Generalized Stochastic Inflation

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Introduction

- Subject of Our Research

  A MONODROMY BRANE INFLATION MODEL, which was set up by E. Silverstein and A. Westphal

  PRD **77** 106006, arXiv 0803.3085)

- The Property of the Model

  Be able to go clear the Lyth Bound

  What is *Lyth Bound*??

  A inflaton ϕ 's variance between end and start of the inflaiton is greater than M_{Pl} , if the gravity wave can be detected in CMB.

  D.H.Lyth PRL **78** 1861(1997)

- Purpose of Our Research

  To investigate a condition whether eternal inflation would occur or not, using the stochastic method to deal in quantum fluctuations of the system.

About Monodromy Model(1)

- Ordinary Brane Inflation

  Inflaton$\phi \longleftrightarrow$ Brane Position

  $\Longrightarrow$ ϕ can move only compactified space

  $\Longrightarrow$Difficult to go clear the Lyth Bound

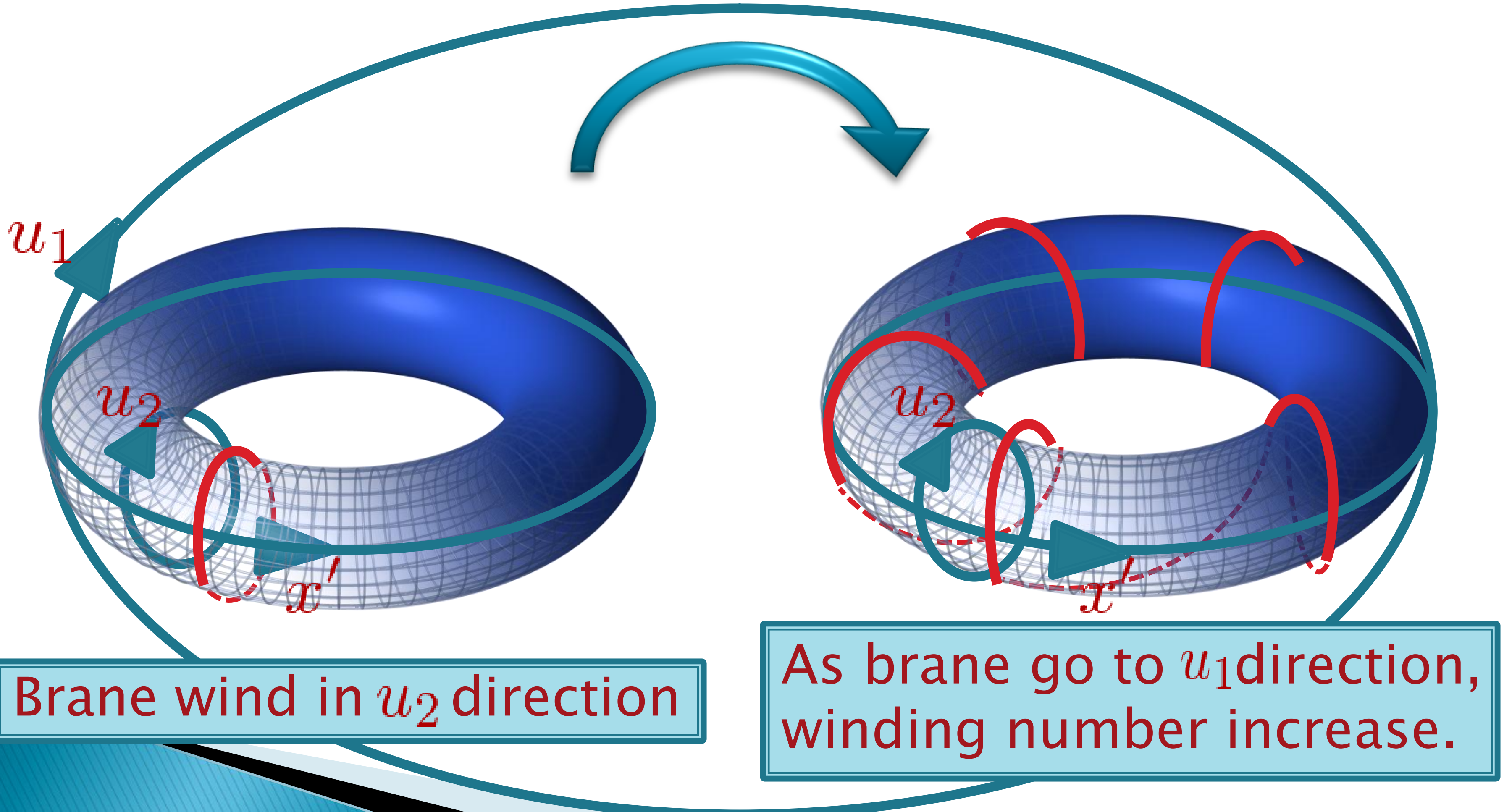
  $\Longrightarrow$Difficult to Explain Our Universe

- Brane Monodromy Inflation Model

  It can retain the Large Field Range by atypical Compactification.

About Monodromy Model(2)

- Simplified Explanation of Monodromy's Topology



About Monodromy Model(3)

- Mechanism of Monodromy Inflation
  - Winding Length $L_{4B} = \sqrt{L_{u_2}^2 + M^2 L_x^2 u_1^2}$
  - Energy $\sim L_{4B}$ × String Tension × Brane Tension

⇒
- The more u_1, The much Energy the brane has.
- Brane can move u_1 direction

→ *Lyth Bound is met!!*

Conclusion

1. Inflation happens while a winding brane moves in u_1 direction.
2. Brane's u_1 position plays Inflaton Field.

About Monodromy Model (4)

- Definition of Physical Variables

L_x, L_{u_1}, L_{u_2} → Size of interior space

$\beta = L_{u_2}/L_{u_1}$ → Ratio of the Size of interior space

M → Winding number as brane go round

g_s → String Coupling

$\frac{1}{\alpha'} = \frac{(2\pi)^7}{2} g^2$ → String Tension

f_6 → Quantum Flux Number

About Monodromy Model (5)

- Dynamical Equations
  - Action

$$S = -\Xi \int d^4x \left(\sqrt{(1+\Theta u_1^2)(1-\Gamma \dot{u}_1^2)} - 1 \right)$$

  - Equation of Motion

$$(1+\Theta u_1^2)\Gamma \ddot{u}_1 = -(1-\Gamma \dot{u}_1^2)\left(3H(1+\Theta u_1^2)\Gamma \dot{u}_1 + \Theta u_1\right)$$

  - Friedmann Equation

$$H^2 = \frac{8\pi\Xi}{3}\left(\sqrt{(1+\Theta u_1^2)/(1-\Gamma \dot{u}_1)} + \sqrt{(1+\Theta u_1^2)(1-\Gamma \dot{u}_1)} - 1 \right)$$

- Effective Physical Variables

$\Gamma = \alpha' L_u^2/\beta$ → Plays as time coordinate

$\Theta = M^2 L_x^2/\beta L_u^2$ → Plays as field's mass ($M \uparrow \Rightarrow \Theta \uparrow$)

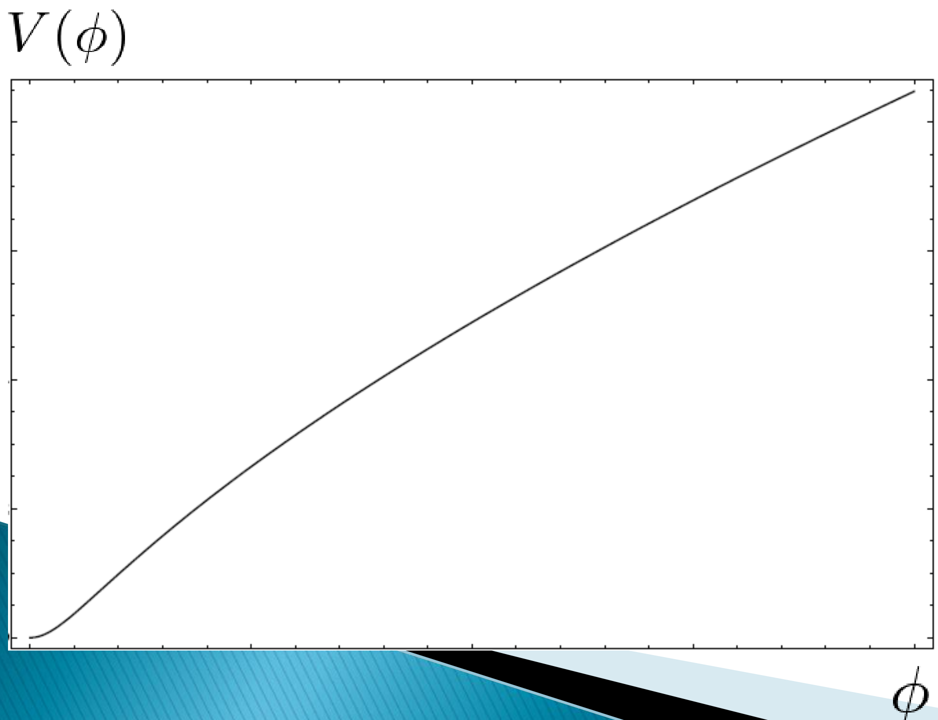
$\Xi = \sqrt{\beta} L_u/(2\pi)^4 g_s \alpha'^2$ → Plays as String Coupling Constant ($g_s \uparrow \Rightarrow \Xi \uparrow$)

About Monodromy Model(6)

- Canonical Approximation

If $\Gamma \dot{u}_1^2 \ll 1$, we can approximate the kinetic term canonically.

$$S_{D4} = \int d^4x \sqrt{-g_4} \left(\frac{1}{2}\dot{\phi}^2 - V(\phi) \right) \qquad V(\phi) \propto \begin{cases} \phi^2 & (u_1 \ll \Theta^{-1/2}) \\ \phi^{2/3} & (u_1 \gg \Theta^{-1/2}) \end{cases}$$



Variable Transformation

- For $u_1 \ll \Theta^{-1/2}$

$$\phi = \frac{L_u^{3/2}}{(2\pi)^2 \sqrt{g_s \alpha'} \beta^{1/4}} u_1$$

- For $u_1 \gg \Theta^{-1/2}$

$$\phi = \frac{L_u^{3/2} \beta^{-1/4}}{3(2\pi)^2 \sqrt{g_s \alpha'}} \left[F^2_{1,\frac{1}{2},\frac{3}{4},\frac{3}{2}} \left(-\frac{M^2 L_x^2}{\beta L_u^2} u_1^2 \right) + 2 \left(1 + \frac{M^2 L_x^2}{\beta L_u^2} u_1^2 \right)^{1/4} \right]$$

About Eternal Inflation (1)

- By Approximation --The Conditions for Eternal Inflation

Condition 1 $\phi^4 > \sqrt{\Gamma/(\Theta\Xi^2)}$

Canonical Approx.

・This means quantum fluctuation $\delta\phi = \dfrac{H}{2\pi}$ is much lager than classical motion $\Delta\phi = \dfrac{|\dot{\phi}|}{H}$

Slow-Roll Approx.

Condition 2 $\phi < 3\Theta/2\Gamma\Xi$

・This means Moduli Stabilization.

Considering these two conditions,
if $\dfrac{\Theta^3}{\Gamma^2\Xi^3} \gg 1$ are met, Eternal Inflation happens !!

Can we deal in quantum effects without any approximations ??

What happens if DBI-Action is influential ??

About Eternal Inflation(2)

Supplement Stochastic Inflation

- Purpose
  - To deal in quantum fluctuations of Inflaton.
  - To acquire Effective theory which isn't so much difficult.
- Method
  - Inflation is so macroscopic phenomenon.
  - Field's Large Scale behavior

⇒ We can divide fields into long wavelength mode and short wavelength mode.

{ Long mode … obeys classical motion.
Short mode … represents quantum fluctuation.

About Eternal Inflation(3)

Supplement Stochastic Inflation

- How should we deal in short wavelength mode?
  - The expectation value of short mode $\sigma(x)$ in Bunch–Davis Vacuum is as follows;

$$\langle\sigma(x_1)\sigma(x_2)\rangle = \frac{H^3}{4\pi^2} j_0(\epsilon a H|x_1 - x_2|)\delta(t_1 - t_2)$$

$j_0(x)$:Spherical Bessel Function

  - Then we can regard short mode as Brownian motion.

⇒ The equation of motion is modified by addition of white noise $n(t)$

K. Nakao, Y. Nambu, M. Sasaki(1988)

About Eternal Inflation(4)

- Numerical Simulation

  Equation of Motion is modified in a way that de-Sitter fluctuation is duplicated if we approximate the kinetic term as canonical.

$$\left(1+\Theta u_1^2\right)\Gamma\ddot{u}_1+\left(1-\Gamma\dot{u}_1^2\right)\left(3H\left(1+\Theta u_1^2\right)\Gamma\dot{u}_1+\Theta u_1\right)$$

$$=\left\{\left(1+\Theta u_1^2\right)\Gamma^2/\Xi^2\right\}^{\frac{1}{4}}H^{\frac{5}{2}}n(t)$$

  We could deal in Non-Canonical Effect with accuracy !!

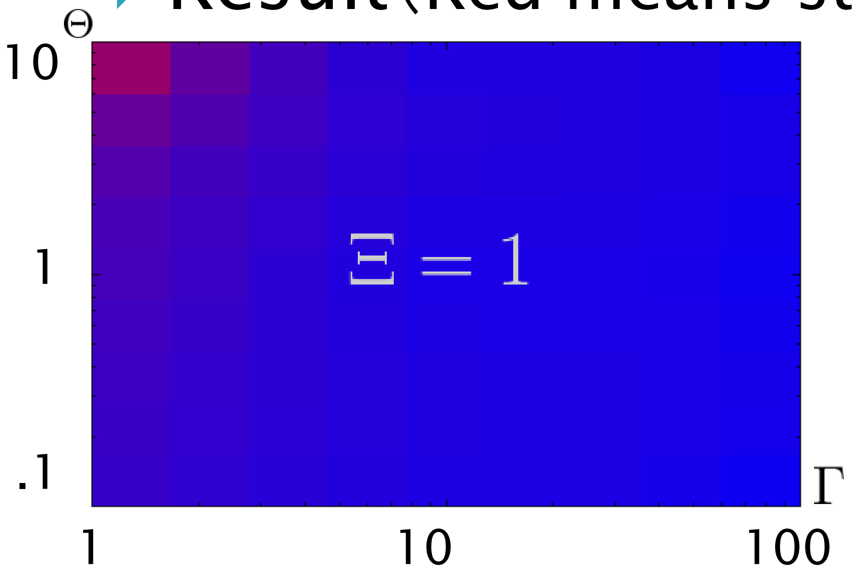
- How determinate ??

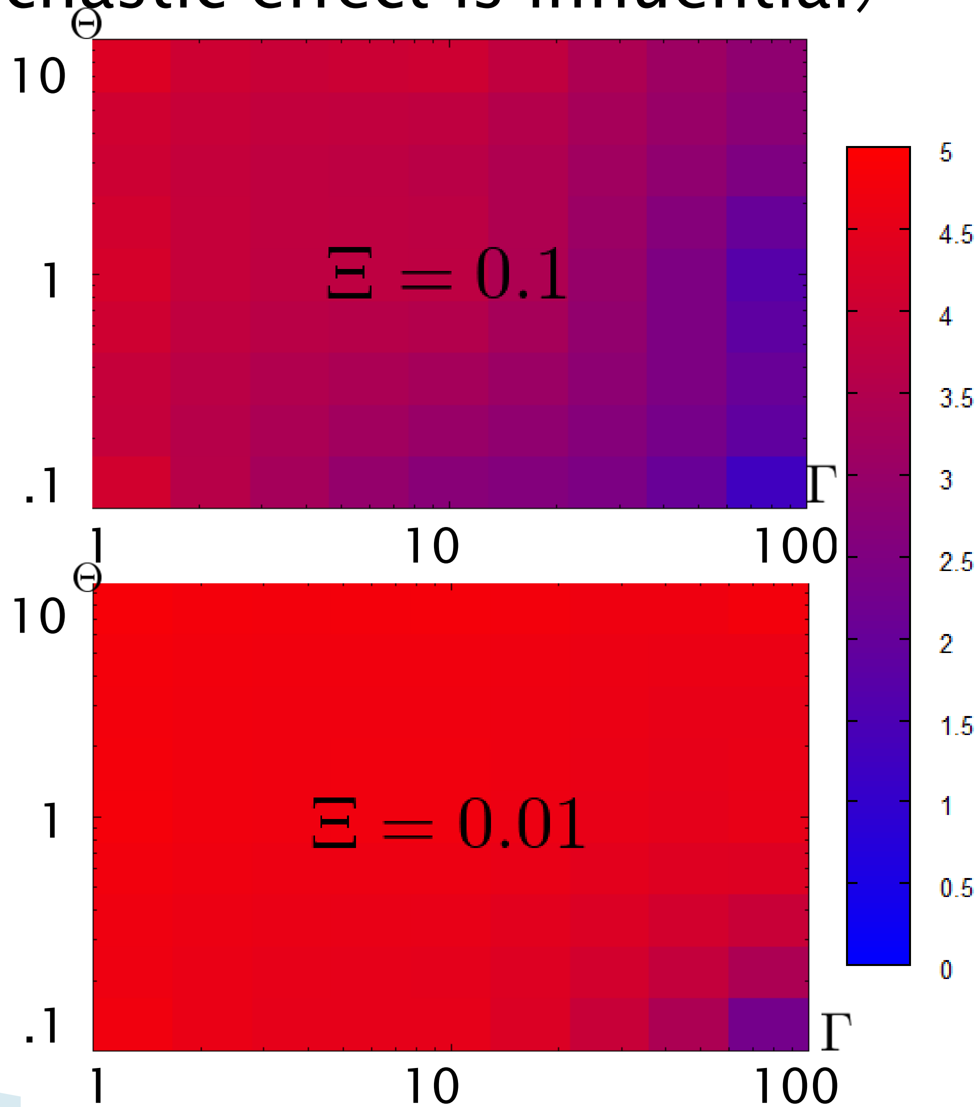
  Place of stochastic solution when classical solution converged at the origin.

$\phi\approx 0$ →Stochastic Effect is negligible $\phi\gg 1$→Stochastic Effect is influential

About Eternal Inflation(5)

- Result(Red means stochastic effect is influential)





If $\frac{\Theta^3}{\Gamma^2 \Xi^3} = \frac{g_s M^6}{(2\pi)^7} \frac{L_x^{10}}{L_{u_1}^3 L_{u_1}^8} \gtrsim 10$,

stochastic effect is influential.

$\Downarrow$

Large M leads Eternal Inflation

Result and Discussions

- About Monodromy Model
  - Very useful model which is go clear Lyth Bound.
  - But it is so strange that the brane has large winding number M

- About Eternal Inflation
  - If $\frac{\Theta^3}{\Gamma^2 \Xi^3} = \frac{g_s M^6}{(2\pi)^7} \frac{L_x^{10}}{L_{u_1}^3 L_{u_1}^8} \gtrsim 10$ is met, we confirmed that eternal inflation occurs!!
  - Considering expansion effect in physical volume, the probability of large M is so high, then we can conclude that ***Monodromy Model is very down-to-earth idea to explain our universe !!***

Appendix Metric of Monodromy

- Details of Model

Extra dimensions are compactified as Nil-manifold in 10 dimensional spacetime (type IIA).

$$\frac{ds^2}{\alpha'} = L_{u_1}^2 du_1^2 + L_{u_2}^2 du_2^2 + L_x \left(dx' + M u_1 du_2\right)^2 \quad x' = x - \frac{M}{2} u_1 u_2$$

Compactification by isometry groups

$$\begin{cases} t_x : (x, u_1, u_2) \simeq (x+1, u_1, u_2) \\ t_{u1} : (x, u_1, u_2) \simeq (x - M/2, u_1 + 1, u_2) \\ t_{u2} : (x, u_1, u_2) \simeq (x + M/2, u_1, u_2 + 1) \end{cases}$$

- Property of model

T2(u2 and x' direction) attach to S1(u1)

Twisted Torus!!