Cosmological dynamics and perturbations in Light mass Galileon models

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Plan

- Introduction to Light Mass Galileon
- Background Cosmology
- First and Second Order Perturbations
**Figure:** Cosmic history. Picture is taken from wfirst.gsfc.nasa.gov.
• Cosmic acceleration $\rightarrow$ Equation of state $\rightarrow$ $w = \frac{\text{Pressure}}{\text{Density}} < -\frac{1}{3}$.

• Observationally $\rightarrow$ $w_{\text{inf}} \approx -1$ and also currently $w_{\text{DE},0} \approx -1$. 
Cosmic acceleration $\implies$ Equation of state $\implies w = \frac{\text{Pressure}}{\text{Density}} < -\frac{1}{3}$.

Observationally $\implies w_{\inf} \approx -1$ and also currently $w_{\text{DE},0} \approx -1$.

**Inflation:**

$$\mathcal{L} = \partial_\mu \phi \partial^\mu \phi + V(\phi). \quad \phi \text{ is a scalar field.}$$
Cosmic acceleration $\implies$ Equation of state $\implies w = \frac{P}{\rho} < -\frac{1}{3}$.

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**Inflation:**

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**Dark Energy:**

Simplest candidate can be $\Lambda$.

Fine tuning problem $\implies \frac{\rho_{\Lambda,\text{obs}}}{\rho_{\Lambda,\text{theo}}} = 10^{-120}$.

Cosmic Coincidence $\implies \rho_{\Lambda} \approx \rho_{m0}$.

Alternatives $\implies$ Make DE dynamical $\implies$ Modification of gravity.
Most general scalar-tensor Lagrangian

\[ \mathcal{L} = \sum_{i=1}^{5} c_i \mathcal{L}_i \]

\[ \mathcal{L}_2 = K(\phi, X), \]
\[ \mathcal{L}_3 = -G_3(\phi, X)\Box \phi, \]
\[ \mathcal{L}_4 = G_4(\phi, X)R + G_4X(\phi, X)\left[ (\Box \phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2 \right], \]
\[ \mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu} \nabla^\mu \nabla^\nu \phi - \frac{1}{6} G_5X(\phi, X)\left[ (\Box \phi)^3 - 3\Box \phi (\nabla_\mu \nabla_\nu \phi)^2 \right. \]
\[ \left. + 2(\nabla_\mu \nabla_\nu \phi)^3 \right], \]

- with \( K \) and \( G_i \) \((i = 3, 4, 5)\) are arbitrary functions of \( \phi \) and \( X = -\frac{1}{2} \partial_\mu \phi \partial^\mu \phi \) and \( G_iX = \partial G_i/\partial X \).
- Second order equation of motion.
Light Mass Galileon

- $K(\phi, X) = X - V(\phi)$, \quad $G_3(\phi, X) = 2X$ \quad and \quad $G_4 = G_5 = 0$.

\begin{align*}
\mathcal{L}_2 &= -\frac{1}{2}(\partial_\mu \phi)^2 - V(\phi), \\
\mathcal{L}_3 &= -(\partial_\mu \phi)^2 \Box \phi,
\end{align*}
Light Mass Galileon

- $K(\phi, X) = X - V(\phi)$, $G_3(\phi, X) = 2X$ and $G_4 = G_5 = 0$.

$$L_2 = -\frac{1}{2}(\partial_\mu \phi)^2 - V(\phi),$$
$$L_3 = - (\partial_\mu \phi)^2 \Box \phi,$$

- If $V(\phi) \sim \phi$ then $\phi$-field preserves galilean shift symmetry in the flat space time

$$\phi \rightarrow \phi + b_\mu x^\mu + c$$
$$\partial_\mu \phi \rightarrow \partial_\mu \phi + b_\mu$$

$\Rightarrow$ Galileon field.

- There are five Lagrangians which preserve the shift symmetry and give second order equation of motion.
Galileon Lagrangian

\[ \mathcal{L} = \sum_{i=1}^{5} c_i \mathcal{L}_i \]

\[ \mathcal{L}_1 = M^3 \phi , \]
\[ \mathcal{L}_2 = \frac{1}{2} (\partial_\mu \phi)^2 , \]
\[ \mathcal{L}_3 = \frac{1}{M^3} (\partial_\mu \phi)^2 \Box \phi , \]
\[ \mathcal{L}_4 = \frac{1}{M^6} \phi_{;\mu} \phi_{;\mu} \left[ 2(\Box \phi)^2 - 2 \phi_{;\mu\nu} \phi_{;\mu\nu} - \frac{1}{2} R \phi_{;\mu} \phi_{;\mu} \right] , \]
\[ \mathcal{L}_5 = \frac{1}{M^9} \phi_{;\mu} \phi_{;\mu} \left[ (\Box \phi)^3 - 3(\Box \phi) \phi_{;\mu\nu} \phi_{;\mu\nu} + 2 \phi_{\mu\nu} \phi_{;\mu\rho} \phi_{;\nu} - 6 \phi_{;\mu} \phi_{;\mu \nu} \phi_{;\nu} G_{\nu \rho} \right] . \]

- Second order equation of motion.
- Can explain late time acceleration of the Universe.
- Local physics is restored through the Vainshtein mechanism.
The Model


Action

\[
S = \int d^4x \sqrt{-g} \left[ \frac{M_{pl}^2}{2} R - \frac{1}{2} (\nabla \phi)^2 \left( 1 + \frac{\alpha}{M^3} \Box \phi \right) - V(\phi) \right] + S_m \left[ \psi_m; e^{2\beta \phi/M_{pl}} g_{\mu\nu} \right]
\]

- EH action is modified with Galileon Lagrangian \( \mathcal{L}^{(2)} \) and \( \mathcal{L}^{(3)} \) and with a potential \( \Rightarrow \) Only \( \mathcal{L}^{(2)} \) and \( \mathcal{L}^{(3)} \) can’t give late time acceleration.
- Potential is added phenomenologically to get acceleration.
- Potential \( \Rightarrow \) breaks shift symmetry even in the flat background.
- Non-linear self interaction term of the galileon field plays the main role to preserve the local physics through Vinshtein mechanism.
\[ V = V_0 \left( \cosh \left( \frac{\alpha \phi}{M_{Pl}} \right) - 1 \right)^p \]

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V(\phi) = \frac{V_0}{2} e^{\frac{\alpha \phi}{M_{Pl}}}, \quad \text{for} \quad \frac{\alpha \phi}{M_{Pl}} \gg 1, \quad \phi > 0

V(\phi) = \frac{V_0}{2} \left( \frac{\alpha \phi}{M_{Pl}} \right)^{2p}, \quad \text{for} \quad \frac{\alpha \phi}{M_{Pl}} \ll 1


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\[ \rho_\phi = \rho_{\phi_0} a^{-6p/(p+1)} \]
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$\rho_\phi = \rho_{\phi 0} a^{-6p/(p+1)}$

$\langle w_\phi \rangle = \left\langle \frac{1}{2} \dot{\phi}^2 - V(\phi) \right\rangle = \frac{p - 1}{p + 1}$


\[ V = V_0 \left( \cosh\left( \frac{\alpha \phi}{M_{Pl}} \right) - 1 \right)^p \]

\[ \alpha = 20 \text{ and } p = 1. \]
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\( \alpha = 20 \) and \( p = 1 \).

\( \alpha = 30 \) and \( p = 0.1 \).
\[ V = V_0 \left( e^{-\mu_1 \phi / M_{Pl}} + e^{-\mu_2 \phi / M_{Pl}} \right) \]


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Figure: $\mu_1 = 20$ and $\mu_2 = -0.1$. 
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Figure: \( \mu_1 = 20 \) and \( \mu_2 = -0.1 \).

Figure: \( \mu_1 = 20 \) and \( \mu_2 = 1 \).
Figure: $\mu_1 = 20$, $\mu_2 = 0.1$ and $\beta = 0.01$. 

Here are the graphs showing the evolution of different parameters against the logarithm of $(1+z)$. The graphs illustrate the changes in density, effective pressure, and density fractions over time. The equations and parameters are used to model the cosmological evolution.
We consider the following metric in the Newtonian gauge

\[ ds^2 = a(\tau)^2 \left[ - (1 + 2\Phi) d\tau^2 + (1 - 2\Psi) d\tilde{x}^2 \right], \]

where \( \tau \) is the conformal time, \( \Phi \) and \( \Psi \) are the scalar perturbations of the metric.

We expand the perturbations in series,

\[
\Phi = \Phi_1 + \frac{1}{2!} \Phi_2 + \frac{1}{3!} \Phi_3 + \cdots, \\
\Psi = \Psi_1 + \frac{1}{2!} \Psi_2 + \frac{1}{3!} \Psi_3 + \cdots,
\]
Equation for the linear density contrast in subhorizon ($k^2 \gg \mathcal{H}^2$) and quasistatic ($|\dot{\phi}| \lesssim \mathcal{H}|\phi| \ll k^2|\phi|$) approximations,

$$\ddot{\delta}_1 + \left(\mathcal{H} + \frac{\beta}{M_{Pl}} \dot{\phi}\right) \dot{\delta}_1 - 4\pi G_{\text{eff}} a^2 \bar{\rho}_m \delta_1 = 0$$

$$G_{\text{eff}} = G \left(1 + \frac{\left(\frac{\alpha}{M^3 a^2 M_{Pl}} \dot{\phi}^2 + 2\beta\right)^2}{2 - \frac{4\alpha}{M^3 a^2} \left(\ddot{\phi} + \mathcal{H} \dot{\phi}\right) - \frac{\alpha^2}{M^6 a^4 M_{Pl}^2} \dot{\phi}^4}\right)$$
Evolution of $G_{\text{eff}}$

Figure: Evolution of $G_{\text{eff}}/G$ with $\mu_1 = 20$, $\mu_2 = 0.5$ and $\beta = 0.5$. 
Using the transformation

\[ \tilde{a} = a e^{\beta \phi / M_{\text{Pl}}} \]

Evolution equation of the density contrast can be written as

\[ \ddot{\delta}_1 + \tilde{H} \dot{\delta}_1 - 4\pi \tilde{G}_{\text{eff}} \tilde{a}^2 \bar{\rho}_m \delta_1 = 0 \]

where

\[ \tilde{H} = \frac{1}{\tilde{a} \text{d} \tau} = \mathcal{H} + \frac{\beta}{M_{\text{Pl}}} \dot{\phi} = \mathcal{H} \frac{\text{d} \ln \tilde{a}}{\text{d} \ln a} \]

\[ \tilde{G}_{\text{eff}} = G_{\text{eff}} e^{-(2\beta / M_{\text{Pl}}) \phi} \]
Growing and decaying modes: Integral solutions

**Growing Mode**

\[
D_+ (\tilde{a}) = \tilde{a}_m^{7/4} e^{-3 \beta \phi_m / 4 M_{Pl}} \left( \frac{A(\phi_m)}{A(\phi_0)} \right)^{1/2} \frac{\gamma(\tilde{a})}{\tilde{a}} \sqrt{\frac{\tilde{\mathcal{H}}_0}{\dot{\tilde{\mathcal{H}}}}} \]

**Decaying Mode**

\[
D_- (\tilde{a}) = \tilde{a}_m^{-3/4} e^{7 \beta \phi_m / 4 M_{Pl}} \left( \frac{A(\phi_m)}{A(\phi_0)} \right)^{1/2} \frac{\gamma(\tilde{a})}{\tilde{a}} \sqrt{\frac{\tilde{\mathcal{H}}_0}{\dot{\tilde{\mathcal{H}}}}} \times \left[ 1 - \frac{5}{2} \frac{1}{\tilde{a}_m A(\phi_m)} \int_{\tilde{a}_m}^{\tilde{a}} \frac{d\tilde{a}'}{\gamma^2(\tilde{a}')} \right] \]
Growing and decaying modes: Integral solutions

Where,

\[ A(\phi) = \frac{d \ln \tilde{a}}{d \ln a} = 1 + \frac{\beta}{M_{Pl}} \frac{d \phi}{d \ln a} \]

And

\[ \gamma^2(\tilde{a}) = e^{-\int_{\tilde{a}_m}^{\tilde{a}} d\tilde{a}' g(\tilde{a}')} \]

Where \( g(\tilde{a}') \) satisfies

\[ \frac{dg(\tilde{a})}{d\tilde{a}} - \frac{1}{2} g^2(\tilde{a}) + 2l(\tilde{a}) = 0 \]

With

\[ l(\tilde{a}) = A(\tilde{a}) + \frac{1}{\tilde{a} \tilde{H}} \frac{d \tilde{H}}{d \tilde{a}} - \frac{1}{4 \tilde{H}^2} \left( \frac{d \tilde{H}}{d \tilde{a}} \right)^2 + \frac{1}{2 \tilde{H}} \frac{d^2 \tilde{H}}{d \tilde{a}^2} \]

\[ A(\tilde{a}) = 4\pi \tilde{G}_{eff} \frac{\tilde{\rho}_m}{\tilde{H}^2} \]
Figure: $\mu_1 = 20$, $\mu_2 = 0.1$ and $\beta = 0, 0.1, 0.2$. 
Figure: $\mu_1 = 20$, $\mu_2 = 0.1$ and $\beta = 0.1$. 
Power spectrum and $f\sigma_8$

Figure: $\mu_1 = 20$, $\mu_2 = 0.1$ and $\Omega_b = 0.04$, $\Omega_m = 0.3$ and $n_s = 0.968$. 
Equation for the second order density contrast,

\[ \ddot{\delta}_2 + \left( \mathcal{H} + \frac{\beta}{M_{\text{Pl}}} \dot{\phi} \right) \dot{\delta}_2 - 4\pi G_{\text{eff}} a^2 \bar{\rho}_m \delta_2 = S_\delta \]

Fourier transform of \( S_\delta \) can be written as

\[ S_\delta(a, \vec{k}) = \int d^3k_1 d^3k_2 \delta^{(3)}(\vec{k} - \vec{k}_1 - \vec{k}_2) \mathcal{K}(a, \vec{k}_1, \vec{k}_2) \delta_1(a, \vec{k}_1) \delta_1(a, \vec{k}_2) \]
Second Order Density Contrast

\[
\delta_2(\bar{a}, \bar{k}) = D_+(\bar{a})\delta_2(\bar{k}) - D_+(\bar{a}) \int_{\bar{a}_m}^{\bar{a}} \frac{D_-(\bar{a}')\hat{S}_\delta(\bar{a}', \bar{k})}{\bar{a}'^2\tilde{H}^2(\bar{a}')W_r(\bar{a}')} d\bar{a}' + D_-(\bar{a}) \int_{\bar{a}_m}^{\bar{a}} \frac{D_+(\bar{a}')\hat{S}_\delta(\bar{a}', \bar{k})}{\bar{a}'^2\tilde{H}^2(\bar{a}')W_r(\bar{a}')} d\bar{a}'
\]

\[
W_r(\bar{a}) = D_+(\bar{a}) \frac{dD_-(\bar{a})}{d\bar{a}} - D_-(\bar{a}) \frac{dD_+(\bar{a})}{d\bar{a}}
\]
Second Order Perturbation

\[ \delta(\bar{a}, \bar{k}) = \delta_1(\bar{a}, \bar{k}) + \frac{1}{2} \delta_2(\bar{a}, \bar{k}) = D_+(\bar{a})\delta_1(\bar{k}) + \int d^3k_1d^3k_2\delta(\bar{k} - \bar{k}_1 - \bar{k}_2) \]

\[ \times \mathcal{F}_2(\bar{a}, \bar{k}_1, \bar{k}_2)\delta_1(\bar{a}, \bar{k}_1)\delta_1(\bar{a}, \bar{k}_2) \]

Second Order Kernel

\[ \mathcal{F}_2(\bar{a}, \bar{k}_1, \bar{k}_2) = \int_{\bar{a}_m}^{\bar{a}} d\bar{a}' \frac{\mathcal{D}(\bar{a}, \bar{a}')\mathcal{K}(\bar{a}', \bar{k}_1, \bar{k}_2)}{2\bar{a}'^2\bar{H}^2(\bar{a}')W_r(\bar{a}')} \]

\[ \mathcal{D}(\bar{a}, \bar{a}') = \frac{D^2_+(\bar{a}')}{D^2_+(\bar{a})} \left( D_-(\bar{a})D_+(\bar{a}') - D_+(\bar{a})D_-(\bar{a}') \right) \]
\[ \langle \delta(\tau, \vec{k})\delta(\tau, \vec{k}')\delta(\tau, \vec{k}'') \rangle = \delta^{(3)}(\vec{k} + \vec{k}' + \vec{k}'') B(\tau, k, k') \]

\[ B(\tau, k, k') = 2\mathcal{F}_2(\vec{k}, \vec{k}') \mathcal{P}(k)\mathcal{P}(k') + \text{cyc} \]

\[ Q = \frac{B(\tau, k, k')}{\mathcal{P}(\tau, k)\mathcal{P}(\tau, k') + \mathcal{P}(\tau, k')\mathcal{P}(\tau, k'')} + \ldots \]
Figure: $\mu_1 = 20$, $\mu_2 = 0.1$, and $k = k' = 0.01\ \text{hMpc}^{-1}$ and $5k = k' = 0.05\ \text{hMpc}^{-1}$ for $\beta = 0, 0.5$ and $\Lambda$CDM.
We have discussed the effect of the conformal coupling at the perturbation level in a tracker scalar field model with a cubic Galileon correction term.

Integral solution of the growing and decaying modes are calculated in the subhorizon approximation.

Effect of the conformal coupling constant on matter power spectrum and bispectrum has been observed.

The power spectrum changes for different conformal constant but there is no significant change in the reduced bispectrum.

Comparison with $f\sigma_8$ data shows that higher values of the conformal constant can be ruled out.
THANK YOU