Non-gaussianity in axion N-flation models

Soo A Kim
Kyung Hee University

Based on arXiv:1005.4410
by SAK, Andrew R. Liddle and David Seery (Sussex),
and earlier papers by SAK and Liddle.
Assisted inflation (Liddle-Mazumdar-Schnook 1998) is the observation that multiple scalar fields can cooperate to drive inflation even if each individually is unable to.

Each field feels the acceleration from its own potential, but the collective Hubble friction from all fields.

N-flation

N-flation (Dimopolos et al 2008) is a realization of assisted inflation using string axions.
Motivations and assumptions

- One motivation for this idea is that sufficient inflation can be obtained with all fields maintaining sub-Planckian values.
- Another is that it may be possible to relate assisted inflation to proper fundamental physics models.

- Focused on adiabatic perturbations.
- Random initial conditions for fields.
- Assumptions we made:
  Horizon crossing and Slow-roll approximations.
**N-flation phenomenology**

The full string axion potential is

\[
V_i = \Lambda_i^4 \left( 1 - \cos \frac{2\pi \phi_i}{f_i} \right)
\]

, where there are \(N_f\) fields with constants \(\Lambda_i\) and \(f_i\). Throughout will ignore possible couplings btw the fields.

This has been extensively explored in the quadratic approximation where all fields are close to their minima, in which case they are simply a set of massive fields with

\[
m_i \equiv \frac{2\pi \Lambda_i^2}{f_i}
\]
Regardless of these choices, the Nflation phenomenology in this approximation is remarkably simple:

- The tensor-to-scalar ratio always equals to the single-field values: \( r = 8/N \) where \( N \) is the number of e-foldings.

- The scalar spectral index cannot exceed the single-field value, equalling it only in the equal-mass case: \( n \leq 1-2/N \).

- The non-gaussianity \( f_{\text{NL}} \) always equals its single-field value: \( f_{\text{NL}} = 2/N \) and hence is unobservably small.
The N-flation model

But..

.. in fact the quadratic approximation to the potential is unlikely to be valid. We should consider the full potential

\[ V_i = \Lambda_i^4 \left( 1 - \cos \frac{2\pi \phi_i}{f_i} \right) \]

Even if the potentials are all taken to be identical, assisted inflation is an attractor solution only if

\[ \frac{d^2V}{d\phi^2} > 0 \] (Calcagni and Liddle 2008), which is not true near the maximum of the potential(s) where the trajectories will diverge.

[ \implies \text{ From now on, called the Naxion model } ]
Naxion equations

With the full potential, the observables can be calculated using the so-called $\delta N$ formalism

$$\mathcal{P}_\zeta = \frac{H_*^2}{4\pi^2} \sum_i N, iN_i = \frac{H_*^2}{8\pi^2 M_P^2} \sum_i \frac{1}{\epsilon_i^*};$$

$$n - 1 = -2\epsilon^* - \frac{8\pi^2}{3H_*^2} \sum_j \frac{\Lambda_j^4}{f_j^2} \frac{1}{\epsilon_j^*} / \sum_i \frac{1}{\epsilon_i^*};$$

$$r = \frac{2}{\pi^2 \mathcal{P}_\zeta M_P^2} = 16 / \sum_i \frac{1}{\epsilon_i^*};$$

$$\frac{6}{5} f_{NL} \sim \frac{\sum_{ij} N, iN, jN, ij}{(\sum_{k} N, kN, k)^2} = \frac{r^2}{128} \sum_i \frac{1}{\epsilon_i^*} \frac{1}{1 + \cos \alpha_i^*},$$

Here $\alpha_i = 2\pi \phi_i / f_i$, $\epsilon_i$ is the slow-roll parameter of each field, derivatives wrt field $i$ and indicated by a comma, and $*$ indicates evaluation at horizon crossing.

$$\epsilon_i \equiv \frac{M_P^2}{2} \left( \frac{V_i'}{V_i} \right)^2$$

$$\mathcal{P}_\zeta \equiv -\dot{H}/H^2$$

$$\approx \Sigma_i (V_i/V)^2 \epsilon_i$$
The number of e-foldings is given by

\[
N_{\text{tot}} \sim \sum_i \left( \frac{f_i}{2\pi M_P} \right)^2 \ln \frac{2}{1 + \cos \alpha_i} \sim \frac{\ln 2}{2\pi^2} \frac{f^2}{M_P^2} N_f
\]

where the last expression calculates the expectation value of the sum under assumption of uniformly-distributed angles \(\alpha_i\). For values of \(f\) of order the Planck mass, sufficient inflation requires a large number of fields, at least hundreds.
Naxion: $n$ and $r$

A similar summation trick, assuming uniformly distributed fields, gives an analytic estimate of the spectral index as

$$\langle n - 1 \rangle \simeq -\frac{5 \ln 2}{N_*}$$

NB: $5 \ln 2 \simeq 3.5$

At the same time, the tensor-to-scalar ratio is highly suppressed by the small $\epsilon_i$ of fields close to the maximum.
Nflation: $n$ and $r$

This plot shows simulations with several different values of $f$. Clearly the dependence on $N^*$ dominates.
Naxion: non-gaussianity

The interesting aspect of the model is the behavior of the non-gaussianity:

\[
\frac{6}{5} f_{\text{NL}} \simeq \frac{\sum_{ij} N_i N_j N_{ij}}{(\sum_k N_k N_k)^2} = \frac{r^2}{128} \sum_i \frac{1}{\epsilon_i^*} \frac{1}{1 + \cos \alpha_i^*},
\]

The sum may be dominated by a small number of fields whose \( \alpha_i \) is very close to \( \pi \). If there are \( \tilde{N} \) fields which dominate with comparable \( \epsilon_i \), there is an approximate form

\[
\frac{6}{5} f_{\text{NL}} \approx \frac{2\pi^2}{N} \left( \frac{M_P}{f} \right)^2,
\]

hence for \( f \) of order \( M_P \) the \( f_{\text{NL}} \) may be of order tens.
Naxion: non-gaussianity

FIG. 1: Model predictions in the parameter space of $f = M_P; N^* = 50$. The black (left) cluster of points takes into account the model parameters (though there is significant uncertainty less than the Planck scale, of order $\bar{P}_N/L$). Although Eq. (7) is singular in the initial conditions, we see that the non-gaussian fraction is reduced. Fig. 3 shows the expected maximum, which is nearly saturated in cases where a single field dominates the summations. In cases of large $e \eta/\bar{P}_N$ and $\alpha/\bar{P}_N$, the non-gaussian signal is enhanced. Indeed, the cooperative effect of the Nflation mechanism does not enhance the non-gaussian signal. Instead, the large $e \eta/\bar{P}_N$ and $\alpha/\bar{P}_N$ that are ejected downhill. This process typically leaves a few potentials that dominate the summations. In this case the maximum achievable value of the non-gaussian signal is $\bar{P}_N/L$, which is almost saturated in some realizations.

The expectations described above are borne out in numerical calculations. In Fig. 1 we show model predictions for various initial angles ranging from 464 to 10,000, all giving similar results.
Interpretation

There is nothing particularly unusual about the predictions of the non-gaussianity in this model; it is in fact what one would get from a single field evolving in the axion potential.

However a single-field model with those parameters would not be satisfactory, as it would not give sufficient inflation and the spectral index would be far from unity.

The scenario works because the assisted inflation mechanism strongly alters the predicted spectral index, but has only a marginal effect on the non-gaussianity.
The axion Nflation model is a simple construction which offers significant non-gaussianity while maintaining viable values of other observables.
Naxion: trispectrum

A similar analysis yields an estimate of the trispectra

\[ \tau_{NL} = \frac{4\pi^4}{N^2} \left( \frac{M_P^4}{f^4} \right) \]

\[ \frac{54}{25} g_{NL} = \frac{8\pi^4}{\bar{N}^2} \left( \frac{M_P^4}{f^4} \right) \]

\[ = \left( \frac{6}{5} f_{NL} \right)^2 \]

As seen in the bispectra plot, there is a large spread of predictions due to the randomness of initial conditions.

However there are predicted correlations within a realization, for instance between \( r \) and \( f_{NL} \).