Primordial Non-Gaussianities from non-attractor inflation Revisited: Gone Signals

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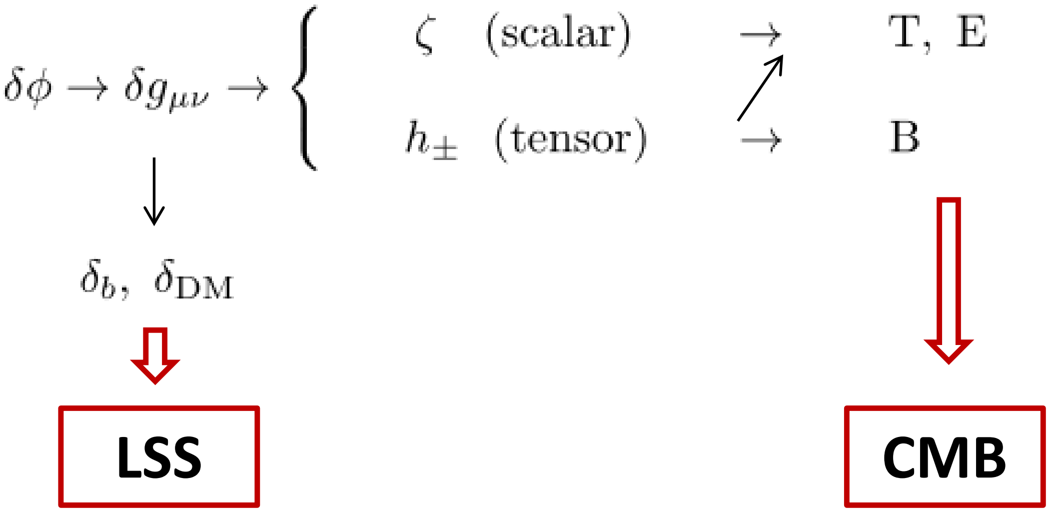
D. Wang, Z. Wang & ***CYF***, in preparation

Inflationary cosmology

- Solved the horizon, flatness, monopole problems

- Primordial fluctuations lead to Large-Scale Structure (LSS) and temperature anisotropies of Cosmic Microwave Background (CMB)



$$\begin{tikzcd}
 & & \zeta\ (\text{scalar}) \arrow[r] & \mathrm{T},\ \mathrm{E} \\
\delta\phi \arrow[r] & \delta g_{\mu\nu} \arrow[ru] \arrow[rd] \arrow[d] & & \\
 & \delta_b,\ \delta_{\mathrm{DM}} & h_\pm\ (\text{tensor}) \arrow[r] \arrow[ruu] & \mathrm{B}
\end{tikzcd}$$

LSS

CMB

- A nearly **scale-invariant** power spectrum of primordial curvature perturbations dominated by **constant mode** at super-Hubble scales of **Gaussian distribution**

Perturbation theory

- Quadratic action

$$S_2 = \int d^3x d\tau z^2[\zeta'^2 - (\partial_i\zeta)^2] \qquad\qquad z^2 = 2\epsilon a^2 = a^2\frac{\dot{\phi}^2}{H^2}$$

- Equation of motion in Fourier space

$$\left(\frac{d}{z^2 d\tau}z^2\frac{d}{d\tau} + k^2\right)\zeta_k = 0$$

- General solution in squeezed limit $|k\tau| \ll 1$)

$$\zeta_k \to \underbrace{C_k}_{\boxed{\text{Constant mode}}} + \underbrace{D_k\int_k^\tau \frac{d\tau}{z^2(\tau)}}_{\boxed{\text{Time varying mode}}}$$

During inflation:

$$a \simeq -1/H\tau$$

Hubble crossing:

$$k = a_k H_k$$

a) If the D mode is decaying, by matching the BD vacuum:

$$C_k \to \zeta_k \to \frac{1}{\sqrt{2k}z_k} \to \frac{H_k}{2\sqrt{\epsilon k^3}} \quad \Longrightarrow$$

Scale invariant if ε is constant

Perturbation theory

b) But if the D mode is growing, by matching the BD vacuum:

$$D_k \int^{\tau_k} \frac{d\tau}{z^2(\tau)} \to D_k \int^{\tau_k} \frac{H^2\tau^2 d\tau}{2\epsilon(\tau)} \to \frac{H_k}{2\sqrt{\epsilon(\tau_k)k^3}}$$

Interestingly, two sides evolve parallel if $\epsilon \sim a^{-6} \sim |H\tau|^6$

$$D_k \sim \frac{3H_k^2}{\sqrt{k^3}} \quad \Rightarrow \quad P_\zeta \equiv \frac{k^3}{2\pi^2}|\zeta_k|^2 \sim \frac{1}{8\pi^2 H^4 \tau^6}$$

Scale invariant but growing at super-Hubble

- This result requires:

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{\dot{\phi}^2}{2H^2} \propto a^{-6} \,, \qquad \eta \equiv \frac{\dot{\epsilon}}{H\epsilon} = -6$$

Thus, not on attractor trajectory, dubbed as ***non-attractor inflation***.

Kinney, 05'; Namjoo, Firouzjahi, Sasaki, 12';
Chen, Firouzjahi, Namjoo, Sasaki, 13'

- It coincides with ***matter bounce***, which was found to yield O(1) non-gaussianity (NG)

Wands, 98'; Finelli & Brandenberger 01'
CYF, Xue, Brandenberger, Zhang, 08';
CYF, 14'; Li, Quintin, Wang, *CYF*, 16'

- Q: can one also get large NG in NAI?

NG Estimator

- To quantify NG, the three point correlator takes:

$$\langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\rangle \equiv (2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\mathcal{B}_\zeta(k_1,k_2,k_3)$$

- At squeezed limit, the local NG is

$$\mathcal{B}_\zeta(k_1,k_2,k_3) = \frac{(2\pi)^4}{k_1^3 k_3^3} P_\zeta(k_1) P_\zeta(k_3) \frac{3}{5} f_{\rm NL}$$

- Since most cubic terms are suppressed by extremely small value of ε, the dominant contribution comes from a field redefinition:

$$\zeta \to \zeta + \frac{\eta}{4}\zeta^2 + \frac{1}{\mathcal{H}}\zeta\zeta' \quad \Rightarrow \quad f_{\rm NL} = -\frac{5}{4}(\eta+4) = \frac{5}{2}$$

- A large amount of NG can be achieved in a pure phase of NAI

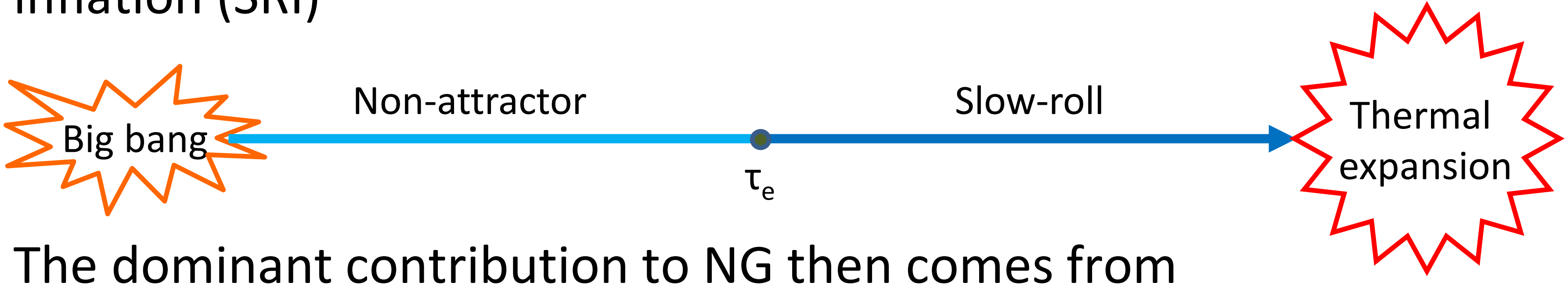
Namjoo, Firouzjahi, Sasaki, 12’

- But, NAI is phenomenologically incomplete. Because, without a regular attractor phase, there are not enough efolds to fit the COBE normalization.

CYF, Gong, Wang, Wang, 16’

From NAI to SRI: instant transition

- A straightforward completion is to introduce a phase of slow-roll inflation (SRI)



- The dominant contribution to NG then comes from

$$S_3 \supset \int d^3x d\tau \frac{a^2\epsilon}{2}\eta'\zeta'\zeta^2$$

- The integration shall stop at the end of the whole inflation

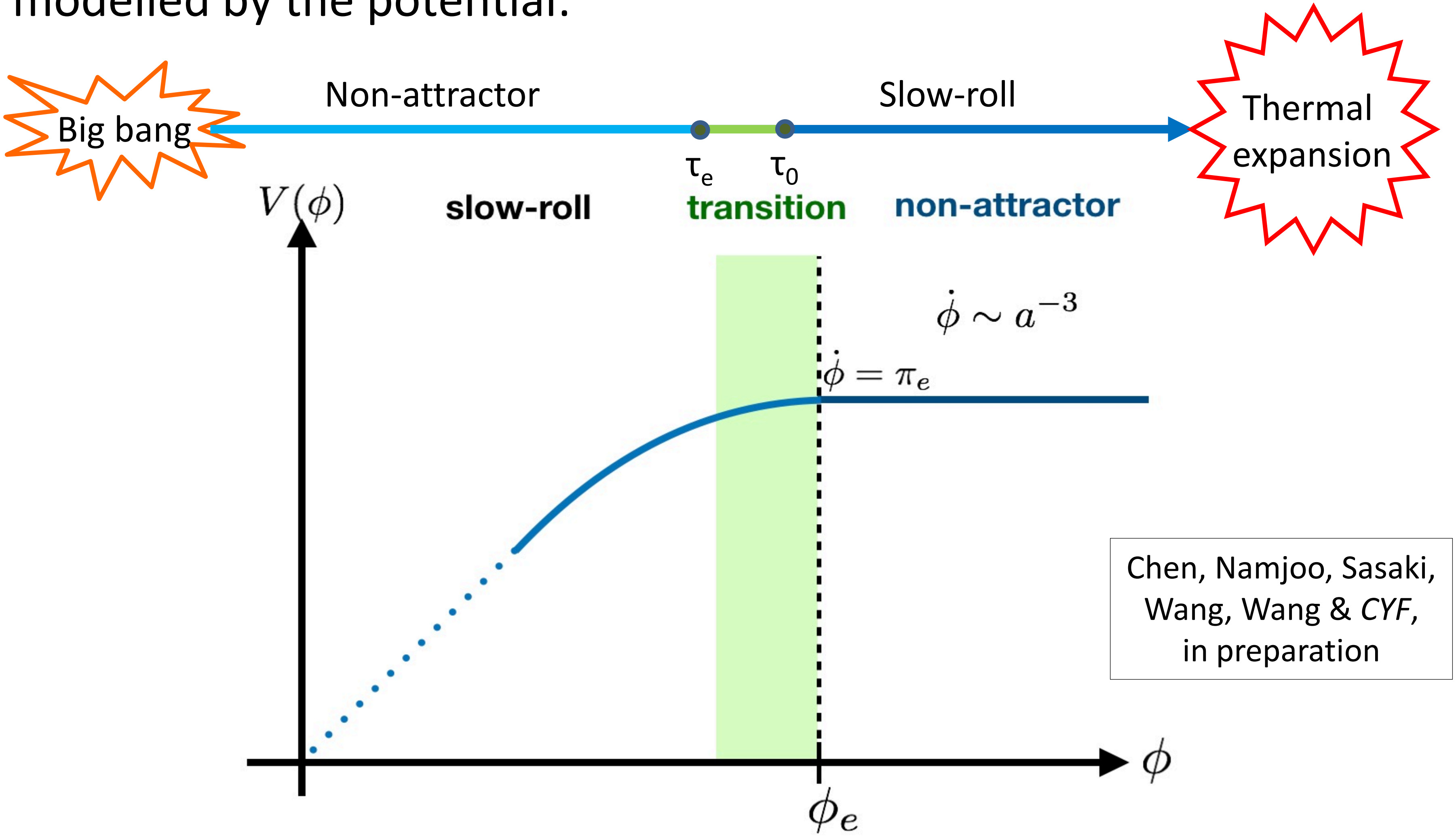
- An instant transition parameterization: $\eta = -6\left[1 - \theta(\tau - \tau_e)\right]$

Namjoo, Firouzjahi, Sasaki, 12'

- In this case, the result f_{NL} = 5/2 is repeated. However, a transition must be smooth in a realistic case.

From NAI to SRI: smooth transition

- A much more realistic case to realize the NAI to SRI transition may be modelled by the potential:



Chen, Namjoo, Sasaki, Wang, Wang & *CYF*, in preparation

From NAI to SRI: detailed analyses

- In the neighborhood of transition, the potential takes

$$V(\phi) = \begin{cases} V_0[1 + \frac{1}{2}\eta_V(\phi - \phi_e)^2] \ , & \phi \leq \phi_e \\ V_0 \ , \quad \phi > \phi_e \end{cases}$$

- Setting N=0 at ϕ_e with $s = \sqrt{9 - 12\eta_V} \simeq 3 - 2\eta_V$ ($|\eta_V| << 1$), there are

$$\pi \equiv \frac{d\phi}{dN} = \pi_e e^{-\frac{3N}{2}} \left[\cosh\left(\frac{s}{2}N\right) - \frac{3}{s}\sinh\left(\frac{s}{2}N\right)\right]$$

$$\epsilon(N) = \frac{\pi_e^2}{2} e^{-3N} \left[\cosh\left(\frac{s}{2}N\right) - \frac{3}{s}\sinh\left(\frac{s}{2}N\right)\right]^2 \qquad \eta(N) = s - 3 - \frac{2s(3+s)}{e^{sN}(s-3) + 3 + s}$$

- In-In calculation in squeezed limit

$$\zeta_k(\tau) = \frac{H(1+ik\tau)}{\sqrt{4\epsilon k^3}} e^{-ik\tau} \qquad \zeta_k'(\tau) = \frac{Hk^2\tau}{\sqrt{4\epsilon k^3}} e^{-ik\tau} - \frac{\eta}{2} \frac{aH^2(1+ik\tau)}{\sqrt{4\epsilon k^3}} e^{-ik\tau}$$

$$\mathcal{B}_\zeta(k_1, k_2, k_3) = \frac{(2\pi)^4}{k_1^3 k_3^3} P_\zeta^2 \int_{\tau_e}^{\tau_0} d\tau \frac{\eta'}{4\sqrt{\epsilon/\epsilon_0}} \left[1 + \frac{\eta}{2}\left(1 - \left(\frac{\tau_0}{\tau}\right)^3\right)\right]$$

$$\Rightarrow \quad \boxed{f_{\rm NL} = -\frac{5}{6}\eta_V \frac{\pi_0}{\pi_e} \ll 1}$$

- Conclusion: large local NG is gone after a smooth transition!

From NAI to SRI: a delta N point of view

- Recall N=0 at ϕ_e and after ϕ_e :

$$\phi = \phi_e + \frac{\pi_e}{s}\left[e^{\frac{s-3}{2}N} - e^{-\frac{s+3}{2}N}\right]$$

$$\pi \equiv \frac{d\phi}{dN} = \pi_e e^{-\frac{3N}{2}}\left[\cosh\left(\frac{s}{2}N\right) - \frac{3}{s}\sinh\left(\frac{s}{2}N\right)\right]$$

- One can derive the e-folding number

  During NAI: $N_i = -\frac{1}{3}\ln\frac{\pi}{\pi_e}$ After NAI: $N_f \simeq \frac{2}{s-3}\ln\left[\frac{\phi_f - \phi_e}{\pi_e(s-3)}\right]$

- The total e-folding number is estimated as

$$N = N_f - N_i \simeq \frac{2}{s-3}\ln\left[\frac{\phi_f - \phi_e}{\pi_e(s-3)}\right] + \frac{1}{3}\ln\frac{\pi}{\pi_e}$$

- The delta N formalism can be derived by varying N with respect to π_e

- Delta N result: $f_{\rm NL} = \frac{5N_{,\phi\phi}}{6{N_{,\phi}}^2} = \frac{5N_{,\pi_e\pi_e}}{6{N_{,\pi_e}}^2} \simeq -\frac{5}{6}\eta_V\frac{3}{2(3-\eta_V)} \ll 1$

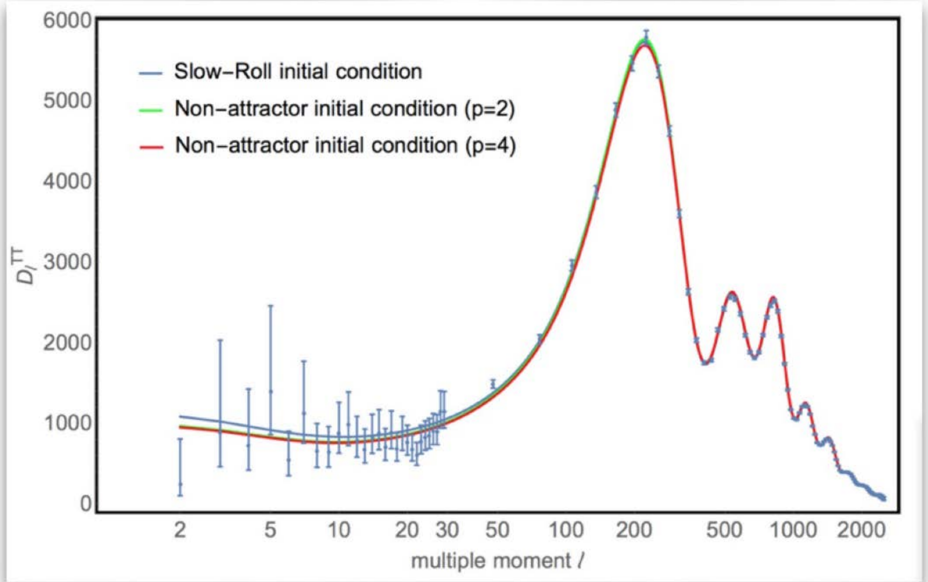
- To leading order, it coincides with the in-in result.

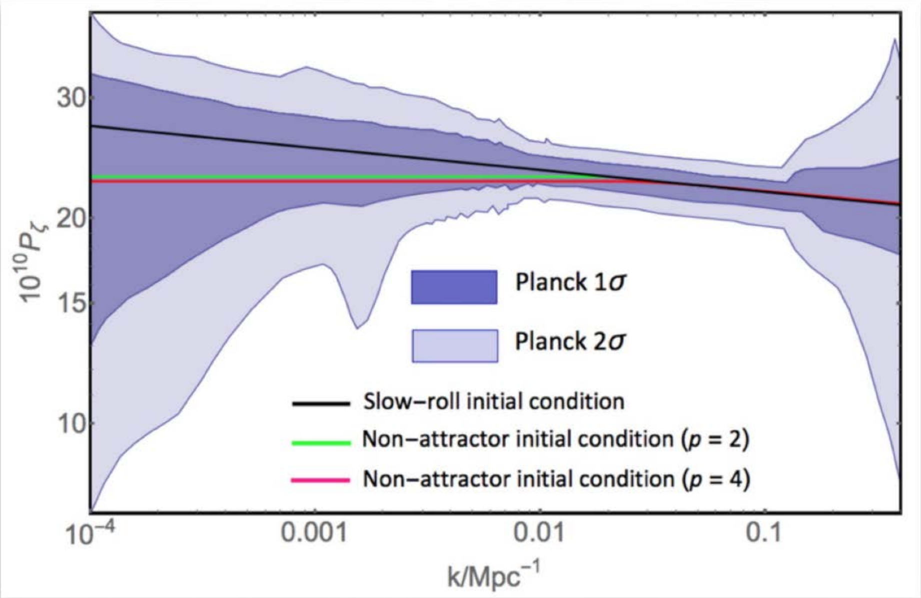
From NAI to SRI: observational feature

- Power spectrum during NAI is very close to scale-invariance due to vanishing epsilon
- Power spectrum during SRI is red tilted in agreement with CMB
- To compare with a red-tilt spectrum, the scale-invariant one can be viewed to be suppressed

  (Jinn-Ouk suppression)
- Thus, their combination shows a relative suppression feature at low ell regime

CYF, Gong, Wang, Wang, 16’





Conclusions

- Non-attractor inflation with a canonical scalar is known to generate large local non-gaussianity due to a large value of eta

- However, this signal is gone if it is connected with a phase of slow-roll inflation smoothly due to a fast damping of eta

- The same result can be confirmed by the delta N formalism

- Power spectrum from NAI to SRI can experience a (relative) suppression at large scales

Outlook

- A more general model in terms of a non-canonical scalar is expected

- Other possible transition processes are necessary to be considered