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Non-linear evolution of matter power spectrum in a closure theory

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Closure Approxmation (CLA)

Taruya, Hiramatsu, ApJ 674 (2008) 617

- An alternative theoretical framework to measure the baryon acoustic oscillation scale to probe the dark energy.
- Based on fluid description of CDM+baryon components.
  - Diagramatic representation of naïve perturbation series.
  - Renormalised expressions of them in terms of three non-perturbative quantities :

    Power spectrum, *Propagator*, *Vertex function*

  - Truncation of the renormalised expressions at 1-loop order + tree-level vertex function

Closed system coupled with power spectrum and propagator

Contents

In this poster, we apply our closure formalism to 1-loop standard perturbation theory (SPT) which can be realised by replacing all quantities in the mode coupling term by linear solutions, and show …

- formalism consistent to 1-loop SPT, and how to solve the closure equation derived in ApJ 674 (2008) 617,
- numerical check to recover the 1-loop SPT results,
- resultant power spectra in
  - time-varying dark energy model
  - DGP model where the Poisson equation has non-linear terms coming from 2^{nd}-order brane bending mode.

Actors

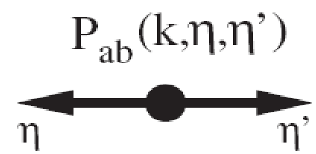
Vector representation of perturbative quantities $\Phi_a(\vec{k}) = \begin{pmatrix} \delta(\vec{k}) \\ -\theta(\vec{k}) \end{pmatrix}$

Irrotational velocity field $\theta = \dfrac{i(\vec{k}\cdot\vec{\mathrm{v}})}{aH}$

- Non-linear power spectrum

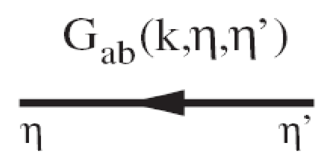
$$\left\langle \Phi_a(\vec{k},\tau)\Phi_b(\vec{k}',\tau)\right\rangle = (2\pi)^3\delta_D(\vec{k}+\vec{k}')\; P_{ab}(k,\tau)$$

$$\left\langle \Phi_a(\vec{k},\tau)\Phi_b(\vec{k}',\tau')\right\rangle = (2\pi)^3\delta_D(\vec{k}+\vec{k}')\; R_{ab}(k,\tau,\tau')$$



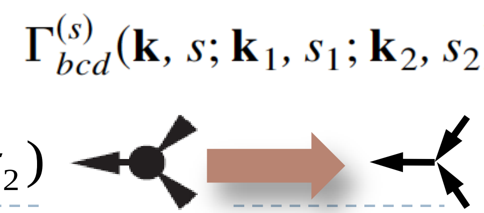
- Propagator

$$\left\langle \frac{\delta\Phi_a(\vec{k};\tau)}{\delta\Phi_b(\vec{k}';\tau')}\right\rangle = \delta_D(\vec{k}-\vec{k}')\; G_{ab}(k\,|\,\tau,\tau')$$



- Non-linear vertex function

$$\left\langle \frac{\delta\Phi_a(\vec{k};\tau)}{\delta\Phi_b(\vec{k}_1;\tau_1)\delta\Phi_c(\vec{k}_2;\tau_2)}\right\rangle = \delta_D(\vec{k}-\vec{k}_1-\vec{k}_2)\; \Gamma_{abc}(\vec{k}_1,\vec{k}_2\,|\,\tau,\tau_1,\tau_2)$$



Replaced (CLA)

Script

- Euler equation + conservation equation yields …

$$\Lambda_{ab} G_{bc}(|\boldsymbol{k}|;\tau,\tau') = \int_{\tau'}^{\tau} d\tau'' M_{as}(|\boldsymbol{k}|;\tau,\tau'') G_{sc}(|\boldsymbol{k}|;\tau'',\tau')$$

$$\Lambda_{ab} R_{bc}(|\boldsymbol{k}|;\tau,\tau') = \int_{\tau_0}^{\tau} d\tau'' M_{as}(|\boldsymbol{k}|;\tau,\tau'') R_{\overline{sc}}(|\boldsymbol{k}|;\tau'',\tau') + \int_{\tau_0}^{\tau'} d\tau'' N_{al}(|\boldsymbol{k}|;\tau,\tau'') G_{cl}(|\boldsymbol{k}|;\tau',\tau'')$$

$$\Sigma_{abcd} P_{cd}(|\boldsymbol{k}|;\tau) = \int_{\tau_0}^{\tau} d\tau'' M_{bs}(|\boldsymbol{k}|;\tau,\tau'') R_{as}(|\boldsymbol{k}|;\tau,\tau'') + \int_{\tau_0}^{\tau} d\tau'' N_{bl}(|\boldsymbol{k}|;\tau,\tau'') G_{al}(|\boldsymbol{k}|;\tau,\tau'') + (a \leftrightarrow b)$$

$$\tau = -\log(1+z)$$

The original closure equation is expressed by different forms. The present symmetric forms are more suitable for numerical calculations than the original ones. In addition, while the original version uses the growth rate as the time, we here use the scale factor in view of the application to modified gravities.

Script

- Left-hand side operator

$$\Lambda_{ab} = \delta_{ab}\frac{\partial}{\partial\tau} + \Omega_{ab}(\tau)$$

$$\Omega_{ab}(\tau) = \begin{pmatrix} 0 & -1 \\ -4\pi G\frac{\rho_m}{H^2}\sigma & 2+\frac{\dot{H}}{H^2} \end{pmatrix}$$

$\sigma(k,\tau)$ represents a correction of the gravity constant appeared in the linear Poisson equation. In the standard theory, $\sigma(k,\tau)=1$ while it is not generally true in modified gravity theories.

- Integral kernels

$$M_{as}(|\boldsymbol{k}|;\tau,\tau'') = 4\int\frac{d^3k'}{(2\pi)^3}\gamma_{apq}(\boldsymbol{k}-\boldsymbol{k}',\boldsymbol{k}')\gamma_{lrs}(\boldsymbol{k}'-\boldsymbol{k},\boldsymbol{k}) \times G_{ql}(|\boldsymbol{k}'|,\tau,\tau'')R_{pr}(|\boldsymbol{k}-\boldsymbol{k}'|;\tau,\tau'')$$

$$N_{al}(|\boldsymbol{k}|;\tau,\tau'') = 2\int\frac{d^3k'}{(2\pi)^3}\gamma_{apq}(\boldsymbol{k}-\boldsymbol{k}',\boldsymbol{k}')\gamma_{lrs}(\boldsymbol{k}-\boldsymbol{k}',\boldsymbol{k}') \times R_{qs}(|\boldsymbol{k}'|,\tau,\tau'')R_{pr}(|\boldsymbol{k}-\boldsymbol{k}'|;\tau,\tau'')$$

In CLA, the vertex functions are replaced by ones with tree-level approximation. Only γ_{112} γ_{121} γ_{222} have non-zero value.

Formal solution

- Complex combination of evolution equations yields…

$$R_{ab}(|\boldsymbol{k}|;\tau,\tau') = G_{ac}(|\boldsymbol{k}|;\tau,\tau_0)G_{bd}(|\boldsymbol{k}|;\tau',\tau_0)P_{cd}(|\boldsymbol{k}|;\tau_0)$$
$$+\int_{\tau_0}^{\tau} d\tau_1 \int_{\tau_0}^{\tau} d\tau_2 G_{ac}(|\boldsymbol{k}|;\tau,\tau_1)G_{bd}(|\boldsymbol{k}|;\tau',\tau_2)N_{cd}(\boldsymbol{k};\tau_2,\tau_1)$$

$$G_{ab}(|\boldsymbol{k}|;\tau,\tau') = g_{ab}(|\boldsymbol{k}|;\tau,\tau') + \int_{\tau'}^{\tau} d\tau''' \int_{\tau'}^{\tau} d\tau'' g_{ac}(|\boldsymbol{k}|;\tau,\tau''')$$
$$\times M_{cs}(|\boldsymbol{k}|;\tau''',\tau'')G_{sb}(|\boldsymbol{k}|;\tau'',\tau')$$

Linearised closure equation = SPT

The closure equation mentioned above has all non-linear contributions in 1-loop level except for the vertex function.

An approximation we can easily take here is to replace all quantities in the right-hand by those obtained in the linear theory.

= 'linearised closure equation'.

In our previous paper, we have proven an important fact :

> The solution of linearised closure equation coincides with the prediction of 1-loop standardperturbation theory (SPT)

Linear solution

- Dropped all right-hand side terms

$$\Lambda_{ab} G^0_{bc}(|\boldsymbol{k}|;\tau,\tau') = 0$$

$$\Lambda_{ab} R^0_{bc}(|\boldsymbol{k}|;\tau,\tau') = 0$$

In the matter dominant Universe, the power spectrum takes the form :

$$R^0_{sc}(|\boldsymbol{k}|;\tau,\tau'') = e^{\tau+\tau''} P_0(k) \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$ + (decaying solution)

where $P_0(k)$ is the linearly extrapolated power spectrum at the present time. We use this (neglecting the decaying solution) as the initial condition of the above equation, and normalise the solution by a given present amplitude, σ_8 .

As for the propagator, the initial condition is given by its definition :

$$G^0_{ab}(|\boldsymbol{k}|;\tau_0,\tau_0) = \delta_{ab}$$

Linearised closure equation

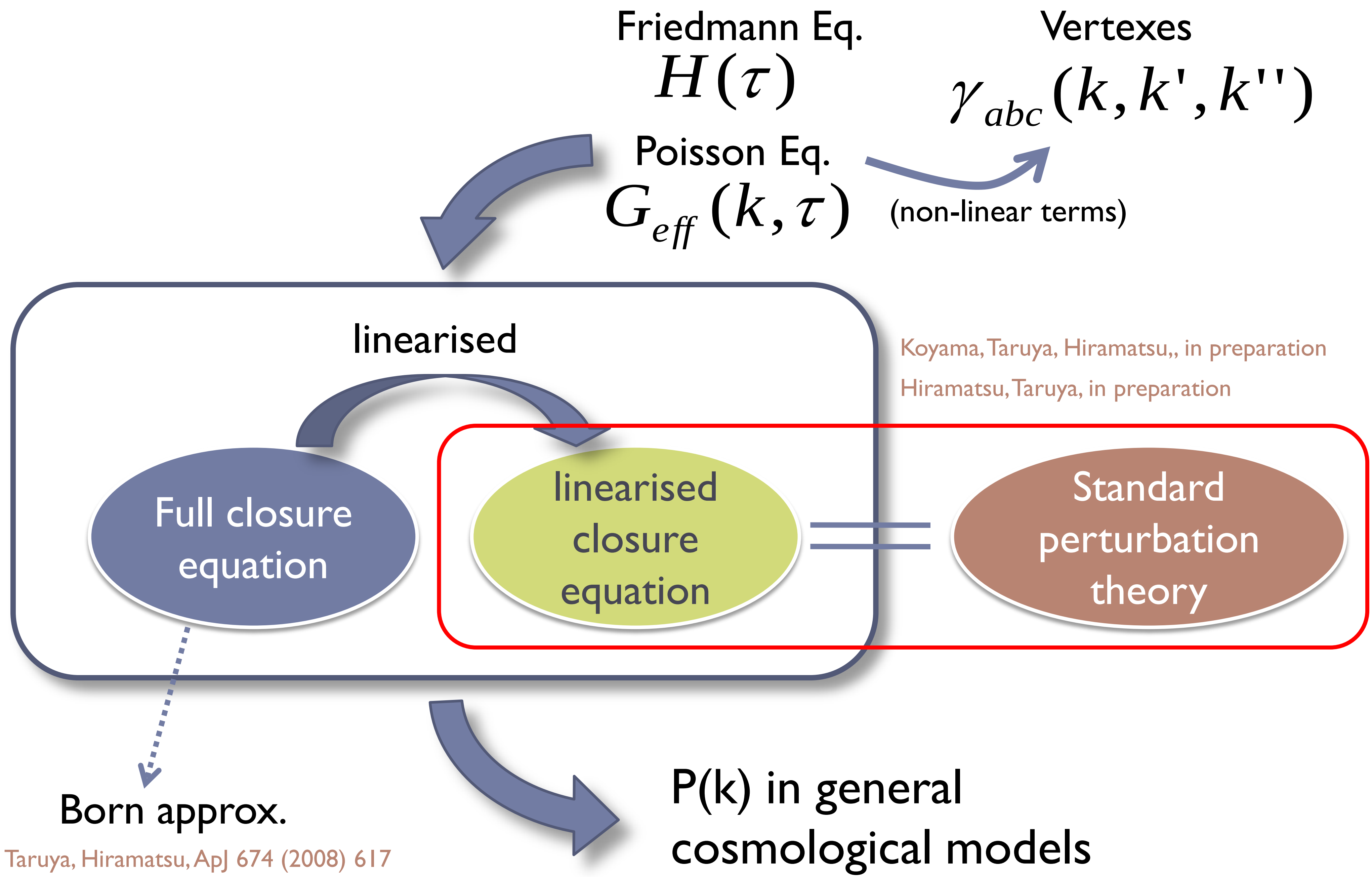
- Replacing all right-hand side quantities by linear ones

$$\Lambda_{ab} G^L_{bc}(|\boldsymbol{k}|;\tau,\tau') = \int_{\tau'}^{\tau} d\tau'' M^L_{as}(|\boldsymbol{k}|;\tau,\tau'') G^0_{sc}(\tau'',\tau')$$

$$\Lambda_{ab} R^L_{bc}(|\boldsymbol{k}|;\tau,\tau') = \int_{\tau_0}^{\tau} d\tau'' M^L_{as}(|\boldsymbol{k}|;\tau,\tau'') R^0_{sc}(|\boldsymbol{k}|;\tau'',\tau') + \int_{\tau_0}^{\tau'} d\tau'' N^L_{al}(|\boldsymbol{k}|;\tau,\tau'') G^0_{cl}(|\boldsymbol{k}|;\tau',\tau'')$$

$$M^L_{as}(|\boldsymbol{k}|;\tau,\tau'') = 4\int \frac{d^3k'}{(2\pi)^3} \gamma_{apq}(\boldsymbol{k}-\boldsymbol{k}',\boldsymbol{k}')\gamma_{lrs}(\boldsymbol{k}'-\boldsymbol{k},\boldsymbol{k}) \times G^0_{ql}(|\boldsymbol{k}'|,\tau,\tau'') R^0_{pr}(|\boldsymbol{k}-\boldsymbol{k}'|;\tau,\tau'')$$

$$N^L_{al}(|\boldsymbol{k}|;\tau,\tau'') = 2\int \frac{d^3k'}{(2\pi)^3} \gamma_{apq}(\boldsymbol{k}-\boldsymbol{k}',\boldsymbol{k}')\gamma_{lrs}(\boldsymbol{k}-\boldsymbol{k}',\boldsymbol{k}') \times R^0_{qs}(|\boldsymbol{k}'|,\tau,\tau'') R^0_{pr}(|\boldsymbol{k}-\boldsymbol{k}'|;\tau,\tau'')$$



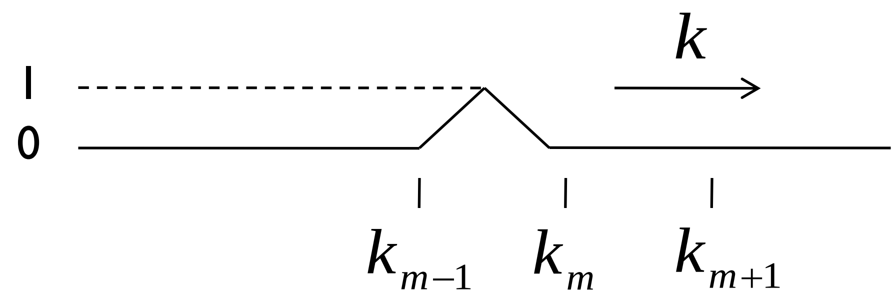
Friedmann Eq.
$H(\tau)$
Poisson Eq.
$G_{eff}(k,\tau)$
Vertexes
$\gamma_{abc}(k,k',k'')$
(non-linear terms)
linearised
Koyama, Taruya, Hiramatsu,, in preparation
Hiramatsu, Taruya, in preparation
Full closure equation
linearised closure equation
Standard perturbation theory
Born approx.
Taruya, Hiramatsu, ApJ 674 (2008) 617
P(k) in general cosmological models

Strategy for numerics

cf. Valageas AA465 (2007) 725

1. Expand G_{ab} and R_{ab} by a set of basis functions of k

$$G_{ab}(k,\tau) = \sum_m G_{ab,m}(\tau) T_m(k); \quad T_m(k) =$$



2. Integrate functions of k appeared in M_{ab} and N_{ab}

3. Replace the differential operator by the central difference

$$\left.\frac{\partial g}{\partial \tau}\right|_{\tau=\tau_n} \approx \frac{g^{(n+1)} - g^{(n-1)}}{2\Delta\tau}$$

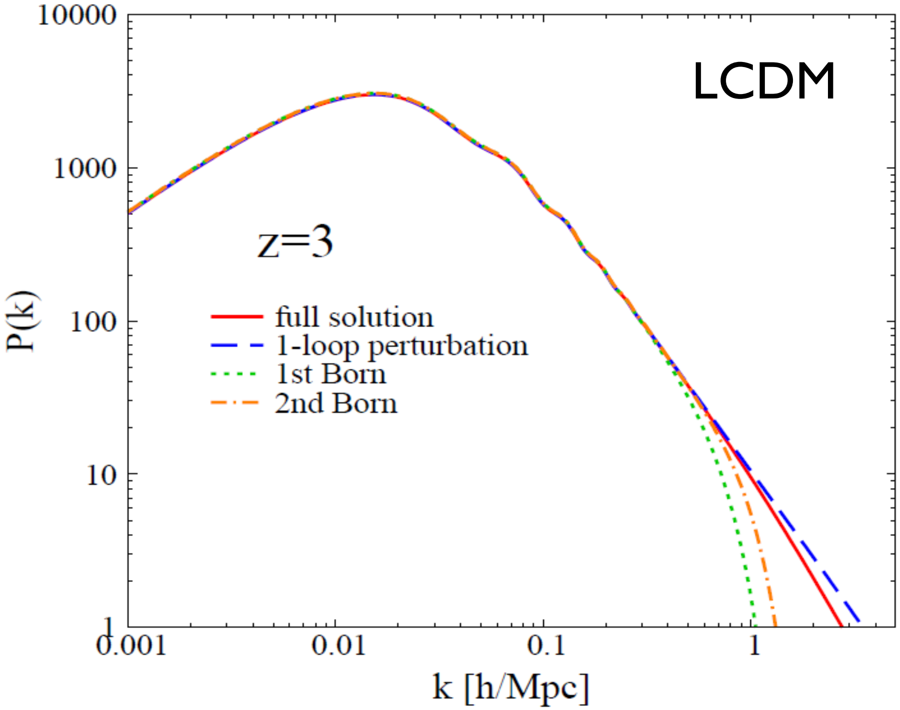
4. Apply the trapezoidal rule to the time-integrations

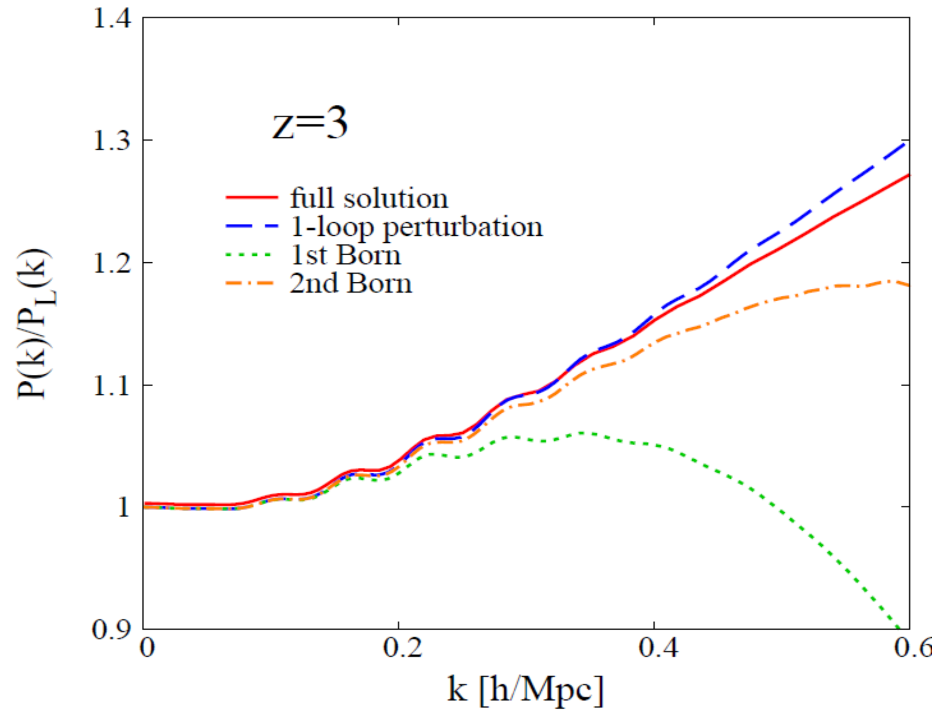
$$\int g(\tau) d\tau \frac{\Delta\tau}{2} \sum_n \left(g^{(n)} + g^{(n+1)}\right)$$

5. Sequentially solve the recursive equations

Result 0 : demonstration of ‘Full’ closure Eq.

- Power spectrum in LCDM



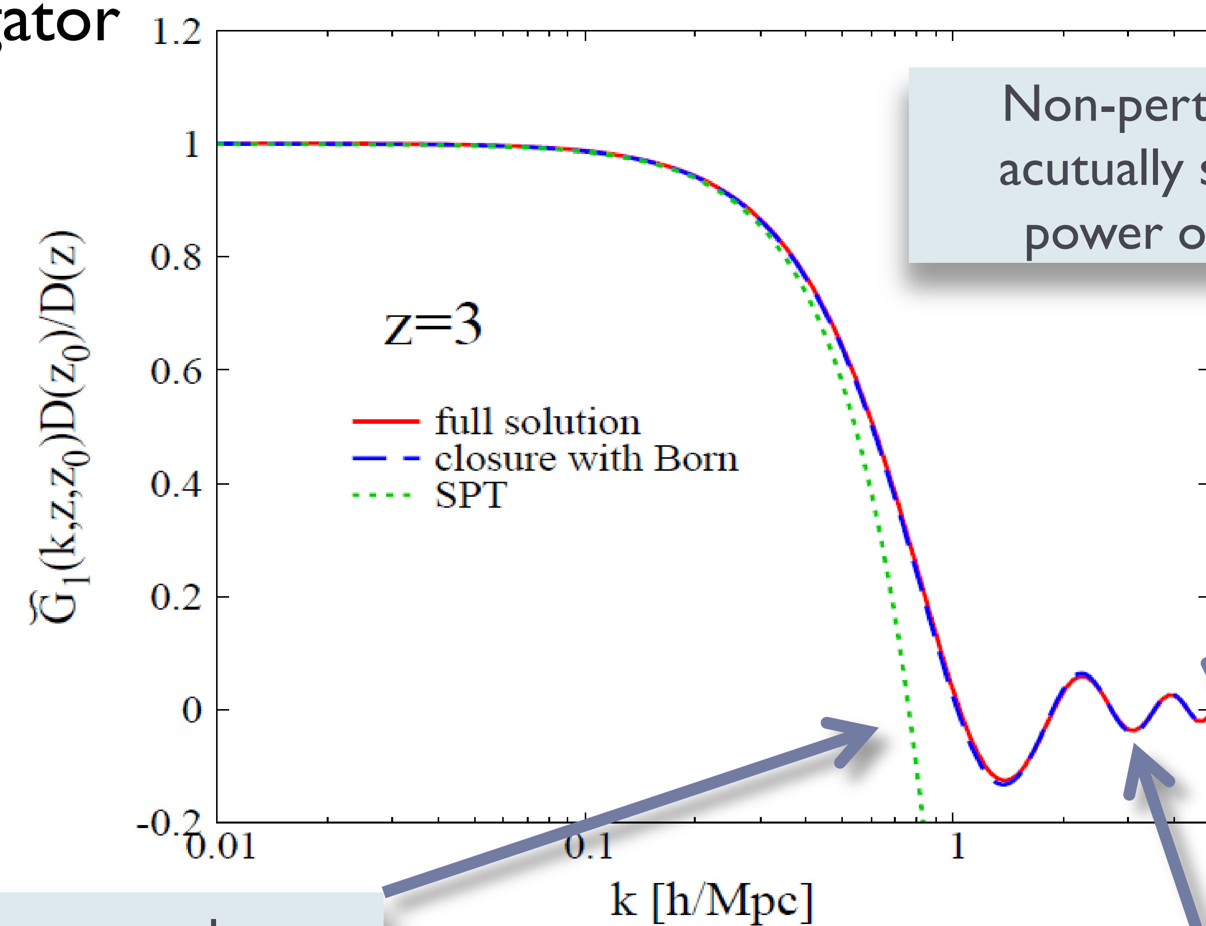


1-loop SPT (blue) yields a little large power because of breakdown of SPT

Born approximation to the formal solution of closure eq. suppresses the small scale power.

Result 0 : demonstration of 'Full' closure Eq.

- Propagator

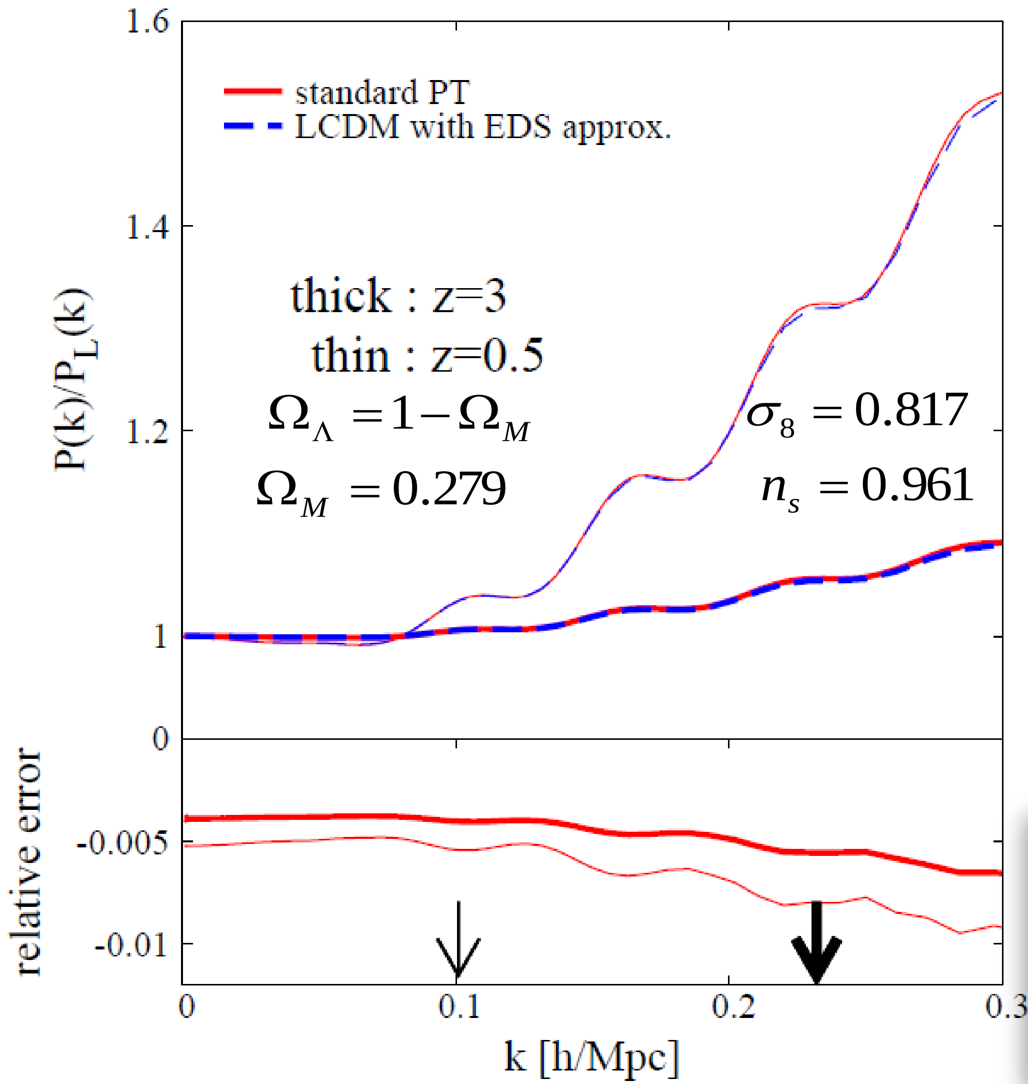


Non-perturbative effect acutually suppresses the power on small scales

Leading to an anomalous enhancement of power

But, there is unphysical oscillation due to truncation at 1-loop

Result I : Recovery of SPT results



SPT frequently uses Einstein-de Sitter approximation : $\delta(\tau,k) = \sum D_L^n(\tau)\delta_n(k)$ which is realised by a small modification of **left-hand side operator as**

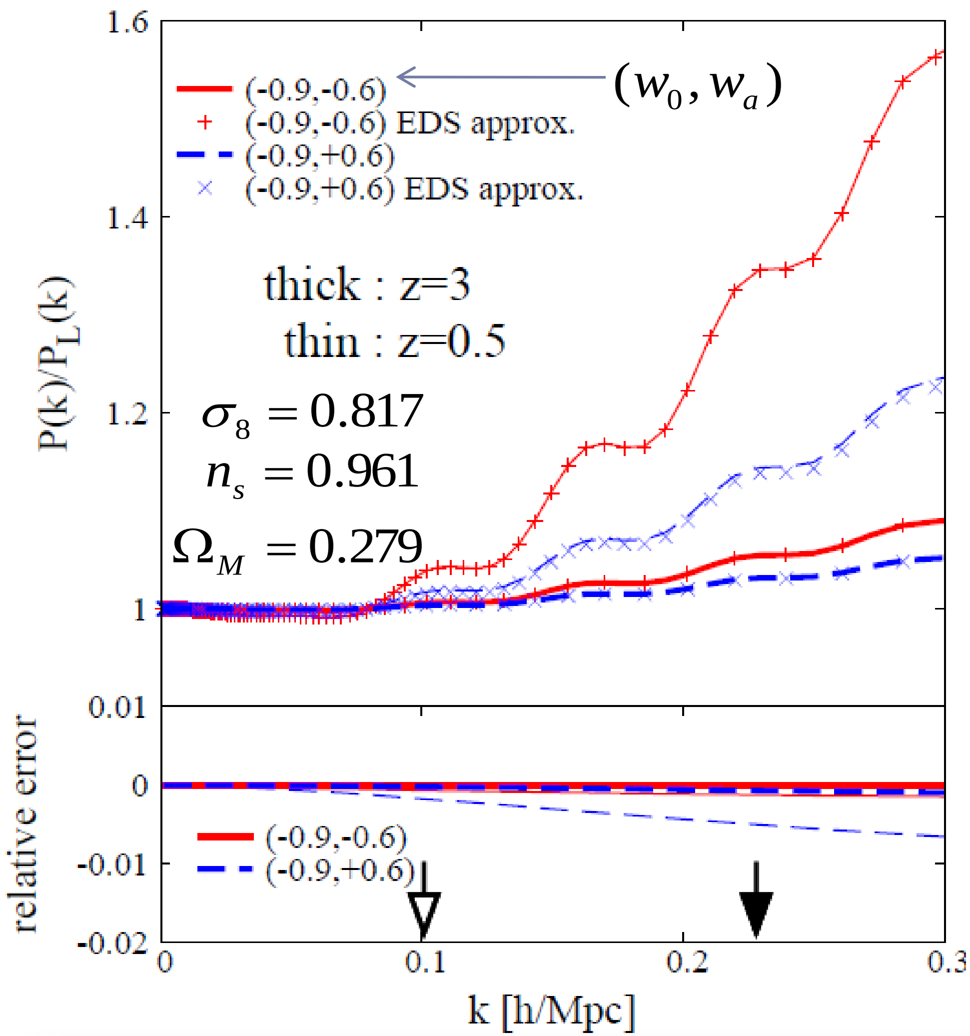
$$\Omega^{EDSapprox.}{}_{ab}(\tau) = \begin{pmatrix} 0 & -1 \\ -\frac{3}{2}f^2 & \frac{f}{2} - \frac{1}{f}\frac{df}{d\tau} \end{pmatrix}$$

$$f(\tau) = \frac{d \log D_L(\tau)}{d\tau}$$

$D_L(\tau)$: Linear growth rate

Our numerical scheme can recover the results of standard perturbation theory with 1-loop corrections.

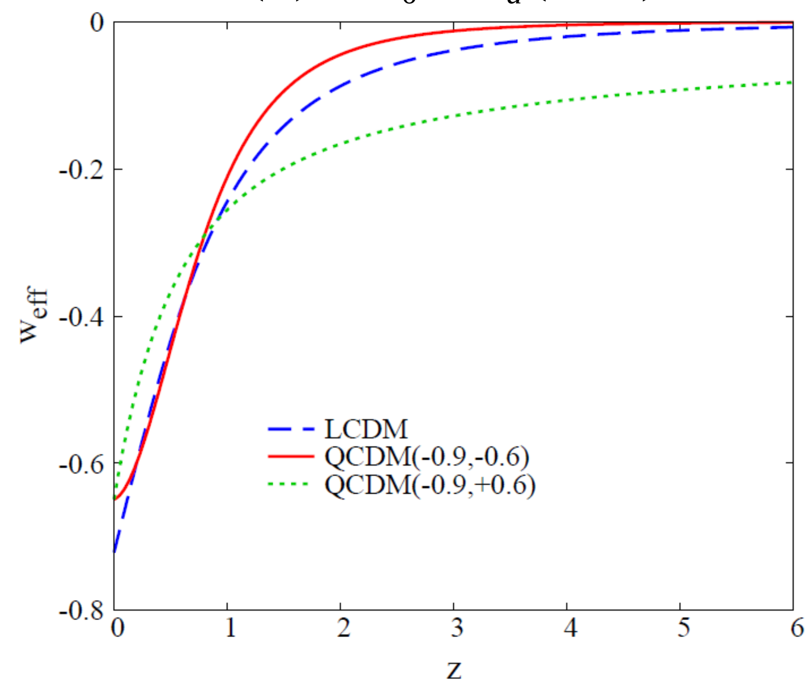
Result II : Validity of EdS approx.



Next we investigate a general dark energy model in which the EOS parameter varies like

Linder PRL 90 (2003) 091301

$$w(a) = w_0 + w_a(1-a)$$



The approximation does not work especially near the present time.

Non-linearity of Poisson equation

- Extra time-dependent vertexes

$$-\frac{k^2}{a^2}\phi = 4\pi G\rho_m\delta + F(\delta)$$

$$\gamma_{211}(\boldsymbol{k}_1,\boldsymbol{k}_2;\tau) \qquad \sigma_{2111}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3;\tau)$$

$$\Lambda_{ab}R_{bc}(|\boldsymbol{k}|;\tau,\tau') = \int_{\tau_0}^{\tau} d\tau'' M_{as}(|\boldsymbol{k}|;\tau,\tau'')R_{\overline{sc}}(|\boldsymbol{k}|;\tau'',\tau')$$

$$+\int_{\tau_0}^{\tau'} d\tau'' N_{al}(|\boldsymbol{k}|;\tau,\tau'')G_{cl}(|\boldsymbol{k}|;\tau',\tau'')$$

$$+3\int \frac{d^3\boldsymbol{k}'}{(2\pi)^3}\sigma_{apqr}(\boldsymbol{k}',\text{-}\boldsymbol{k}',\boldsymbol{k};\tau)P_{pq}(|\boldsymbol{k}'|;\tau)P_{rc}(|\boldsymbol{k}|;\tau,\tau')$$

Non-linearity of Poisson equation

- Dvali-Gabadazde-Porratti braneworld model

$$ds^2 = -(1+2\Psi)N^2dt^2 + A^2(1+2\Phi)\delta_{ij}dx^i dx^j + 2r_c\varphi_{,i}dx^i dy + (1+2\Gamma)dy^2$$

Brane is at y=0

Brane bending mode

mode parameter

Poisson eq. $$\frac{2}{a^2}\nabla^2\Psi = \kappa^2\rho\delta + \frac{1}{a^2}\nabla^2\varphi$$

Expanded up to 2nd order $$\frac{3\beta}{a^2}\nabla^2\varphi + \frac{r_c}{a^4}\left[(\nabla^2\varphi)^2 - (\nabla_i\nabla_j\varphi)^2\right] = \kappa^2\delta\rho$$

$$\beta = 1 \pm 2Hr_c\left(1 + \frac{\dot{H}}{3H^2}\right)$$

Non-linear Poisson eq. $$\frac{1}{a^2}\nabla^2\Psi = \frac{\kappa^2}{2}\left(1 + \frac{1}{3\beta}\right)\delta\rho + O(\delta\rho^2)$$

Friedmann eq. $$H^2 \pm \frac{H}{r_c} = \frac{\kappa^2\rho}{3}$$

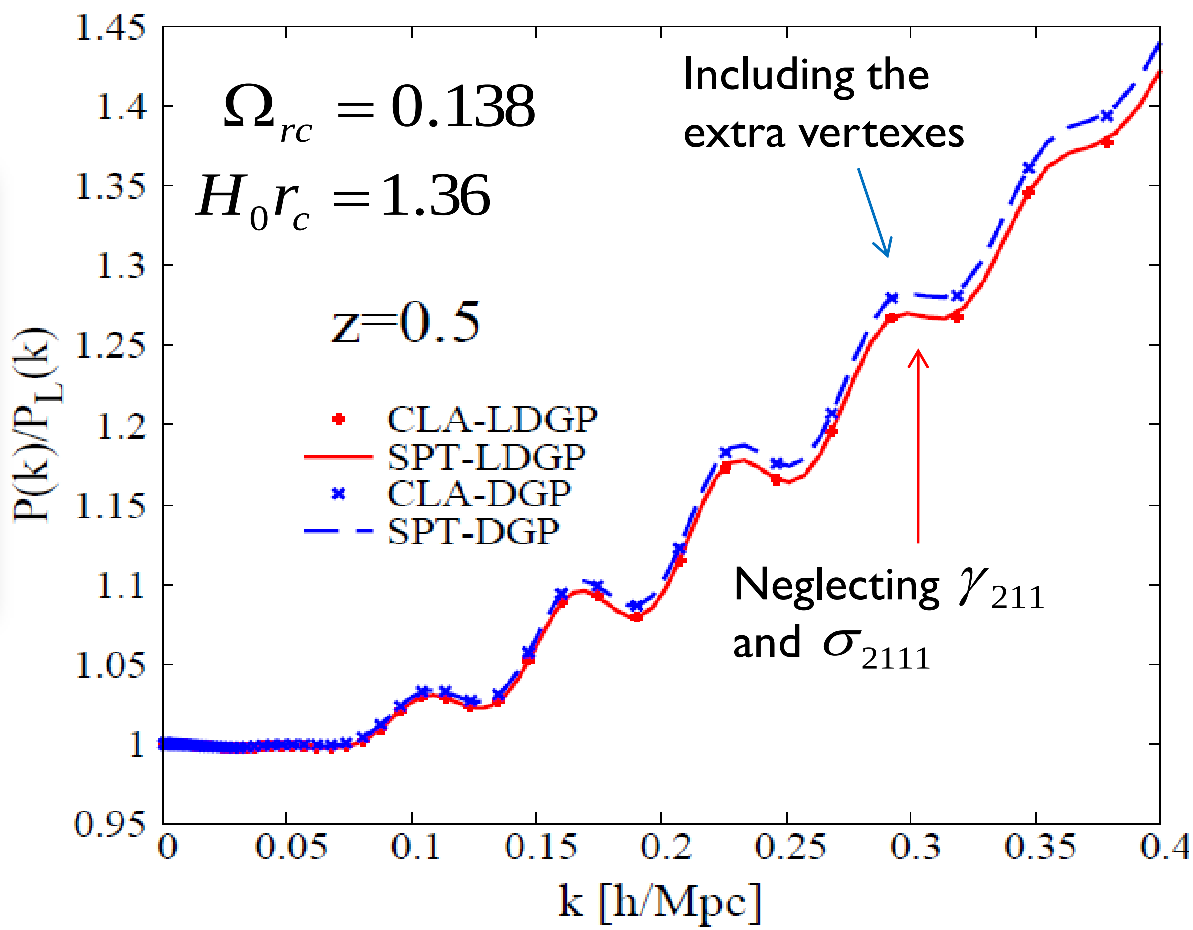
$$\gamma_{211}(\boldsymbol{k}_1, \boldsymbol{k}_2; \tau) = -\frac{(Hr_C \pm 1)^2}{6\beta^3}\left(1 - \frac{(\boldsymbol{k}_1 \cdot \boldsymbol{k}_2)^2}{k_1^2 k_2^2}\right) > 0$$

$$\sigma_{2111}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3; \tau) = \frac{Hr_c(Hr_c \pm 1)^3}{27\beta^5} \times \frac{k^2k'^2 - (\boldsymbol{k} \cdot \boldsymbol{k}')^2}{k^2k'^2}\left[2 - \frac{(\boldsymbol{k} \cdot (\boldsymbol{k}' + \boldsymbol{k}))^2}{k'^2|\boldsymbol{k}' + \boldsymbol{k}|^2} - \frac{(\boldsymbol{k} \cdot (\boldsymbol{k}' - \boldsymbol{k}))^2}{k'^2|\boldsymbol{k}' - \boldsymbol{k}|^2}\right] < 0$$

Result III : Power spectrum in DGP model

- Self-acceleration branch

γ_{211} gives a positive (dominant) contribution to mode couplings. In contrast, σ_{2111} gives a negative (sub-dominant) one. As a result, the power on small scales enhances about 3%.



Summary

We have focused the following fact :

Linearised closure equation = 1-loop standard perturbation theory

The confronting issue was …

Directly solve the (non-linear) simultaneous integro-partial-differential equation

This challenging task has provided ...

- We checked the recovery of 1-loop SPT calculation.
- As a demonstration, numerical solution of 'full' closure equation was presented. non-perturbative effects beyond 1-loop SPT.
- We found the Einstein-de Sitter approximation woks well even beyond LCDM model at small z.
- Finally we have demonstrated our method by applying to DGP model in which Poisson equation becomes to depend on wave number. This demonstration makes it clear that our unified method is capable of wide application.