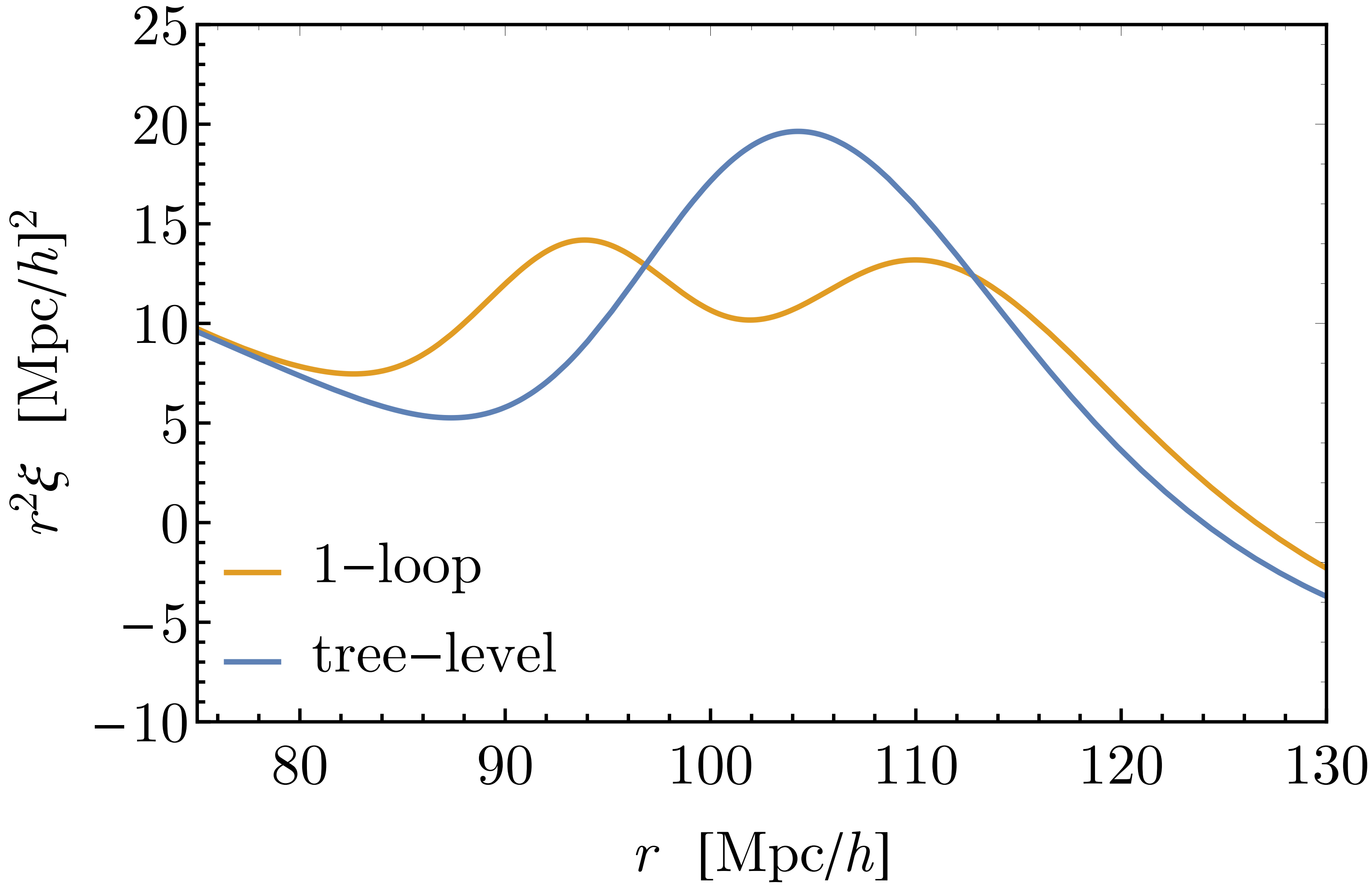

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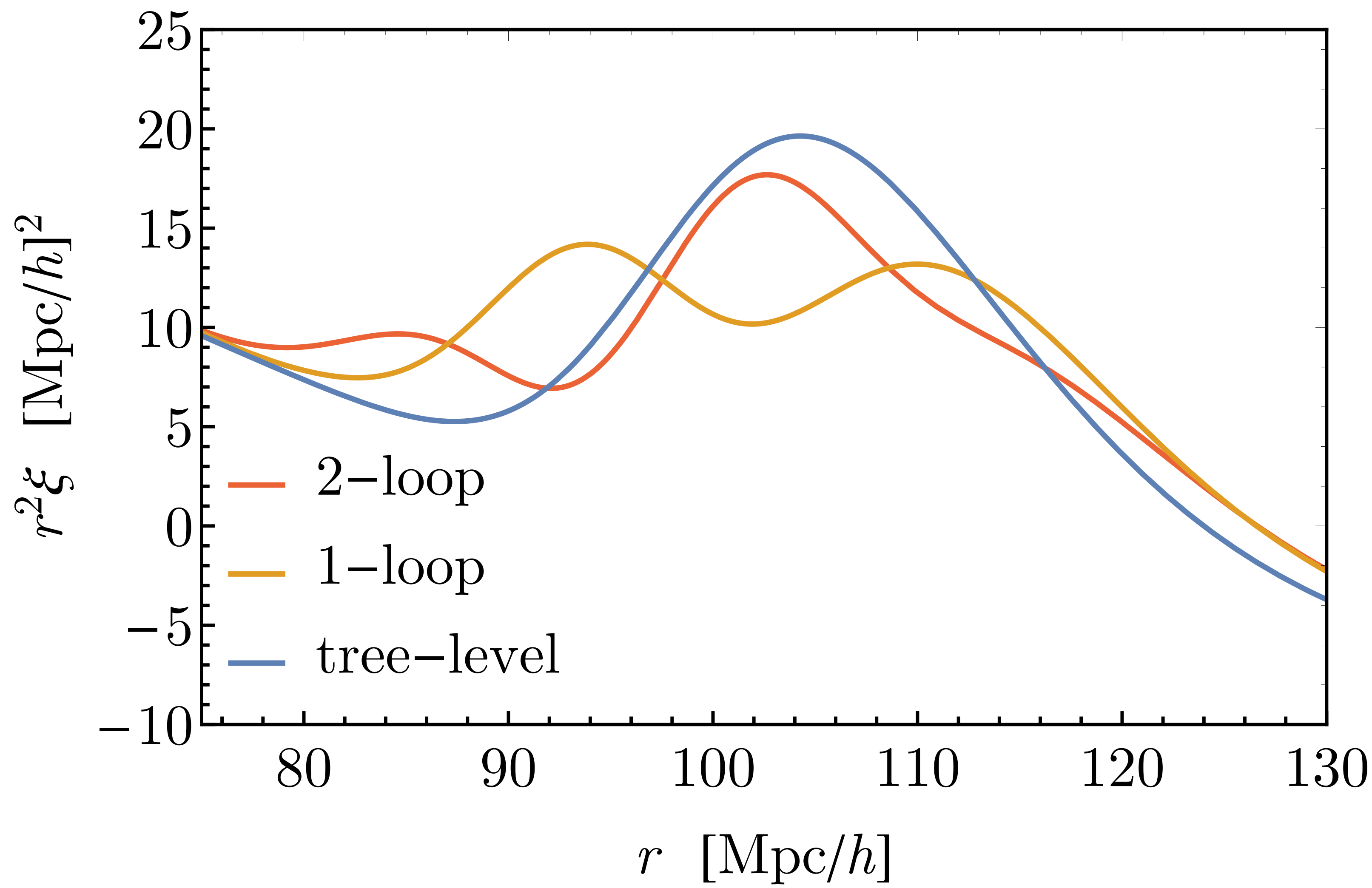

Why SPT fails?

The wiggly component (BAO) receive large infrared (IR-enhanced) contribution from loop integrals

$$P^w_{1-\mathrm{loop}}(k) \sim k^2 P^w_{\mathrm{lin}}(k) \int_{\ell^{-1}_{\mathrm{BAO}} \lesssim p \lesssim k} \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{P_{\mathrm{lin}}(p)}{p^2}$$
$$= P^w_{\mathrm{lin}}(k) \epsilon_{s_<}$$

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$$= P^{w}_{\mathrm{lin}}(k) \epsilon_{s_<}$$

and $\epsilon_{s_<} \approx 1$

Bad for doing PT! Better not to expand

To fix just resum

- Lagrangian PT (à la Zeldovich)

  short modes should not be resummed,
  calculations are more cumbersome, especially EFT,
  Fourier space is numerically more challenging

- IR-resummation (à la EPT) Senatore, Zaldarriaga '15

  numerically more demanding

- Consistency relations (using EP) Baldauf et. al '15

  split P_{lin} into smooth and wiggly components,
  NLO corrections

- Time-Sliced PT (à la QFT) Blas et al. '15

  need to split P_{lin} into smooth and wiggly components
  manually check the IR-enhanced contribution
  resummation relies on separation of scales
  calculations are more cumbersome, especially NLO and UV

IR-resummation

The idea is to resum IR displacement modes

$$K_0(\boldsymbol{k}, \boldsymbol{q}) \equiv \exp\left[-\frac{1}{2} k_i k_j A_{ij}^{\mathrm{IR}}(\boldsymbol{q}) \qquad\qquad\qquad\qquad \right]$$

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$$\sim \epsilon_{s<}$$

IR-resummation

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$$K_0(\boldsymbol{k},\boldsymbol{q}) \equiv \exp\left[-\frac{1}{2}k_i k_j A^{\rm IR}_{ij}(\boldsymbol{q}) + \frac{i}{6}k_i k_j k_k B^{\rm IR}_{ijk}(\boldsymbol{q})\right]$$

$\underbrace{k_i k_j A^{\rm IR}_{ij}(\boldsymbol{q})}_{\sim\, \epsilon_{s<}}$

$\sim \epsilon_{s<}$

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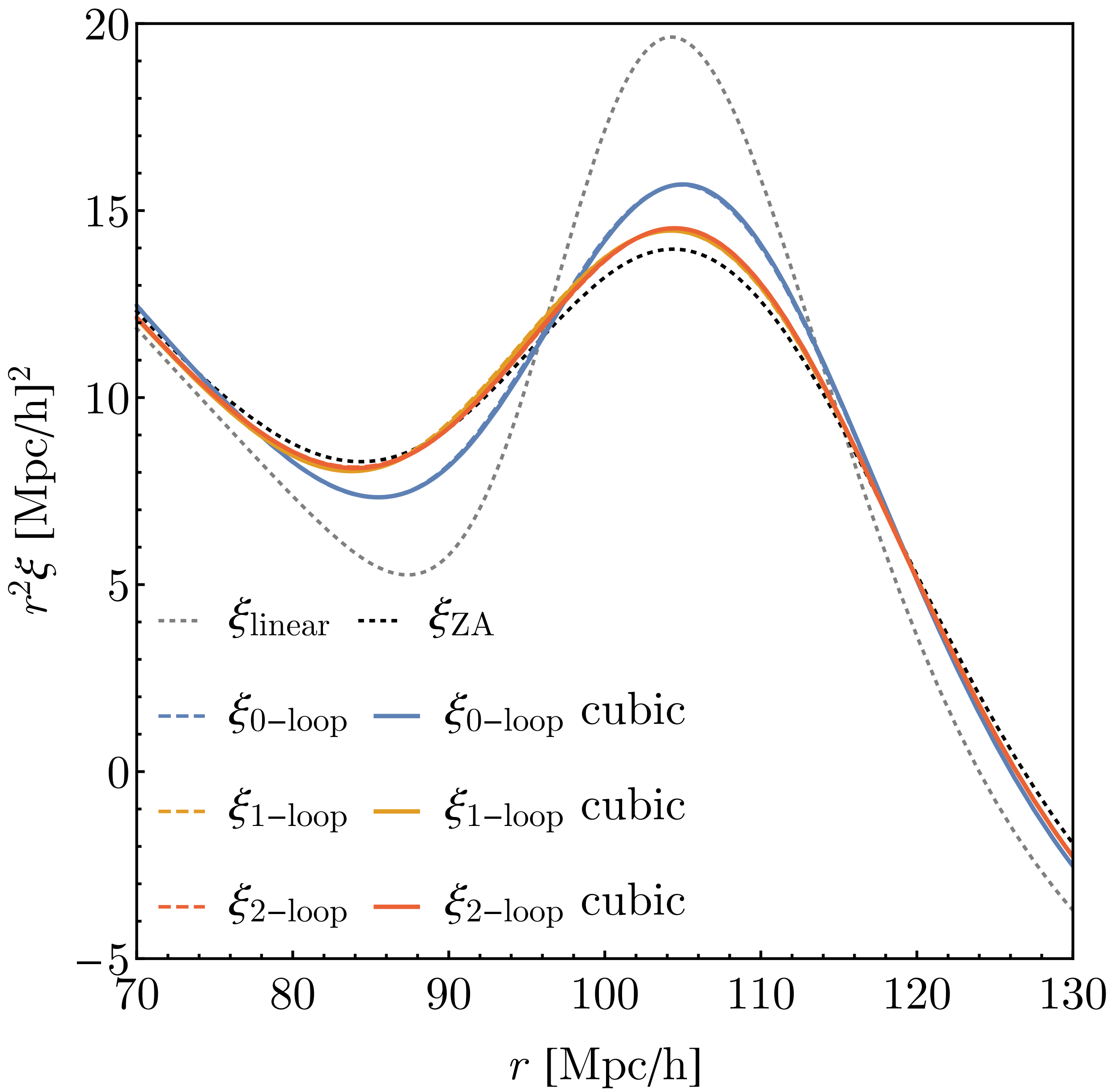
$\sim \epsilon_{s<}$

For example at tree-level

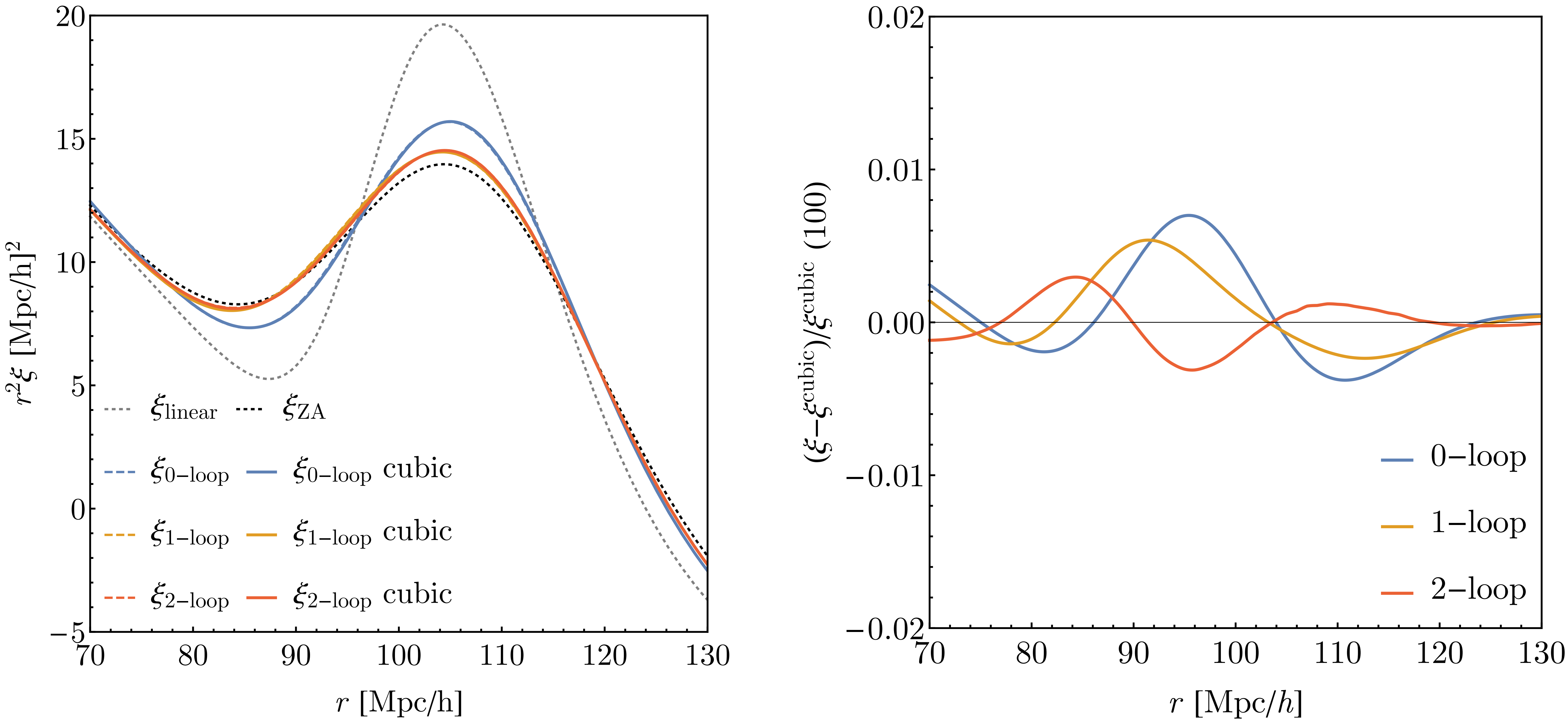
$$\xi_{\rm tree}(r) = \frac{1}{|A_{ij}(\boldsymbol{r})|^{1/2}}\int {\rm d}^3 q\ \xi_{\rm tree}^{\rm E}(q)\ \exp\left[-\frac{1}{2}(\boldsymbol{r}-\boldsymbol{q})_i A_{ij}^{-1}(\boldsymbol{r})(\boldsymbol{r}-\boldsymbol{q})_j\right].$$

IR-resummation with NLO terms



Senatore, Trevisan: arXiv 1710.02178

IR-resummation with NLO terms



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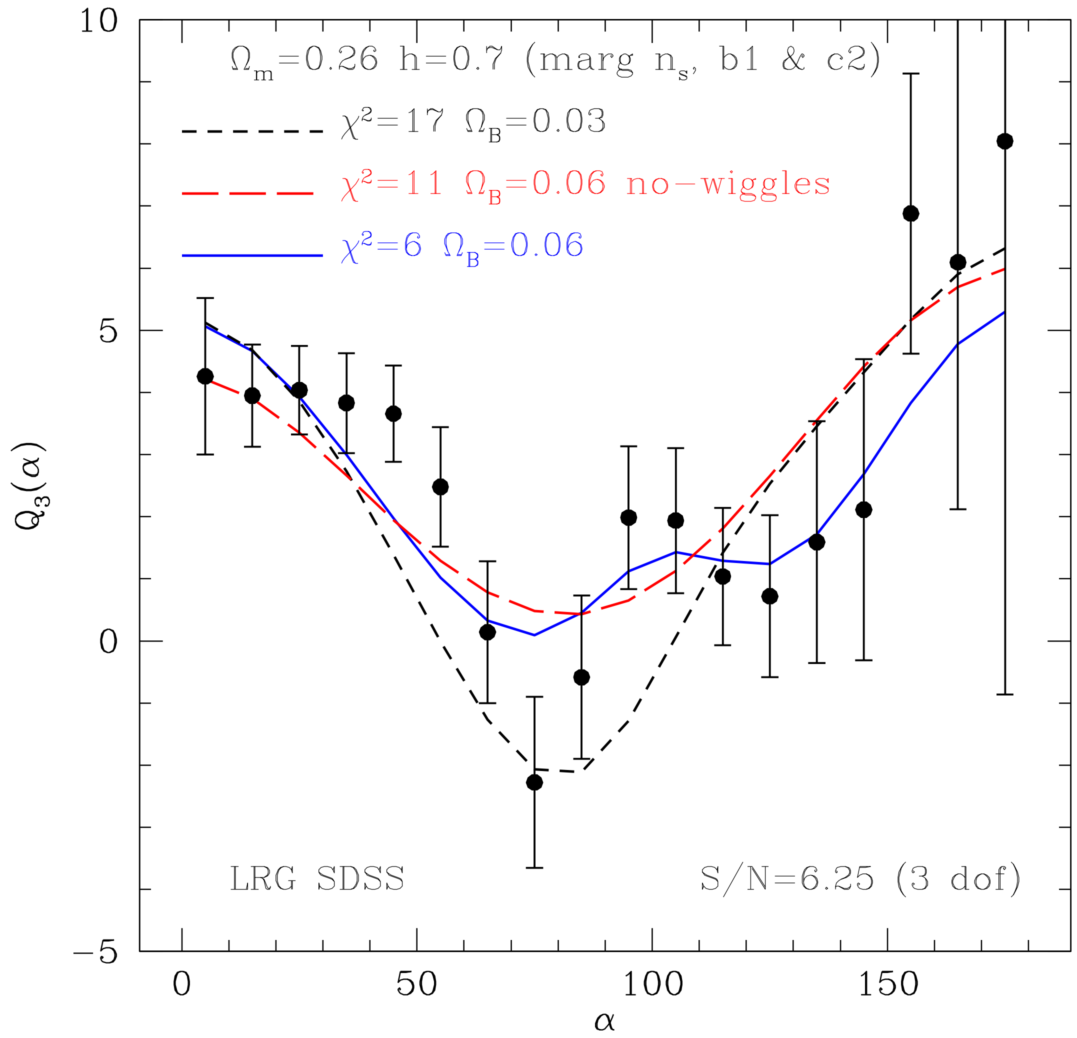
Detection of the BAO in the 3-PF

Gaztanaga et. al 0807.2448



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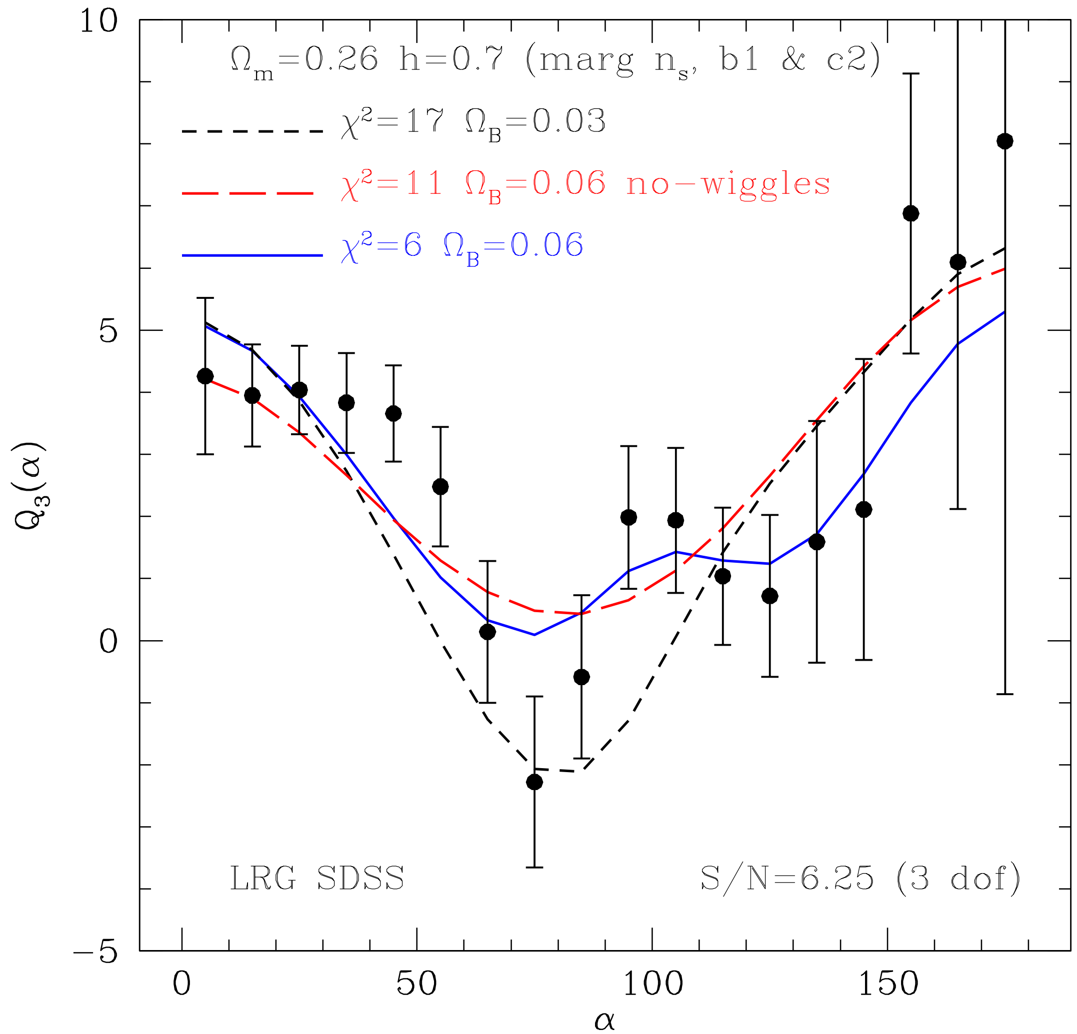
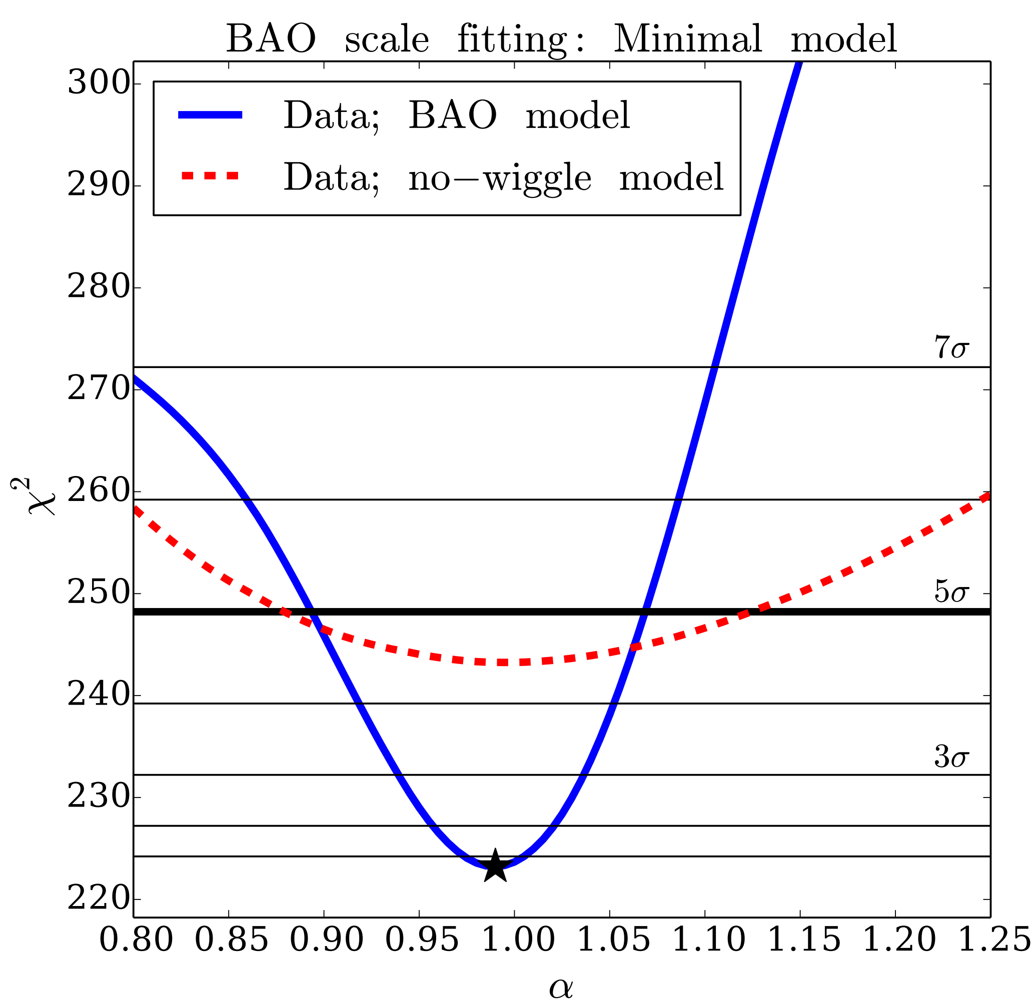


Slepian et al. 1607.06097



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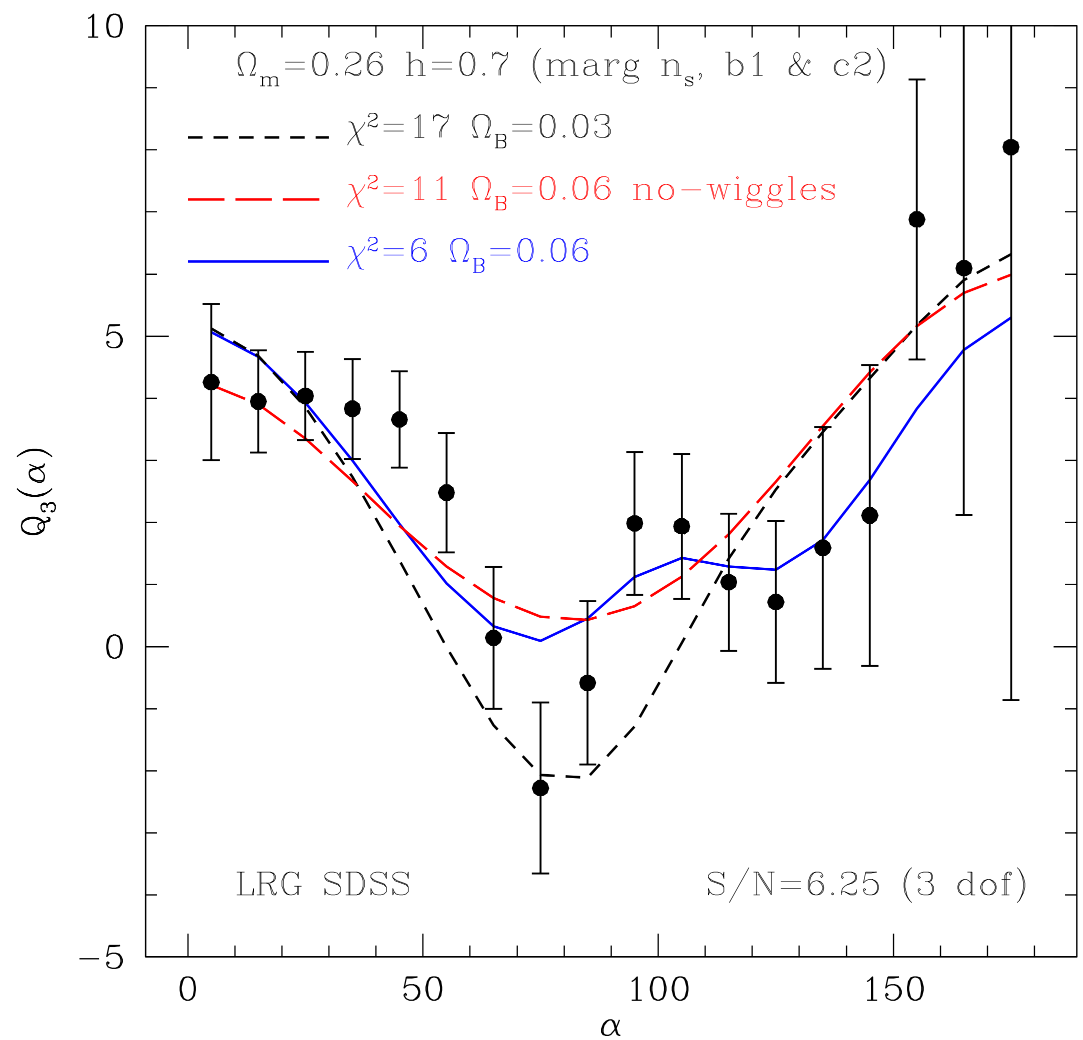

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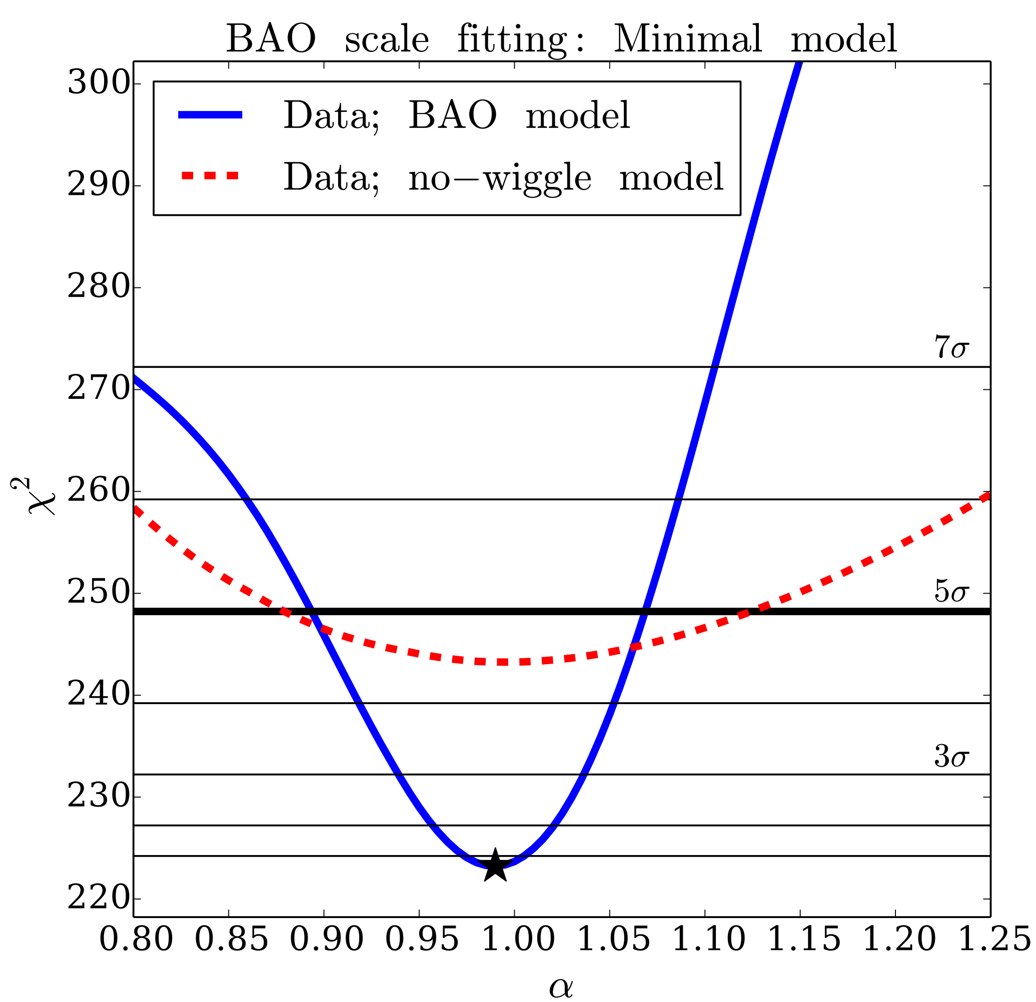
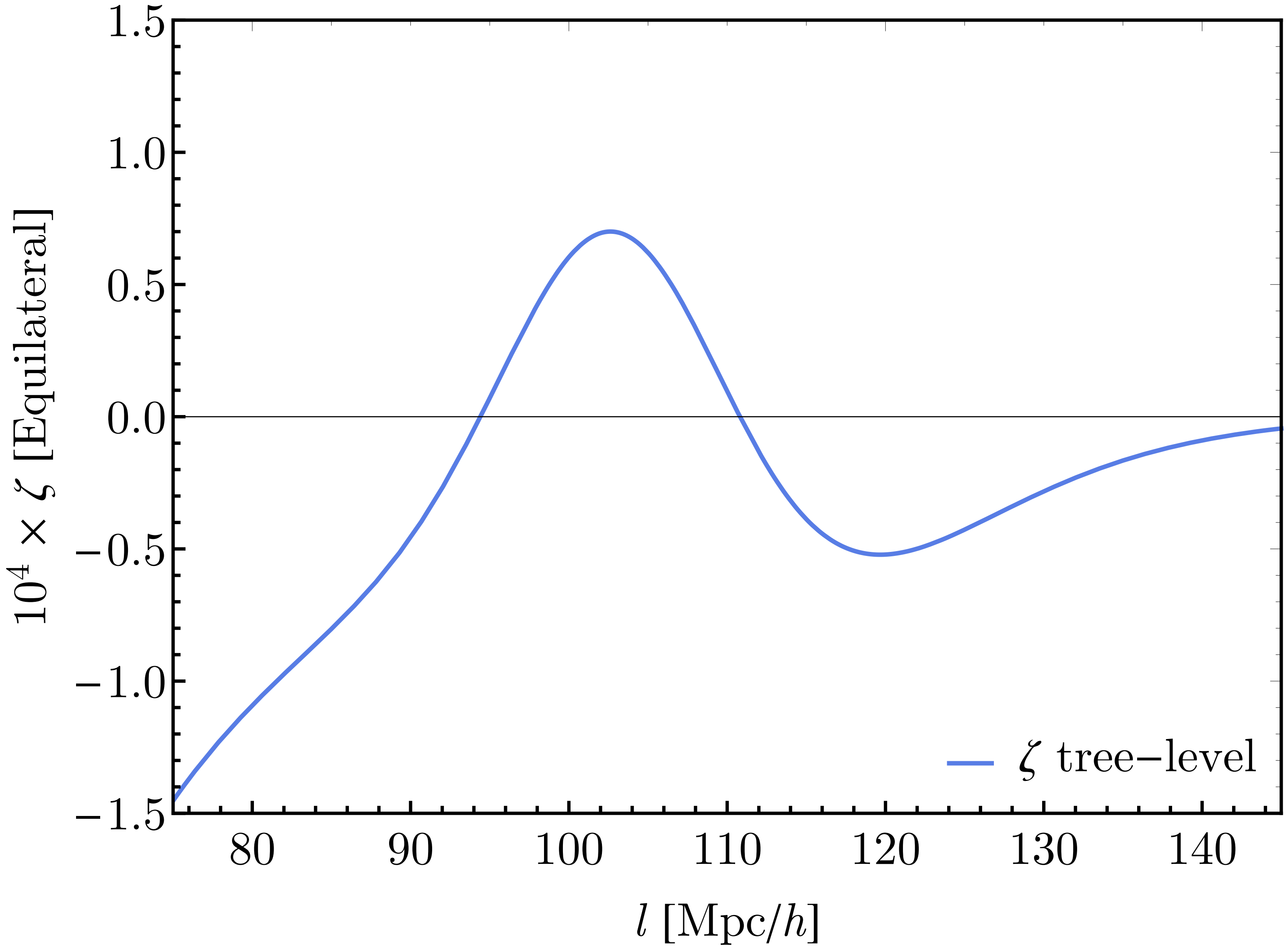


Analyses use $P_{\rm phys}(k) = [P(k) - P_{\rm nw}(k)] \exp\left[-k^2\Sigma_{\rm nl}^2/2\right] + P_{\rm nw}(k),$

plugged into the tree-level SPT 3-PF

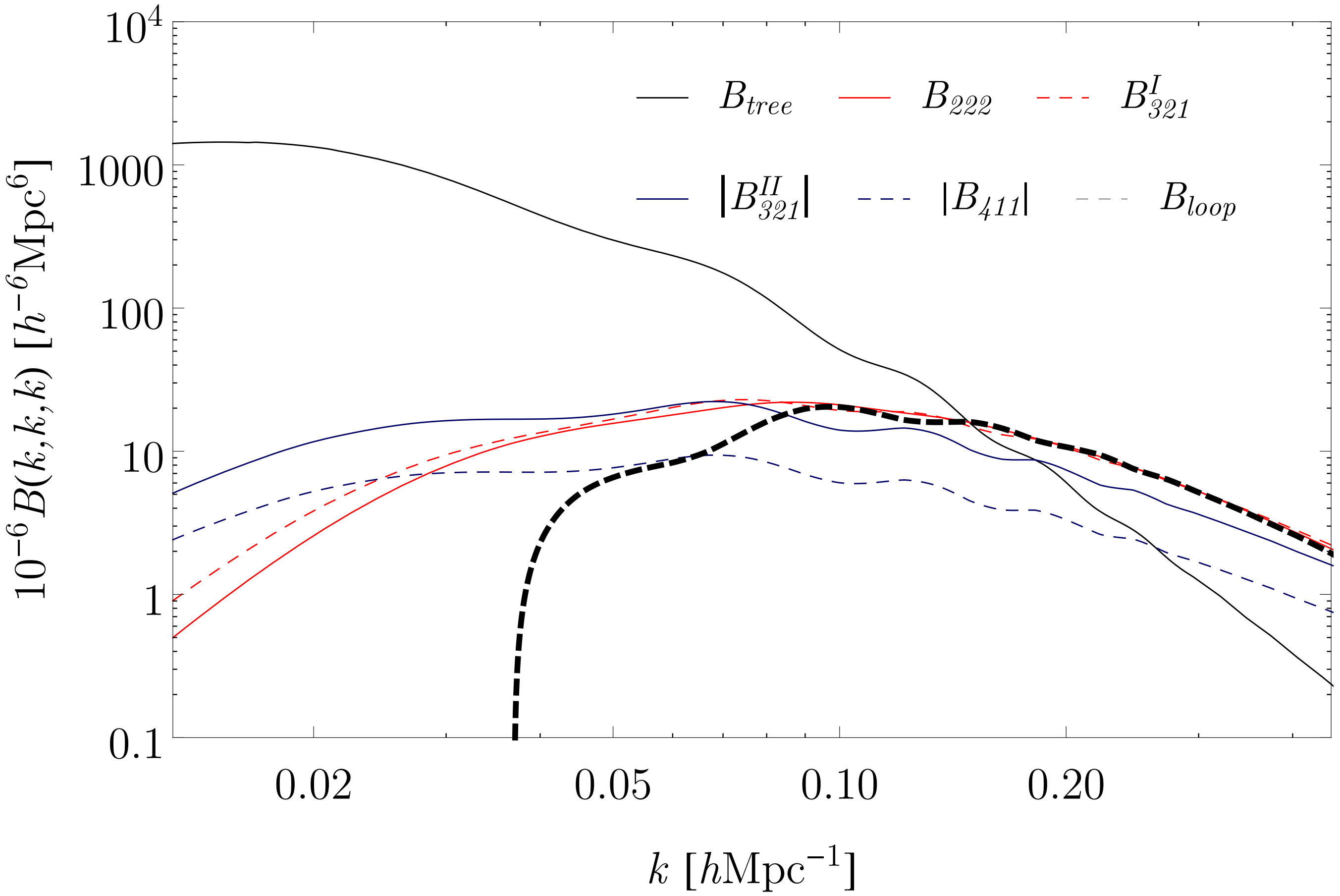
SPT predictions for the 3-PF

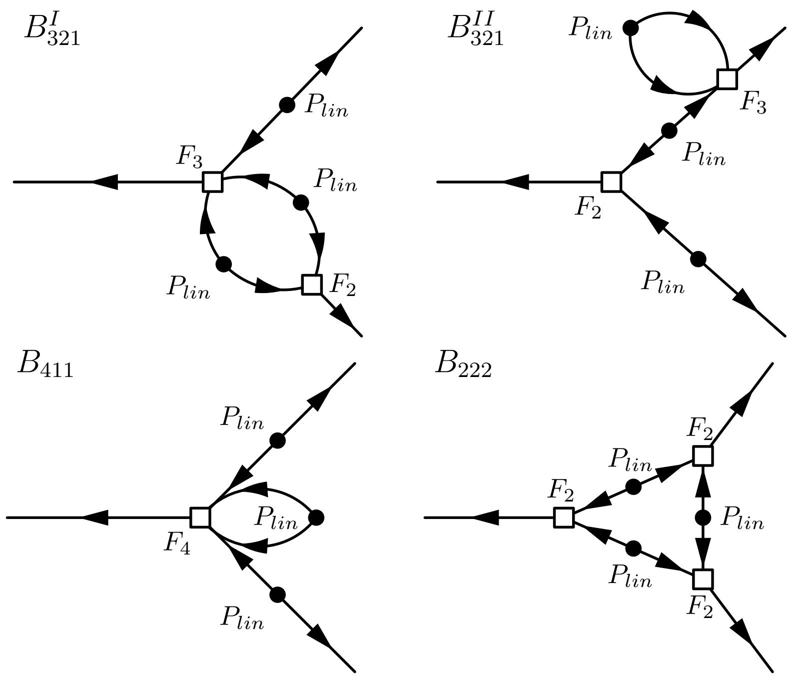
Equilateral triangles



SPT predictions for the 3-PF

Baldauf et.al 1406.4135





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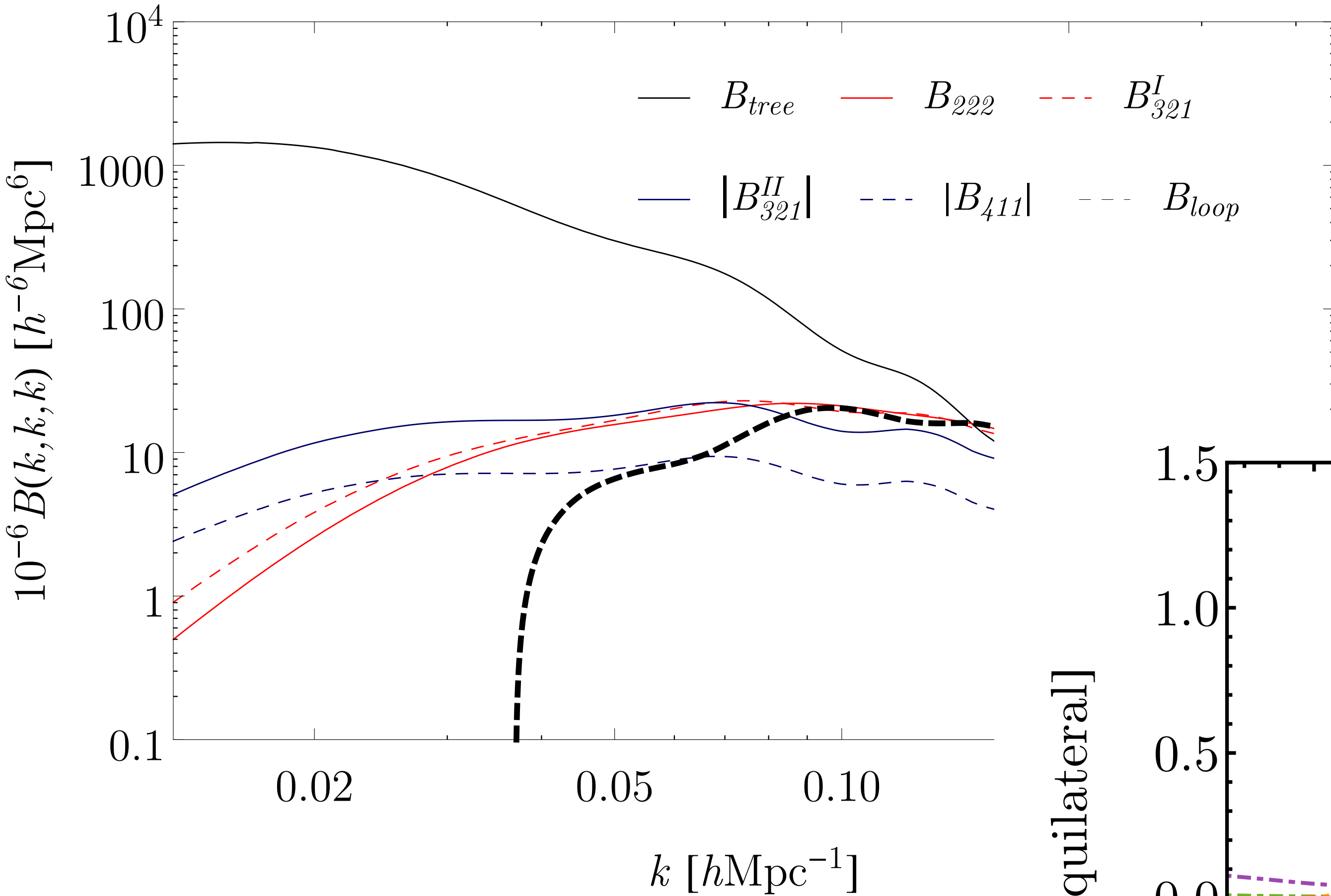
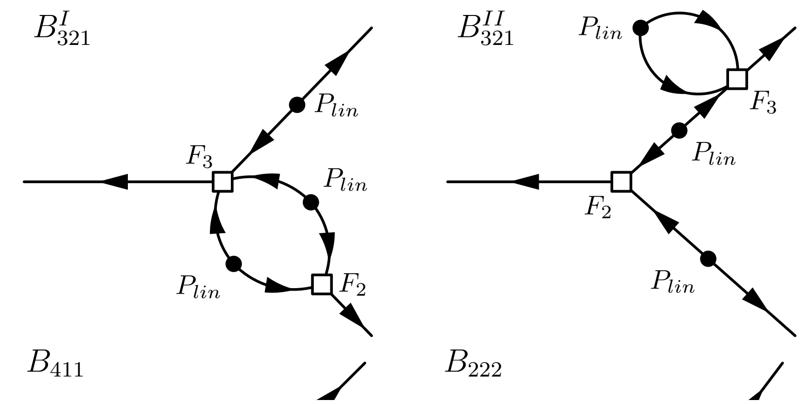
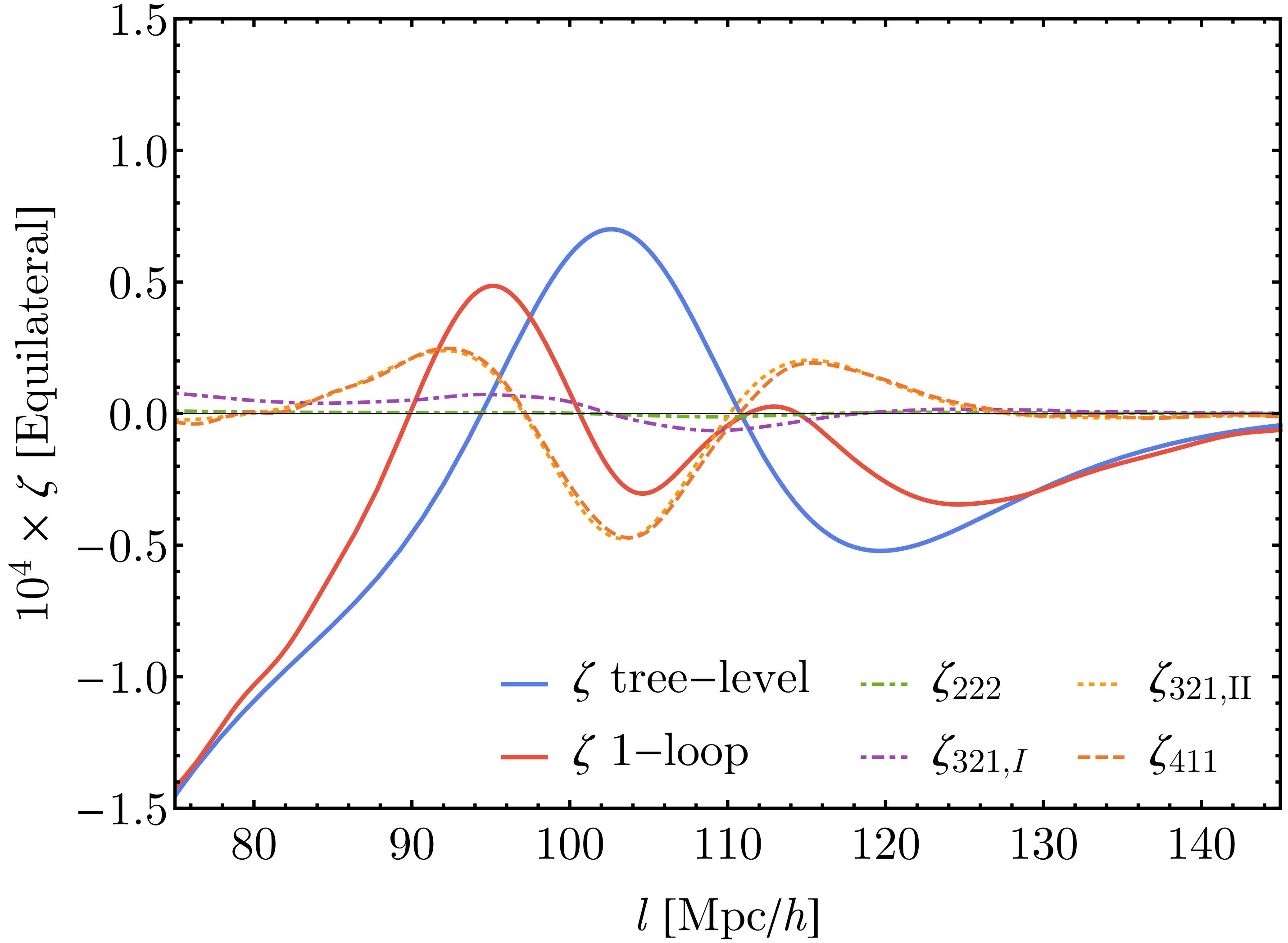






IR-resummation for the 3-PF

Similar to the resummation of the 2PF

For example at tree-level

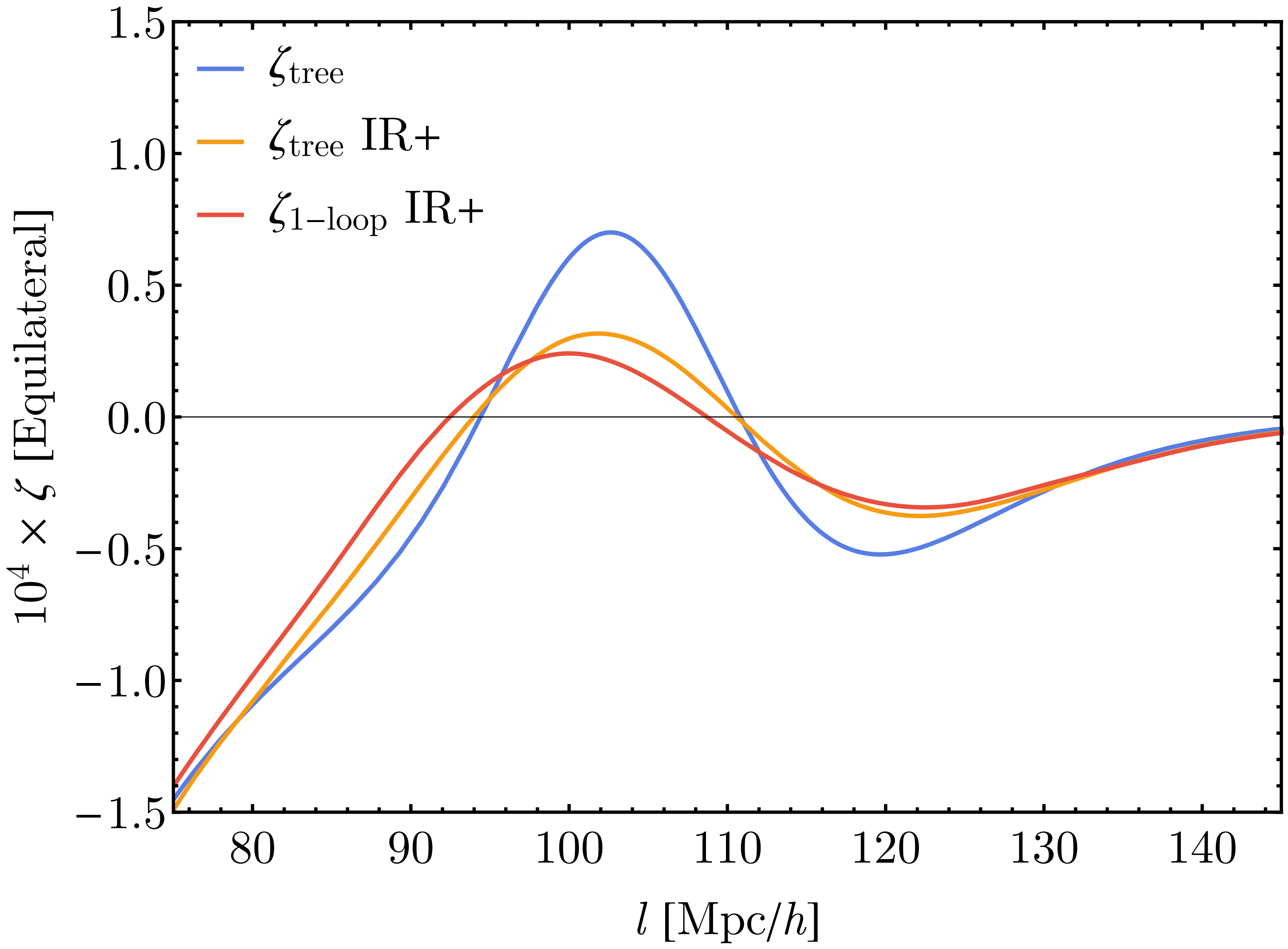
$$\zeta^{\mathrm{IR+}}_{tree}(\vec{x}_1, \vec{x}_2, \vec{x}_3) = \int \mathrm{d}^6 r \, \zeta^{\mathrm{E}}_{tree}(\boldsymbol{r}) G(\boldsymbol{x}, \boldsymbol{x} - \boldsymbol{r})$$

Gaussian-like kernel induced by long displacement modes

Scoccimarro, Trevisan: in preparation

IR-resummation for the 3-PF

Similar to the resummation of the 2PF



Scoccimarro, Trevisan: in preparation

Conclusions

- A simpler approx. for the IR-resummation
- Although IR-enhanced, IR mode-coupling leads to <1% in the 2-PF
- IR-resummation fixes also the 3-PF
- Tree-level and 1-loop already agree quite well
- Analysis of 2+3 PF may be an alternative to reconstruction

Thanks!

A brief excursus on Lagrangian PT (LPT)

Instead of using comoving coordinates, use fluid coordinates

$$\vec{x}(\vec{q},t) = \vec{q} + \vec{s}(\vec{q},t)$$

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to obtain the 2-PF

$$P(k) = \int \mathrm{d}^3 q_{12} \, e^{-i\vec{k}\cdot\vec{q}_{12}} \left\langle e^{-i\vec{k}\cdot(\vec{s}(q_1) - \vec{s}(q_2))} \right\rangle$$

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The linear order solution for the displacement field is

$$\vec{s}(p) \simeq \vec{s}_1(p) = i\frac{\vec{p}}{p^2}\delta_{\rm lin}(p),$$

and leads to the Zel'dovich approximation:

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and leads to the Zel'dovich approximation:

$$P(k) = \int \mathrm{d}^3 q_{12}\, e^{-i\vec{k}\cdot\vec{q}_{12}} e^{-\frac{1}{2}k_i k_j \langle s_i s_j \rangle (q_{12})}$$

where

$$\langle s_i s_j \rangle \sim \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{P_{\rm lin}(p)}{p^2}$$

Why SPT fails?

Lets go back to the 1-loop expression

$$P_{1-\mathrm{loop}}(k) \sim \frac{1}{2}\int_{p\ll\Lambda} \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{(\boldsymbol{p}\cdot\boldsymbol{k})^2}{p^4}\left[P_{\mathrm{lin}}(|\boldsymbol{k}-\boldsymbol{p}|)+P_{\mathrm{lin}}(|\boldsymbol{k}+\boldsymbol{p}|)-2P_{\mathrm{lin}}(|\boldsymbol{k}|)\right]P_{\mathrm{lin}}(p).$$

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$$P_{\text{lin}} \propto k^n$$

For a smooth component $\sim P_{\text{lin}}(k)\dfrac{p^2}{k^2},$

Very long modes ($p \ll k$) do not contribute to the loop

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$$P_{\text{lin}} \propto \sin(k/k_{osc})$$

For the BAO $\sim P^{w}_{\text{lin}}(k)\left(\cos\left(p\ell_{\text{BAO}}\right)-1\right)$

So there is an IR-enhancement for modes $\ell_{\text{BAO}}^{-1} \lesssim p \lesssim k$

IR-resummation

LPT calculations involve the average of an exponential

$$P(k) = \int \mathrm{d}^3 q_{12}\, e^{-i\vec{k}\cdot\vec{q}_{12}} \left\langle e^{-i\vec{k}\cdot(\vec{s}(q_1)-\vec{s}(q_2))} \right\rangle$$

which can be done as

$$\left\langle e^{-i\boldsymbol{k}\cdot\boldsymbol{\Delta}(\boldsymbol{q})} \right\rangle = \exp\left[\sum_{n=1}^{\infty} \frac{(-i)^n}{n!} \left\langle (\boldsymbol{k}\cdot\boldsymbol{\Delta}(\boldsymbol{q}))^n \right\rangle_c \right] = K(\boldsymbol{k},\boldsymbol{q}),$$

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Once expanded to some order *N* in P_{lin}

LPT = SPT

IR-resummation

and the idea is to resum IR modes ($\sim \epsilon_{s<}$)

$X|_N$
means up to order *N*

K_0 contains only
IR-displacements

$$K^{\mathrm{IR+}}|_N = K_0 \cdot \left.\frac{K}{K_0}\right|_N$$

$$= \sum_{j=0}^{N} R|_{N-j} \cdot K_j$$

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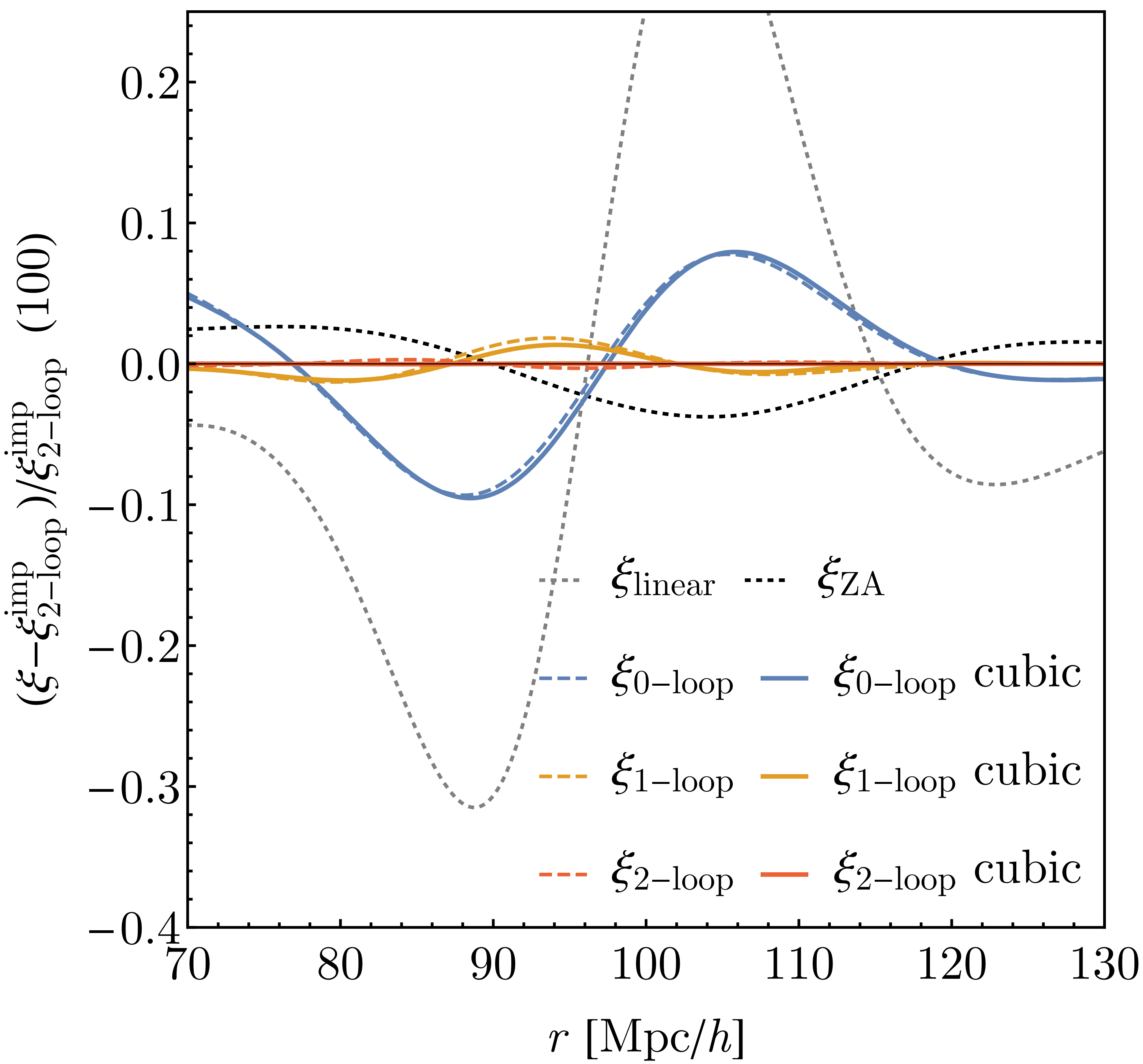
to get*

$$P(k) = \int \mathrm{d}^3 q \, e^{-i\boldsymbol{q}\cdot\boldsymbol{k}} \sum_{j=0}^{N} R(\boldsymbol{k},\boldsymbol{q})||_{N-j} \cdot \xi^{\mathrm{E}}_{||j}(q)$$

*we actually use a much simpler and intelligible approximation wrt the original paper which is parametrically justified

$$\zeta(\vec{x}_1, \vec{x}_2, \vec{x}_3) = \frac{10}{7}\xi_2(\vec{x}_{12})\xi_2(\vec{x}_{13}) + \nabla_i^{-1}\xi_2(\vec{x}_{12})\nabla_i\xi_2(\vec{x}_{13}) + \nabla_i\xi_2(\vec{x}_{12})\nabla_i^{-1}\xi_2(\vec{x}_{13})$$
$$+ \frac{4}{7}\nabla_i\nabla_j^{-1}\xi_2(\vec{x}_{12})\nabla_i\nabla_j^{-1}\xi_2(\vec{x}_{13}) + \text{cyc.}$$

Back-up



0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.4
70
80
90
100
110
120
130
r [Mpc/h]
$(\xi - \xi^{\rm imp}_{2\text{-loop}})/\xi^{\rm imp}_{2\text{-loop}}$ (100)
$\xi_{\rm linear}$
$\xi_{\rm ZA}$
$\xi_{0\text{-loop}}$
$\xi_{0\text{-loop}}$ cubic
$\xi_{1\text{-loop}}$
$\xi_{1\text{-loop}}$ cubic
$\xi_{2\text{-loop}}$
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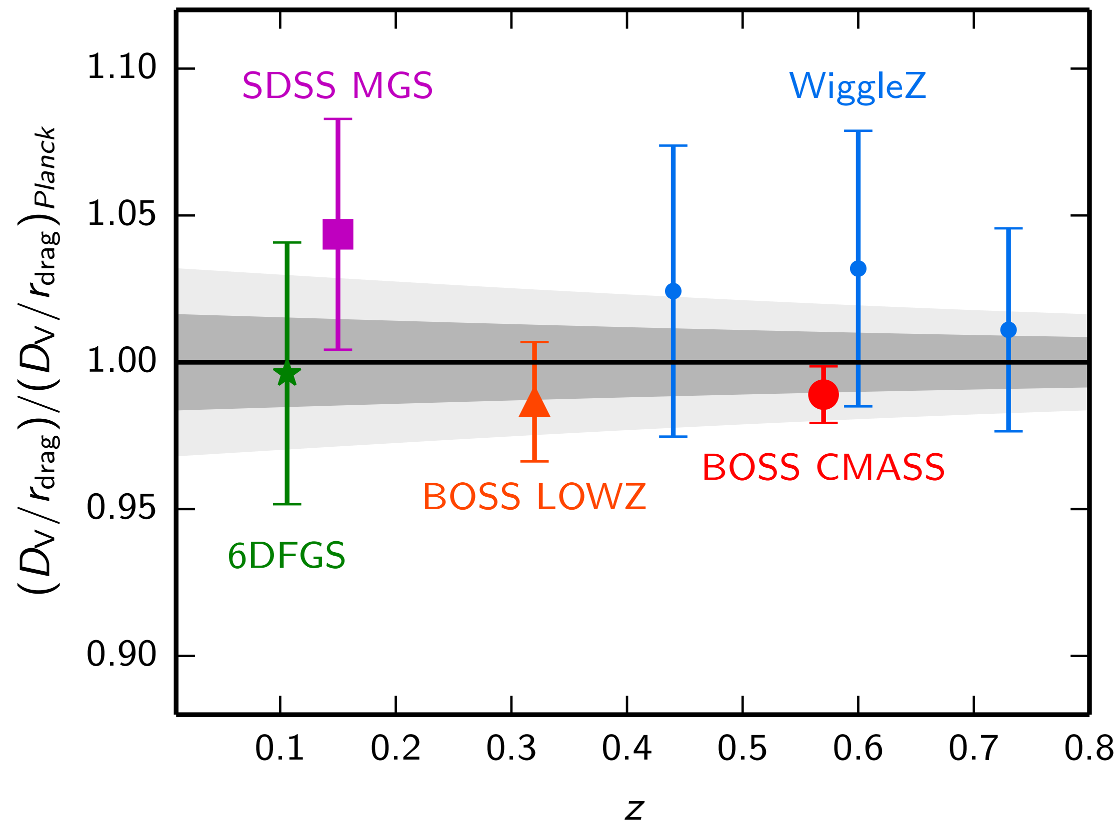
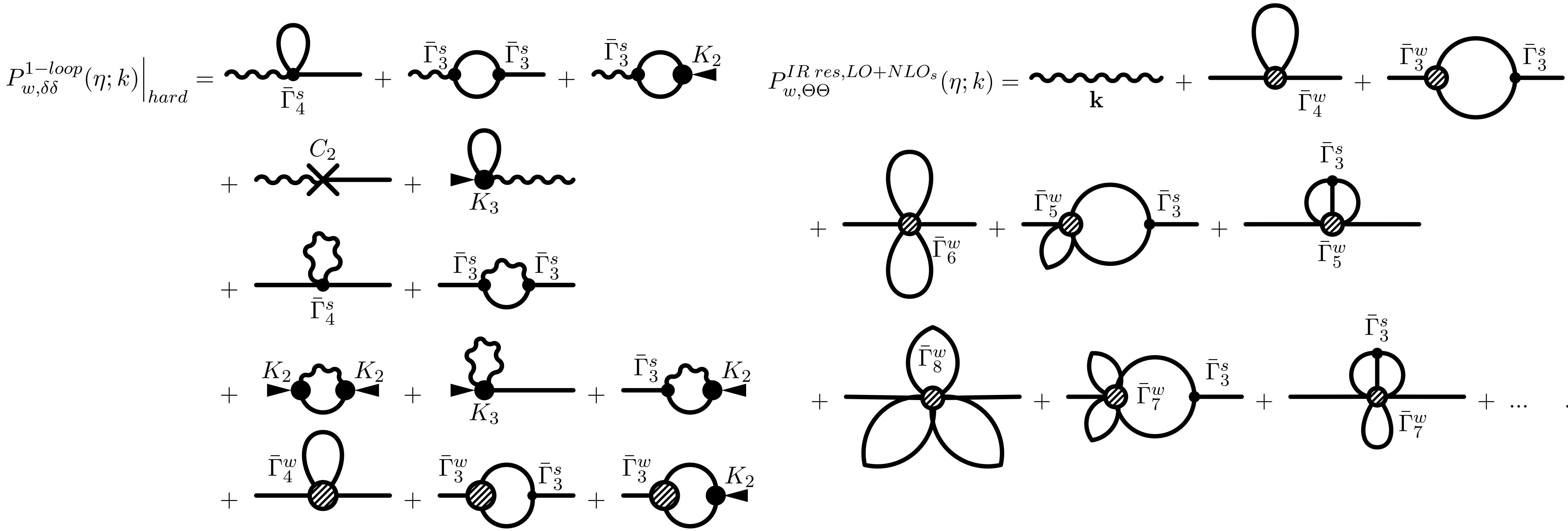


Fig. 14. Acoustic-scale distance ratio $D_V(z)/r_{drag}$ in the base ΛCDM model divided by the mean distance ratio from *Planck* TT+lowP+lensing. The points with $1\,\sigma$ errors are as follows: green star (6dFGS, Beutler et al. 2011); square (SDSS MGS, Ross et al. 2015); red triangle and large circle (BOSS "LOWZ" and CMASS surveys, Anderson et al. 2014); and small blue circles (WiggleZ, as analysed by Kazin et al. 2014). The grey bands show the 68 % and 95 % confidence ranges allowed by *Planck* TT+lowP+lensing.

$$D_V(z) = \left[(1+z)^2 D_A^2(z) \frac{cz}{H(z)}\right]^{1/3}.$$

1 loop TSPT



The perturbative solution parametrically goes as

$$s_n \sim s_1 \delta_1^{n-1}$$