

Higgs- R^2 Inflation

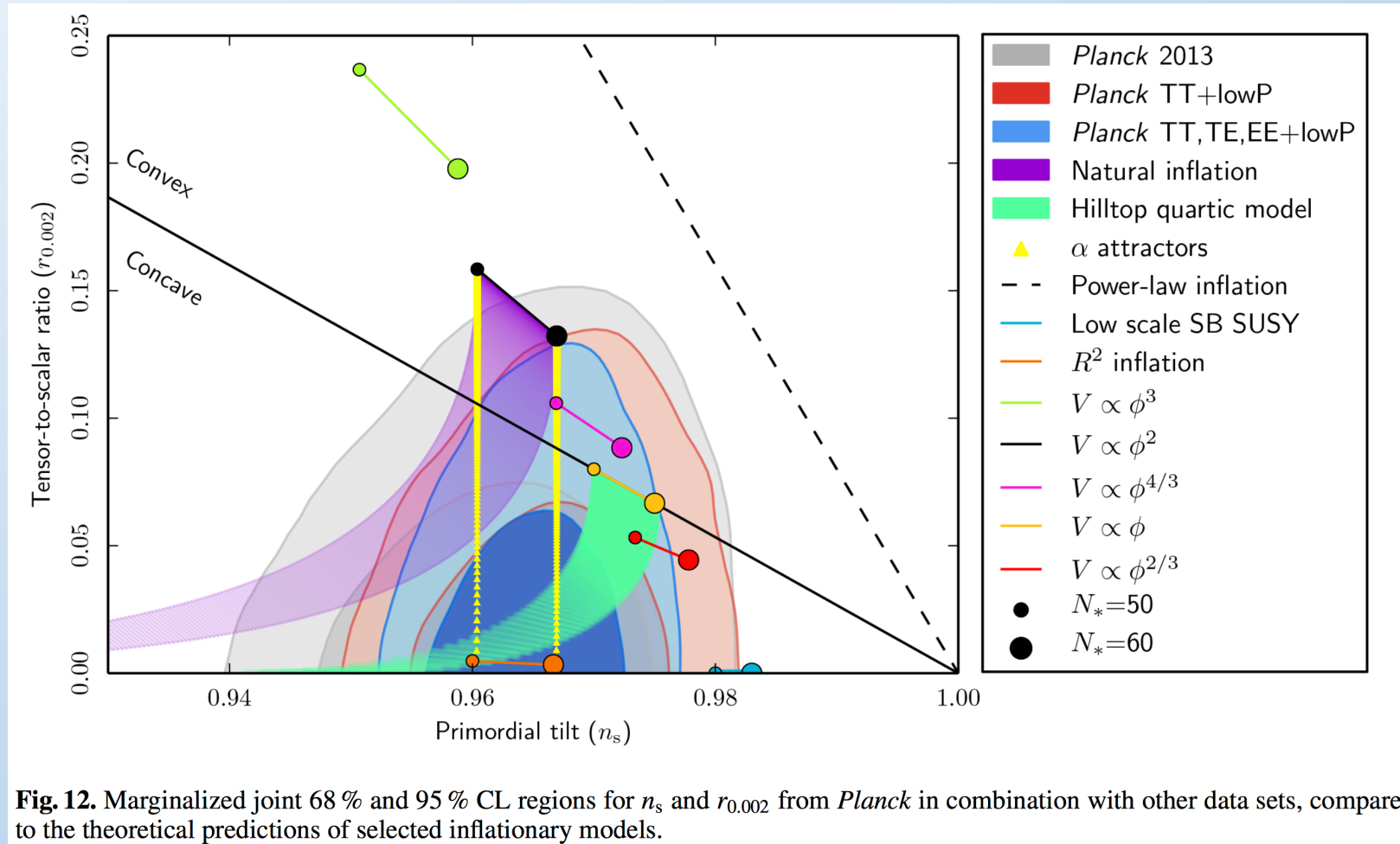
SPEAKER: Minxi He (RESCEU, UTOKYO)
(mean c her)

COLLABORATOR: Jun'ichi Yokoyama

OUTLINE

- MOTIVATION
- MODEL AND FORMALISM
- COMPARISON WITH EXPERIMENTS
- FUTURE WORK

MOTIVATION



MODEL AND FORMALISM

R^2 inflation + Higgs inflation

A. A. Starobinsky, Phys. Lett. B 91, 99 (1980)

F. L. Bezrukov, M. E. Shaposhnikov, Phys.Lett.B659:703-706,2008

$$S = \int d^4x \sqrt{-\hat{g}} \left[\frac{M_p^2}{2} \hat{R} + \frac{M_p^2}{12M^2} \hat{R}^2 + \frac{1}{2} \xi \chi^2 \hat{R} - \frac{1}{2} \hat{g}^{\mu\nu} \hat{\nabla}_\mu \chi \hat{\nabla}_\nu \chi - \frac{\lambda}{4} \chi^4 \right]$$

R^2 term

Non-minimal coupling

Higgs potential

$$F(\chi, R) \equiv \frac{M_p^2}{2} \hat{R} + \frac{1}{2} \xi \chi^2 \hat{R} + \frac{M_p^2}{12M^2} \hat{R}^2 - \frac{\lambda}{4} \chi^4$$

Y. Ema, arXiv: 1701.07665[hep-ph]

Y-C. Wang, T. Wang, arXiv: 1701.06636v2[gr-qc]

MODEL AND FORMALISM

Define a new field

$$\sqrt{\frac{2}{3}} \frac{\psi}{M_p} \equiv \ln \left(\frac{2}{M_p^2} \left| \frac{\partial F}{\partial R} \right| \right)$$

K. Maeda, Phys. Rev. D **39**, 3159

Conformal transformation
from Jordan frame to Einstein
frame

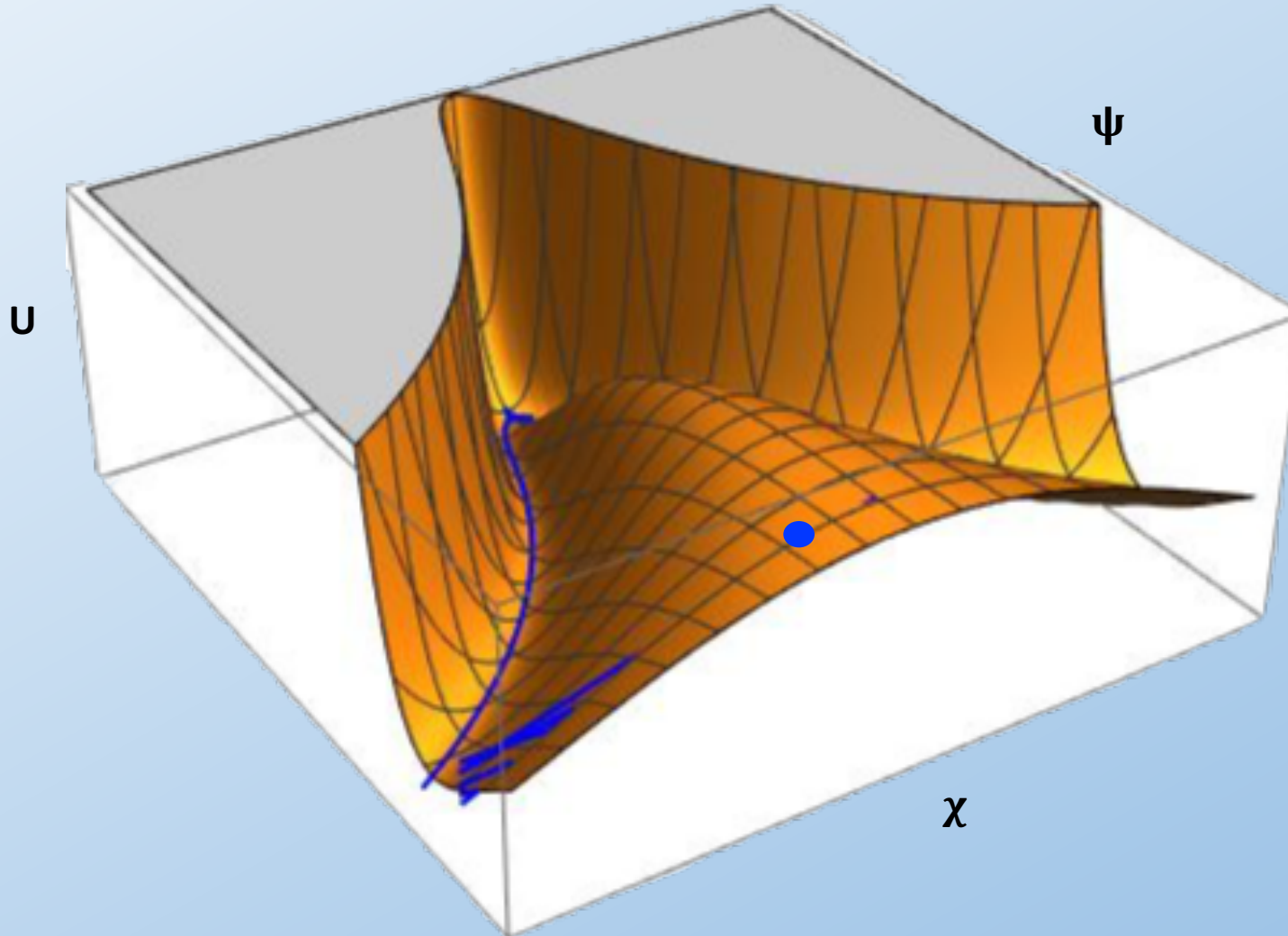
$$g_{\mu\nu}(x) = e^{\sqrt{\frac{2}{3}} \frac{\psi(x)}{M_p}} \hat{g}_{\mu\nu}(x)$$

$$S = \int d^4x \sqrt{-g} \left[\frac{M_p^2}{2} R - \frac{1}{2} g^{\mu\nu} \nabla_\mu \psi \nabla_\nu \psi - \frac{1}{2} e^{-\sqrt{\frac{2}{3}} \frac{\psi}{M_p}} g^{\mu\nu} \nabla_\mu \chi \nabla_\nu \chi - U(\psi, \chi) \right]$$

$$U(\psi, \chi) \equiv \frac{\lambda}{4} \chi^4 e^{-2\sqrt{\frac{2}{3}} \frac{\psi}{M_p}} + \frac{3}{4} M_p^2 M^2 e^{-2\sqrt{\frac{2}{3}} \frac{\psi}{M_p}} \left(e^{\sqrt{\frac{2}{3}} \frac{\psi}{M_p}} - 1 - \frac{1}{M_p^2} \xi \chi^2 \right)^2$$

MODEL AND FORMALISM

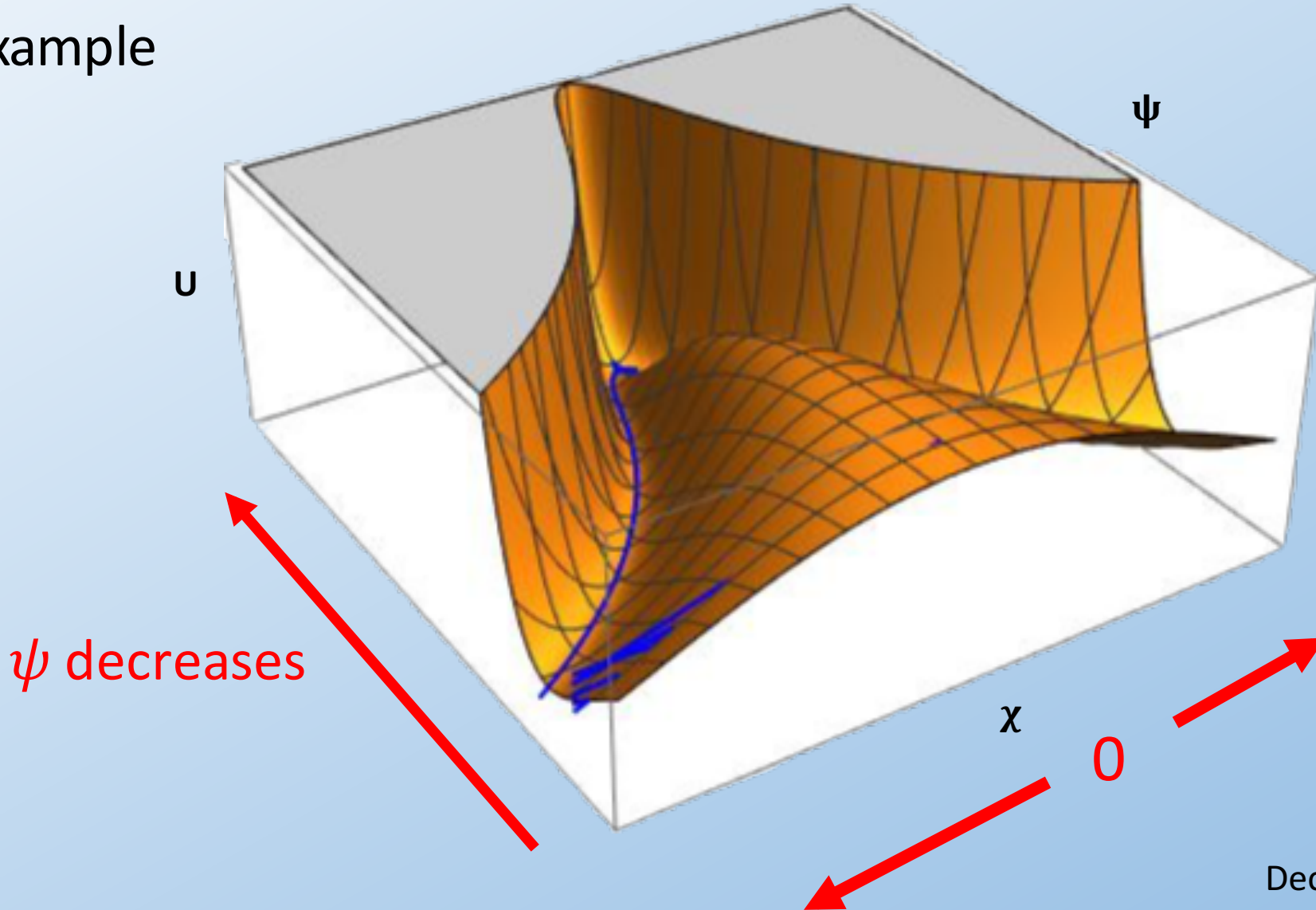
An example



$$\lambda = 0.01$$
$$\xi = 1000$$
$$M = 10^{-5}$$

MODEL AND FORMALISM

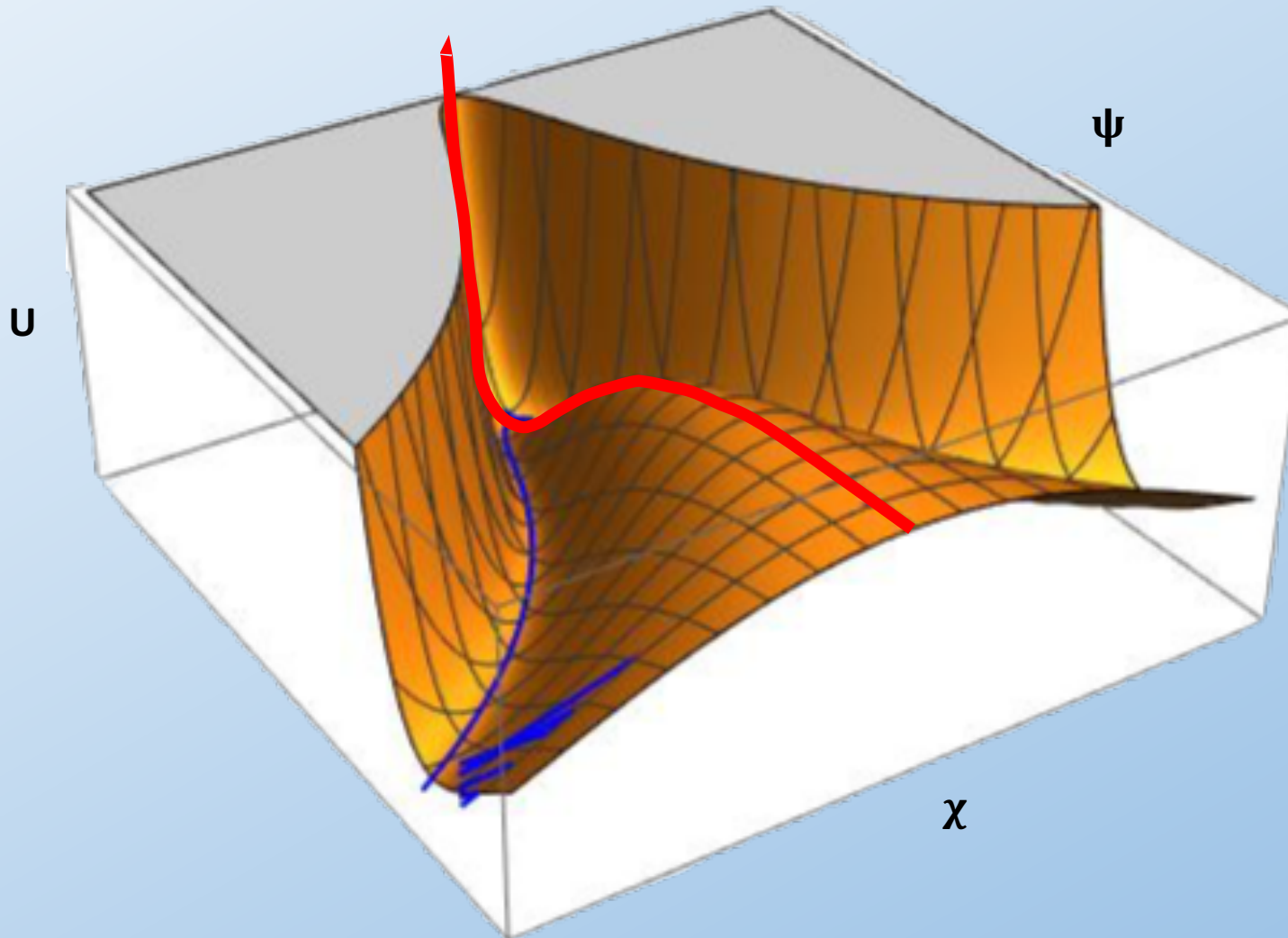
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MODEL AND FORMALISM

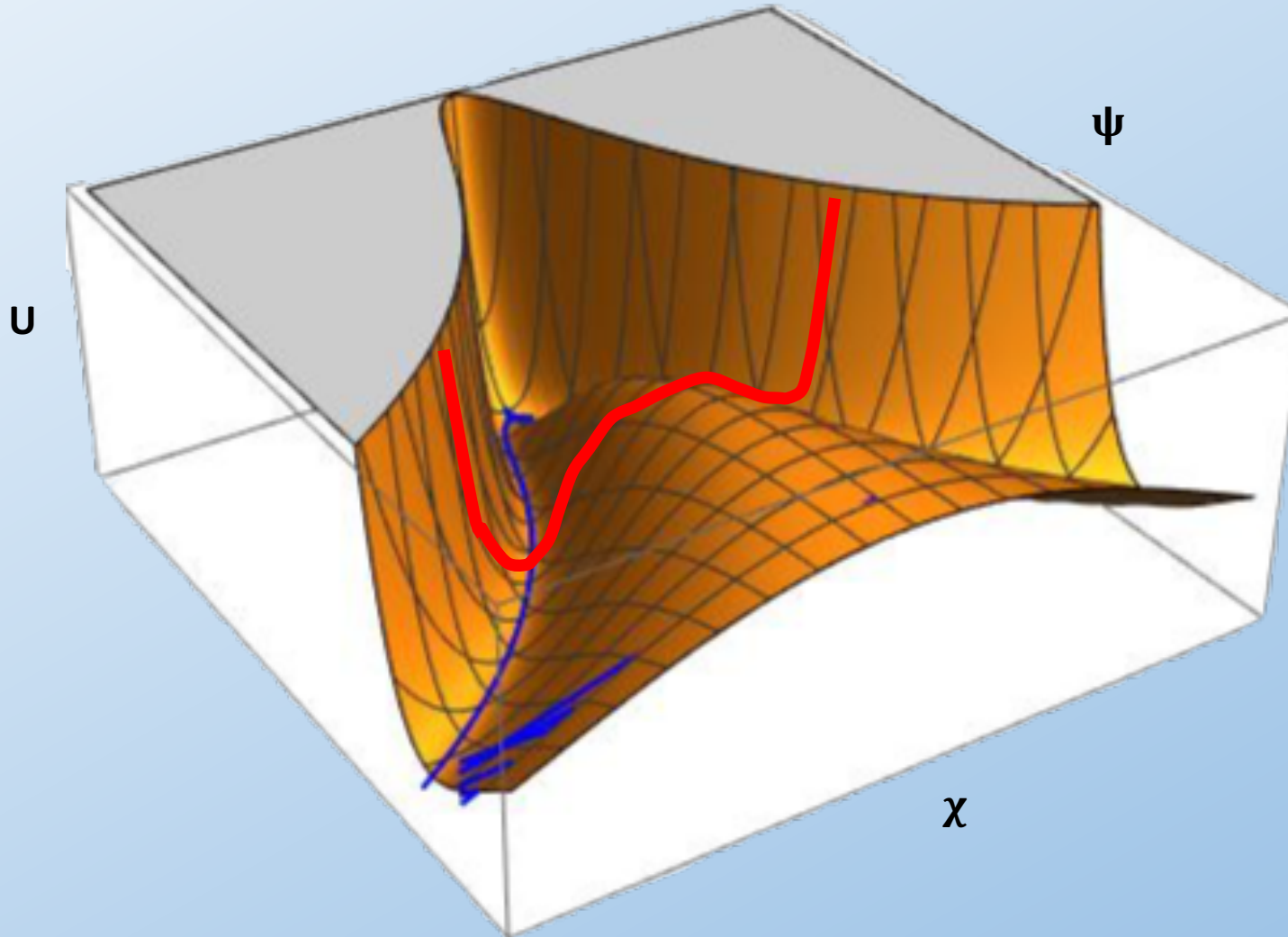
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MODEL AND FORMALISM

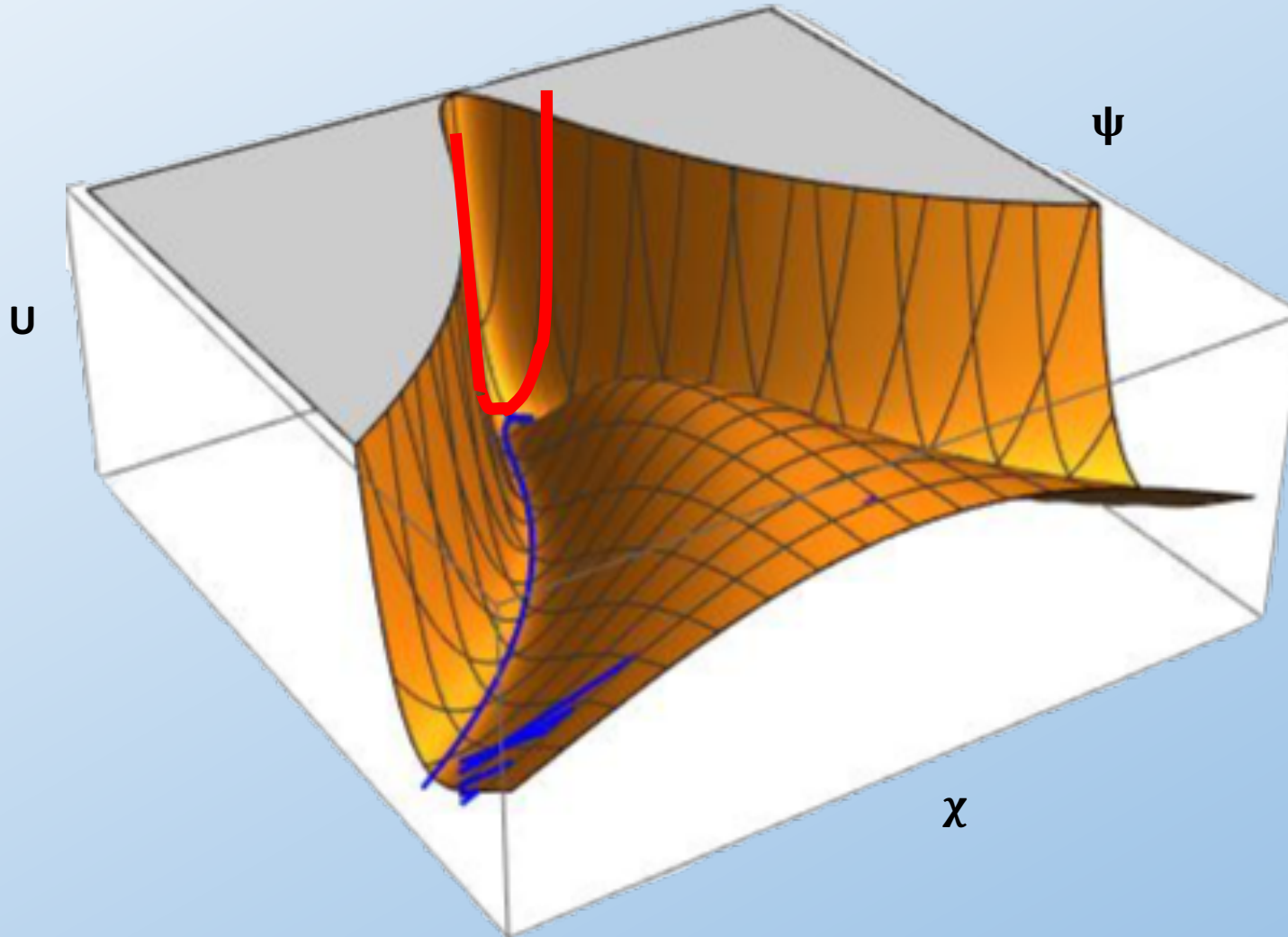
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MODEL AND FORMALISM

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MODEL AND FORMALISM

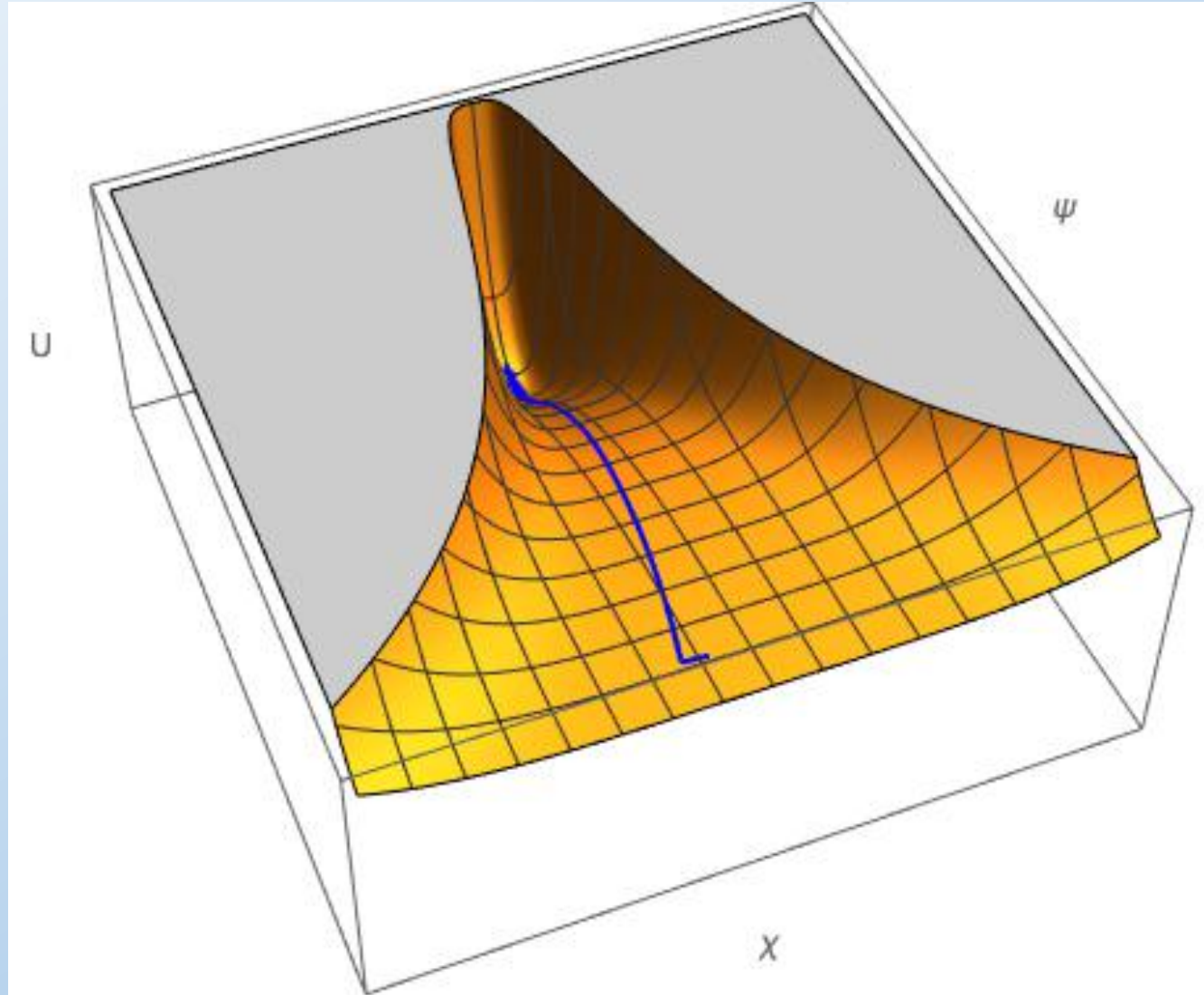
Pure Higgs

$$F(\chi, R) \equiv \frac{M_p^2}{2} \hat{R} + \frac{1}{2} \xi \chi^2 \hat{R} + \frac{M_p^2}{12M^2} \hat{R}^2 - \frac{\lambda}{4} \chi^4$$

$$\sqrt{\frac{2}{3}} \frac{\psi}{M_p} \equiv \ln \left(\frac{2}{M_p^2} \left| \frac{\partial F}{\partial R} \right| \right)$$

MODEL AND FORMALISM

An example



$$\lambda = 0.01$$
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MODEL AND FORMALISM

Rewrite the action in a more compact form as

$$S = \int d^4x \sqrt{-g} \left[\frac{M_p^2}{2} R - \frac{1}{2} h_{ab} g^{\mu\nu} \partial_\mu \phi^a \partial_\nu \phi^b - U(\phi) \right]$$

where $a, b = 1, 2$, $\phi^1 = \psi$, $\phi^2 = \chi$

$$h_{11} = 1, \quad h_{22} = e^{-\sqrt{\frac{2}{3}} \frac{\psi}{M_p}}, \quad h_{12} = h_{21} = 0$$

We will set $M_p = 1$ below.

MODEL AND FORMALISM

Equations of motion for $\phi(\mathbf{x}, t) = \phi_0(t) + \delta\phi(\mathbf{x}, t)$

$$\frac{D\dot{\phi}_0^a}{dt} + 3H\dot{\phi}_0^a + h^{ab}U_{,b} = 0$$

$$\text{where } \frac{D}{dt} \equiv \dot{\phi}_0^a \nabla_a$$

$$\frac{D^2\delta\phi_{\mathbf{k}}^a}{dt^2} + 3H\frac{D\delta\phi_{\mathbf{k}}^a}{dt} - R^a{}_{bcd}\dot{\phi}_0^b\dot{\phi}_0^c\delta\phi_{\mathbf{k}}^d + \frac{k^2}{a^2}\delta\phi_{\mathbf{k}}^a + U^{;a}{}_{;b}\delta\phi_{\mathbf{k}}^b = \frac{1}{a^3}\frac{D}{dt}\left(\frac{a^3}{H}\dot{\phi}_0^a\dot{\phi}_0^b\right)h_{bc}\delta\phi_{\mathbf{k}}^c$$

M. Sasaki, E. Stewart, Prog.Theor.Phys.95:71-78,1996

- Geodesic equation of the field space within an expanding universe with potential $U(\psi, \chi)$
- Equations of geodesic deviation.

A. Achúcarro et al, Phys.Rev.D84:043502,2011

MODEL AND FORMALISM

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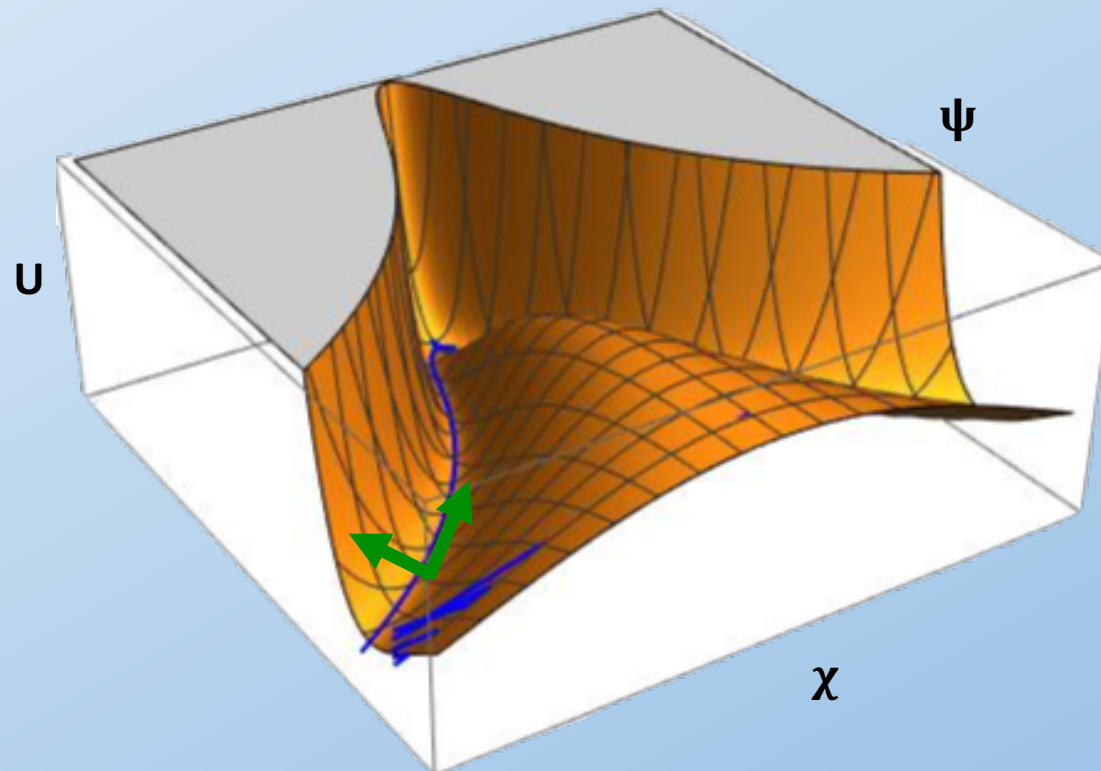
M. Sasaki, E. Stewart, Prog.Theor.Phys.95:71-78,1996

- Geodesic equation of the field space within an expanding universe with potential $U(\psi, \chi)$
- Equations of geodesic deviation. \longrightarrow
 - Not rolling in the local minimum
 - Not rolling along geodesics

A. Achúcarro et al, Phys.Rev.D84:043502,2011

MODEL AND FORMALISM

- Mass hierarchy, slow-roll regime
- Decomposition into two directions, T^a and $-\dot{\theta}N^a \equiv D_t T^a$.
(Also new slow-roll parameters.)



MODEL AND FORMALISM

$$\mathcal{R} \propto T_a \delta\phi^a$$

$$\mathcal{F} \propto N_a \delta\phi^a$$

$$S_2 = \frac{1}{2} \int a^3 \left[\frac{\dot{\phi}_0^2}{H^2} \dot{\mathcal{R}}^2 - \frac{\dot{\phi}_0^2}{H^2} \frac{(\nabla \mathcal{R})^2}{a^2} + \dot{\mathcal{F}}^2 - \frac{(\nabla \mathcal{F})^2}{a^2} - \underline{M_{\text{eff}}^2} \mathcal{F}^2 - 4\dot{\theta} \frac{\dot{\phi}_0^2}{H} \dot{\mathcal{R}} \mathcal{F} \right]$$

Effective mass including $\dot{\theta}$ & U_{NN} which is much larger than the H^2 .

- Integrating out the high energy part
- Slow-roll regime where the heavy direction is determined by the light direction

$$S_{\text{eff}} = \frac{1}{2} \int a^3 \frac{\dot{\phi}_0^2}{H^2} \left[\frac{\dot{\mathcal{R}}^2}{\underline{c_s^2(k)}} - \frac{k^2 \mathcal{R}^2}{a^2} \right]$$

MODEL AND FORMALISM

Mukhanov-Sasaki equation

$$v_k'' + (\underline{c_s^2} k^2 - \frac{z''}{z}) v_k = 0$$

Modified speed of sound
but still close to 1 during
slow-roll regime

$$c_s^{-2} = 1 + \frac{4\dot{\theta}^2}{\frac{k^2}{a^2} + U_{NN} + \epsilon H^2 R - \dot{\theta}^2}$$

Mode function

$$v_k = \frac{e^{-ic_s k \tau}}{\sqrt{2c_s k}} \left(1 - \frac{i}{c_s k \tau}\right)$$

Power spectrum at
large scale

$$\mathcal{P}_{\mathcal{R}}(k) \approx \frac{H^2}{4c_s k^3} \frac{1}{\epsilon}$$

ϵ has contribution
from both fields

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{\dot{\phi}_0^2}{2H^2}$$

MODEL AND FORMALISM

Scalar index	$n_s - 1 = 2\underline{\eta_{ }} - 4\epsilon$	Second slow-roll parameter in the tangent direction
Tensor-to-scalar ratio	$r = 16\epsilon\underline{c_s}$	Correction from speed of sound

COMPARISON WITH EXPERIMENTS

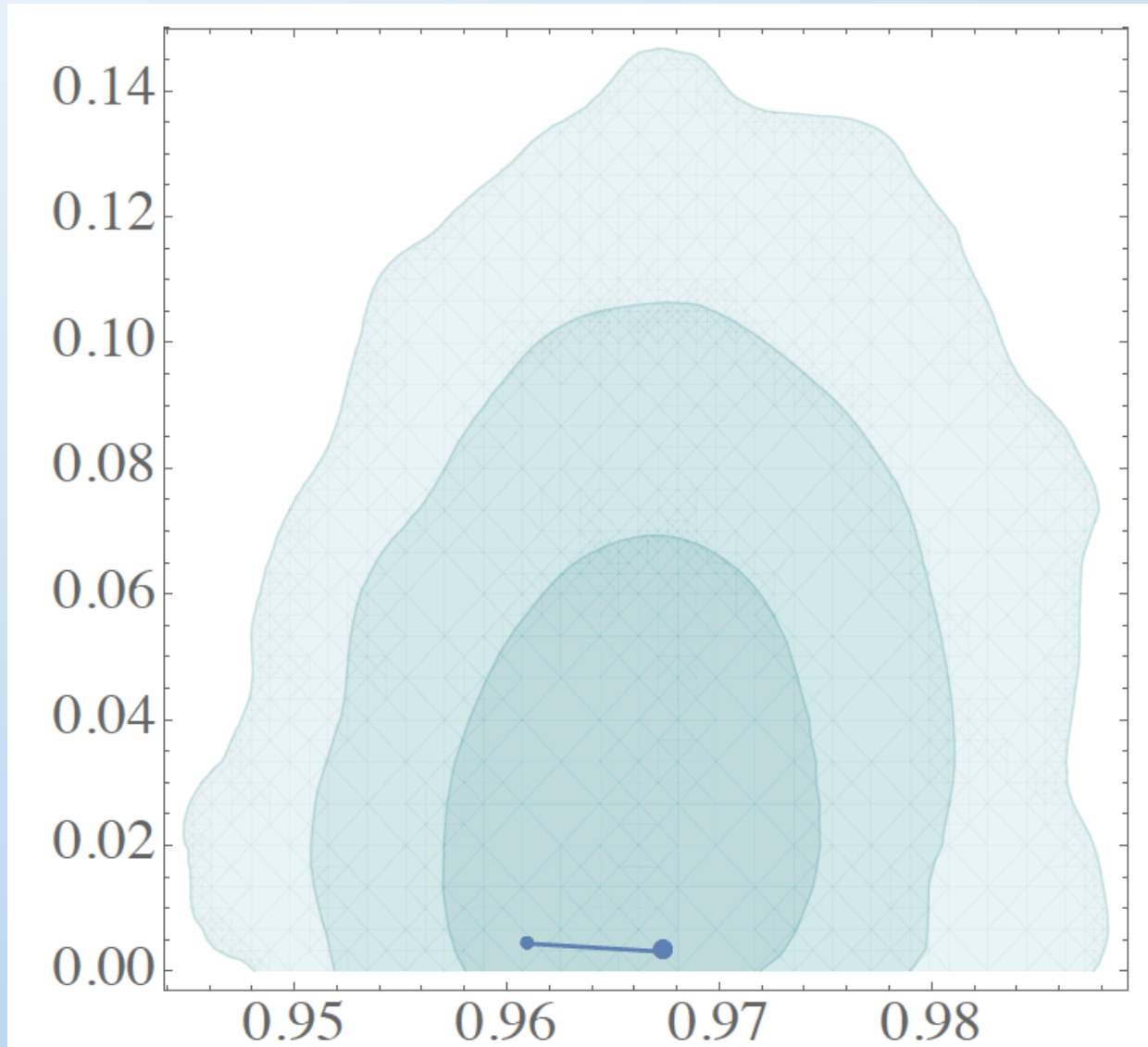
Fix $\lambda = 0.01$; (Then we have two free parameters, ξ and M .)

$$\psi_0 = 5.7; \chi_0 = 0.01; \psi'_0 = 0; \chi'_0 = 0;$$

$$c_s \approx 1;$$

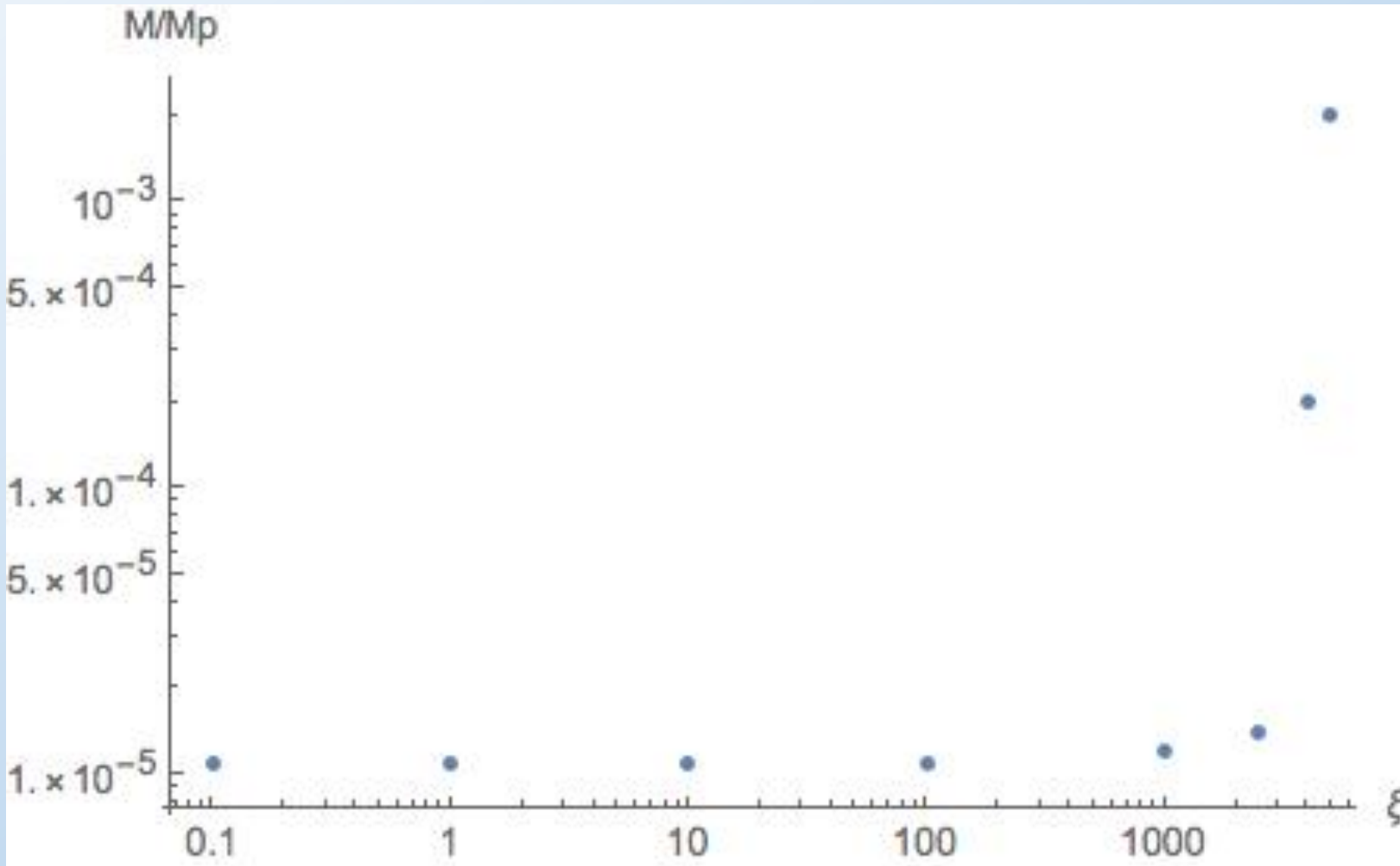
Requiring the amplitude of curvature perturbations to be 2×10^{-9} .

COMPARISON WITH EXPERIMENTS



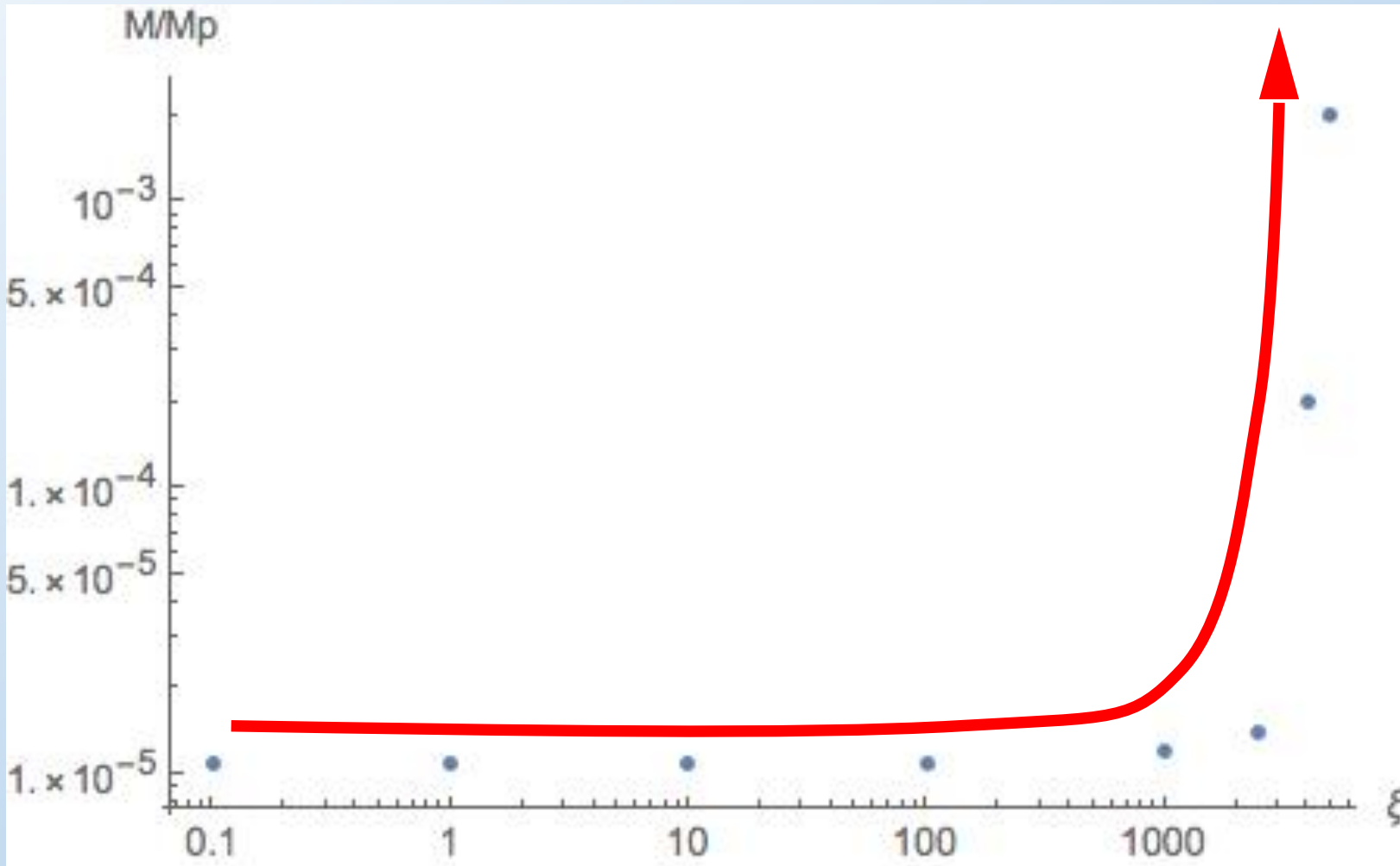
- Confined amplitude

COMPARISON WITH EXPERIMENTS



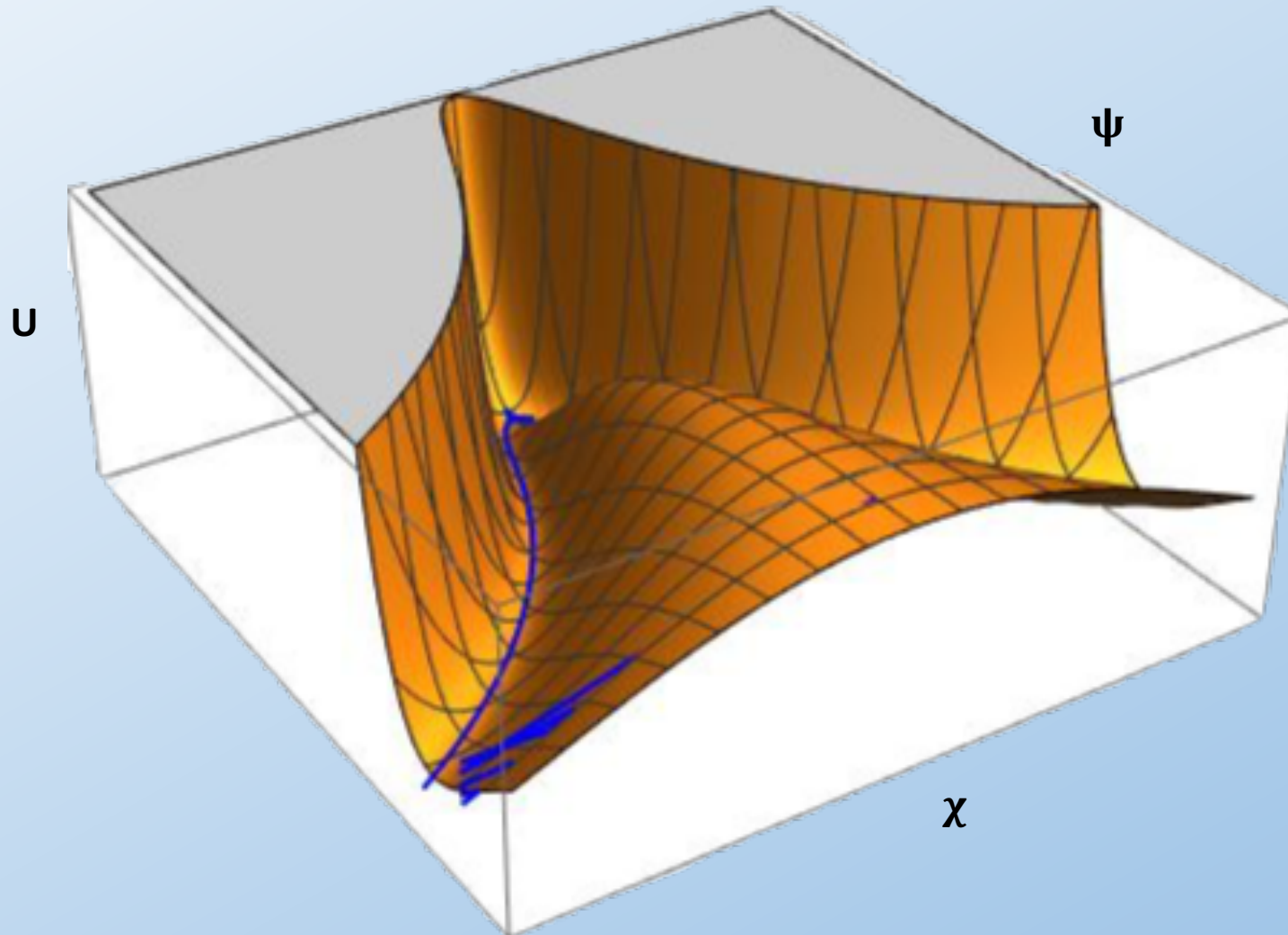
- Relation between the two parameters given by confined amplitude

COMPARISON WITH EXPERIMENTS



- Relation between the two parameters given by confined amplitude

COMPARISON WITH EXPERIMENTS



- ξ & effective mass of Higgs field & shape of potential

COMPARISON WITH EXPERIMENTS

- One possible way to understand this: Large ψ suppresses those terms with higher order of $e^{-\sqrt{\frac{2}{3}}\psi}$

Approximately $H \propto U(\psi, \chi) \approx \frac{3}{4} M^2 (\underbrace{1}_{\text{red}} - \underbrace{\xi \chi^2 e^{-\sqrt{\frac{2}{3}}\psi}}_{\text{green}})^2$

- Competition between **unity** and **the second term**

COMPARISON WITH EXPERIMENTS

- Another possible way to understand this: thanks to Prof. Starobinsky's useful comment

$$S = \int d^4x \sqrt{-\hat{g}} \left[\frac{M_p^2}{2} \hat{R} + \frac{M_p^2}{12M^2} \hat{R}^2 + \frac{1}{2} \xi \chi^2 \hat{R} - \frac{1}{2} \hat{g}^{\mu\nu} \hat{\nabla}_\mu \chi \hat{\nabla}_\nu \chi - \frac{\lambda}{4} \chi^4 \right]$$

- Ricci scalar R , second derivatives of the metric
- Derivatives on χ through integration by parts
- Extra contributions to the kinetic term of χ , much larger than the original one for large χ
- Non-dynamical Higgs field, constraint on χ and R

→
$$S = \int d^4x \sqrt{-\hat{g}} \left[\frac{M_p^2}{2} \hat{R} + \left(\frac{M_p^2}{12M^2} + \frac{\xi^2}{4\lambda} \right) \hat{R}^2 \right]$$

COMPARISON WITH EXPERIMENTS

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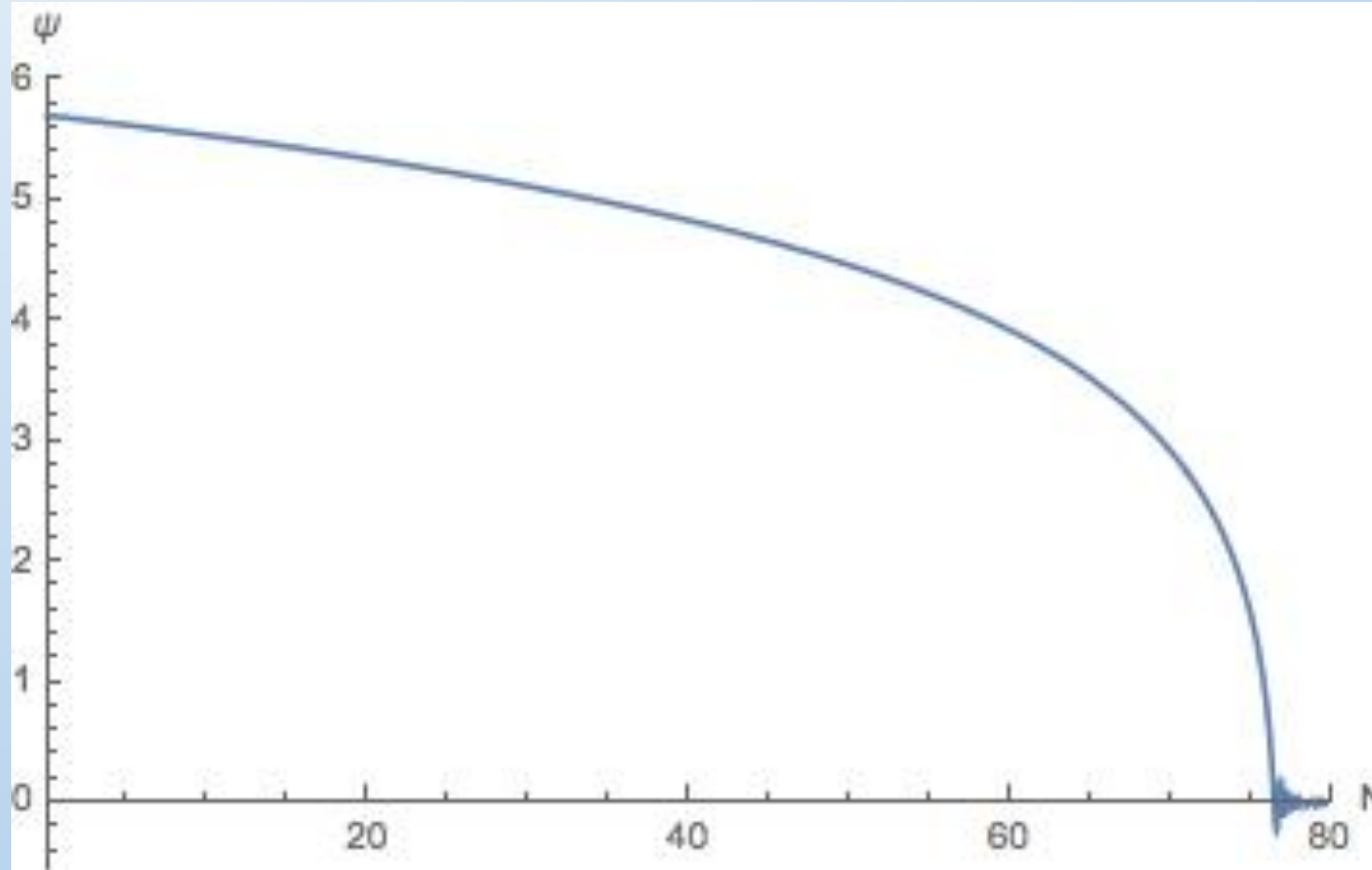
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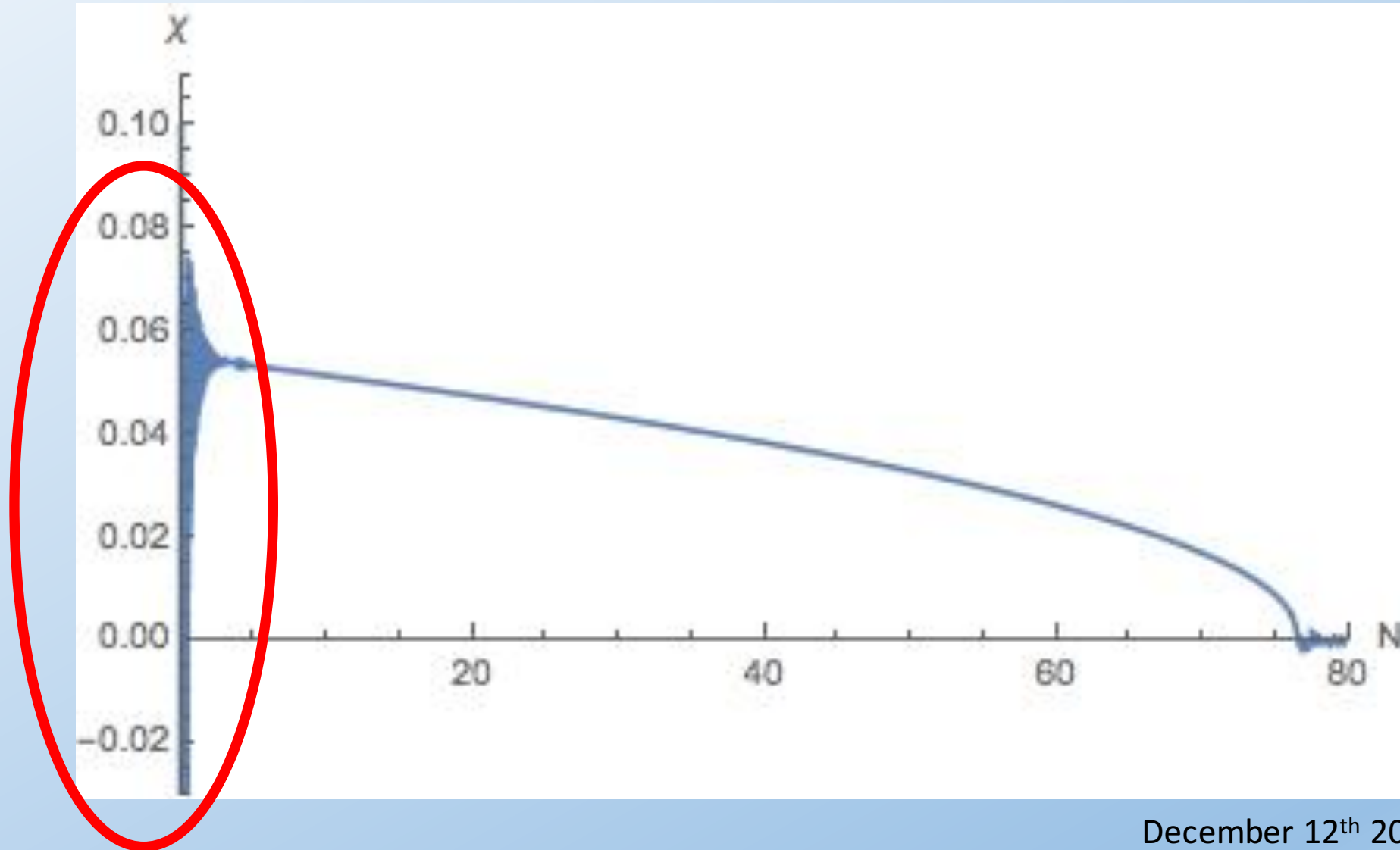
→
$$M_{eff}^2 \equiv \frac{M^2}{1 + \frac{3\xi^2 M^2}{\lambda M_p^2}}$$

COMPARISON WITH EXPERIMENTS

e.g. $\xi = 1000$; $M = 10^{-5}$.



COMPARISON WITH EXPERIMENTS



FUTURE WORK

- This work is on going. Mass hierarchy is considered here so that one of the fields dominates the inflation. The final goal is to consider the regime where both fields are of same importance to see whether there are more interesting features appear on power spectrum and bispectrum, isocurvature perturbations, etc.
- Find out different behaviors of the fields and predictions in different regions of parameter space, e.g. the correction from the speed of sound.
- Also it is worth considering the links with primordial black holes, reheating, etc.

Higgs- R^2 Inflation



THANK YOU FOR LISTENING!

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