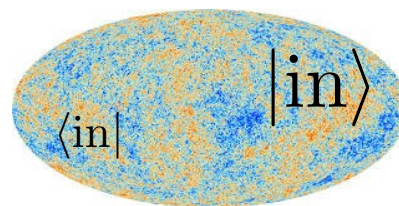


CosPA2017 @ YITP, Kyoto

# Inflationary fluctuations with phase transitions



based on [1704.05026]

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The University of Tokyo

in collaboration with  
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Yi Wang & Siyi Zhou  
(Hong Kong University of Science & Technology)

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu}$$
$$H^2 = \frac{8\pi G}{3} \left[ \frac{1}{2}\dot{\phi}^2 + V(\phi) \right]$$



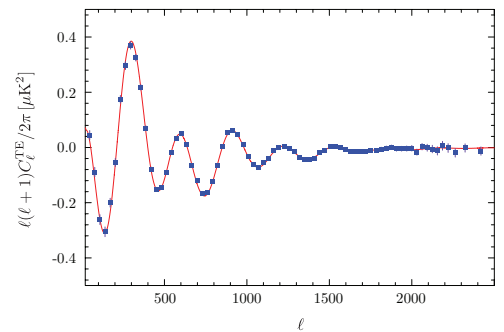
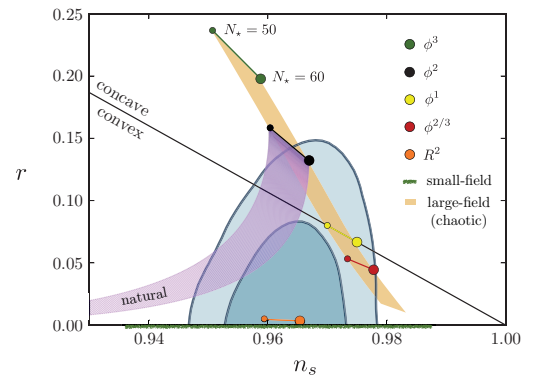
**RESCEU**

東京大学大学院理学系研究科附属ビッグバン宇宙国際研究センター  
Research Center for the Early Universe

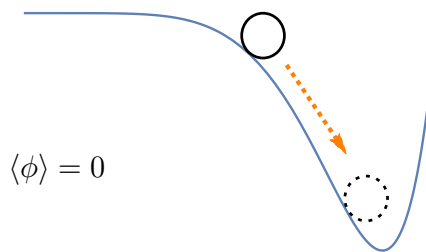
# What is inflation all about?

$$a(t) \sim e^{Ht}$$

- The initial conditions of Big Bang cosmology.
- The generation of primordial density fluctuations.
- The small deviation from scale-invariant primordial power spectrum.
- The existence of acoustic oscillation peaks.
- Current status: reported by inflationary speakers!



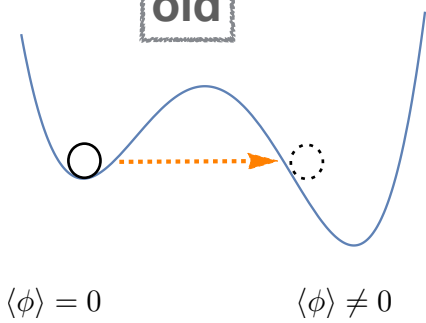
# What is inflation all about?



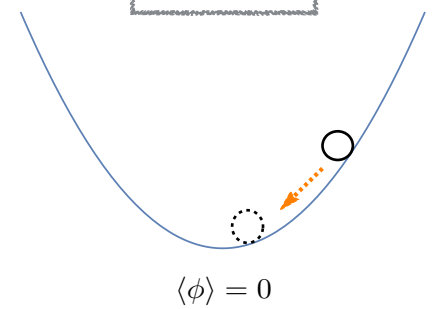
old

new

chaotic

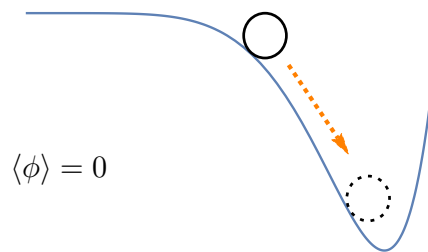


$\langle \phi \rangle \neq 0$



# What is inflation all about?

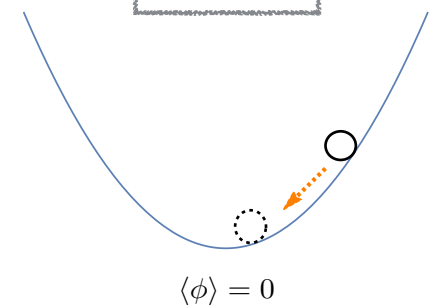
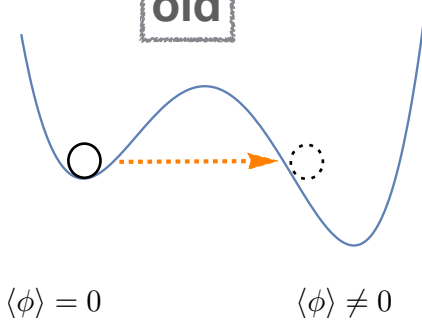
➤ The transition of **vev** plays a fundamental role in all inflation scenarios.



old

new

chaotic

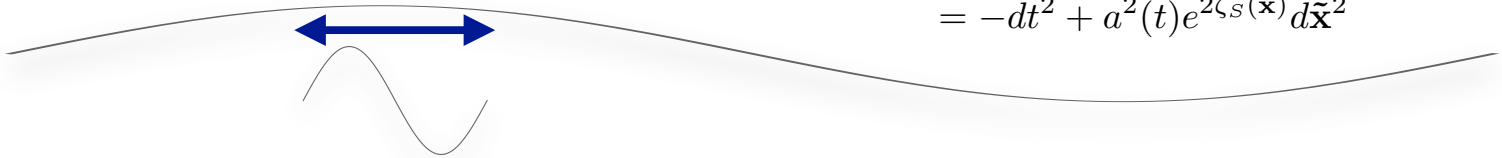




# The single-field consistency relation

Maldacena (2003)

$$\begin{aligned}
 ds^2 &= -dt^2 + a^2(t)e^{2\zeta_S(\mathbf{x})+2\zeta_L} d\mathbf{x}^2 \\
 &= -dt^2 + a^2(t)e^{2\zeta_S(\tilde{\mathbf{x}})} d\tilde{\mathbf{x}}^2
 \end{aligned}$$



de Putter et al. [1610.00785]

The dilatation transformation

$$\langle \zeta(k_1)\zeta(k_2) \rangle_{\zeta_L} = e^{-(n_s-1)\zeta_L} \langle \zeta(\tilde{k}_1)\zeta(\tilde{k}_2) \rangle_0$$

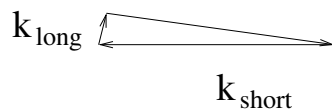
The squeezed bispectrum

$$\lim_{k_3 \rightarrow 0} \langle \zeta(k_1)\zeta(k_2)\zeta(k_3) \rangle = -(n_s - 1) \langle \zeta(k_S)\zeta(k_S) \rangle \langle \zeta(k_L)\zeta(k_L) \rangle$$

- The curvature perturbation in single-clock inflation is conserved.
- The squeezed limit of bispectrum is suppressed by spacetime symmetry.

# Cosmological collider

— probing signals of massive fields during inflation

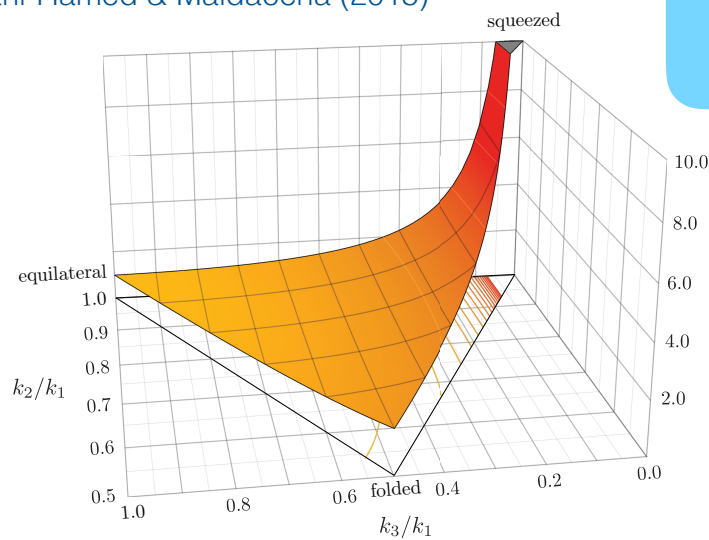


Assassi, Baumann & Green (2012)

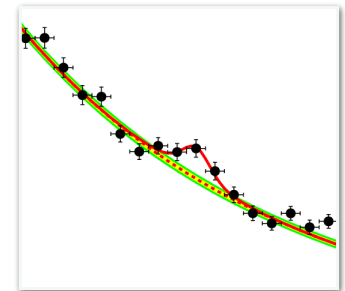
Arkani-Hamed & Maldacena (2015)

The squeezed bispectrum

$$\frac{\langle \zeta \zeta \zeta \rangle}{\langle \zeta \zeta \rangle_{\text{short}} \langle \zeta \zeta \rangle_{\text{long}}} \sim \epsilon \sum_i w_i \left( \frac{k_{\text{long}}}{k_{\text{short}}} \right)^{\Delta_i}$$



$B_{\mathcal{R}}(k_1, k_2, k_3)$



More in Yi Wang's talk!

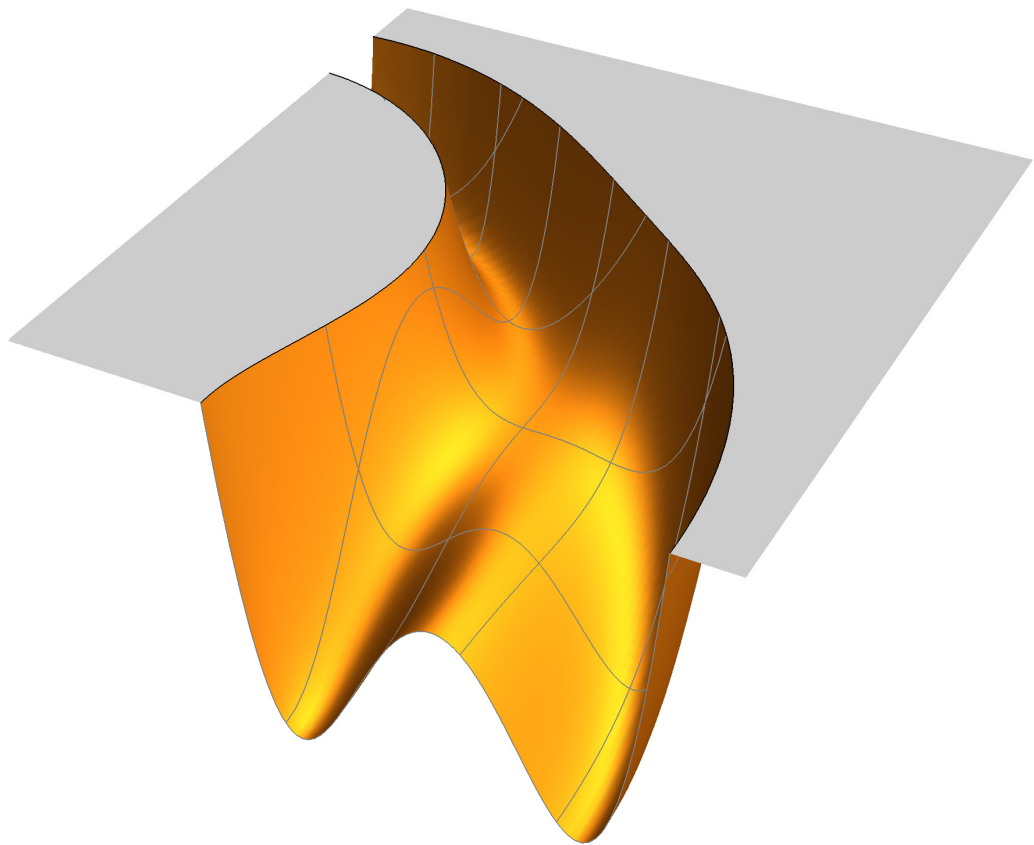
# Cosmological collider

— probing signals of massive fields during inflation

**Steps towards new discovery:** [Chen, Wang & Xianyu \(2016,2017a,b\)](#)

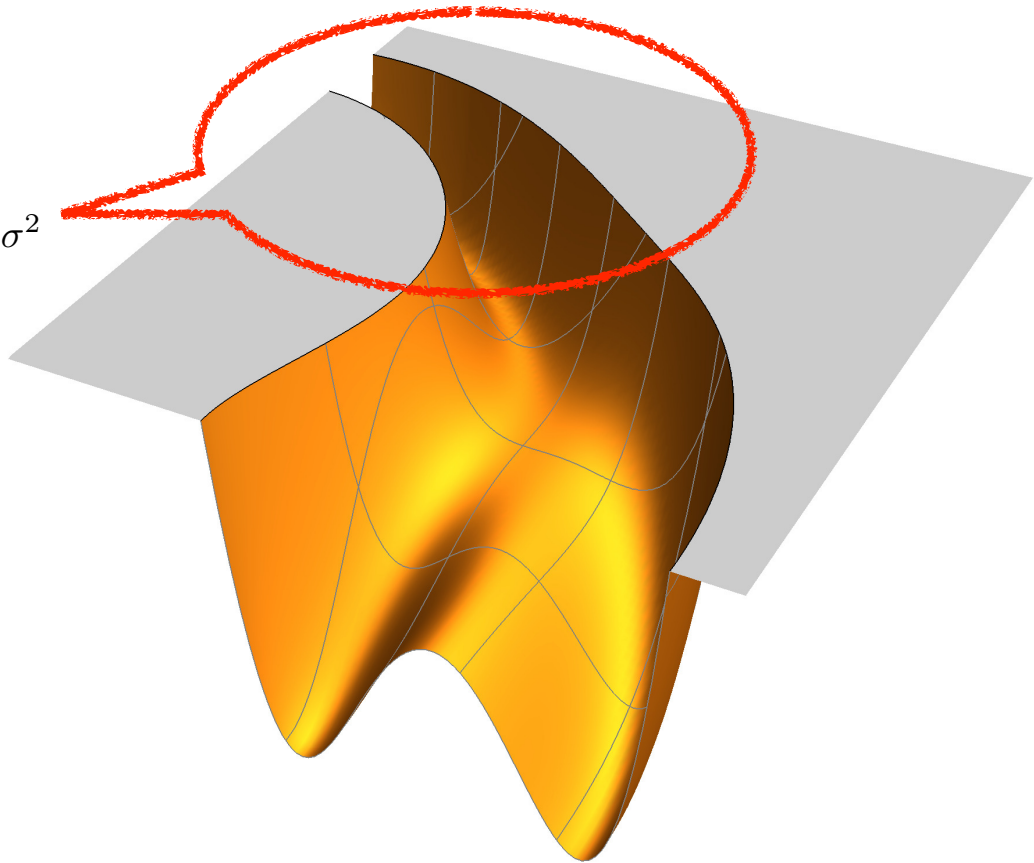
1. To work out the background signals during inflation.
- ✓ 2. To figure out how new particles enter the bispectrum.

# Inflation with phase transitions



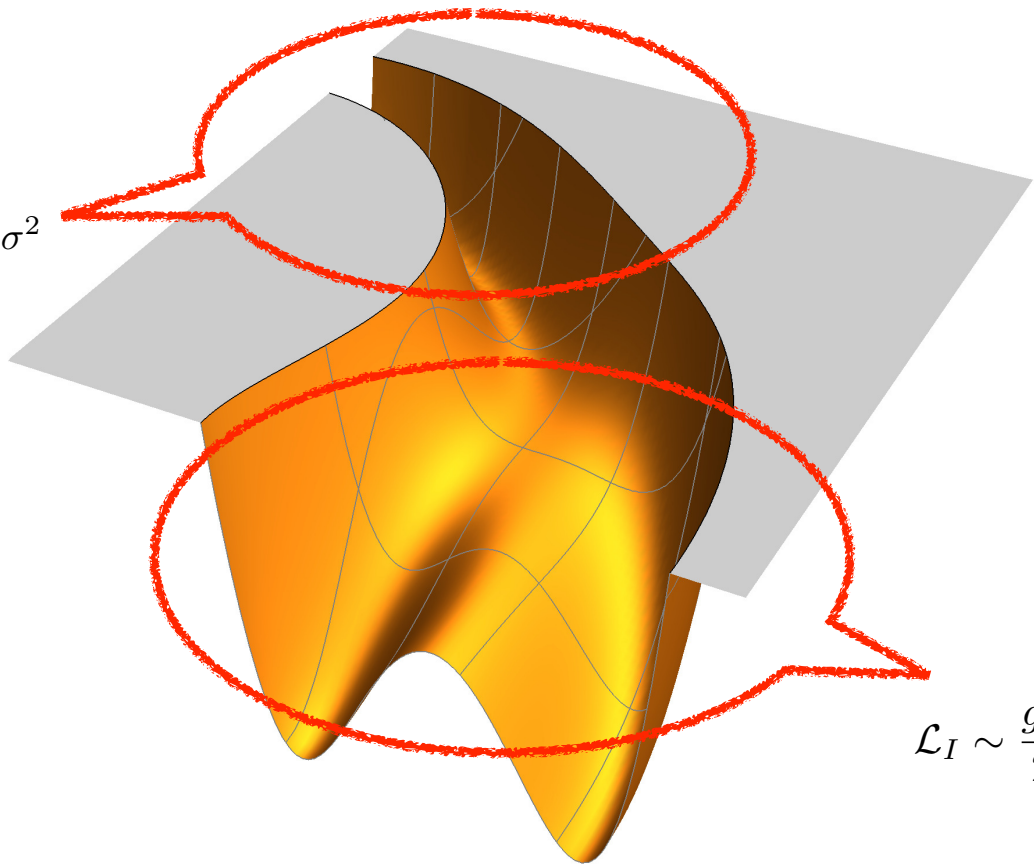
# Inflation with phase transitions

$$\mathcal{L}_I \sim \frac{\mu}{R^2} (\partial\phi)^2 \sigma^2$$



# Inflation with phase transitions

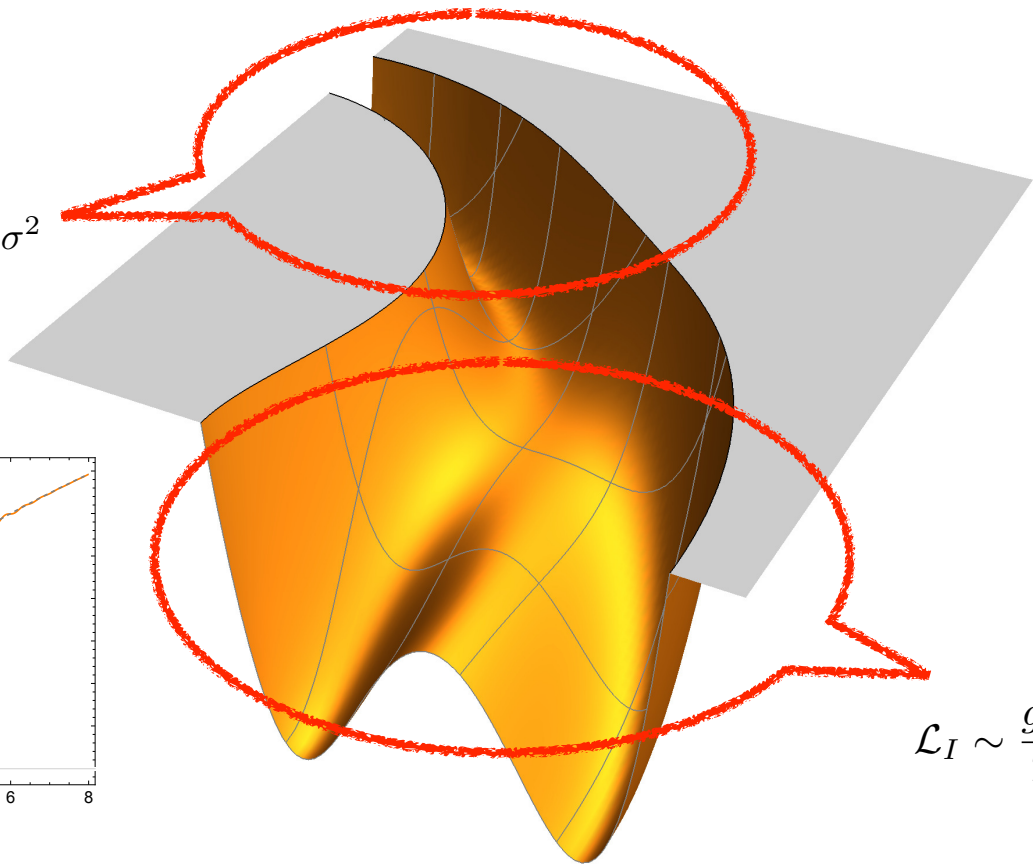
$$\mathcal{L}_I \sim \frac{\mu}{R^2} (\partial\phi)^2 \sigma^2$$



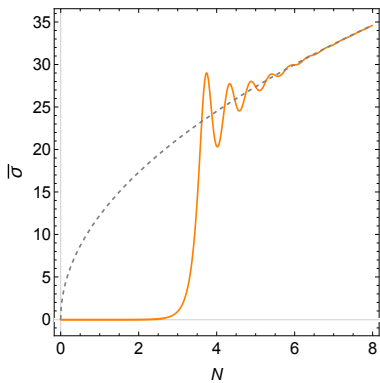
$$\mathcal{L}_I \sim \frac{g^2}{2} \phi^2 \sigma^2$$

# Inflation with phase transitions

$$\mathcal{L}_I \sim \frac{\mu}{R^2} (\partial\phi)^2 \sigma^2$$



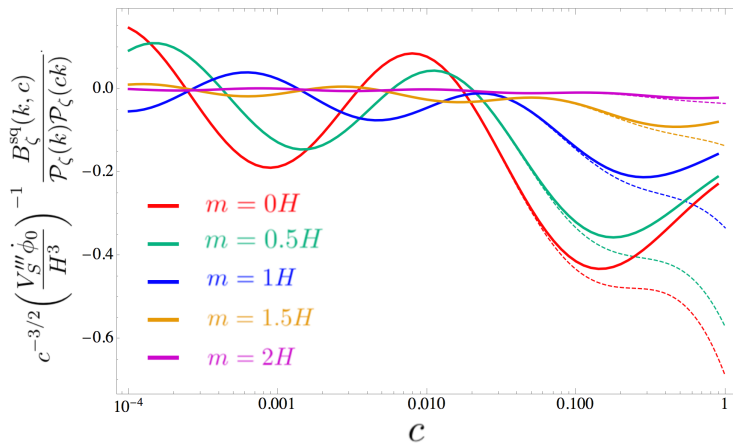
$$\mathcal{L}_I \sim \frac{g^2}{2} \phi^2 \sigma^2$$



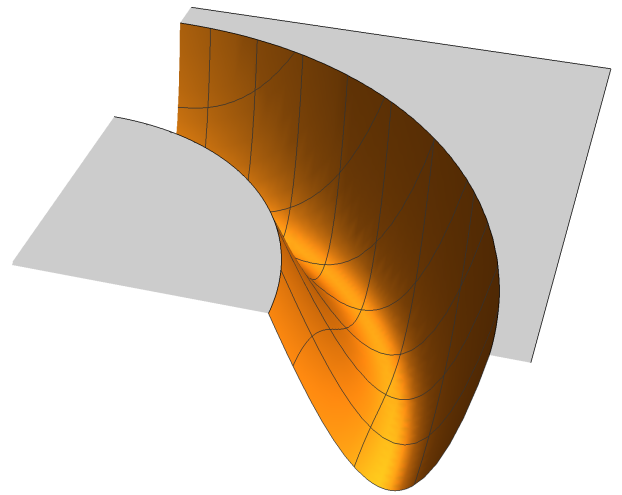
# Inflation with a turn

$$\mathcal{L}_I \sim \frac{\mu}{R^2} (\partial\phi)^2 \sigma^2$$

► Signals of massive fields in squeezed bi-spectrum



Chen & Wang (2009)



An, McAneny, Ridgway & Wise [1706.09971]

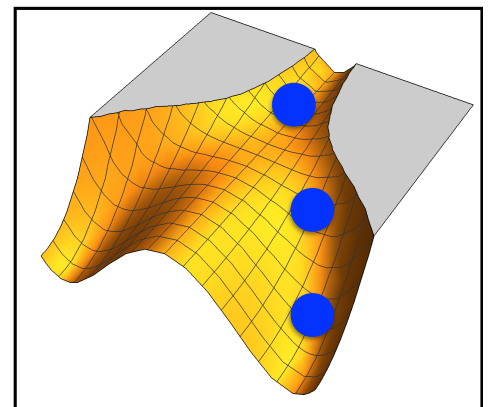
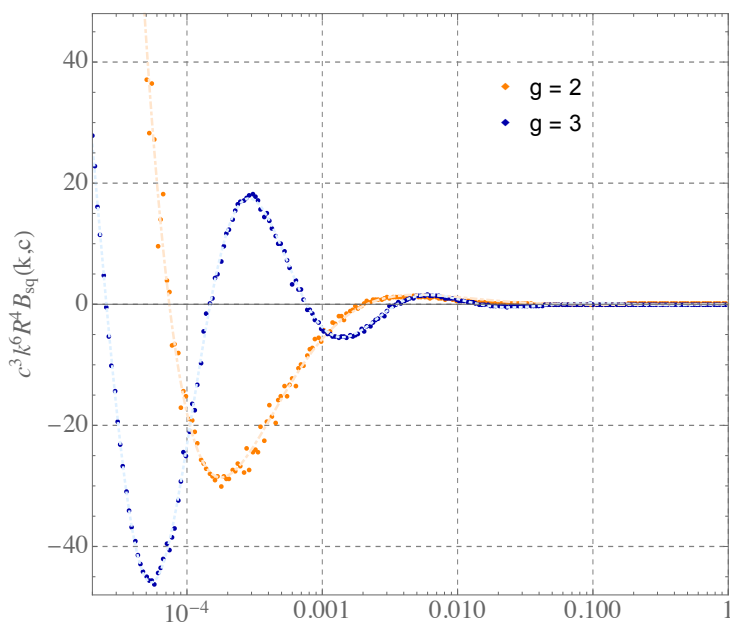


# Inflation with a waterfall

$$\mathcal{L}_I \sim \frac{g^2}{2} \phi^2 \sigma^2$$

Linde (1993)

Gong & Sasaki (2010)



Wang, YPW, Yokoyama & Zhou [in preparation]

squeezed  $\longleftrightarrow$  equilateral  
 $c = k/k_c$

## - Methods -

- **Cosmological in-in formalism:**

- Perturbative interactions (gravitational or derivative couplings)
- Standard initial states (the Bunch-Davies vacuum)

- **Effective field theory (equation-of-motion approach):**

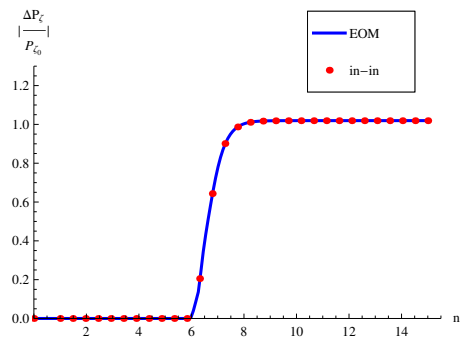
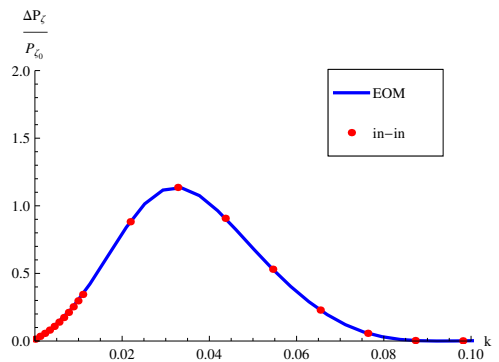
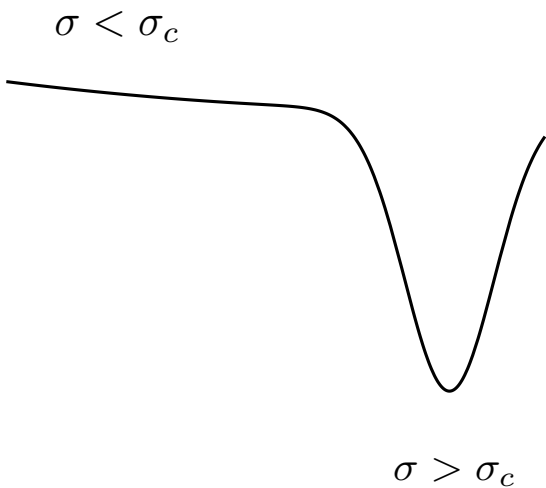
- Non-perturbative regime
- Mixed initial vacuum states

# Phase transition triggered by slow-rolling

Chen, Namjoo & Wang [1505.03955]

A modified potential:

$$V(\phi, \sigma) = V_{\text{slow-roll}}(\phi) + V_0 \left[ 1 - e^{-\sigma^2/\sigma_c^2} \right] + \frac{1}{2} m_0^2 \sigma^2$$



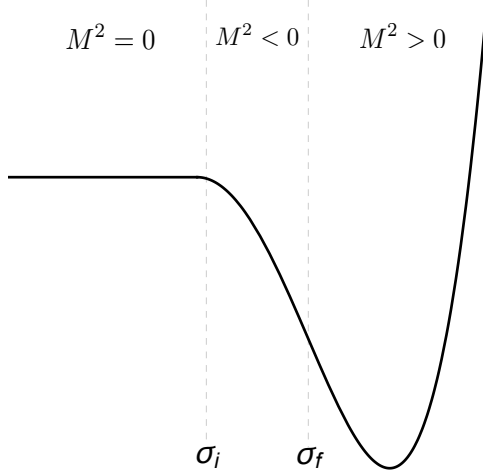
The two approaches matched at tree level.

# Phase transition triggered by quantum fluctuations

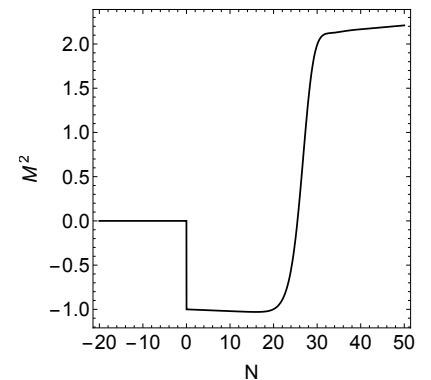
YPW & Yokoyama [1704.05026]

A step potential:

$$V_1(\sigma) = \begin{cases} \frac{\lambda}{4} v^4, & \sigma < 0, \\ \frac{\lambda}{4} (\sigma^2 - v^2)^2, & \sigma \geq 0, \end{cases}$$



$$M^2 = \frac{1}{H^2} \frac{\partial^2 V(\sigma)}{\partial \sigma^2}$$



The critical value of classical evolution:  $\sigma_i \geq 3H_i^3 / (2\pi\lambda v^2)$

$$\Delta N = N_f - N_i = \frac{3}{M^2} \ln \left( \frac{2\pi}{3\sqrt{3}\lambda} M^3 \right) \quad \text{classical}$$

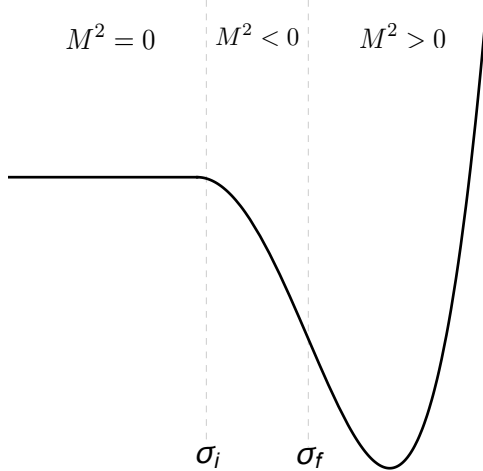
The duration of the growing phase:

$$\Delta N = \frac{3}{2M^2} \ln \left[ \frac{M^6(2 + 4\pi^2/\lambda)}{27 + 2M^6} \right] \quad \text{stochastic}$$

# Phase transition triggered by quantum fluctuations

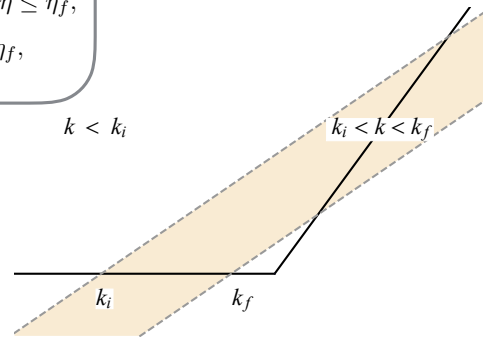
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$$u_k'' + \left( k^2 - \frac{l^2 - 1/4}{\eta^2} \right) u_k = 0,$$

$$l = \begin{cases} 3/2, & \eta \leq \eta_i, \\ \sqrt{9/4 + M_-^2}, & \eta_i < \eta \leq \eta_f, \\ \sqrt{9/4 - M_+^2}, & \eta > \eta_f, \end{cases}$$



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classical

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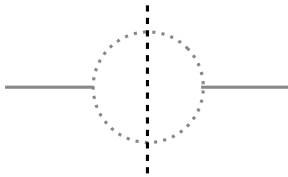
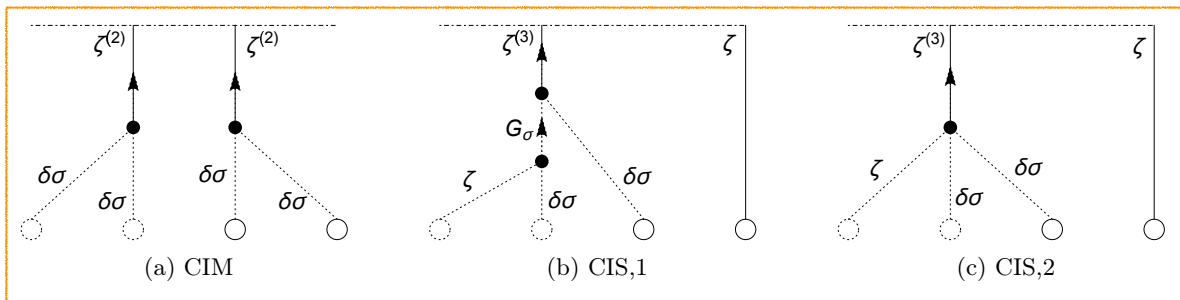
stochastic

One-loop channels:

$$\langle \zeta^2 \rangle_{\text{CIM}} = - \int^\eta d\eta_1 \int^\eta d\tilde{\eta}_1 \left\langle \left[ H_I^{(3)}(\eta_1), \zeta(\eta) \right] \left( \left[ H_I^{(3)}(\tilde{\eta}_1), \zeta(\eta) \right] \right)^\dagger \right\rangle,$$

$$\langle \zeta^2 \rangle_{\text{CIS},1} = -2 \operatorname{Re} \left[ \int^\eta d\eta_2 \int_2^\eta d\eta_1 \left\langle \left[ H_I^{(3)}(\eta_1), \left[ H_I^{(3)}(\eta_2), \zeta(\eta) \right] \right] \zeta(\eta) \right\rangle \right],$$

$$\langle \zeta^2 \rangle_{\text{CIS},2} = -2 \operatorname{Im} \left[ \int^\eta d\eta_1 \left\langle \left[ H_I^{(4)}(\eta_1), \zeta(\eta) \right] \zeta(\eta) \right\rangle \right].$$



$$M^2 < 0$$

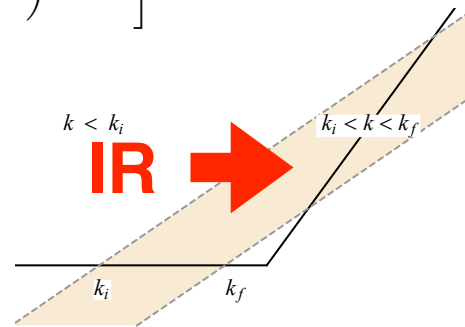
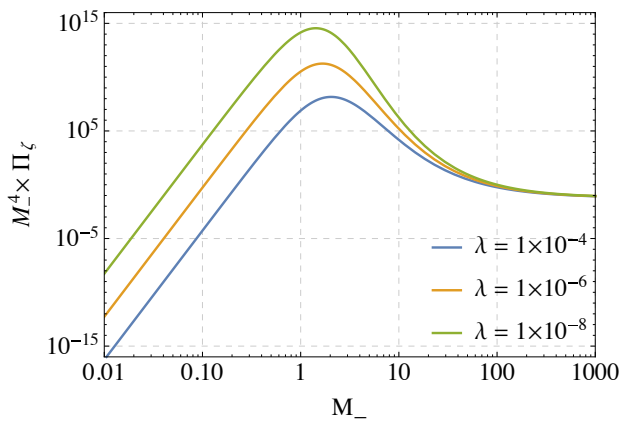
➤ The dominant decay channel changed!

## Results of bilinear correlators

Loop corrections from phase transitions:

$$\langle \zeta^2 \rangle \sim \frac{H^2}{\epsilon M_p^2} \left[ 1 + c_* \frac{H^2}{\epsilon M_p^2} \epsilon^2 M^4 \Pi_\zeta(a) + \left( c_* \frac{H^2}{\epsilon M_p^2} \epsilon^2 M^4 \Pi_\zeta(a) \right)^2 + \dots \right]$$

YPW & Yokoyama [1704.05026]



Loop corrections from spectator fields may be more important than the previous conclusion!

# Messages (for the moment)

- The transition of vacuum expectation values (**vevs**) of scalar fields play a fundamental role in all inflation scenarios.
- Primordial signals of massive fields in the cosmological collider are enhanced by a waterfall phase transition.
- Loop corrections from spectator fields are never large, if they always stay in one stable vacuum during inflation.
- IR loop corrections are enhanced by phase transition with a growing **vev** (a tachyonic phase).