Effective Stress-energy Tensor and Gravitational Radiation from Binary Systems in Horndeski Theory

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Introduction

- Scalar-tensor theory $(g_{\mu\nu}, \phi)$: Brans-Dicke theory, $f(R)$ gravity, Einstein-dilaton-Gauss-Bonnet gravity (EdGB), **Horndeski theory**...
- Effective stress-energy tensor: flat background
- Nordtvedt effect: violation of Strong Equivalence Principle (vSEP)
- Shapiro time delay
- Gravitational Waves (GWs) detected:

  NS-NS GW170817 (GRB 170817A: $|\frac{v_{GW}-c}{c}| \lesssim 10^{-15}$ [3])
- Previous constraints:
  1. Creminelli & Vernizzi [4]: $\partial G_5/\partial X = 0$ and $2\partial G_4/\partial X + \partial G_5/\partial\phi = 0$ (Effective Field Theory of Dark Energy)
  2. Ezquiaga & Zumalacárregui [5]: $\partial G_4/\partial X \approx 0$ and $G_5 \approx$ constant
  3. and other constraints on subclasses [6, 7, 8, 9, 10, 11, 12, 13, 14]...

Horndeski Theory

- Action [15]:

$$S = \int \mathrm{d}^4x\sqrt{-g}(\mathscr{L}_2 + \mathscr{L}_3 + \mathscr{L}_4 + \mathscr{L}_5) + S_m[g_{\mu\nu}, \psi_m],$$

$$\mathscr{L}_2 = G_2(\phi, X),\ \mathscr{L}_3 = -G_3(\phi, X)\Box\phi,$$

$$\mathscr{L}_4 = G_4(\phi, X)R + G_{4X}[(\Box\phi)^2 - (\phi_{;\mu\nu})^2],$$

$$\mathscr{L}_5 = G_5(\phi, X)G_{\mu\nu}\phi^{;\mu\nu} - \frac{G_{5X}}{6}[(\Box\phi)^3 - 3(\Box\phi)(\phi_{;\mu\nu})^2 + 2(\phi_{;\mu\nu})^3].$$

- $\phi_{;\mu} = \nabla_\mu\phi$, $X = -\phi_{;\mu}\phi^{;\mu}/2$, $\phi_{;\mu\nu} = \nabla_\nu\nabla_\mu\phi$, $\Box\phi = g^{\mu\nu}\phi_{;\mu\nu}$, $(\phi_{;\mu\nu})^2 = \phi_{;\mu\nu}\phi^{;\mu\nu}$ and $(\phi_{;\mu\nu})^3 = \phi_{;\mu\nu}\phi^{;\mu\rho}\phi^{;\nu}{}_{;\rho}$
- Subclasses:

	G_2	G_3	G_4	G_5
BD	$2\omega X/\phi$	0	ϕ	0
$f(R)$	$f(\phi) - \phi f'(\phi)$	0	$f'(\phi)$	0
EdGB	$X+8\xi^{(4)}X^2(3-\ln X)$	$4\xi^{(3)}X(7-3\ln X)$	$\frac{1}{2}+4\xi^{(2)}X(2-\ln X)$	$-4\xi^{(1)}\ln X$
SSHT	$G_2(X)$	$G_3(X)$	$G_4(X)$	$G_5(X)$

Horndeski Theory: Matter Action for Systems of Stars

- Matter action according to Eardley's prescription [16]:

$$S_m = -\sum_a \int m_a(\phi)\mathrm{d}\tau_a$$

- $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ and $\phi = \phi_0 + \varphi$
- Expand $m_a(\phi)$:

$$m_a(\phi) = m_a \left[1 + \frac{\varphi}{\phi_0}s_a - \frac{1}{2}\left(\frac{\varphi}{\phi_0}\right)^2 (s_a' - s_a^2 + s_a) + \cdots\right].$$

- $m_a = m_a(\phi_0)$, and sensitivities $\rightsquigarrow$ vSEP

$$s_a = \frac{\mathrm{d}\ln m_a(\phi)}{\mathrm{d}\ln\phi}\bigg|_{\phi_0}, \quad s_a' = -\frac{\mathrm{d}^2\ln m_a(\phi)}{\mathrm{d}(\ln\phi)^2}\bigg|_{\phi_0}.$$

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Main Packages

- *xCore*
- *xPerm*
- *xTensor*
- *xCoba*

Applications

- *xPert*
- *Harmonics*
- *Invar*
- *Spinors*

Contributed

- *xPrint*

xAct: Efficient tensor computer algebra for the Wolfram Language

José M. Martín-García, GPL 2002-2016

Main collaborators: Alfonso García-Parrado, Alessandro Stecchina, Barry Wardell, Cyril Pitrou, David Brizuela, David Yllanes, Guillaume Faye, Leo Stein, Renato Portugal, Teake Nutma, Thomas Bäckdahl.

Introduction

xAct is a suite of free packages for tensor computer algebra in *Mathematica*. *xAct* implements state-of-the-art algorithms for fast manipulations of indices and has been modelled on the current geometric approach to General Relativity. It is highly programmable and configurable. Since its first public release in March 2004, *xAct* has been intensively tested and has solved a number of hard problems in GR.

There are four packages acting as a kernel for the rest:

- *xCore*: generic programming tools
- *xPerm*: manipulation of large groups of permutations
- *xTensor*: abstract tensor computations, the flagship of the system
- *xCoba*: component tensor computations

Application packages include:

- *xPert*: high-order perturbation theory in GR
- *Harmonics*: tensor spherical harmonics
- *Invar*: polynomial invariants of the Riemann tensor
- *Spinors*: spinor computations in GR

Contributed packages:

Linearized Equations of Motion

- $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ and $\phi = \phi_0 + \varphi$
- At the zeroth order: $g_{\mu\nu} = \eta_{\mu\nu}$ and $\phi = \phi_0$

$$\Rightarrow G_{2(0,0)} = 0, \;\; G_{2(1,0)} = 0$$

- $$\boxed{f_{(m,n)} = \frac{\partial^{m+n} f(\phi, X)}{\partial\phi^m \partial X^n}\Big|_{\phi=\phi_0, X=0}}$$
- Linearized EoMs:

$$\Box\tilde{h}_{\mu\nu} = -\frac{T^{(1)}_{\mu\nu}}{G_{4(0,0)}}, \quad (\Box - m_s^2)\varphi = \frac{T^{(1)}_*}{2G_{4(0,0)}\zeta}.$$

$$\tilde{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h - \frac{G_{4(1,0)}}{G_{4(0,0)}}\eta_{\mu\nu}\varphi$$

$$m_s^2 = -G_{2(2,0)}/\zeta, \quad \zeta = G_{2(0,1)} - 2G_{3(1,0)} + 3G^2_{4(1,0)}/G_{4(0,0)}$$

$$T^{(1)}_* = G_{4(1,0)}T^{(1)} - 2G_{4(0,0)}\left(\frac{\partial T}{\partial\phi}\right)^{(1)}$$

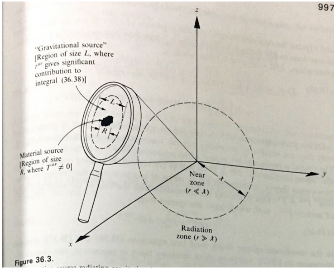
Effective Stress-energy Tensor

- The methods to obtain the effective stress-energy tensor for GWs: Landau-Lifshitz pseudo-tensor [17], the second variation [18], Noether's theorem [19], *Isaacson* [1, 2]...
- Take the second order Einstein's equation, with the first order equations substituted in
- Take the terms quadratic in $\tilde{h}_{\mu\nu}$ and φ, then average their sums over several wavelengths
- Effective Stress-energy Tensor, gauge invariant:

$$T_{\mu\nu}^{\mathrm{GW}} = \Big\langle \overbrace{\frac{1}{2}G_{4(0,0)}\Big(\partial_\mu\tilde{h}_{\rho\sigma}\partial_\nu\tilde{h}^{\rho\sigma} - \frac{1}{2}\partial_\mu\tilde{h}\partial_\nu\tilde{h} - \partial_\mu\tilde{h}_{\nu\rho}\partial_\sigma\tilde{h}^{\sigma\rho} - \partial_\nu\tilde{h}_{\mu\rho}\partial_\sigma\tilde{h}^{\sigma\rho}\Big)}^{\sim \text{GR}}$$
$$+\zeta\partial_\mu\varphi\partial_\nu\varphi \leftarrow \text{the contribution of the scalar field}$$
$$+G_{4(1,0)}(m_s^2\varphi\tilde{h}_{\mu\nu} + \partial_\mu\varphi\partial^\rho\tilde{h}_{\rho\nu} + \partial_\nu\varphi\partial^\rho\tilde{h}_{\rho\mu} - \eta_{\mu\nu}\partial_\sigma\varphi\partial_\rho\tilde{h}^{\rho\sigma})\Big\rangle\,.$$

Near Zone Solutions

Figure 36.3, Page 997,
MTW [20]



- $$\Box \tilde{h}_{\mu\nu} = -\frac{T^{(1)}_{\mu\nu}}{G_{4(0,0)}}, \quad (\Box - m_s^2)\varphi = \frac{T^{(1)}_{*}}{2G_{4(0,0)}\zeta}.$$

$$\varphi(t,\vec{x}) = \frac{1}{8\pi G_{4(0,0)}\zeta}\sum_a \frac{m_a S_a}{r_a} e^{-m_s r_a}$$

$$h_{00} = \frac{1}{8\pi G_{4(0,0)}}\sum_a \frac{m_a}{r_a}\left(1 + \frac{G_{4(1,0)}}{G_{4(0,0)}\zeta} S_a e^{-m_s r_a}\right),$$

$$h_{jk} = \frac{\delta_{jk}}{8\pi G_{4(0,0)}}\sum_a \frac{m_a}{r_a}\left(1 - \frac{G_{4(1,0)}}{G_{4(0,0)}\zeta} S_a e^{-m_s r_a}\right),$$

$$h_{0j} = 0,$$

- $S_a = G_{4(1,0)} - \dfrac{2G_{4(0,0)}}{\phi_0} s_a$

Near Zone Solutions

A single mass M:

- $g_{00} = -1 + 2G_{\rm N}(r)\dfrac{M}{r} + \cdots ,$
- "Newton's constant": $G_{\rm N}(r) = \dfrac{1}{16\pi G_{4(0,0)}}\left(1 + \dfrac{G_{4(1,0)}}{G_{4(0,0)}\zeta} S_M e^{-m_s r}\right)$
- $g_{jk} = \delta_{jk}\left(1 + 2\gamma(r) G_{\rm N}(r)\dfrac{M}{r}\right) + \cdots ,$
- "PPN parameter": $\gamma(r) = \dfrac{G_{4(0,0)}\zeta - G_{4(1,0)} S_M e^{-m_s r}}{G_{4(0,0)}\zeta + G_{4(1,0)} S_M e^{-m_s r}}$

$$S_M = G_{4(1,0)} - \frac{2G_{4(0,0)}}{\phi_0} s_M$$

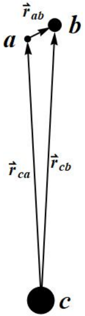
Nordtvedt Effect

- The polarization of the moon's orbit [21, 22, 23, 24]:
- The relative acceleration:

$$
\begin{aligned}
a^j_{ab} \approx & -\frac{m_a + m_b}{16\pi G_{4(0,0)}} \frac{\hat{r}^j_{ab}}{r^2_{ab}} \left[1 + \frac{S_a S_b}{G_{4(0,0)}\zeta}(1 + m_s r_{ab}) e^{-m_s r_{ab}}\right] \\
& - \frac{m_c}{16\pi G_{4(0,0)}} \left(\frac{\hat{r}^j_{ac}}{r^2_{ac}} - \frac{\hat{r}^j_{bc}}{r^2_{bc}}\right) \\
& + \boxed{\frac{S_c}{8\pi G_{4(0,0)}\phi_0\zeta} \frac{m_c \hat{r}^j_{ac}}{r^2_{ac}} (s_a - s_b)(1 + m_s r_{ac}) e^{-m_s r_{ac}}},
\end{aligned}
$$

- "Nordtvedt parameter":

$$
\eta_{\mathrm{N}} = \frac{S_c}{8\pi G_{4(0,0)}\phi_0\zeta}(1 + m_s r_{ac}) e^{-m_s r_{ac}}
$$



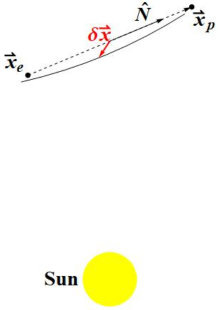
Nordtvedt Effect - Lunar Laser Ranging Exp.'s

- The lunar laser ranging experiment gave [25]

$$\eta_{\rm N}^{\rm obs.} = (0.6 \pm 5.2) \times 10^{-4} = \delta \pm \epsilon.$$

- $|\eta_{\rm N} - \delta| < 2\epsilon$ at 95% confidential level
- $\left| \frac{G_{4(1,0)}(1 + m_s r)}{8\pi G_{4(0,0)}\phi_0 \zeta} e^{-m_s r} - \delta \right| < 2\epsilon$ at $r = 1$ AU.
- EdGB: $\left| -\frac{s_\odot}{4\pi \phi_0^2} - \delta \right| < 2\epsilon \Rightarrow \phi_0 > 9.01 \sqrt{s_\odot / G_{\rm N}}$ with $s_\odot \lesssim 10^{-4}$
- SSHT (covariant Galileon, Fab Four...) trivially satisfies this constraint since $G_{4(1,0)} = 0$ and $s_a = 0$ [26]

Shapiro Time Delay



- 4 velocity of the photon: $u^\mu = u^0(1, \vec{v})$

$$-1 + h_{00} + (\delta_{jk} + h_{jk})v^j v^k = 0,$$

- $\hat{N} \cdot \frac{\mathrm{d}\delta\vec{x}}{\mathrm{d}t} = -\frac{M}{8\pi G_{4(0,0)} r(t)}$ where $r(t) = |\vec{x}(t)|$
- Shapiro time delay [27]:

$$\begin{aligned}\delta t =& 2\int_{t_e}^{t_p} \hat{N} \cdot \frac{\mathrm{d}\delta\vec{x}}{\mathrm{d}t}\mathrm{d}t \\ =& 2G_{\mathrm{N}} M_{\mathrm{K}}(1+\gamma(r)) \ln \frac{(r_e + \hat{N}\cdot\vec{x}_e)(r_p - \hat{N}\cdot\vec{x}_p)}{r_b^2},\end{aligned}$$

where $r_e = |\vec{x}_e|$, $r_p = |\vec{x}_p|$ and $r_b = |\hat{N} \times \vec{x}_e|$ is the impact parameter of the photon relative to the source.

Shapiro Time Delay - Cassini Time Delay Data

- In 2002, the Cassini spacecraft measured the Shapiro time delay effect in the solar system by radio tracking [28].
- $\gamma_{\text{meas.}} = 1 + (2.1 \pm 2.3) \times 10^{-5} = 1 + \delta \pm \epsilon$
- At 95% C.L., $|\gamma(r) - \gamma_{\text{meas.}}| < 2\epsilon$:

$$\frac{G_{4(1,0)}^2}{G_{4(0,0)}\zeta} < \frac{2\epsilon - \delta}{2 + \delta - 2\epsilon} e^{m_s r},$$

  at $r = 1$ AU.
- EdGB and SSHT trivially satisfy this constraint as $G_{4(1,0)} = 0$.

Gravitational Radiation

Energy loss at (*massless*)

$$\dot{E} \approx -\frac{G_{4(0,0)}}{2} r^2 \int \left\langle \partial_0 \tilde{h}^{\mathrm{TT}}_{jk} \partial_0 \tilde{h}^{jk}_{\mathrm{TT}} \right\rangle \mathrm{d}\Omega - \zeta r^2 \int \langle \partial_0 \varphi \partial_0 \varphi \rangle \, \mathrm{d}\Omega$$

- Energy lost through the spin-2 wave:

$$\tilde{h}_{jk}(t, \vec{x}) = \frac{1}{8\pi G_{4(0,0)} r} \frac{\mathrm{d}^2 I_{jk}}{\mathrm{d}t^2} \sim O(v^4),$$

- Effective one-body problem:

$$a^j = -\frac{\varsigma m}{16\pi G_{4(0,0)}} \frac{\hat{r}^j_{12}}{r^2_{12}}$$

with $\varsigma = 1 + \frac{S_1 S_2}{G_{4(0,0)}\zeta}$ and $m = m_1 + m_2$

- $\dot{E}_2 = -(1-e^2)^{-7/2}\left(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4\right)\frac{32}{5}\frac{\varsigma^3 \mu^2 m^3}{(16\pi G_{4(0,0)})^4 a^5}$ with e the eccentricity, a the semi-major axis and μ the reduced mass

Gravitational Radiation

$$
\begin{aligned}
\Box\varphi =& \frac{T_*^{(1)}}{2G_{4(0,0)}\zeta} + \frac{G_{4(1,0)}T^{(2)}}{2G_{4(0,0)}\zeta} - \frac{1}{\zeta}\left(\frac{\partial T}{\partial \phi}\right)^{(2)} + \left[\frac{\varphi T_*^{(1)}}{G_{4(0,0)}\zeta} + (\partial_\mu\varphi)(\partial^\mu\varphi)\right]\Bigg(-\frac{G_{4(1,0)}}{2G_{4(0,0)}} \\
&+ \frac{3G_{4(1,0)}^3}{2G_{4(0,0)}^2\zeta} - \frac{G_{2(1,1)}}{2\zeta} + \frac{G_{3(2,0)}}{\zeta} - 3\frac{G_{4(1,0)}G_{4(2,0)}}{G_{4(0,0)}\zeta}\Bigg) + \frac{G_{4(1,0)}}{G_{4(0,0)}}(\partial_\mu\varphi)\partial^\mu\varphi - \frac{\tilde{h}T_*^{(1)}}{4G_{4(0,0)}\zeta} \\
&+ \frac{\varphi T^{(1)}}{2G_{4(0,0)}\zeta}\left(G_{4(2,0)} - \frac{G_{4(1,0)}^2}{G_{4(0,0)}}\right) - \varphi^2\frac{G_{2(3,0)}}{2\zeta} + \\
&\left[\frac{(T_*^{(1)})^2}{4G_{4(0,0)}^2\zeta^3} - \frac{(\partial_\mu\partial_\nu\varphi)(\partial^\mu\partial^\nu\varphi)}{\zeta}\right]\left(G_{3(0,1)} - 3\frac{G_{4(0,1)}G_{4(1,0)}}{G_{4(0,0)}} + 3\frac{G_{4(1,0)}G_{5(1,0)}}{G_{4(0,0)}} - 3G_{4(1,1)}\right) \\
&+ \tilde{h}_{\mu\nu}\partial^\mu\partial^\nu\varphi + \frac{T_{\mu\nu}^{(1)}\partial^\mu\partial^\nu\varphi}{G_{4(0,0)}\zeta}(G_{4(0,1)} - G_{5(1,0)}) \Rightarrow O(v^6).
\end{aligned}
$$

Gravitational Radiation

$$\varphi = \frac{f_0}{r} + \frac{f_1}{r}(\hat{n}\cdot\vec{v}) + \frac{f_2}{r}(\hat{n}\cdot\vec{v})^2 + \frac{f_3}{r}\frac{(\hat{n}\cdot\vec{r}_{12})^2}{r_{12}^3} + \frac{f_4}{r}v^2 + \frac{f_5}{rr_{12}} + f_6\frac{r_{12}}{r}$$

$$f_0 = \frac{m_1 S_1 + m_2 S_2}{8\pi G_{4(0,0)}\zeta}, \quad f_1 = -\frac{\mu}{4\pi\phi_0\zeta}(s_1 - s_2), \quad f_2 = \frac{\mu\Gamma}{8\pi G_{4(0,0)}\zeta},$$

$$f_3 = -\frac{\varsigma\mu m\Gamma}{128\pi^2 G_{4(0,0)}^2\zeta}, \quad f_4 = -\frac{\mu\Gamma}{16\pi G_{4(0,0)}\zeta}, \quad f_6 = -\frac{\mu m G_{2(3,0)} S_1 S_2}{128\pi^2 G_{4(0,0)}^2\zeta^3},$$

$$f_5 = -\frac{\mu m\Gamma'}{64\pi^2 G_{4(0,0)}^2\zeta} + \frac{\mu m\Gamma'}{32\pi^2 G_{4(0,0)}^2\zeta^2}\left(G_{4(2,0)} - \frac{G_{4(1,0)}^2}{G_{4(0,0)}}\right) + \frac{\mu m}{64\pi^2 G_{4(0,0)}^2\zeta^2}\times$$
$$\left[\left(\frac{3G_{4(1,0)}^3}{2G_{4(0,0)}^2} - \frac{G_{2(1,1)}}{2} + G_{3(2,0)} - \frac{3G_{4(1,0)}G_{4(2,0)}}{G_{4(0,0)}}\right)\frac{2S_1 S_2}{\zeta} + \frac{S_1' S_2 + S_2' S_1}{\phi_0}\right],$$

and

$$S_a' = G_{4(1,0)} s_a - \frac{2G_{4(0,0)}}{\phi_0}(s_a^2 - s_a - s_a'),$$

$$\Gamma = G_{4(1,0)} - \frac{2G_{4(0,0)}}{\phi_0}\frac{m_2 s_1 + m_1 s_2}{m}, \quad \Gamma' = G_{4(1,0)} - \frac{G_{4(0,0)}}{\phi_0}(s_1 + s_2).$$

Gravitational Radiation

$$\dot{E}_0 = -(1-e^2)^{-7/2}\Bigg\{\frac{\zeta\varsigma^3 m^3}{120(16\pi)^2 G_{4(0,0)}^3 a^5}\Big[15(e^2+4)e^2 f_4^2 + 10(e^2+4)e^2 f_2 f_4$$
$$+ (6e^4+36e^2+8)f_2^2\Big] + \frac{\zeta\varsigma^2 m^2}{1920\pi G_{4(0,0)}^2 a^5}\Big[-5a(1-e^2)(2+e^2)f_1^2$$
$$+ (3e^4+36e^2+16)f_2 f_3 - 5(e^2+4)e^2 f_2 f_5 + 20a^2 e^2(1-e^2)^2 f_2 f_6$$
$$- 5e^2(e^2+4)f_3 f_4 - 15e^2(e^2+4)f_4 f_5 + 60a^2 e^2(1-e^2)^2 f_4 f_6\Big]$$
$$+ \frac{\zeta\varsigma m}{480 G_{4(0,0)} a^5}\Big[(15e^4+108e^2+32)f_3^2 + 15e^2(e^2+4)f_5^2$$
$$+ 10e^2(e^2+4)f_3 f_5 - 120a^4\left(1-1/\sqrt{1-e^2}\right)(1-e^2)^4 f_6^2$$
$$- 120a^2 e^2(1-e^2)^2 f_5 f_6 - 40a^2 e^2(1-e^2)^2 f_3 f_6\Big]\Bigg\}.$$

Gravitational Radiation

$$
\begin{aligned}
\frac{\dot{T}}{T} =& -\frac{3}{2}\frac{\dot{E}_0+\dot{E}_2}{E} \\
=& -(1-e^2)^{-7/2}\Bigg\{\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right)\frac{96}{5}\frac{\varsigma^2\mu m^2}{(16\pi G_{4(0,0)})^3 a^4} \\
&+\frac{\zeta\varsigma^2 m^2}{640\pi\mu G_{4(0,0)}^2 a^4}\Big[15(e^2+4)e^2 f_4^2+10(e^2+4)e^2 f_2 f_4 \\
&+(6e^4+36e^2+8)f_2^2\Big]+\frac{\zeta\varsigma m}{40\mu G_{4(0,0)} a^4}\Big[-5a(1-e^2)(2+e^2)f_1^2 \\
&+(3e^4+36e^2+16)f_2 f_3-5(e^2+4)e^2 f_2 f_5+20a^2 e^2(1-e^2)^2 f_2 f_6 \\
&-5e^2(e^2+4)f_3 f_4-15e^2(e^2+4)f_4 f_5+60a^2 e^2(1-e^2)^2 f_4 f_6\Big] \\
&+\frac{\pi\zeta}{10\mu a^4}\Big[(15e^4+108e^2+32)f_3^2+15e^2(e^2+4)f_5^2 \\
&+10e^2(e^2+4)f_3 f_5-120a^4\left(1-1/\sqrt{1-e^2}\right)(1-e^2)^4 f_6^2 \\
&-120a^2e^2(1-e^2)^2 f_5 f_6-40a^2e^2(1-e^2)^2 f_3 f_6\Big]\Bigg\},
\end{aligned}
$$

Gravitational Radiation

A binary system, in circular motion:

- $\dot{E}_2 = -\frac{32}{5}\frac{\varsigma^3\mu^2 m^3}{(16\pi G_{4(0,0)})^4 r_{12}^5}$, r_{12} = distance between two stars
- $\dot{E}_0 = -\frac{1}{12\pi}\frac{\varsigma^2\mu^2 m^2(s_1 - s_2)^2}{(16\pi G_{4(0,0)})^2\phi_0^2\zeta r_{12}^4} - \frac{16}{15}\frac{\varsigma^3\mu^2 m^3\Gamma^2}{(16\pi)^4 G_{4(0,0)}^5\zeta r_{12}^5}$
- The fractional period derivative is

$$\frac{\dot{T}}{T} = -\frac{\varsigma\mu m(s_1 - s_2)^2}{64\pi^2 G_{4(0,0)}\phi_0^2\zeta r_{12}^3} - \frac{16}{5}\frac{\varsigma^2\mu m^2\Gamma^2}{(16\pi)^3 G_{4(0,0)}^4\zeta r_{12}^4} - \frac{96}{5}\frac{\varsigma^2\mu m^2}{(16\pi G_{4(0,0)})^3 r_{12}^4}.$$

Gravitational Radiation

- $|\dot{T}_{\text{pred.}} - \dot{T}_{\text{obs.}}| < 2\sigma$ at 95% C.L. with σ the uncertainty in $T_{\text{obs.}}$
- PSR J1738+0333 [29]:

Eccentricity e	$(3.4 \pm 1.1) \times 10^{-7}$
Orbital period T (days)	0.354 790 739 8724(13)
Period derivative $\dot{T}_{\text{obs}}$	$(-25.9 \pm 3.2) \times 10^{-15}$
Pulsar mass $m_1(M_\odot)$	$1.46^{+0.06}_{-0.05}$
Companion mass $m_2(M_\odot)$	$0.181^{+0.008}_{-0.007}$

- EdGB: $S = \int \mathrm{d}^4x\sqrt{-g}[R/2 + X + \xi(\phi)\mathcal{G}]$ with $\mathcal{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$

$$\phi_0 > 29.8/\sqrt{G_{\rm N}}$$

- SSHT: $G_i = G_i(X)$, $i = 2, 3, 4, 5$. E.g., covariant Galileon, Fab Four

$$3.00 \times 10^{-10}/G_{\rm N} < G_4(0) < 4.07 \times 10^{-10}/G_{\rm N}$$

with $G_4(0) = G_4|_{X=0}$

Summary

- Effective stress-energy tensor
- Nordtvedt effect: $\left|\frac{G_{4(1,0)}(1+m_s r)}{8\pi G_{4(0,0)}\phi_0\zeta}e^{-m_s r}-\delta\right|<2\epsilon$
- Shapiro time delay: $\frac{G_{4(1,0)}^2}{G_{4(0,0)}\zeta}<\frac{2\epsilon-\delta}{2+\delta-2\epsilon}e^{m_s r}$
- EdGB: $\phi_0>29.8/\sqrt{G_\mathrm{N}}$
- SSHT: $3.00\times10^{-10}/G_\mathrm{N}<G_4(0)<4.07\times10^{-10}/G_\mathrm{N}$

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