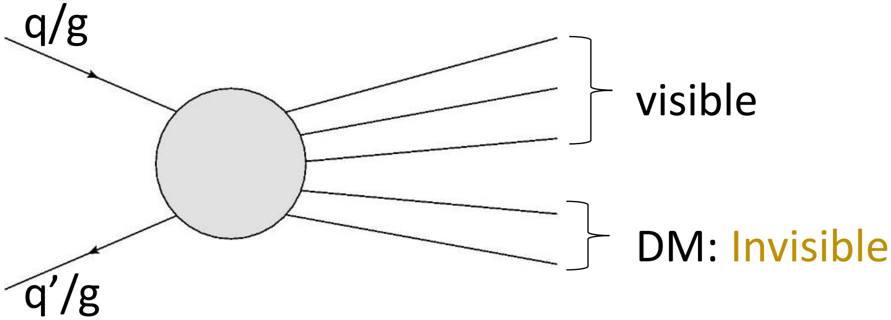
Spectral Decomposition of Missing Transverse Energy at Hadron Colliders

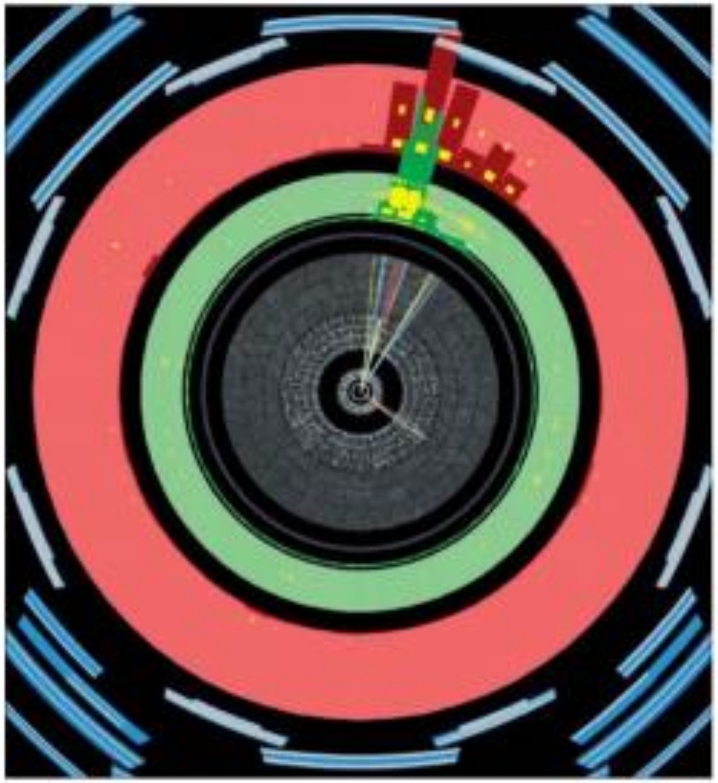
Tae Hyun Jung (IBS-CTPU)

arXiv: 1706.04512

In collaboration with Kyu Jung Bae and Myeonghun Park

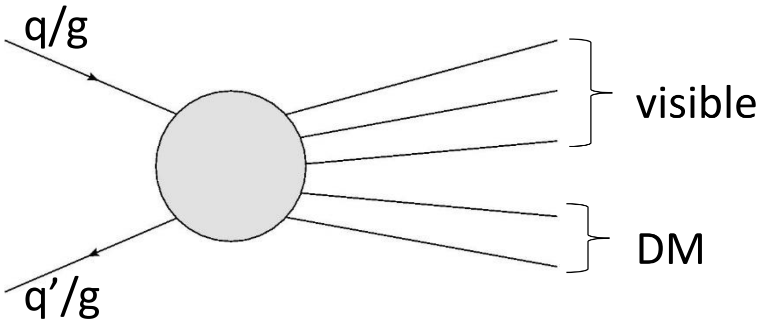
Dark Matter Signal at Hadron Colliders





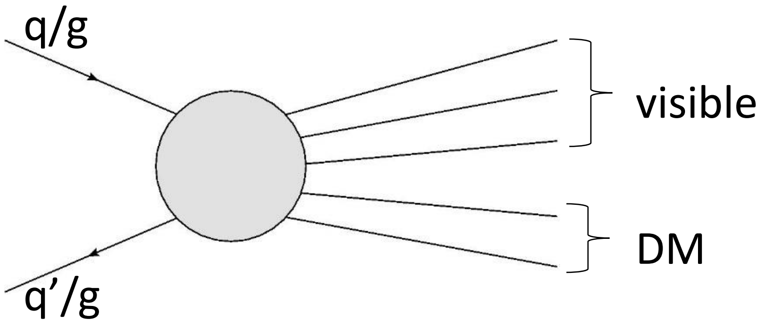
Mono-jet event from ATLAS (credit: CERN courier)

Dark Matter Signal at Hadron Colliders



Mono-jet event from ATLAS (credit: CERN courier)

Dark Matter Signal at Hadron Colliders



Missing **Transverse** Energy

$$\not{E}_T = |\sum_{\text{vis}} \vec{p}^{\,T}_{\text{vis}}| \qquad \vec{p}_T^{\,(\text{inv})} = -\sum_{\text{vis}} \vec{p}^{\,T}_{\text{vis}}$$

Mono-jet event from ATLAS (credit: CERN courier)

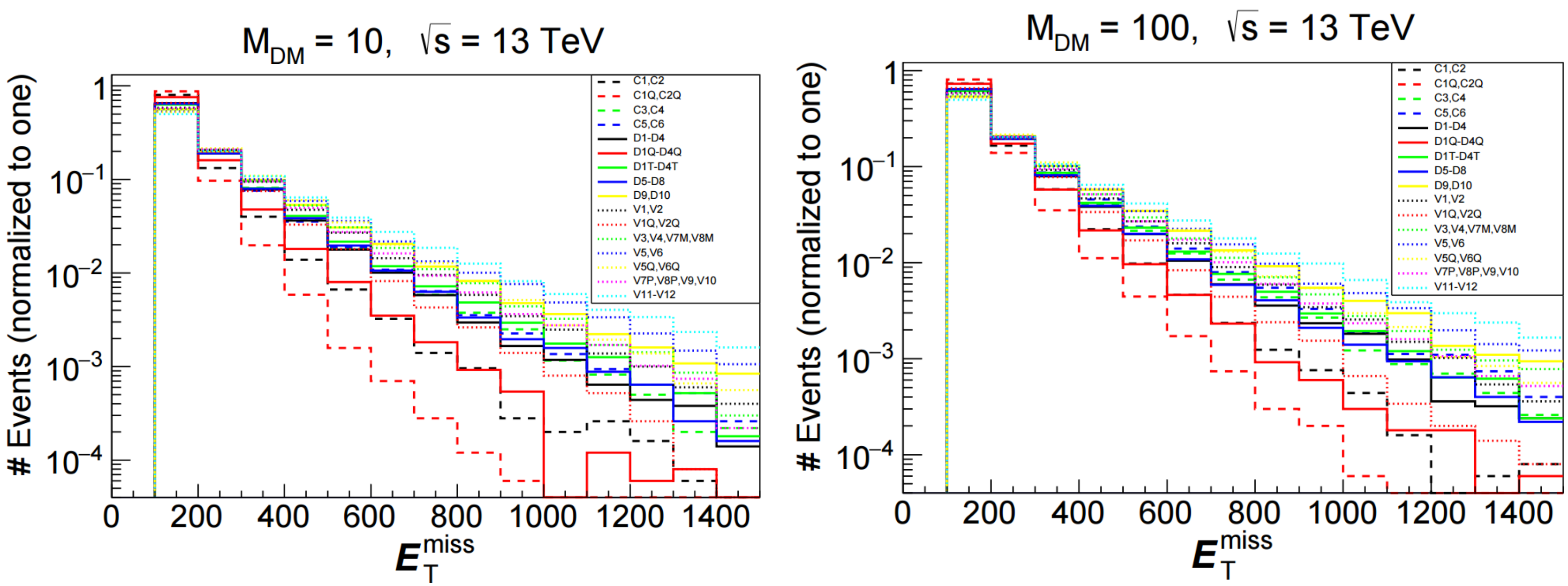
Dark Matter Signal at Hadron Colliders

- What we will see at hadron colliders is…

Dark Matter Signal at Hadron Colliders

- What we will see at hadron colliders is…

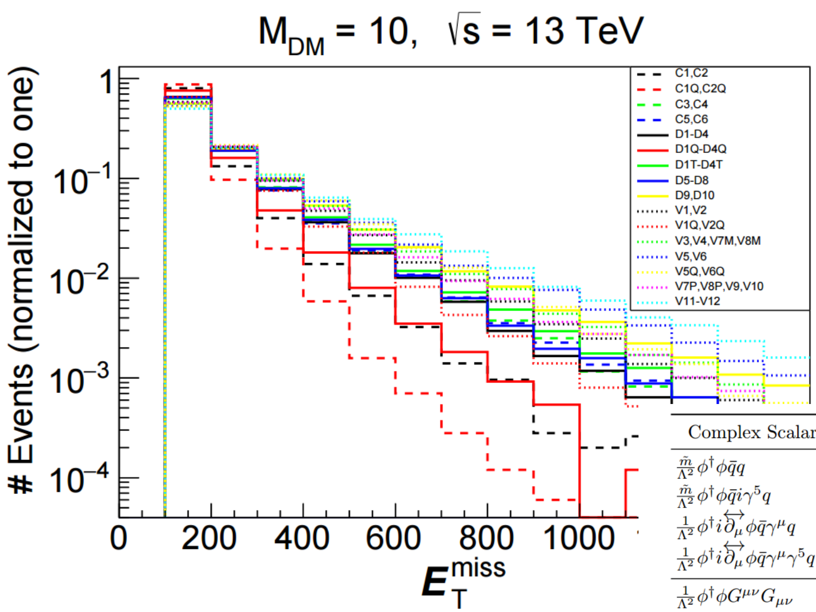
A. Belyaev, L. Panizzi, A. Pukhov, M. Thomas, arXiv:1610.07545



Dark Matter Signal at Hadron Colliders

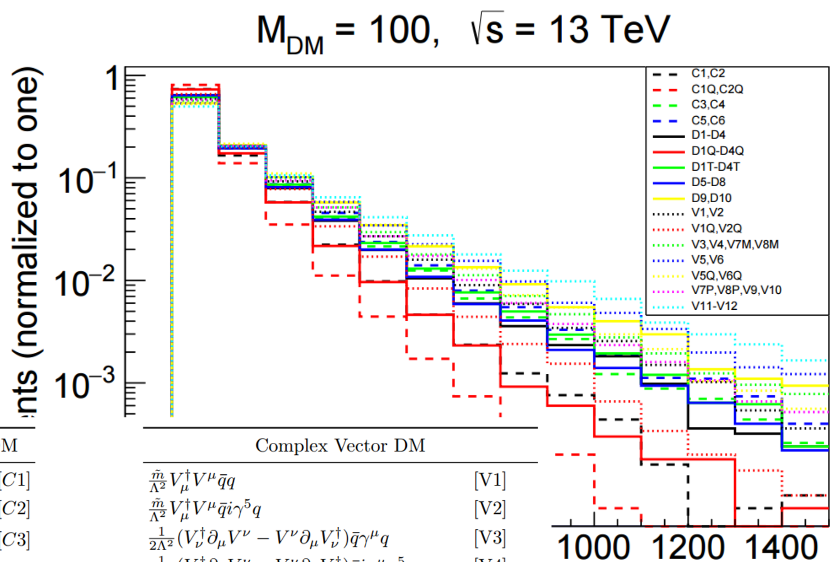
- What we will see at hadron colliders is…

A. Belyaev, L. Panizzi, A. Pukhov, M. Thomas, arXiv:1610.07545



NO direct information!

… mass? spin? interaction?



Complex Scalar DM	
$\frac{\tilde{m}}{\Lambda^2}\phi^\dagger\phi\bar{q}q$	[C1]
$\frac{\tilde{m}}{\Lambda^2}\phi^\dagger\phi\bar{q}i\gamma^5 q$	[C2]
$\frac{1}{\Lambda^2}\phi^\dagger i\overleftrightarrow{\partial_\mu}\phi\bar{q}\gamma^\mu q$	[C3]
$\frac{1}{\Lambda^2}\phi^\dagger i\overleftrightarrow{\partial_\mu}\phi\bar{q}\gamma^\mu\gamma^5 q$	[C4]
$\frac{1}{\Lambda^2}\phi^\dagger\phi G^{\mu\nu}G_{\mu\nu}$	[C5]
$\frac{1}{\Lambda^2}\phi^\dagger\phi\tilde{G}^{\mu\nu}G_{\mu\nu}$	[C6]

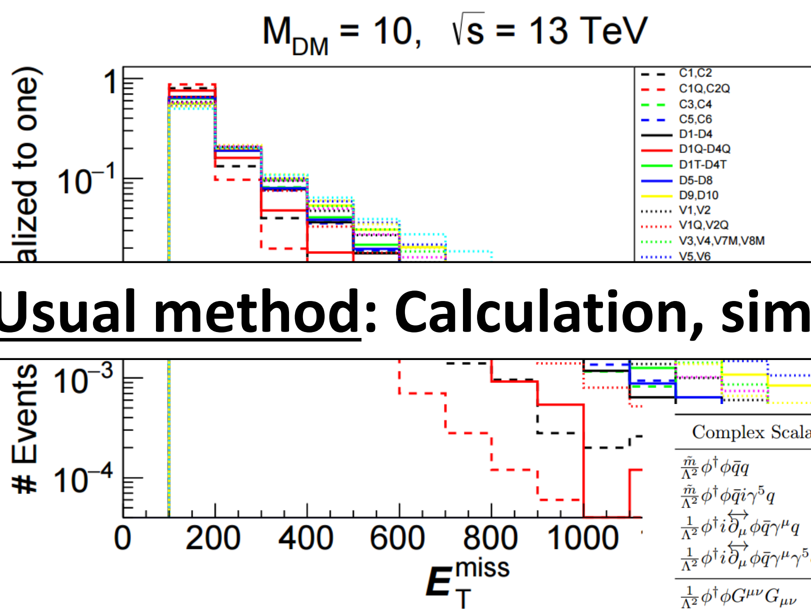
Dirac Fermion DM	
$\frac{\tilde{m}}{\Lambda^3}\bar{\chi}\chi\bar{q}q$	[D1]
$\frac{\tilde{m}}{\Lambda^3}\bar{\chi}i\gamma^5\chi\bar{q}q$	[D2]
$\frac{\tilde{m}}{\Lambda^3}\bar{\chi}\chi\bar{q}i\gamma^5 q$	[D3]
$\frac{\tilde{m}}{\Lambda^3}\bar{\chi}\gamma^5\chi\bar{q}\gamma^5 q$	[D4]
$\frac{1}{\Lambda^2}\bar{\chi}\gamma^\mu\chi\bar{q}\gamma_\mu q$	[D5]
$\frac{1}{\Lambda^2}\bar{\chi}\gamma^\mu\gamma^5\chi\bar{q}\gamma_\mu q$	[D6]
$\frac{1}{\Lambda^2}\bar{\chi}\gamma^\mu\chi\bar{q}\gamma_\mu\gamma^5 q$	[D7]
$\frac{1}{\Lambda^2}\bar{\chi}\gamma^\mu\gamma^5\chi\bar{q}\gamma_\mu\gamma^5 q$	[D8]
$\frac{1}{\Lambda^2}\bar{\chi}\sigma^{\mu\nu}\chi\bar{q}\sigma_{\mu\nu}q$	[D9]
$\frac{1}{\Lambda^2}\bar{\chi}\sigma^{\mu\nu}i\gamma^5\chi\bar{q}\sigma_{\mu\nu}q$	[D10]

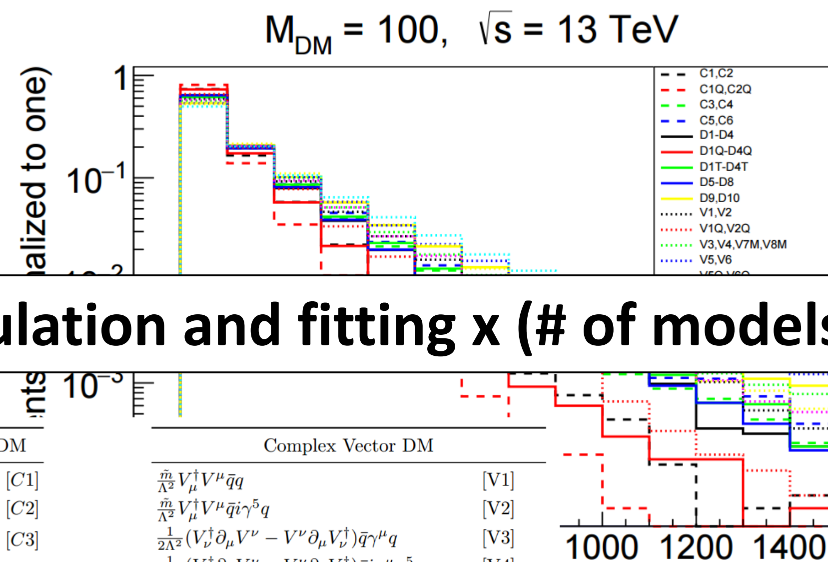
Complex Vector DM	
$\frac{\tilde{m}}{\Lambda^2}V^\dagger_\mu V^\mu\bar{q}q$	[V1]
$\frac{\tilde{m}}{\Lambda^2}V^\dagger_\mu V^\mu\bar{q}i\gamma^5 q$	[V2]
$\frac{1}{2\Lambda^2}(V^\dagger_\nu\partial_\mu V^\nu - V^\nu\partial_\mu V^\dagger_\nu)\bar{q}\gamma^\mu q$	[V3]
$\frac{1}{2\Lambda^2}(V^\dagger_\nu\partial_\mu V^\nu - V^\nu\partial_\mu V^\dagger_\nu)\bar{q}i\gamma^\mu\gamma^5 q$	[V4]
$\frac{\tilde{m}}{\Lambda^2}V^\dagger_\mu V_\nu\bar{q}i\sigma^{\mu\nu}q$	[V5]
$\frac{\tilde{m}}{\Lambda^2}V^\dagger_\mu V_\nu\bar{q}\sigma^{\mu\nu}\gamma^5 q$	[V6]
$\frac{1}{2\Lambda^2}(V^\dagger_\nu\partial^\nu V_\mu + V^\nu\partial^\nu V^\dagger_\mu)\bar{q}\gamma^\mu q$	[V7P]
$\frac{1}{2\Lambda^2}(V^\dagger_\nu\partial^\nu V_\mu - V^\nu\partial^\nu V^\dagger_\mu)\bar{q}i\gamma^\mu q$	[V7M]
$\frac{1}{2\Lambda^2}(V^\dagger_\nu\partial^\nu V_\mu + V^\nu\partial^\nu V^\dagger_\mu)\bar{q}\gamma^\mu\gamma^5 q$	[V8P]
$\frac{1}{2\Lambda^2}(V^\dagger_\nu\partial^\nu V_\mu - V^\nu\partial^\nu V^\dagger_\mu)\bar{q}i\gamma^\mu\gamma^5 q$	[V8M]
$\frac{1}{2\Lambda^2}\epsilon^{\mu\nu\rho\sigma}(V^\dagger_\nu\partial_\rho V_\sigma + V_\nu\partial_\rho V^\dagger_\sigma)\bar{q}\gamma_\mu q$	[V9P]
$\frac{1}{2\Lambda^2}\epsilon^{\mu\nu\rho\sigma}(V^\dagger_\nu\partial^\nu V_\mu - V^\nu\partial^\nu V^\dagger_\mu)\bar{q}i\gamma_\mu q$	[V9M]
$\frac{1}{2\Lambda^2}\epsilon^{\mu\nu\rho\sigma}(V^\dagger_\nu\partial_\rho V_\sigma + V_\nu\partial_\rho V^\dagger_\sigma)\bar{q}\gamma_\mu\gamma^5 q$	[V10P]
$\frac{1}{2\Lambda^2}\epsilon^{\mu\nu\rho\sigma}(V^\dagger_\nu\partial^\nu V_\mu - V^\nu\partial^\nu V^\dagger_\mu)\bar{q}i\gamma_\mu\gamma^5 q$	[V10M]
$\frac{1}{\Lambda^2}V^\dagger_\mu V^\mu G^{\rho\sigma}G_{\rho\sigma}$	[V11]
$\frac{1}{\Lambda^2}V^\dagger_\mu V^\mu\tilde{G}^{\rho\sigma}G_{\rho\sigma}$	[V12]

Dark Matter Signal at Hadron Colliders

- What we will see at hadron colliders is…

A. Belyaev, L. Panizzi, A. Pukhov, M. Thomas, arXiv:1610.07545





<u>Usual method</u>: Calculation, simulation and fitting x (# of models)

NO direct information!
… mass? spin? interaction?

Complex Scalar DM	
$\frac{\tilde{m}}{\Lambda^2}\phi^\dagger\phi\bar{q}q$	$[C1]$
$\frac{\tilde{m}}{\Lambda^2}\phi^\dagger\phi\bar{q}i\gamma^5 q$	$[C2]$
$\frac{1}{\Lambda^2}\phi^\dagger i\overleftrightarrow{\partial_\mu}\phi\bar{q}\gamma^\mu q$	$[C3]$
$\frac{1}{\Lambda^2}\phi^\dagger i\overleftrightarrow{\partial_\mu}\phi\bar{q}\gamma^\mu\gamma^5 q$	$[C4]$
$\frac{1}{\Lambda^2}\phi^\dagger\phi G^{\mu\nu}G_{\mu\nu}$	$[C5]$
$\frac{1}{\Lambda^2}\phi^\dagger\phi\tilde{G}^{\mu\nu}G_{\mu\nu}$	$[C6]$

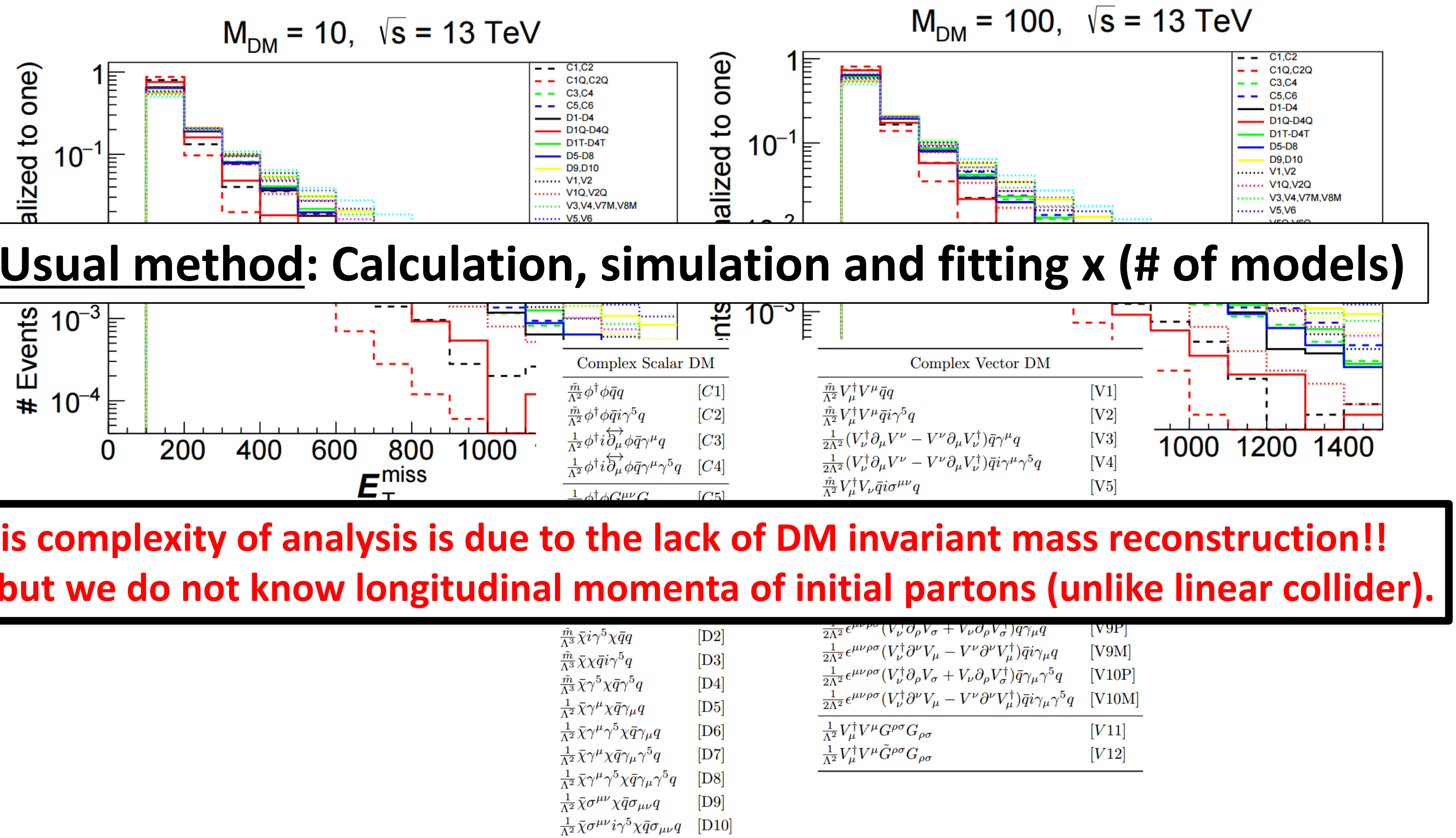
Dirac Fermion DM	
$\frac{\tilde{m}}{\Lambda^3}\bar{\chi}\chi\bar{q}q$	[D1]
$\frac{\tilde{m}}{\Lambda^3}\bar{\chi}i\gamma^5\chi\bar{q}q$	[D2]
$\frac{\tilde{m}}{\Lambda^3}\bar{\chi}\chi\bar{q}i\gamma^5 q$	[D3]
$\frac{\tilde{m}}{\Lambda^3}\bar{\chi}\gamma^5\chi\bar{q}\gamma^5 q$	[D4]
$\frac{1}{\Lambda^2}\bar{\chi}\gamma^\mu\chi\bar{q}\gamma_\mu q$	[D5]
$\frac{1}{\Lambda^2}\bar{\chi}\gamma^\mu\gamma^5\chi\bar{q}\gamma_\mu q$	[D6]
$\frac{1}{\Lambda^2}\bar{\chi}\gamma^\mu\chi\bar{q}\gamma_\mu\gamma^5 q$	[D7]
$\frac{1}{\Lambda^2}\bar{\chi}\gamma^\mu\gamma^5\chi\bar{q}\gamma_\mu\gamma^5 q$	[D8]
$\frac{1}{\Lambda^2}\bar{\chi}\sigma^{\mu\nu}\chi\bar{q}\sigma_{\mu\nu}q$	[D9]
$\frac{1}{\Lambda^2}\bar{\chi}\sigma^{\mu\nu}i\gamma^5\chi\bar{q}\sigma_{\mu\nu}q$	[D10]

Complex Vector DM	
$\frac{\tilde{m}}{\Lambda^2}V_\mu^\dagger V^\mu\bar{q}q$	[V1]
$\frac{\tilde{m}}{\Lambda^2}V_\mu^\dagger V^\mu\bar{q}i\gamma^5 q$	[V2]
$\frac{1}{2\Lambda^2}(V_\nu^\dagger\partial_\mu V^\nu - V^\nu\partial_\mu V_\nu^\dagger)\bar{q}\gamma^\mu q$	[V3]
$\frac{1}{2\Lambda^2}(V_\nu^\dagger\partial_\mu V^\nu - V^\nu\partial_\mu V_\nu^\dagger)\bar{q}i\gamma^\mu\gamma^5 q$	[V4]
$\frac{\tilde{m}}{\Lambda^2}V_\mu^\dagger V_\nu\bar{q}i\sigma^{\mu\nu}q$	[V5]
$\frac{\tilde{m}}{\Lambda^2}V_\mu^\dagger V_\nu\bar{q}\sigma^{\mu\nu}\gamma^5 q$	[V6]
$\frac{1}{2\Lambda^2}(V_\nu^\dagger\partial^\nu V_\mu + V^\nu\partial^\nu V_\mu^\dagger)\bar{q}\gamma^\mu q$	[V7P]
$\frac{1}{2\Lambda^2}(V_\nu^\dagger\partial^\nu V_\mu - V^\nu\partial^\nu V_\mu^\dagger)\bar{q}i\gamma^\mu q$	[V7M]
$\frac{1}{2\Lambda^2}(V_\nu^\dagger\partial^\nu V_\mu + V^\nu\partial^\nu V_\mu^\dagger)\bar{q}\gamma^\mu\gamma^5 q$	[V8P]
$\frac{1}{2\Lambda^2}(V_\nu^\dagger\partial^\nu V_\mu - V^\nu\partial^\nu V_\mu^\dagger)\bar{q}i\gamma^\mu\gamma^5 q$	[V8M]
$\frac{1}{2\Lambda^2}\epsilon^{\mu\nu\rho\sigma}(V_\nu^\dagger\partial_\rho V_\sigma + V_\nu\partial_\rho V_\sigma^\dagger)\bar{q}\gamma_\mu q$	[V9P]
$\frac{1}{2\Lambda^2}\epsilon^{\mu\nu\rho\sigma}(V_\nu^\dagger\partial^\nu V_\mu - V^\nu\partial^\nu V_\mu^\dagger)\bar{q}i\gamma_\mu q$	[V9M]
$\frac{1}{2\Lambda^2}\epsilon^{\mu\nu\rho\sigma}(V_\nu^\dagger\partial_\rho V_\sigma + V_\nu\partial_\rho V_\sigma^\dagger)\bar{q}\gamma_\mu\gamma^5 q$	[V10P]
$\frac{1}{2\Lambda^2}\epsilon^{\mu\nu\rho\sigma}(V_\nu^\dagger\partial^\nu V_\mu - V^\nu\partial^\nu V_\mu^\dagger)\bar{q}i\gamma_\mu\gamma^5 q$	[V10M]
$\frac{1}{\Lambda^2}V_\mu^\dagger V^\mu G^{\rho\sigma}G_{\rho\sigma}$	$[V11]$
$\frac{1}{\Lambda^2}V_\mu^\dagger V^\mu\tilde{G}^{\rho\sigma}G_{\rho\sigma}$	$[V12]$

Dark Matter Signal at Hadron Colliders

- What we will see at hadron colliders is…

A. Belyaev, L. Panizzi, A. Pukhov, M. Thomas, arXiv:1610.07545



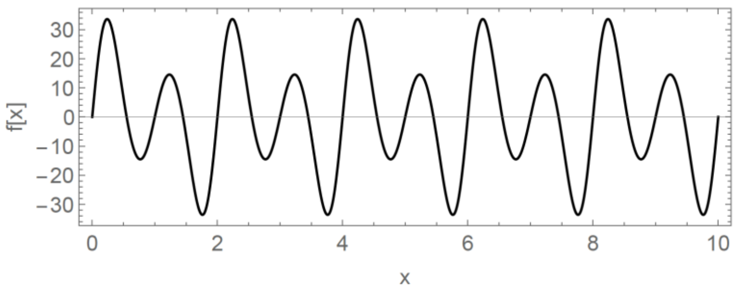
<u>Usual method</u>: Calculation, simulation and fitting x (# of models)

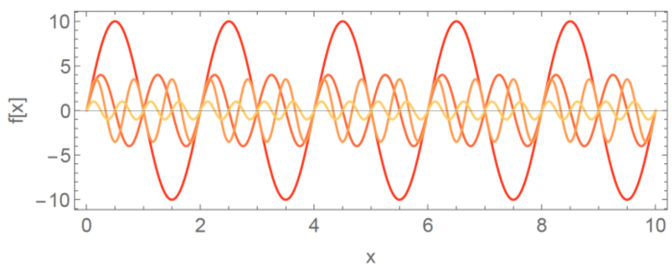
This complexity of analysis is due to the lack of DM invariant mass reconstruction!!
… but we do not know longitudinal momenta of initial partons (unlike linear collider).

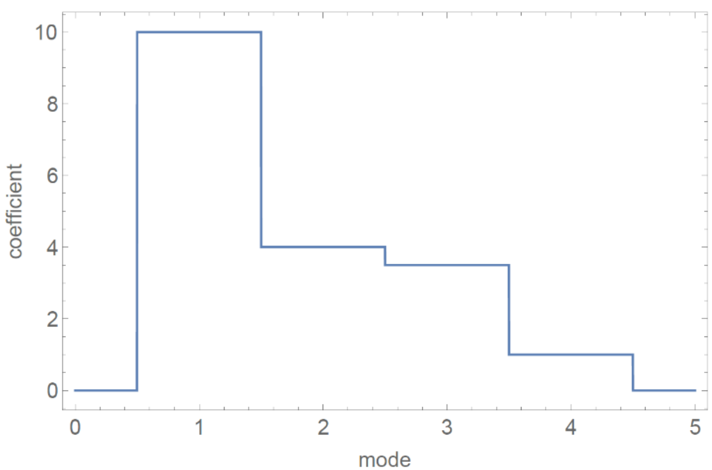
Spectral Decomposition

Spectral Decomposition

- The idea is similar to the Fourier transformation.







Basis functions: sin(nπx/L) w/ n= integer

Coefficients: amplitude of each mode

$$f(x) = \sum_n c_n \sin(n\pi x/L)$$

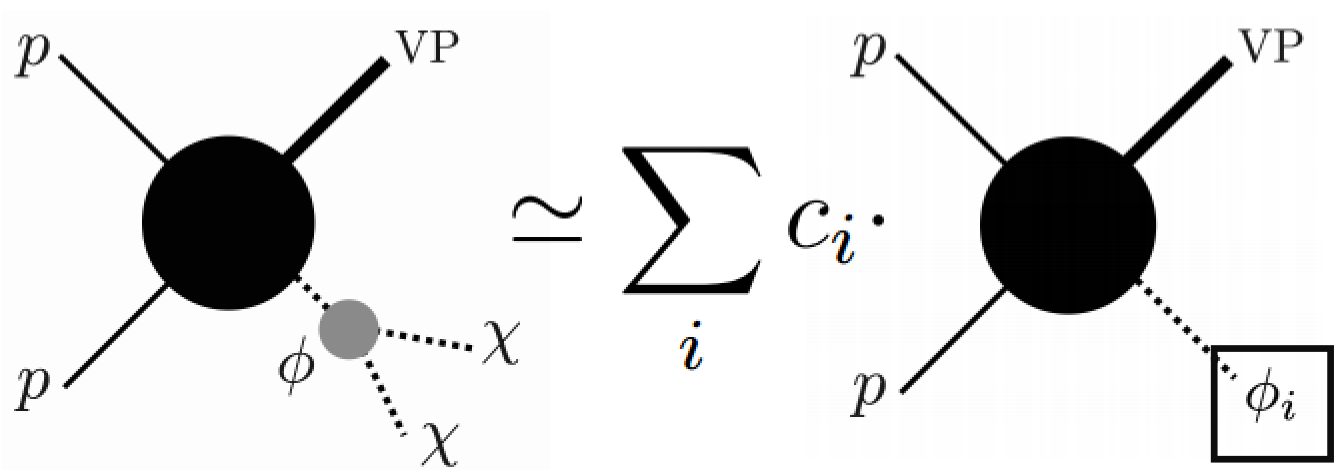
Spectral Decomposition

$$\frac{\mathrm{d}\sigma^{\mathrm{exp}}(X)}{\mathrm{d}X} \simeq \sum_{i=1}^{N} c_i \underbrace{\left(\frac{1}{\mathcal{N}_i} \frac{\mathrm{d}\sigma_{\phi_i}(X)}{\mathrm{d}X} \right)}_{=\text{basis functions}}$$

cf. Fourier transformation

$$f(x) = \sum_n c_n \sin(n\pi x/L)$$

X: a set of collider observables including MET



MET dist. of Physical process

MET dist. of Virtual Mediator production

Virtual mass: $\{m_{\chi\chi}^{(i)}\}$ given by hand

Spectral Decomposition

$$\frac{\mathrm{d}\sigma^{\exp}(X)}{\mathrm{d}X} \simeq \sum_{i=1}^{N} c_i \underbrace{\left(\frac{1}{\mathcal{N}_i}\frac{\mathrm{d}\sigma_{\phi_i}(X)}{\mathrm{d}X}\right)}_{=\text{basis functions}}$$

LHS: from the experiment
Basis functions: from the calculation
Coefficients: from chi-square fitting

$$\chi^2 = \sum_{X_{\text{bin}}} \frac{\left(\text{Ex}(X_{\text{bin}}) - \text{SM}(X_{\text{bin}}) - \sum_{i=1}^{N} c_i F_i(X_{\text{bin}})\right)^2}{\text{Ex}(X_{\text{bin}})}$$

$$F_i(X_{\text{bin}}) = \frac{L}{\mathcal{N}_i}\int_{X \in X_{\text{bin}}} \mathrm{d}X \frac{\mathrm{d}\sigma_{\phi_i}(X)}{\mathrm{d}X}$$: binned basis functions

- What is the physical meaning of coefficients?

Physical meaning of c_i is the DM invariant mass distribution.

$$c_i \simeq \frac{\mathrm{d}\sigma_{\mathrm{full}}(m_{\chi\chi})}{\mathrm{d}m_{\chi\chi}} \Delta m_{\chi\chi}^{(i)}$$

Spectral Decomposition: MET space ⟶ DM inv. mass space

I will skip the proof.

Equivalently, c_i can be related to **Kallen-Lehmann spectral density**.

$$c_i = 2m_{\chi\chi}^{(i)}\,\Delta m_{\chi\chi}^{(i)}\,\mathcal{N}_i\,\rho_{\phi\to\chi\chi}(m_{\chi\chi}^{(i)}, M_\phi)$$

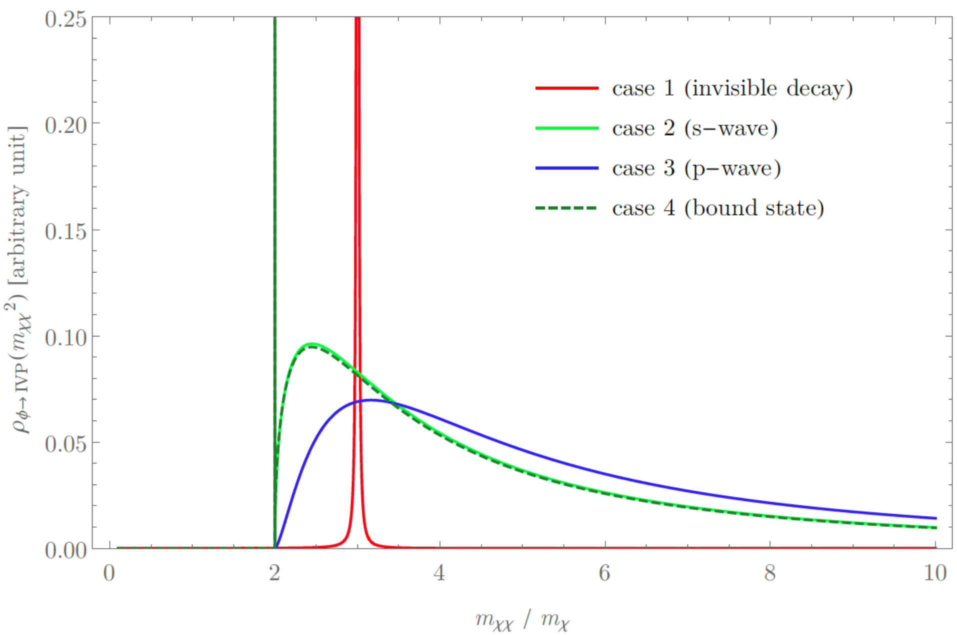
Theoretically, $\rho_{\phi\to\chi\chi}(m_{\chi\chi}^{(i)}, M_\phi) = \frac{1}{\pi}\left|G_\phi(m_{\chi\chi}^{(i)}, M_\phi)\right|^2 m_{\chi\chi}^{(i)}\Gamma_{\phi_i\to\chi\chi}(m_{\chi\chi}^{(i)})$

Two merits of spectral density

1. **Universality** (important in our formalism)
2. Easy to **characterize** dark sector.

Spectral Density (characterization)

	Mediator	Interaction
Case 1	On-shell($M_\phi > 2m_\chi$)	Resonance
Case 2	Off-shell($M_\phi < 2m_\chi$)	S-wave
Case 3	Off-shell($M_\phi < 2m_\chi$)	P-wave
Case 4	Off-shell($M_\phi < 2m_\chi$)	DM bound state (dark long range force)



Summary of our method

Lagrangian: $\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \underbrace{\mathcal{L}_{\mathrm{med-SM}} + \mathcal{L}_{\mathrm{med}}}_{\to \text{basis functions}} + \underbrace{\mathcal{L}_{\mathrm{med-DM}} + \mathcal{L}_{\mathrm{DM}}}_{\to \text{spectral density}}$

1. Fix $\mathcal{L}_{\mathrm{med-SM}} + \mathcal{L}_{\mathrm{med}}$ and calculate basis functions.
2. Obtain coefficients c_i by fitting the signal (experiment).

$$\boxed{\frac{\mathrm{d}\sigma^{\exp}(X)}{\mathrm{d}X} \simeq \sum_{i=1}^{N} c_i \underbrace{\left(\frac{1}{\mathcal{N}_i}\frac{\mathrm{d}\sigma_{\phi_i}(X)}{\mathrm{d}X}\right)}_{=\text{basis functions}}}$$

$$\chi^2 = \sum_{X_{\mathrm{bin}}} \frac{\left(\mathrm{Ex}(X_{\mathrm{bin}}) - \mathrm{SM}(X_{\mathrm{bin}}) - \sum_{i=1}^{N} c_i F_i(X_{\mathrm{bin}})\right)^2}{\mathrm{Ex}(X_{\mathrm{bin}})}$$

3. Find $\mathcal{L}_{\mathrm{med-DM}} + \mathcal{L}_{\mathrm{DM}}$ that matches with c_i obtained in step 2.

Numerical Example
(Mono-jet channel w/ a Simplified Model)

- Simplified DM model

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_{\rm med-SM} + \mathcal{L}_{\rm med} + \mathcal{L}_{\rm med-DM} + \mathcal{L}_{DM}$$

$$\left.\begin{aligned} \mathcal{L}_{\rm med} &= \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}M_\phi^2\phi^2 \\ \mathcal{L}_{\rm med-SM} &= \frac{k}{\Lambda}\phi G^{a\mu\nu}G^a_{\mu\nu} \end{aligned}\right\} \to \text{basis functions}$$

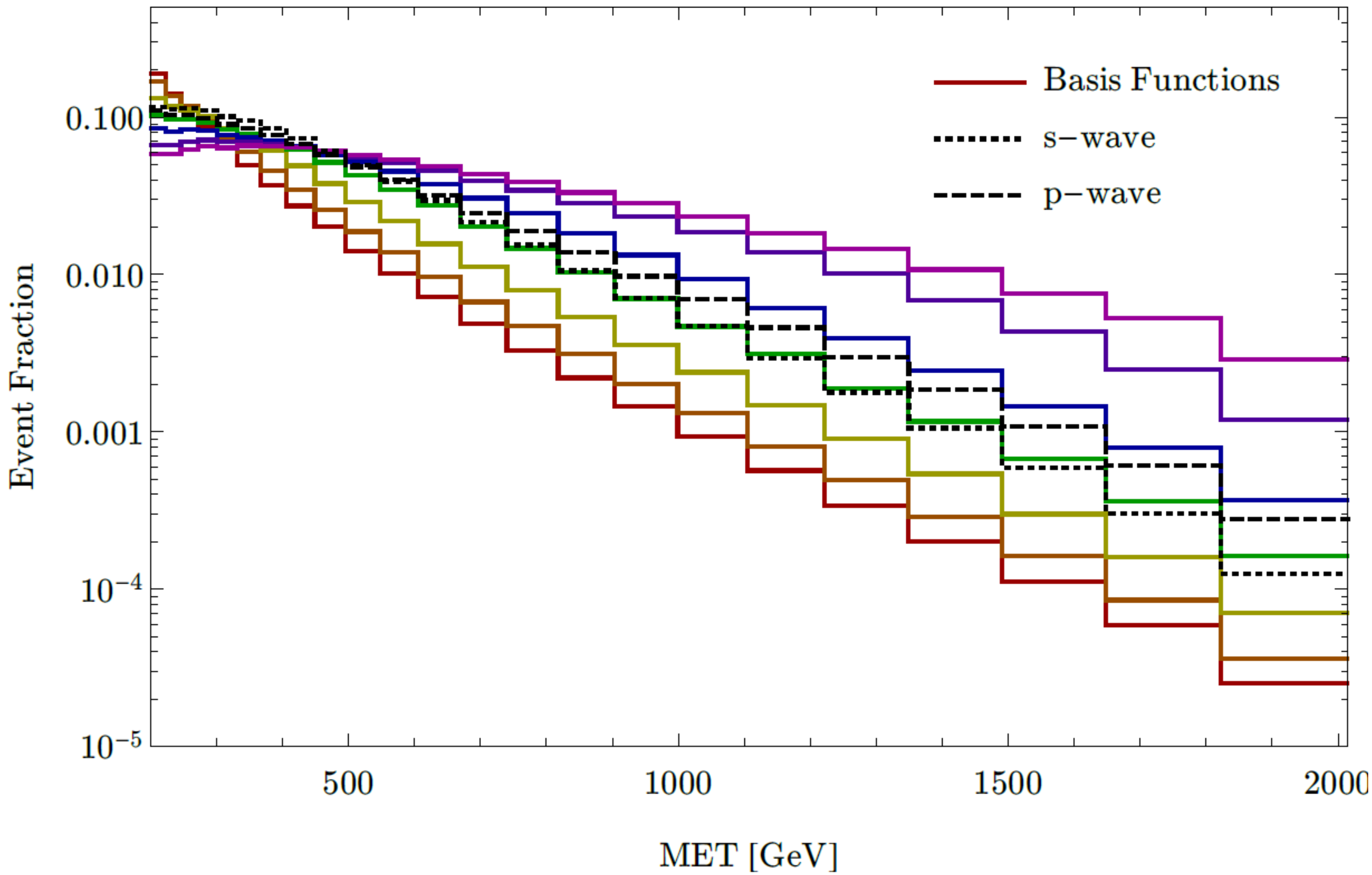
$$\left.\begin{aligned} \mathcal{L}^{(s)}_{\rm DM} &= \frac{1}{2}\partial_\mu s\partial^\mu s - \frac{1}{2}m_s^2 s^2 \\ \mathcal{L}^{(s)}_{\rm med-DM} &= \frac{1}{2}m_s g_s \phi s^2 \end{aligned}\right\} \to \text{scalar dark matter}$$

$$\left.\begin{aligned} \mathcal{L}^{(f)}_{\rm DM} &= \bar{\chi}(i\partial\!\!\!/ - m_\chi)\chi \\ \mathcal{L}^{(f)}_{\rm med-DM} &= \frac{1}{2}\phi\bar{\chi}\chi \end{aligned}\right\} \to \text{fermionic dark matter}$$

[blue box] : Assumption

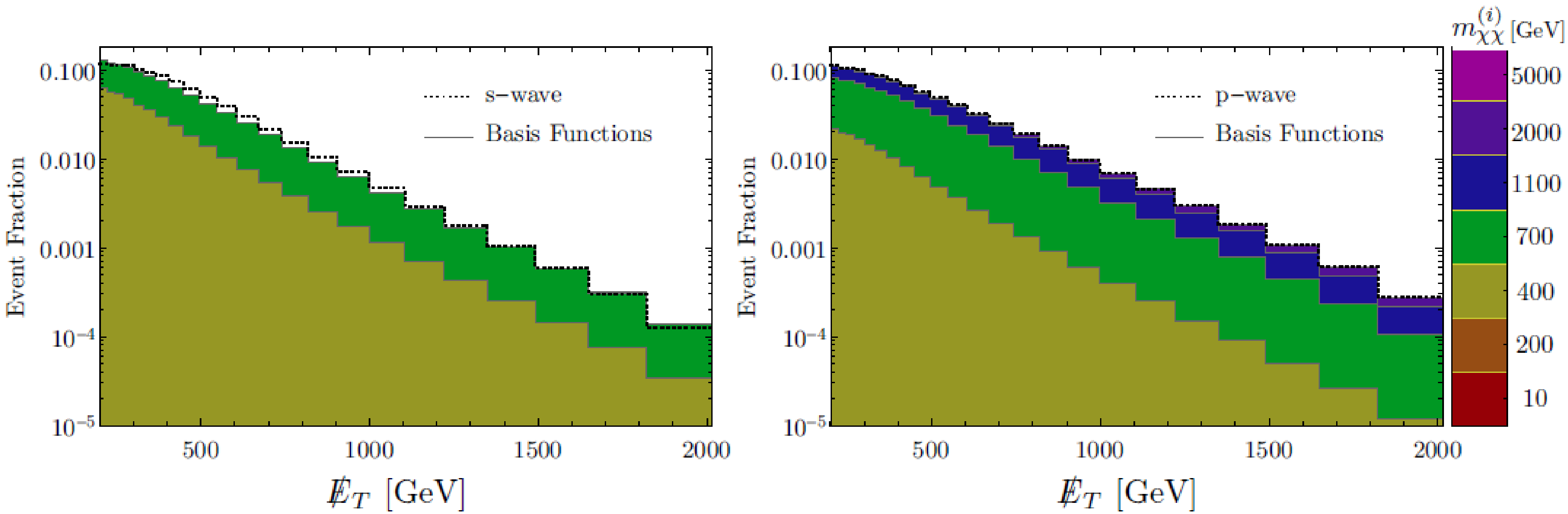
[green box] : Information we will obtain from data (not assumptions, but results)

Example (step 1: calculate basis functions)



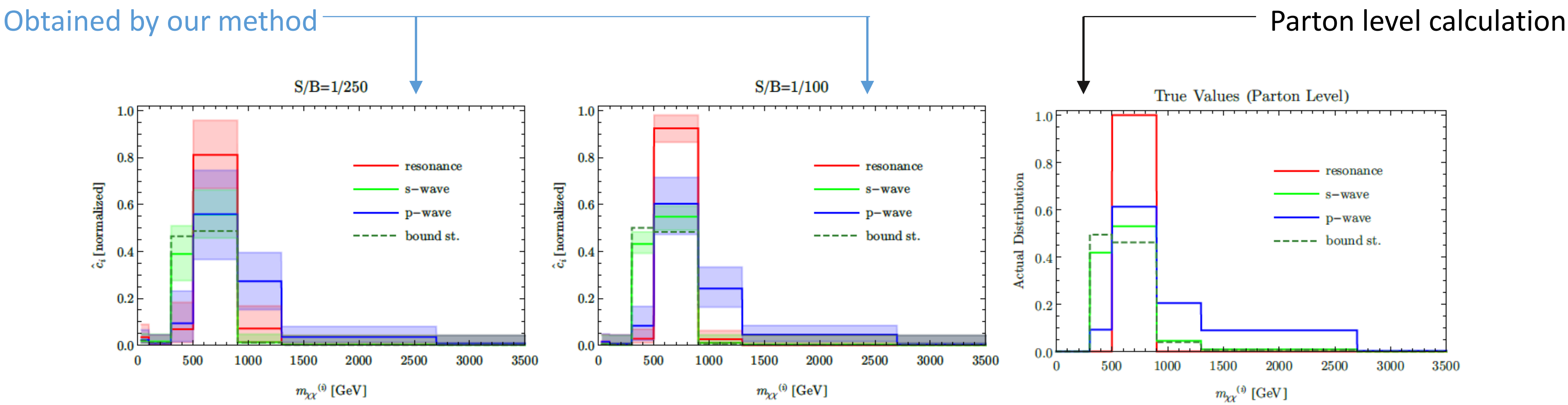
- Colored curves: basis functions
- $\{m_{\chi\chi}^{(i)}\}$={10, 200, 400, 700, 1100, 2000, 5000} GeV.

Example (step 2: obtain coefficients by fitting)



- Signal: dotted lines (left: s-wave, right: p-wave, DM mass= 200 GeV)
- Basis functions: shaded by color
- Coefficients: obtained by chi square fitting = area of each colored region = DM inv. dist.

Example (step 3: the interpretation of results)



- Location of threshold = $2m_x > M_{mediator}$
- Slope at the threshold⇒ interaction between DM and mediator
- Peak position (red): On-shell mass of mediator which decays invisibly
- And the existence of bound state changes the first bin.

	Interaction
Case 1	Resonance ($M_\phi > 2m_\chi$)
Case 2	S-wave ($M_\phi < 2m_\chi$)
Case 3	P-wave ($M_\phi < 2m_\chi$)
Case 4	bound state ($M_\phi < 2m_\chi$)

Conclusion

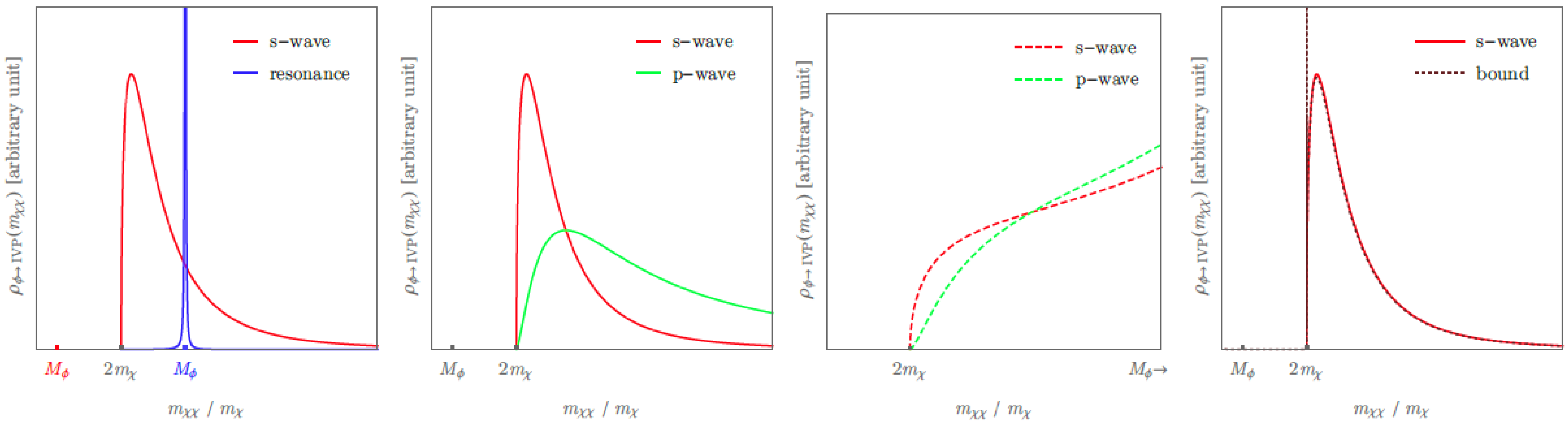
We can obtain DM invariant mass distribution even at hadron colliders by using the spectral decomposition.

From the obtained DM invariant mass distribution, we can easily extract DM information at hadron colliders.

Thank you!

Back Up

More examples of spectral densities



luminosity function

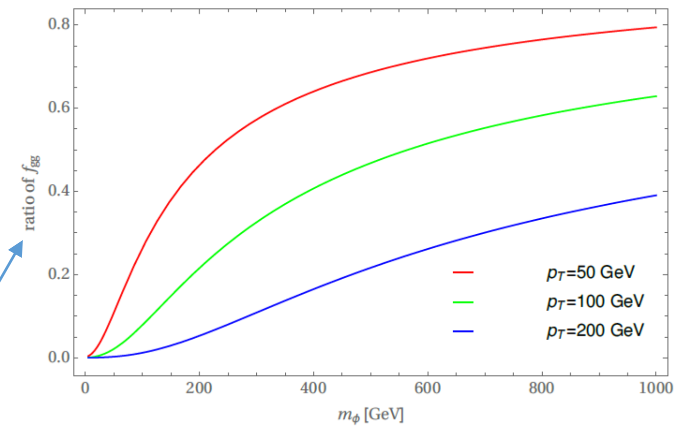
$$\sigma_\phi = \sum_{i,j} \int dx_1 dx_2 P_i(x_1) P_j(x_2) \hat{\sigma}_\phi^{(ij)}$$

$$= \sum_{i,j} \int dm_{ij} dy \frac{dL_{ij}}{dm_{ij} dy} \hat{\sigma}_\phi^{(ij)}$$

$$\frac{dL_{ij}}{dm_{ij} dy} = \frac{2m_{ij}}{S} P_i\left(\frac{m_{ij}}{\sqrt{S}} e^{y}\right) P_j\left(\frac{m_{ij}}{\sqrt{S}} e^{-y}\right)$$

$$\frac{d\sigma_\phi}{d\not{p}_T} = \sum_{i,j} \int_{\not{p}_T + \sqrt{\not{p}_T^2 + m_\phi^2}}^{\sqrt{S}} dm_{ij} f_{ij}(m_{ij}) \frac{d\hat{\sigma}_\phi^{(ij)}}{d\not{p}_T}$$

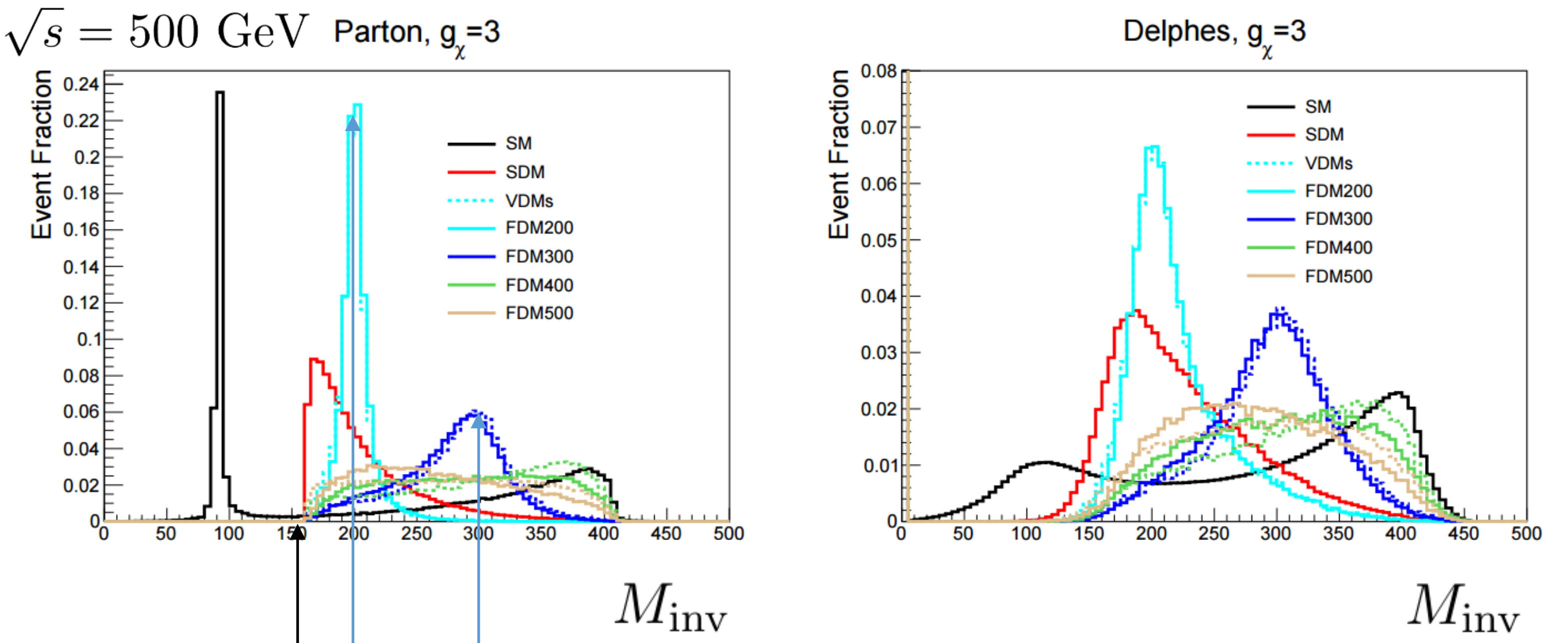
$$\frac{\frac{d\sigma_\phi}{d\not{p}_T}(\not{p}_T, m_{ij}, m_\phi)}{\frac{d\sigma_\phi}{d\not{p}_T}(0, m_{ij}, m_\phi)} \sim \frac{f_{ij}(\not{p}_T + \sqrt{\not{p}_T^2 + m_\phi^2})}{f_{ij}(m_\phi)}$$



Strategy at linear colliders

- Example : mono-Z(to jj) channel

T. Kamon, P. Ko, J. Li, arXiv:1705.02149



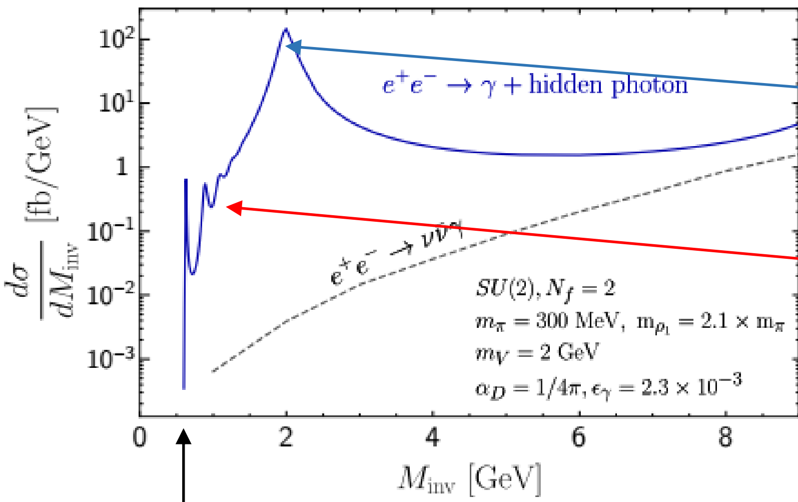
BW Peak : On-shell mediator is produced and decays invisibly.

Location of threshold = $2m_x > M_{mediator}$
Slope at the threshold ⇒ interaction between DM and mediator

Strategy at linear colliders

- Example : mono-photon w/ confining dark sector

Y. Hochberg, E. Kuflik, H. Murayama, arXiv:1512.07917, 1706.05008



Peak position : On-shell Mediator decays to dark matter particles.

Many bumps near the threshold indicates confinement of dark sector. (e.g. SIMP)

Threshold : Mediator is lighter than $2m_{DM}$.
Location of the threshold = $2m_{DM}$
Slope at the threshold ⇒ interaction between DM and mediator

Missing Transverse Energy