CMB Polarization in Einstein-Aether Theory

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Introduction

- Two Big Mysteries of Cosmology
  → Dark Energy & Dark Matter

⇒ The Mysteries of Gravity
= Modification of Gravity at long-distance scale?

- Modification of Gravity
  = Adding the extra degree of freedom

  ✓ scalar : F(R)-gravity, Galileon theory…
  ✓ tensor : massive gravity, bi-gravity theory…
  ✓ vector : ???

  ↳ Einstein-Aether Theory!

Einstein-Aether Theory

✓ Action = Einstein-Hilbert + Fixed Norm Vector Field with a Lorentz Violating VEV. [Jacobson and Mattingly (1999)]

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,[R+\mathcal{L}_A] + \int d^4x\sqrt{-g}\,\mathcal{L}_m$$

$$\left(\begin{array}{r}\mathcal{L}_A = -[c_1\nabla_a A^b\nabla^a A_b + c_2\nabla_a A^a\nabla_b A^b + c_3\nabla_a A^b\nabla_b A^a \\ +c_4 A^a A^b\nabla_a A^c\nabla_b A_c] + \lambda(A^a A_a - 1)\end{array}\right)$$

** $A_a \to \partial_a\phi$: correspondence with Horava-Lifshitz gravity.

✓ Stability Analysis of Perturbation [Armendariz-Picon et al. (2010)]

scalar sector : $-2 \leq c_1 + c_4 \leq 0,\ \ c_1 + c_2 + c_3 < 0$

tensor sector : $c_1 + c_3 > -1$

vector sector : $2c_1 \leq (c_1 + c_3)^2(1 + c_1 + c_3)$

What is the observational consequence of this new transverse vector degree?

➡ CMB B-mode Polarization!

CMB B-mode Polarization

✓B-mode is the curl-like component of CMB polarization.

✓ Now, so many projects are attempting to detect the CMB B-mode polarization ➡ exciting era for B-mode physics!

✓ Possible origin of B-mode Polarization

- ✓ Tensor : Primordial Gravitational Waves (PGW)
- ✓ Vector : Source models such as cosmic strings or other defects

** In GR without sources, vector mode only decays…

✓ Vector modes are more efficient to produce B-mode polarization than tensor modes. [Hu and White (1997)]

⬄ difficult for usual vector models to make a definite prediction.

Aether Field Perturbation can be generated in a same way as PGW during inflation and the subsequent evolution is clearly understood!

Our Works

- ✓ Formulate the linear perturbation of the vector mode in Einstein Aether theory using the covariant formalism.
- ✓ Derive the initial condition in the early Radiation Dominated stage considering the ordinary matter and Aether.
- ✓ Calculate the CMB B-mode Power spectrum due to the vector mode numerically using modified CAMB code
- ✓ Understand the shape of the spectrum in an analytic way.

Basic Quantities for Aether Field

✓ Energy-Momentum Tensor : $T^{(A)}_{ab} = -\frac{1}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L}_A)}{\delta g^{ab}}$

$$T^{(A)}_{ab} = \nabla_c \left(J_{(a}{}^{c} A_{b)} - J^{c}{}_{(a} A_{b)} - J_{(ab)} A^{c} \right) + Y_{ab} + \frac{1}{2} g_{ab} \mathcal{L}_A + \lambda A_a A_b + c_4 A^c A^d (\nabla_c A_a)(\nabla_d A_b)$$

$$\left(\begin{aligned} J^{a}{}_{b} &= -[c_1 \nabla^a A_b + c_2 \delta^a_b \nabla_c A^c + c_3 \nabla_b A^a + c_4 A^a A^c \nabla_c A_b] \\ Y_{ab} &= -c_1 [(\nabla_c A_a)(\nabla^c A_b) - (\nabla_a A_c)(\nabla_b A^c)] \end{aligned} \right)$$

✓ Equation of motion for Aether Field :

$$c_1 \nabla_c \nabla^c A_a + c_2 \nabla_a \nabla_b A^b + c_3 \nabla_b \nabla_a A^b + c_4 [\nabla_b (A^b A^c \nabla_c A_a) - A^b (\nabla_b A_c)(\nabla_a A^c)] = -\lambda A_a$$

✓ Fixed Norm Constraint : $A_a A^a = 1$

✓ Einstein equation : $G_{ab} = T^{(A)}_{ab} + 8\pi G T^{(m)}_{ab}$

✓ Useful Parameter Definition

$$c_{13} = c_1 + c_3, \quad c_{14} = c_1 + c_4, \quad \alpha = c_1 + 3c_2 + c_3$$

Background Cosmology

[Carroll and Lim (2004)]

Flat FRW Background : $ds^2 = a^2(\eta)[d\eta^2 - d\vec{x}^2], \quad A^a = \left(\frac{1}{a}, 0, 0, 0\right)$

Friedman equations : $\mathcal{H} = \frac{8\pi G_{cos}}{3} a^2 \rho, \quad \mathcal{H}' = -\frac{4\pi G_{cos}}{3} a^2 (\rho + 3p)$

$$G_{cos} = \frac{G}{1 - (c_1 + 3c_2 + c_3)/2}$$

Effect of Aether can be incorporated into the cosmological Gravitational Constant (G.C.).

Note that the fundamental G.C. , which appears in the analysis of perturbations, is different from the G.C. of the background cosmology.

** The G.C. that the local experiment measures is $G_N = \frac{G}{1 + (c_1 + c_4)/2}$

Constraint from BBN : $\left|\frac{G_{cos} - G_N}{G_N}\right| < 0.1$

Primordial Spectrum of Aether Vector Perturbation

[Armendariz-Picon et al. (2010)]

Second order action without matter

$$S^{(2)} = \int d\eta d^3x \frac{(-c_{14})}{16\pi G}\left[\xi'^2 - c_v^2 \partial_i \xi \partial^i \xi - \frac{\alpha}{c_{14}}(\mathcal{H}^2 - \mathcal{H}')\xi^2\right]$$

$$\left(\xi = aV \quad \text{Vector Perturbation for Aether Field} \qquad c_v^2 = \frac{c_1}{c_{14}}\left[1 - \frac{c_{13}^2}{2c_1(1+c_{13})}\right] \right)$$

- ✓ During inflation $a \propto (-\eta)^q$, we have the positive frequency solution,

$$V = \frac{\sqrt{8\pi G}}{2}\sqrt{\frac{\pi}{-c_{14}}}\frac{\sqrt{-\eta}}{a}H^{(1)}_{\nu/2}(-c_v k\eta) \qquad \left(\nu = \sqrt{1 - 4q(q+1)\frac{\alpha}{c_{14}}} \right)$$

- ✓ Defining the dimensionless power spectrum as $\langle |V(k,\eta)|^2 \rangle \equiv \frac{2\pi^2}{k^3}\mathcal{P}_V(k,\eta)$, we can derive

$$\mathcal{P}_V(k,\eta) = \mathcal{A}_V(\eta) \times \left(\frac{k}{k_0}\right)^{3-\nu}$$

** The amplitude and tilt depends on the details of the inflation ($\nu = 1$ for dS inflation).

Covariant Formalism

[Hawking (1966), Olsen (1976), …]

- ✓ 1+3 Decomposition using
  - ✓ Fundamental observer : $u_a \quad (u_a u^a = 1)$
  - ✓ Projection tensor : $h_{ab} = g_{ab} - u_a u_b$

Merits
- Gauge invariant
- Non-linear effect
- CAMB !

- ✓ 10 Fundamental Variables
  - ✓ Energy-Momentum Tensor

$$\rho = T_{ab}u^a u^b,\ p = -\frac{1}{3}h^{ab}T_{ab},\ q_a = T_{bc}h^b_a u_b,\ \pi_{ab} = \left[h_{(a}{}^c h_{b)}{}^d - \frac{1}{3}h^{cd}h_{ab}\right]T_{cd}$$

  - ✓ Covariant Derivative of 4-velocity

$$q_a^{(i)} = \left(\rho^{(i)} + p^{(i)}\right)v_a^{(i)}$$

$$K_a = \dot{u}_a = u_b \nabla^b u_a,\ \Theta = D^a u_a,\ \sigma_{ab} = D_{\langle a}u_{b\rangle},\ \omega_{ab} = D_{[a}u_{b]}$$

  - ✓ Decomposition of Weyl Tensor $E_{ab} = C_{acbd}u^c u^d,\ H_{ab} = \frac{1}{2}\epsilon_{acd}C^{cd}{}_{be}u^e$

** In Background FRW universe, only $\Theta,\ \rho,\ p$ have non-vanishing values.

- ✓ Projective Covariant Derivative

$$\dot{Q}^{a\cdots}{}_{b\ldots} = u^c \nabla_c Q^{a\cdots}{}_{b\ldots}$$

$$D_c Q^{a\cdots}{}_{b\ldots} = h^f{}_c h^a{}_d \cdots h^e{}_b \cdots \nabla_f Q^{d\cdots}{}_{e\ldots}$$

Basic Equations

- From Ricci identities and contracted Bianchi identities using Einstein equation, we obtain the following equations.

Constraint eqs.

$$D^a\sigma_{ab} - \frac{1}{2}\mathrm{curl}\Omega_a - \frac{2}{3}D_b\Theta - \kappa q_b = 0$$

$$D^a E_{ab} - \kappa\left(\frac{\Theta}{3}q_b + \frac{1}{3}D_b\rho + \frac{1}{2}D^a\pi_{ab}\right) = 0$$

$$D^a H_{ab} - \frac{1}{2}\kappa\left[(\rho+p)\Omega_b + \mathrm{curl}q_b\right] = 0$$

$$H_{ab} - \mathrm{curl}\sigma_{ab} + \frac{1}{2}D_{\langle a}\Omega_{b\rangle} = 0$$

Evolution eqs.

$$\dot{\Omega}_a + \frac{2}{3}\Theta\Omega_a = \mathrm{curl}K_a$$

$$\dot{\sigma}_{ab} + \frac{2}{3}\Theta\sigma_{ab} = -E_{ab} - \frac{1}{2}\kappa\pi_{ab}$$

$$\dot{E}_{ab} + \Theta E_{ab} = \mathrm{curl}H_{ab} + \frac{\kappa}{2}\left[\dot{\pi}_{ab} - (\rho+p)\sigma_{ab} + \frac{\Theta}{3}\pi_{ab}\right]$$

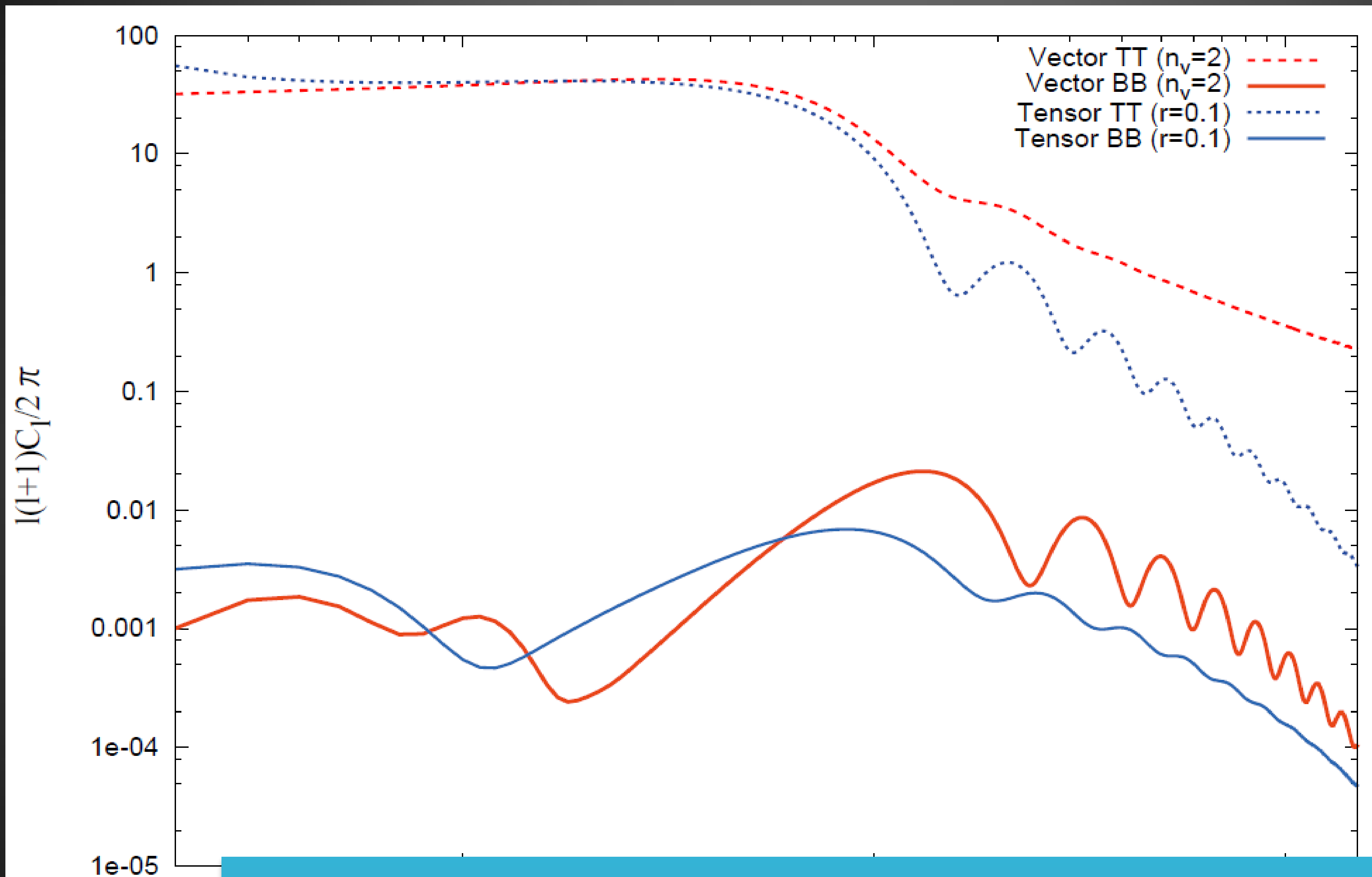
$$\dot{H}_{ab} + \Theta H_{ab} = -\mathrm{curl}E_{ab} - \frac{\kappa}{2}\mathrm{curl}\pi_{ab}$$

Conservation of Energy-Momentum Tensor

$$\dot{\rho} + \Theta(\rho+p) + D^a q_a = 0 \qquad \dot{q}_a + \frac{4}{3}\Theta q_a + (\rho+p)K_a - D_a p + D^b\sigma_{ab} = 0$$

** In the above equations, we have dropped the higher order terms (left only the linear quantity around the flat FRW universe).

Numerical Result from Modified CAMB



$$\mathcal{P}_V = \mathcal{A}_k \left(\frac{k}{k_0}\right)^{n_v}$$

$$\mathcal{P}_T \propto k^0$$

$c_{13} = -0.3$

$c_1 = -0.1$

$c_{14} = -0.2$

$\alpha = 0.2$

It is possible for vector BB to surpass the tensor BB respecting the existing observational constraint from the TT amplitude.

Analytic Approach -1-

Photon Multipole Equations in covariant formalism

$$I'_\ell = -k\frac{\ell}{2\ell+1}\left[\frac{\ell+2}{\ell+1}I_{\ell+1} - I_{\ell-1}\right] - \dot{\tau}\left(I_\ell - \frac{4}{3}\delta_{\ell 1}v - \frac{2}{15}\zeta\delta_{\ell 2}\right) + \frac{8}{15}k\sigma\delta_{\ell 2}$$

$$E'_\ell = -\frac{(\ell+3)(\ell+2)\ell(\ell-1)}{(\ell+1)^3(2\ell+1)}kE_{\ell+1} - \frac{\ell}{2\ell+1}kE_{\ell-1} - \frac{1}{\ell(\ell+1)}kB_\ell - \dot{\tau}\left(E_\ell - \frac{2}{15}\zeta\delta_{\ell 2}\right)$$

$$B'_\ell = -\frac{(\ell+3)(\ell+2)\ell(\ell-1)}{(\ell+1)^3(2\ell+1)}kB_{\ell+1} + \frac{\ell}{2\ell+1}kB_{\ell-1} - \frac{2}{\ell(\ell+1)}kE_\ell - \dot{\tau}B_\ell$$

➡ Integral Solution for B-mode Polarization

$$B_\ell(\eta_0) = -\frac{\ell-1}{\ell+1}\int^{\eta_0} d\eta\,\dot{\tau}e^{-\tau}\frac{\ell j_\ell[k(\eta_0-\eta)]}{k(\eta_0-\eta)}\zeta$$

$$\zeta \equiv \frac{3}{4}I_2 - \frac{9}{2}E_2$$

Note that $\dot{\tau}e^{-\tau} \sim \delta(\eta - \eta_{rec})$, it is important to know ζ at the recombination epoch.

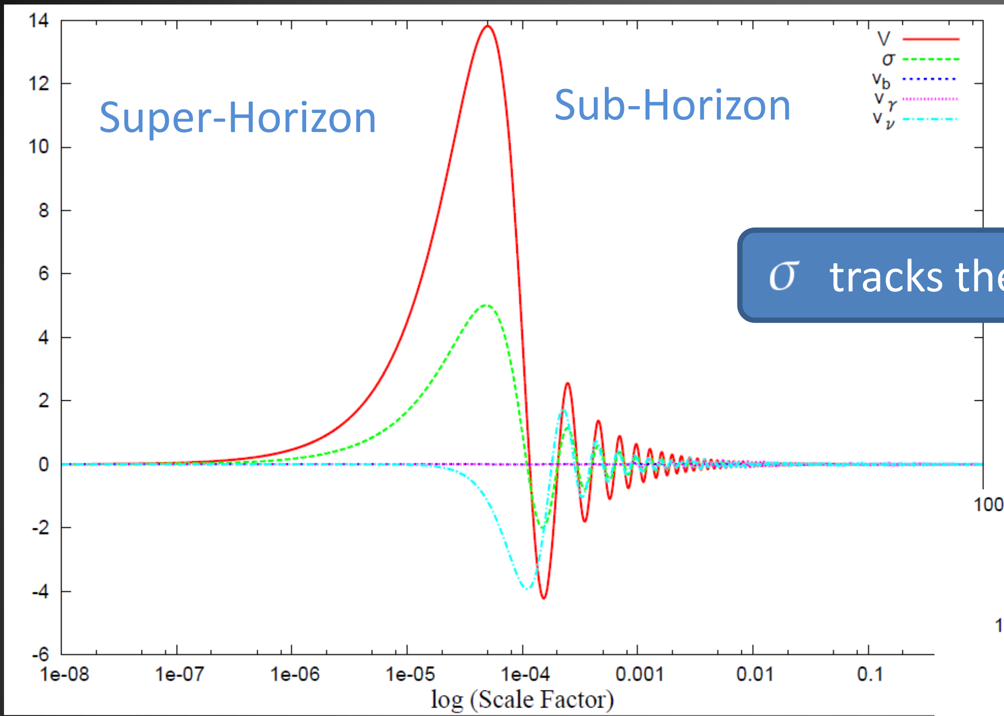
➡ Tight-Coupling Expansion

Evolution of Variables

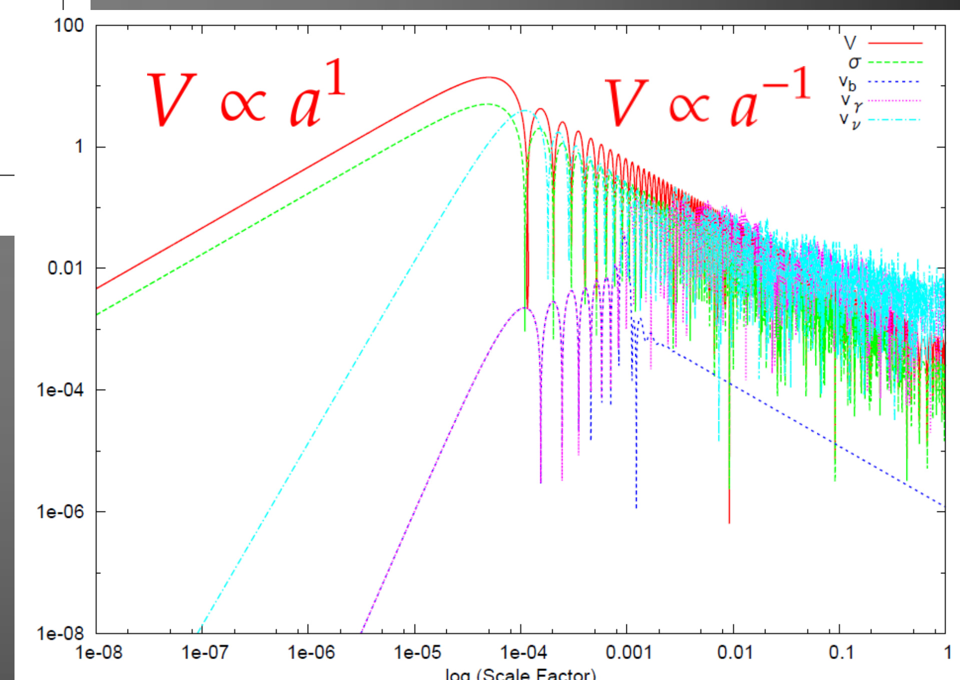
$\alpha = -c_{14}$

$k = 0.1[\mathrm{Mpc}^{-1}]$

σ tracks the evolution of V



Aether vs Scale-factor
in normal plot



Aether vs Scale-factor
in log plot

Summary and Discussion

- ✓ We have computed the power spectrum of CMB B-mode Polarization in the Einstein-Aether Theory.
- ✓ To do that, we have employed the covariant formalism and derived the regular initial conditions.
- ✓ The shape of the spectrum is clearly understood in an analytic way using tight-coupling expansion.
- ✓ More quantitative argument for whether the spectrum can be observed or not and the search for the consistent parameter range considering other experiments will be future works.
- ✓ Are there any cosmological Aether effects? (e.g. Weak Lensing?)