BLACK HOLE REMNANTS, PRE INFLATION MATTER ERA AND CMB POWER SPECTRUM

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Introduction and Outline

- Micro black holes in the pre-inflation era $(10^{-43} - 10^{-37} s)$

  → matter-dominated universe

  → influence on the evolution of a scalar field

  → becomes visible in the CMB power spectrum as Suppression of the large scale multipole (quadrupole) moments

Nucleation of Micro Black Holes: Gross, Perry, Jaffe (1982); Kapusta (1984): gravitational instabilities of flat space. Expression for the probability of spontaneous formation (bubbling) of black holes out of the gravitational (metrical) instabilities of spacetime.

$$\Gamma_N(\Theta) = \frac{1}{15 \cdot 8\pi^2} \, \Theta^{-\frac{167}{45}} \, \exp\left(-\frac{1}{16\pi\Theta^2}\right)$$

Generalized Uncertainty Principle and Black Hole Remnants

GUP: String theory

$$\Delta x \geq \frac{\hbar}{2\Delta p} + 2\beta\, \ell_{4n}^2 \frac{\Delta p}{\hbar},$$

Heisenberg argument: the smallest detail theoretically detectable with a beam of photons of energy *E* is roughly given by

$$\delta x \simeq \frac{\hbar c}{2E}.$$

GUP version of the standard Heisenberg formula

$$\delta x \simeq \frac{\hbar c}{2E} + \beta \ell_p \frac{E}{\mathcal{E}_p}.$$

Uncertainty in photon position just outside a BH

$$\delta x \simeq 2\mu R_S = 2\mu \ell_p m$$

Equipartition law: energy of unpolarized photons of outgoing Hawking radiation

$$E_\epsilon \simeq k_B T\,.$$

Mesuring all temperatures in Planck units as $\Theta = T/T_p$, we have

$$2m = \frac{1}{2\pi\Theta} + \zeta\, 2\pi\Theta$$

where $\zeta = \beta/\pi^2$

Assume $\zeta \approx 1$

→→→

$$\Theta_{\max} = \frac{1}{2\pi\sqrt{\zeta}}$$

$$m_{\min} = \sqrt{\zeta}$$

Minimum Mass &

Maximum Temperature

→→ ! **BLACK HOLE REMNANTS !**

Once nucleated, even after BH evaporation, REMNANTS stay there [GUP !]. They **are sufficient to put the Universe in matter era until the onset of Inflation**

Equation of motion

Pre inflationary Universe containing:

Matter (nucleated micro black holes)(A), Radiation (B),

Constant Vacuum Energy (= responsible for the inflation) (C)

Flat FRW metric: $ds^2 = -c^2dt^2 + a(t)^2 (dr^2 + r^2 d\Omega^2)$.

Einstein equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \kappa\left(\frac{A}{a^3} + \frac{B}{a^4} + C\right)$$

Matter era Condition

$$\frac{B}{A\,a} \ll 1$$

Pre inflation radiation era solution

$$a(t) = \left(\frac{B}{C}\right)^{1/4}\left[\sinh\left(2\sqrt{\kappa C}\;t\right)\right]^{1/2}$$

$a(t) \sim t^{1/2}$

A = O

Pre inflation matter era solution

$$a(t) = \left(\frac{A}{C}\right)^{1/3}\left[\sinh\left(\frac{3}{2}\sqrt{\kappa C}\;t\right)\right]^{2/3}$$

$a(t) \sim t^{2/3}$

B = O

Time in Planck units:

$\tau = t / t_p$

Constants A , B:

$$A = \rho_m(\tau_c)\cdot a^3(\tau_c),$$
$$B = \rho_r(\tau_p)\cdot a^4(\tau_p),$$

Numerical simulation: computation of A, B

Assuming ρ_{rad} = Planck density at $\tau \approx 1\, t_p$ ➔

B = 1

Adiabatically expanding Universe:

Nucleations rate as functions of time

$$\Gamma_{N,r}(\tau) = \frac{1}{15 \cdot 8\pi^2} \cdot \tau^{167/90} e^{-\tau/16\pi}$$

CUTOFFS:

From GUP: $m \approx 1 \Rightarrow \tau \approx 160$ (not enough to avoid BH overlapping)

From **HOLOGRAPHIC PRINCIPLE**:

$$S[L(B)] \leqslant k_B \frac{A(B)}{4\ell_p^2}$$ ➔ $$S_{bh}(\tau) \leqslant S_{HS}(\tau)$$ ➔

BH Nucleation effective at

$\tau_c \approx 990\, t_p$

- Universe starts at $\tau = 1$ in Radiation dominated era;

  and so evolves until $\tau_c \approx 990$

- BH nucleation starts at $\tau_c \approx 990$, goes on for $\approx 10\, t_p$, then is exponentially suppressed

- Inflation starts at $\tau \approx 10^6 - 10^7\, t_p$

- About $N = 10^4$ **BHs are produced**. The average mass is $m(\tau_c) = 2.5\, M_p$. They evaporate down to $\approx 1\, M_p$ in about $10^4\, t_p$

Matter density at end of BH Nucleation era:

$$\rho_m(\tau_c) \sim \frac{10^4 \text{ black holes}}{R_H^3(\tau_c)} \sim \frac{10^4 \sqrt{\zeta}\, \epsilon_p}{10^9\, V_p}$$

➔

$$A = \frac{10^4 \sqrt{\zeta}\, \epsilon_p}{10^9\, V_p} \cdot 10^{9/2} \sim 10^{-1/2} \sqrt{\zeta}\, \frac{\epsilon_p}{V_p}$$

$$\frac{3}{2} \frac{B}{A \cdot a(\tau)} = \frac{1}{10^{-1/2} \cdot a(\tau)} \sim 10^{-1} - 10^{-3} \ll 1$$

Condition for matter dominance: SATISFIED !

Influences of a pre-inflation matter era on scalar field fluctuations

Scalar field fluctuations: $\Phi(t,\vec{y}) = \Phi_0(t,\vec{y}) + \varphi(t,\vec{y})$

Equation of motion: $\Box\varphi(t,\vec{y}) = 0$ $\quad \ddot{\phi}_k(t) + 3\frac{\dot{a}}{a}\dot{\phi}_k(t) + \left(\frac{k^2}{a^2}\right)\phi_k(t) = 0$

THE RE-ENTERING K-MODES: k-modes leaving the horizon just at the onset of inflation, are just now re-entering our Hubble radius. They bring imprints of a possible pre-inflation (matter) era.

Mode of largest visible perturbation

$$k_{\mathrm{min}} \simeq a\,H$$

Pre-inflation radiation era

$$k_{\mathrm{min}} = a\sqrt{\kappa}\left(\frac{B}{a^4} + C\right)^{1/2}$$

Pre-inflation matter era

$$k_{\mathrm{min}} = a\sqrt{\kappa}\left(\frac{A}{a^3} + C\right)^{1/2}$$

ANALYTICAL COMPUTATION:

Solve EoM for ϕ(a,k) k - parameter. Express a=a(k)

Construct Primordial Power Spectrum of the quantum fluctuations of the field ϕ

$$P(k) = k^3 |\phi(a(k), k)|^2$$

P(k) feeds CMBFAST code → → → CMB anisotropy power spectrum

EoM for ϕ(a,k) in **pre inflation matter** era

$$\phi_k'' + \frac{1}{a}\left(\frac{4Ca^3 + \frac{5}{2}A}{Ca^3 + A}\right)\phi_k' + \left(\frac{k^2}{\kappa a(Ca^3 + A)}\right)\phi_k = 0$$

For comparison we consider also ϕ(a,k) in **pre inflation radiation** era

$$\phi_k'' + \frac{2}{a}\left(\frac{Ca^4}{Ca^4 + B} + 1\right)\phi_k' + \left(\frac{k^2}{\kappa(Ca^4 + B)}\right)\phi_k = 0$$

- Boundary conditions in full inflationary era:

From Last WMAP data: almost **scale invariant** (flat), slightly tilted, primordial power spectrum P(k)

$$P(k) \sim k^{n_s - 1}$$

with

$$n_s = 0.963 \pm 0.012 \; (68\% \; CL) \, .$$

Therefore the field ϕ must behave as

$$|\phi(a(k), k)| \sim \frac{k^{\frac{1}{2}(n_s - 1)}}{k^{3/2}}$$

for large k.

- WKB solutions of equation of motion:

Pre inflation matter era

$$\phi_k(a) = \frac{2\sqrt[4]{\kappa C^2}\left[c_+(k)e^{iG(a)} + c_-(k)e^{-iG(a)}\right] e^{-i\pi/4}}{\left[32\kappa C^2 a^6 - 16k^2 C a^4 + 9\kappa A^2\right]^{1/4} \cdot \exp[A/(4Ca^3)]}$$

$$a(k) \simeq \frac{k}{\sqrt{\kappa C}} - \frac{A\kappa}{2k^2}$$

Arbitrary constants c(k) can be chosen so that

$$\left[c_+(k)e^{iG(a(k))} + c_-(k)e^{-iG(a(k))}\right] \sim k^{(n_s-1)/2}$$

Analogous procedure for the pre inflation radiation era

Qualitative plots for $P(k)$ (A = B = C = 1)

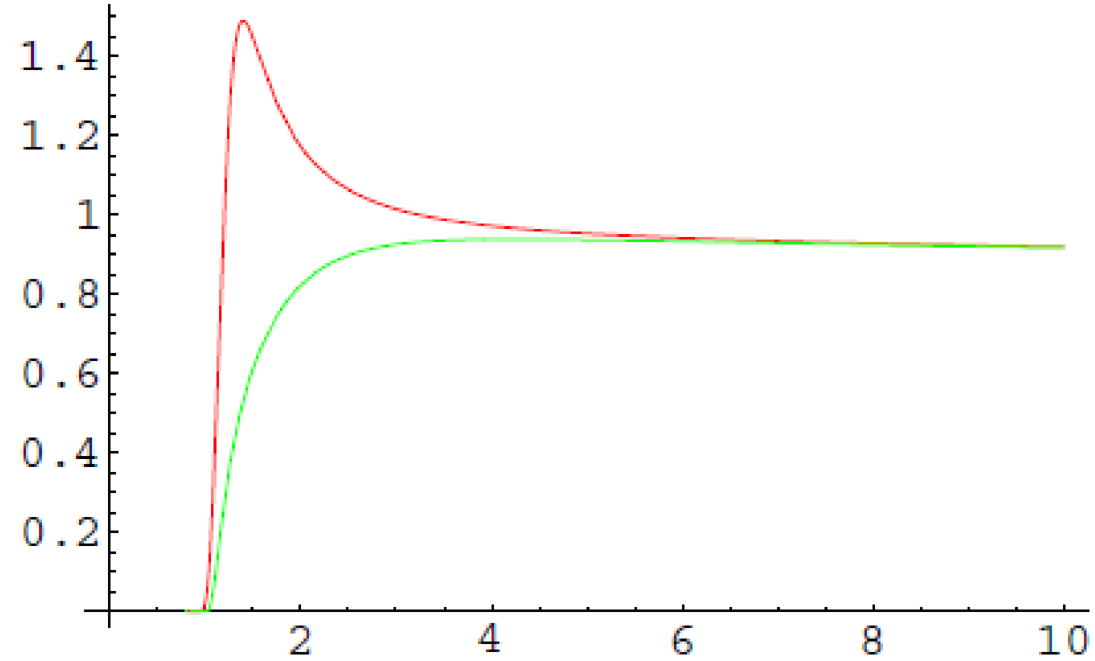


FIG. 14: Primordial power spectra $P(k)$ versus k, for pre-inflation matter (red line) and radiation (green line) eras.

EXACT NUMERICAL COMPUTATION OF THE PRIMORDIAL POWER SPECTRUM

- **MATTER ERA**: Friedmann parameters

$$A = 10^{-1/2}\,\frac{\epsilon_p}{V_p}$$

Condition for the onset of inflation → Computation of C :

$$C = \frac{A}{2\,a_{inf}^3} = \frac{1}{2}\,10^{-11}\,\frac{\epsilon_p}{V_p}$$

$$a_{inf} = 1\cdot(10^3)^{-1/6}\cdot(10^6)^{2/3} = 10^{7/2}$$

Numerical solution for $\phi(\,a(k\,i\,),\,k\,i)$ and $P\,(k\;\;i)$:
just a COLLECTION of DATA POINTS

Fitting Function

$$P(k) = a - \frac{b}{1+\frac{k^2}{c}} + \frac{d}{1+\frac{k^4}{e}} - \frac{f}{1+\frac{k^6}{g}}$$

to feed CMBFAST

parameter	a	b	c	d
value	$2.205\cdot10^{-12}$	$3.233\cdot10^{-12}$	0.03	$2.578\cdot10^{-12}$

parameter	e	f	g
value	$1.680\cdot10^{-9}$	$1.593\cdot10^{-12}$	$6.584\cdot10^{-14}$

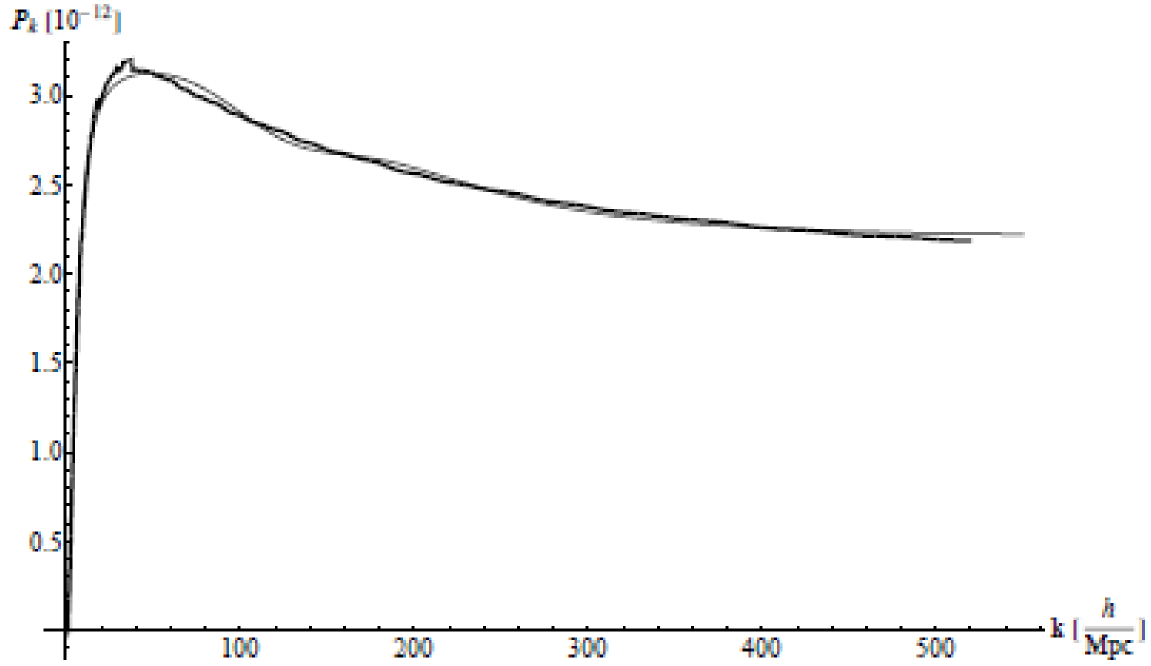


FIG. 15: The numerical solution for the primordial power spectrum (dashed line) and its fitting function (full line) in the pre-inflation matter era scenario.

- Radiation era with totally evaporating black holes (NO GUP !)

Friedmann parameters:

$$B = 10^{2/3} \sqrt{\zeta} \, \frac{\epsilon_p}{V_p}$$

$$\zeta \sim 1$$

$$a(\tau_{inf}) = 10^{25/6} \sim 10^{4.16}$$

$$C \simeq \frac{B}{a_{inf}^4} \sim 10^{-16}$$

Fitting Function:

$$P(k) = a + b \cdot \mathrm{Arctan}(c \cdot k) + d \cdot \mathrm{Arctan}^2(c \cdot k)$$

parameter	a	b	c	d
value	$-3.701 \cdot 10^{-18}$	$1.261 \cdot 10^{-16}$	3.116	$-5.935 \cdot 10^{-17}$

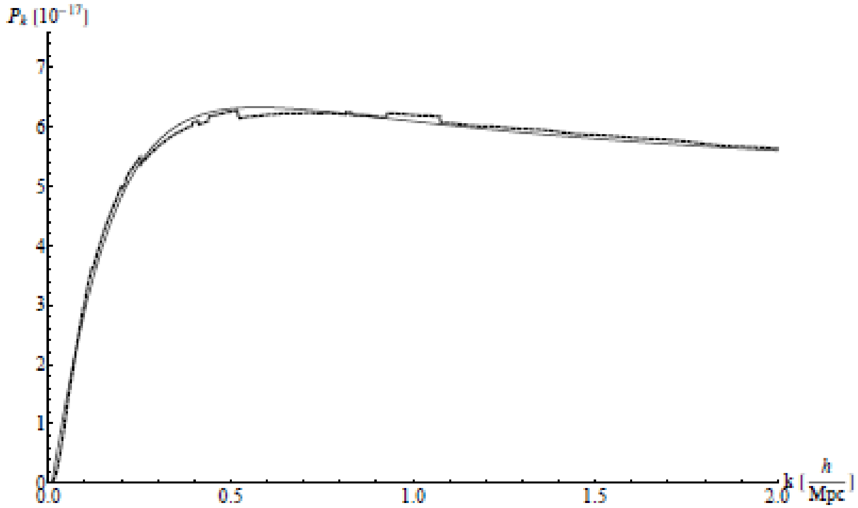


FIG. 16: The numerical solution for the primordial power spectrum (dashed line) and its fitting function (full line) in the pre-inflation radiation era scenario, with completely evaporating black holes (no GUP active).

- Radiation era without black holes

$$a(\tau_{inf}) = 10^{7/2}$$

$$B = \rho_p(\tau_p) \cdot a^4(\tau_p) = 1\,\frac{\epsilon_p}{V_p}$$

$$C = \frac{B}{a_{inf}^4} \sim 10^{-14}$$

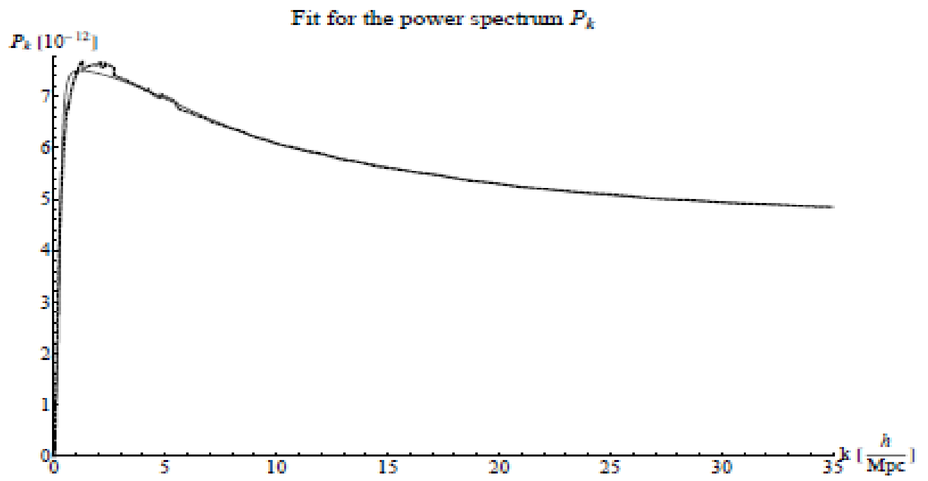


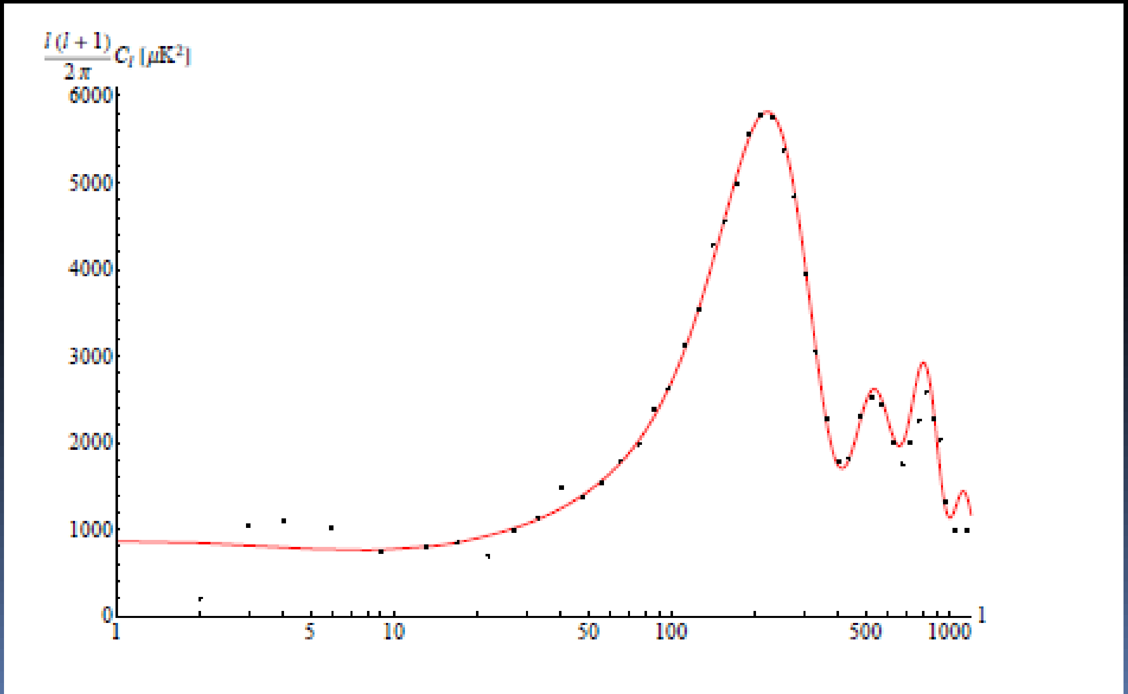
FIG. 17: The numerical solution for the primordial power spectrum (dashed line) and its fitting function (full line) in the pre-inflation radiation era scenario without black holes.

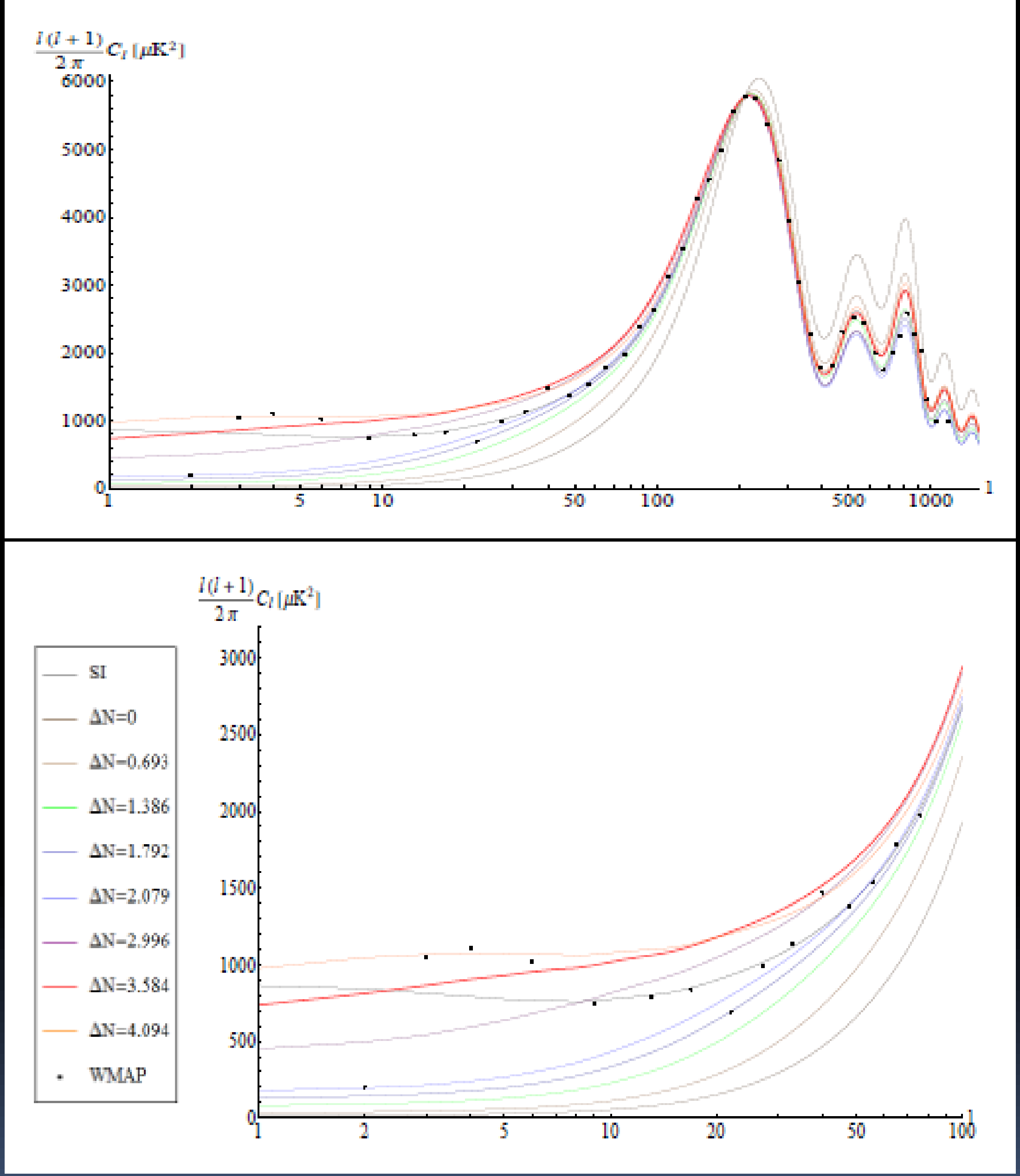
CMB POWER SPECTRUM

- Primordial power spectra (with GUP, without GUP, without black holes) → CMBFAST code [Seljak, Zaldarriaga 1996] → CMB temperature anisotropy spectrum.
- Compare with WMAP 7 year data, and with

  the standard CMB spectrum of standard inflationary ΛCDM
- ΛCDM model: anomalies in the suppressed quadrupole moment:

  the $l = 2$ mode is very low in comparison to the CMB spectrum .





The CMB power spectrum for a pre-inflation matter era, for various cases of ΔN *e-folds* added. Overall view and zoom.

- In the numerical computation for CMB power spectrum, THE TOTAL NUMBER OF E-FOLDS OF INFLATION (from when the mode k_i left the horizon, to the end of inflation) CAN BE VARIED [nobody knows EXACTLY when inflation started].

$$N_{tot} = N(k_0) + ln\left(\frac{k_0}{k_i}\right)$$

- k_0 is the currently largest mode within the horizon

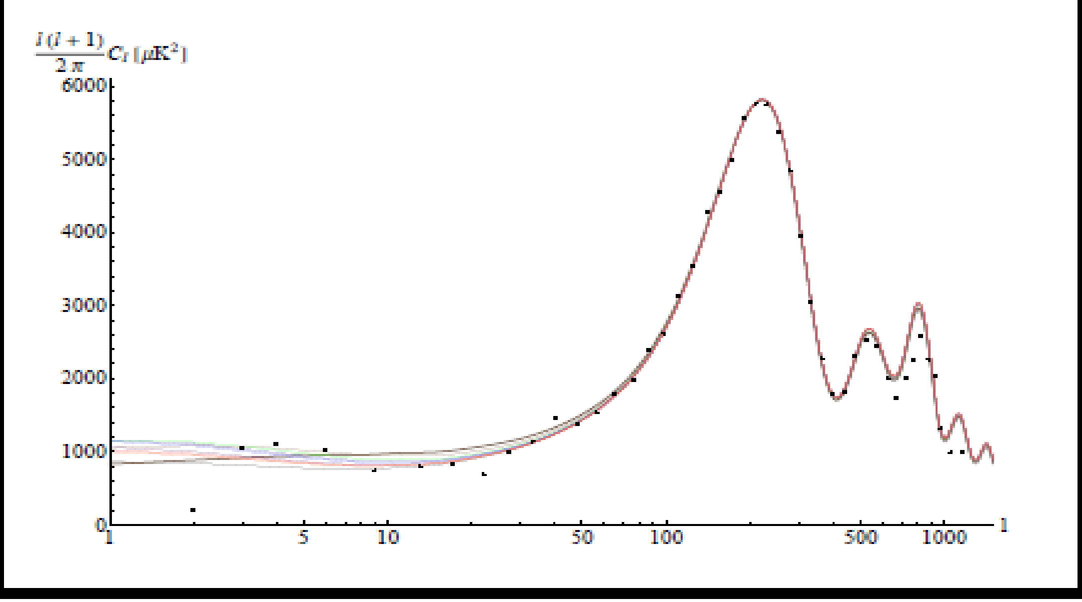
$$k_0 = 0.002\, hMpc^{-1}$$

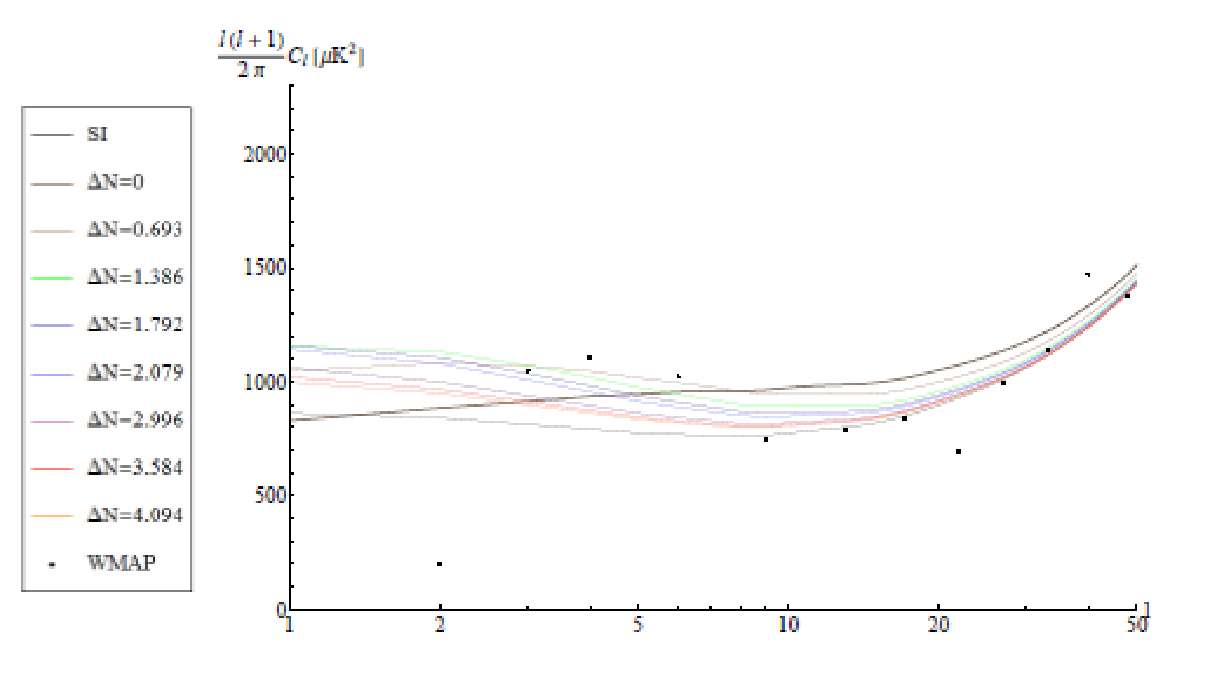
Observations constrain the number $N(k_0)$

no information on the number of e-folds ΔN of inflation before k_0 exited the horizon during inflation. The constraint on $N(k_0)$ is

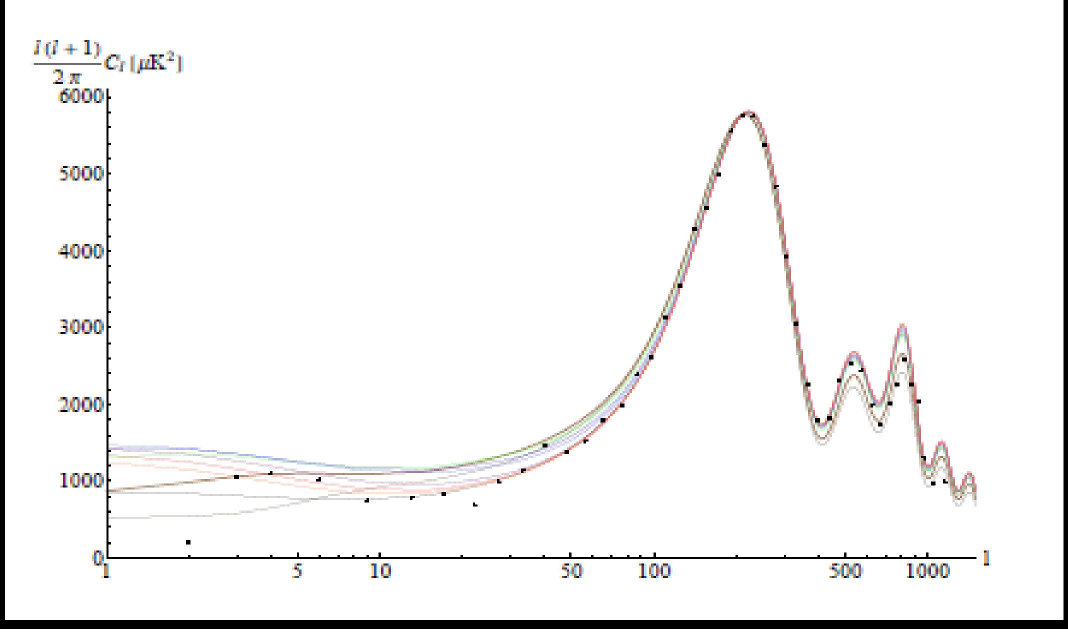
$$N(k_0) = 54 \pm 7$$

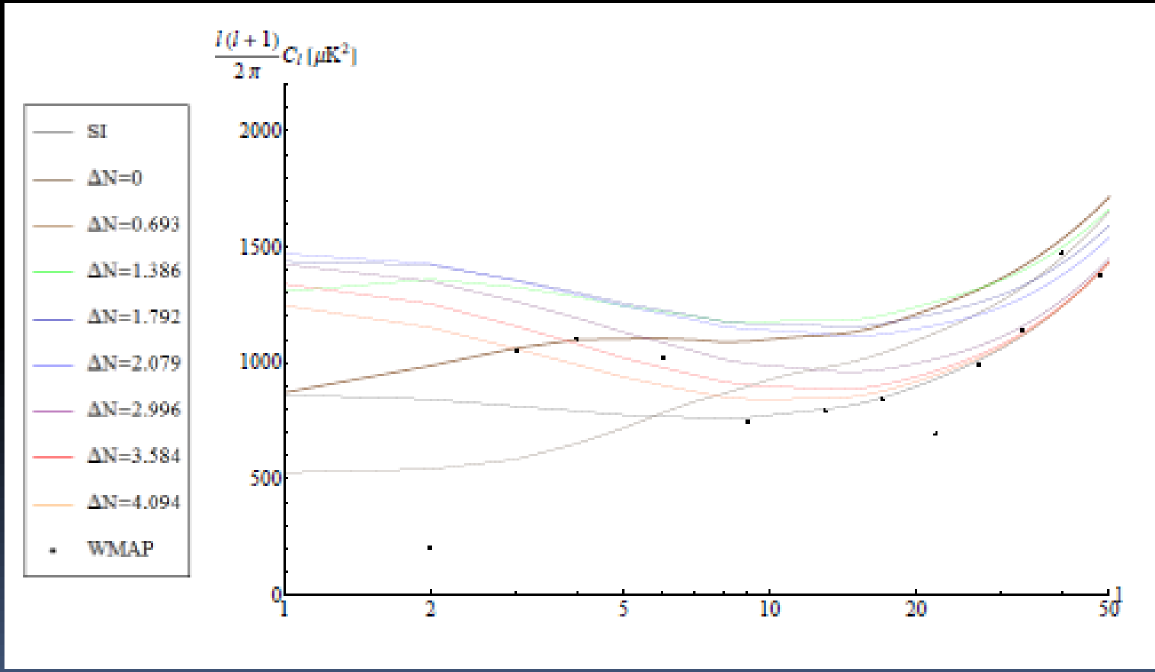
So, varying k_i is equivalent to adding e-folds ΔN to the experimentally constrained number $N(k_0) = 54 \pm 7$.





The CMB power spectrum for a pre-inflation radiation era (NO GUP), for various cases of ΔN *e-folds* added. Overall view and zoom.





The CMB power spectrum for a pre-inflation radiation era without any black holes, for various cases of Δ*N*. Overall view and zoom.

Conclusions and outlook

- Investigated effects of pre inflation era on CMB power spectrum
- BH nucleation induces a pre inflation matter dominated era
- Computed (analytically and numerically) the power spectrum of primordial fluctuations of a scalar field Φ living in this scenario
- The primordial power spectrum processed by CMBFAST code to yield the CMB temperature anisotropy power spectrum, compared with observations
- Alternative scenarii investigated: pre inflation radiation dominated era (with no remnants or with no BH at all)
- **The pre inflation matter model seems to be the only one, among those studied, able to capture and describe the *l = 2* mode suppression, although the radiation model still presents a better fitting of the data at high *l* values**.