HOLOGRAPHIC MEASURE OF THE MULTIVERSE

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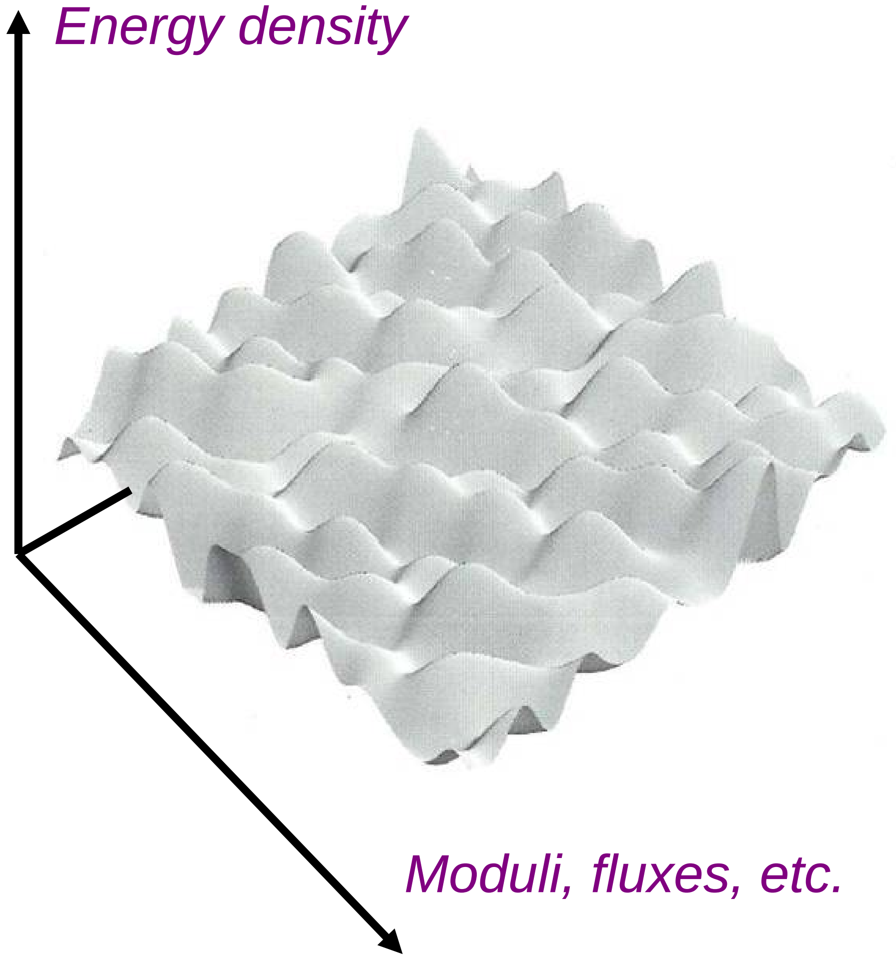
“String theory landscape”

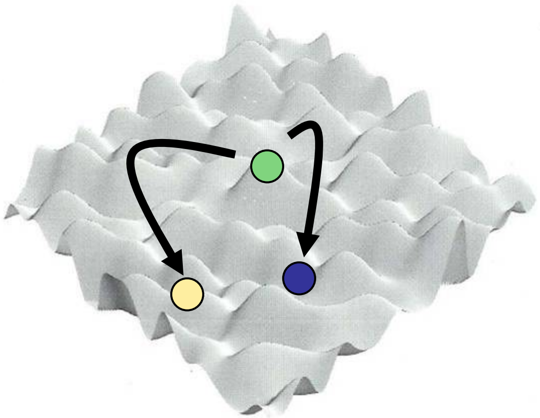


String theory suggests $\sim 10^{500}$ different vacua with different low-energy physics.

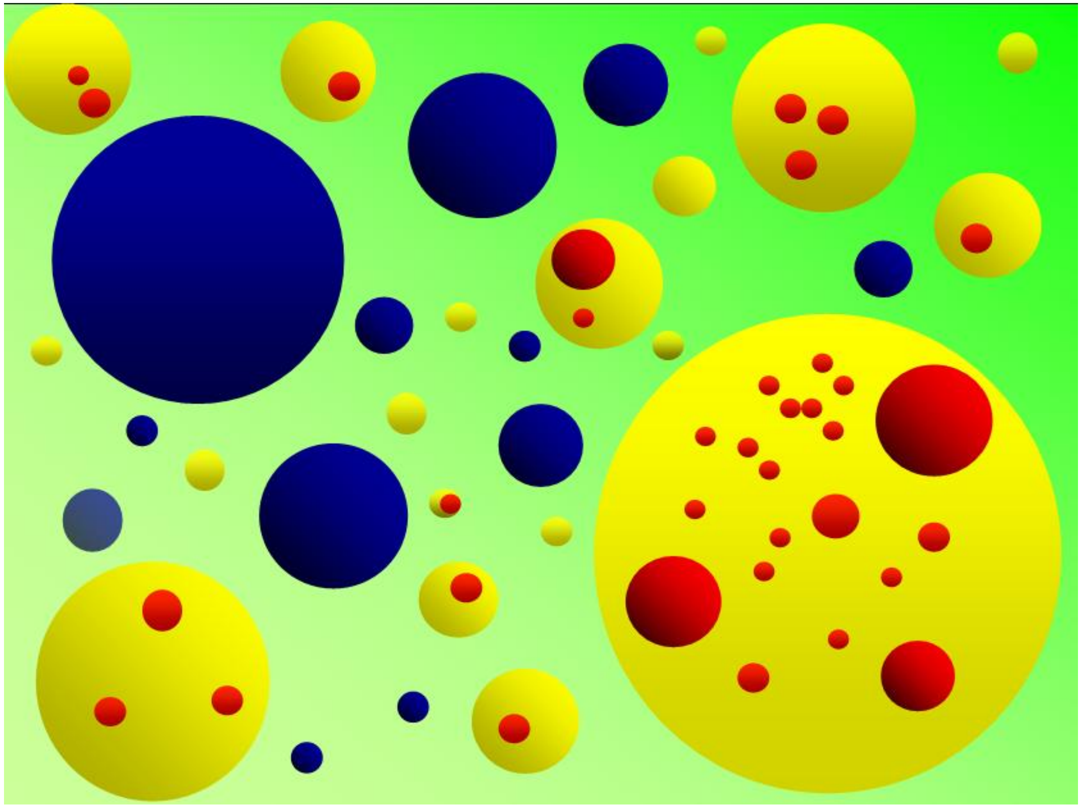
Bousso & Polchinski (2000)

Susskind (2003)

Eternally inflating "multiverse"

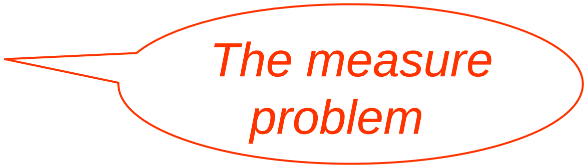


Bubbles of all types will be produced.



Everything that can happen will happen an infinite number of times.
Need a cutoff to compare these infinities.

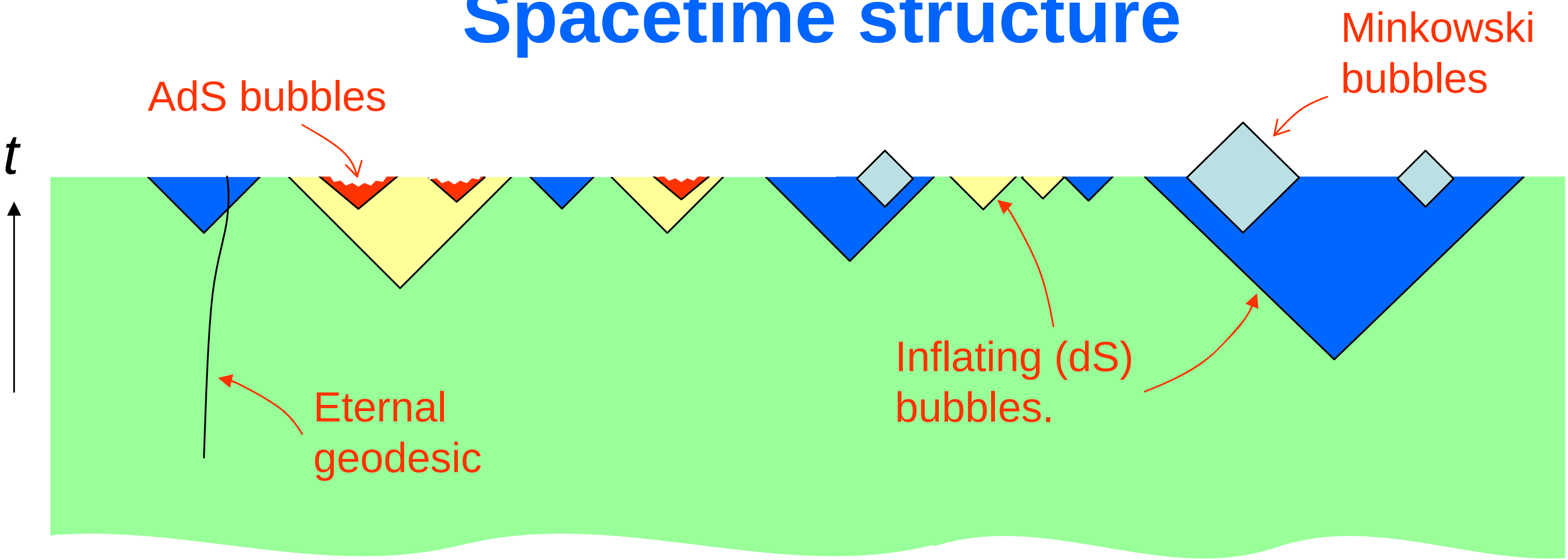
Results are strongly cutoff-dependent.



This talk:

- Spacetime structure.
- Some measure proposals and their problems.
- Measure from fundamental theory?

Spacetime structure

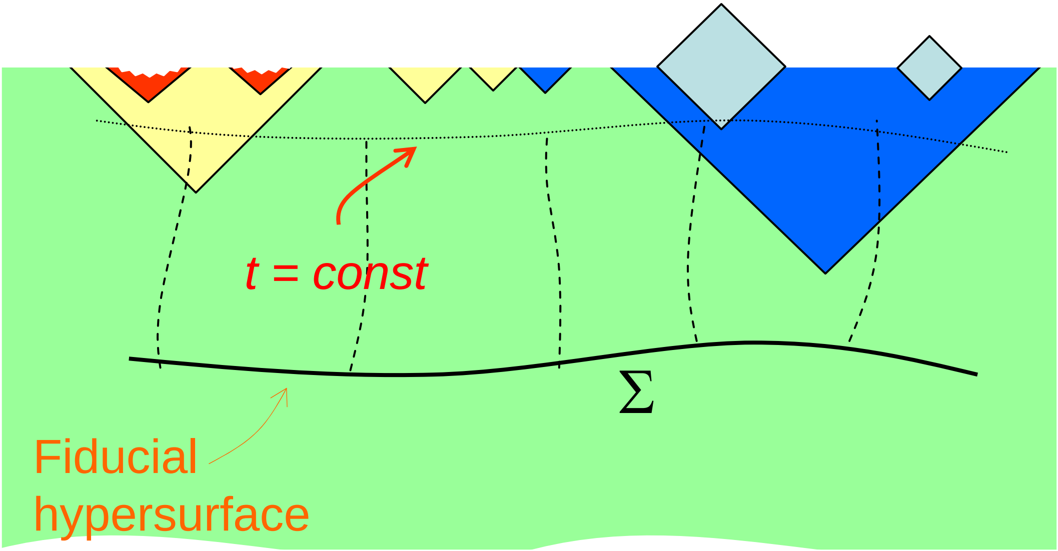


- Bubbles nucleate and expand at nearly the speed of light.
- Inflating and terminal bubbles
- Eternal geodesics

MEASURE PROPOSALS AND THEIR PROBLEMS

Global time cutoff:

Count only events that occurred before some time *t*.



Garcia-Bellido, Linde & Linde (1994); A.V. (1995)

Possible choices of t :

(i) proper time $t = \tau$ along geodesics orthogonal to Σ ;

(ii) scale-factor time, $t = a$.

$t \to \infty$ ⟹ steady-state evolution.

The distribution does not depend on the choice of Σ
-- but depends on what we use as *t*.

Proper time cutoff leads to "youngness paradox"

Linde & Mezhlumian (1996), Guth (2001), Tegmark (2004), Bousso, Freivogel & Yang (2007)

Volume in regions of any kind grows as

$$V_j \propto e^{\kappa\tau}, \quad \kappa \sim H_{\max} \sim M_{Pl}.$$

Driven by fastest-expanding vacuum

Observers who evolve faster by $\Delta\tau$ are rewarded by a huge volume factor.

$$\Delta\tau = 1\,\mathrm{Gyr} \quad \Longrightarrow \quad \exp(\kappa\,\Delta\tau) = \exp(10^{60})$$

We should have evolved at a much earlier cosmic epoch.

Ruled out.

Scale-factor cutoff

De Simone, Guth, Salem & A.V. (2008)

Growth of volume: $V_j \propto a^{\gamma}, \quad \gamma \approx 3.$

$$(3-\gamma) \propto \lambda_{\min}$$

← decay rate of the slowest-decaying vacuum

A mild youngness bias: for $\Delta\tau = 1\,\mathrm{Gyr}$ the volume is enhanced only by ~ 20%.

This is OK.

“Stationary” measure

Linde (2007)

Pocket-based measure

Garriga, Schwartz-Perlov, A.V. & Winitzki (2005)
Easther, Lim & Martin (2005)

Large inflation inside bubbles is rewarded:

$$P_j \propto Z_j^3$$

← expansion factor during inflation

⇨ ***“Q catastrophe”***

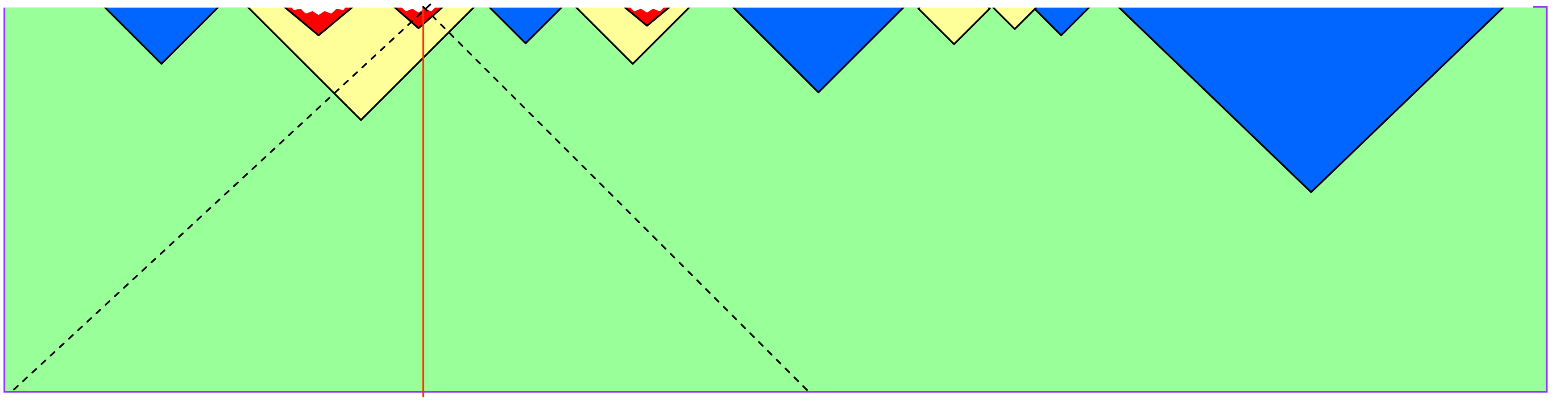
Feldstein, Hall & Watari (2005)
Garriga & A.V. (2006)
Graesser & Salem (2007)

Parameters correlated with the shape of the inflaton potential are driven to extreme values.

In particular the amplitude of density perturbations *Q*.

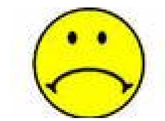
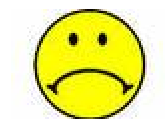
Causal patch measure

Bousso (2006)
Susskind (2007)



Include only observations in spacetime region accessible to a single observer.

Depends on the initial state.

	Youngness paradox	Q catastrophe	Dependence on initial state
Proper time cutoff			
Scale factor cutoff			
Pocket-based measure			
Stationary measure			
Causal patch measure			

Probability distribution for Λ in scale factor cutoff measure

De Simone, Guth, Salem & A.V. (2008)

$$dV(\Lambda) = C(\Lambda) d\Lambda \, a^{\gamma-1} da$$

Volume thermalized in scale factor interval *da*.

$\gamma \approx 3$

Assumptions:

- $C(\Lambda) \approx const$ in the range of interest.
- Number of observers is proportional to the fraction of matter clustered in large galaxies ($M \geq 10^{12} M_\odot$).
- Observers do their measurements at a fixed time $\Delta\tau = 5\,\mathrm{Gyr}$ after galactic halo collapse.

$$P(\Lambda) \propto \int^{a_c} da \, a^{\gamma-1} f \, [\tau(a_c / a) - \Delta\tau]$$

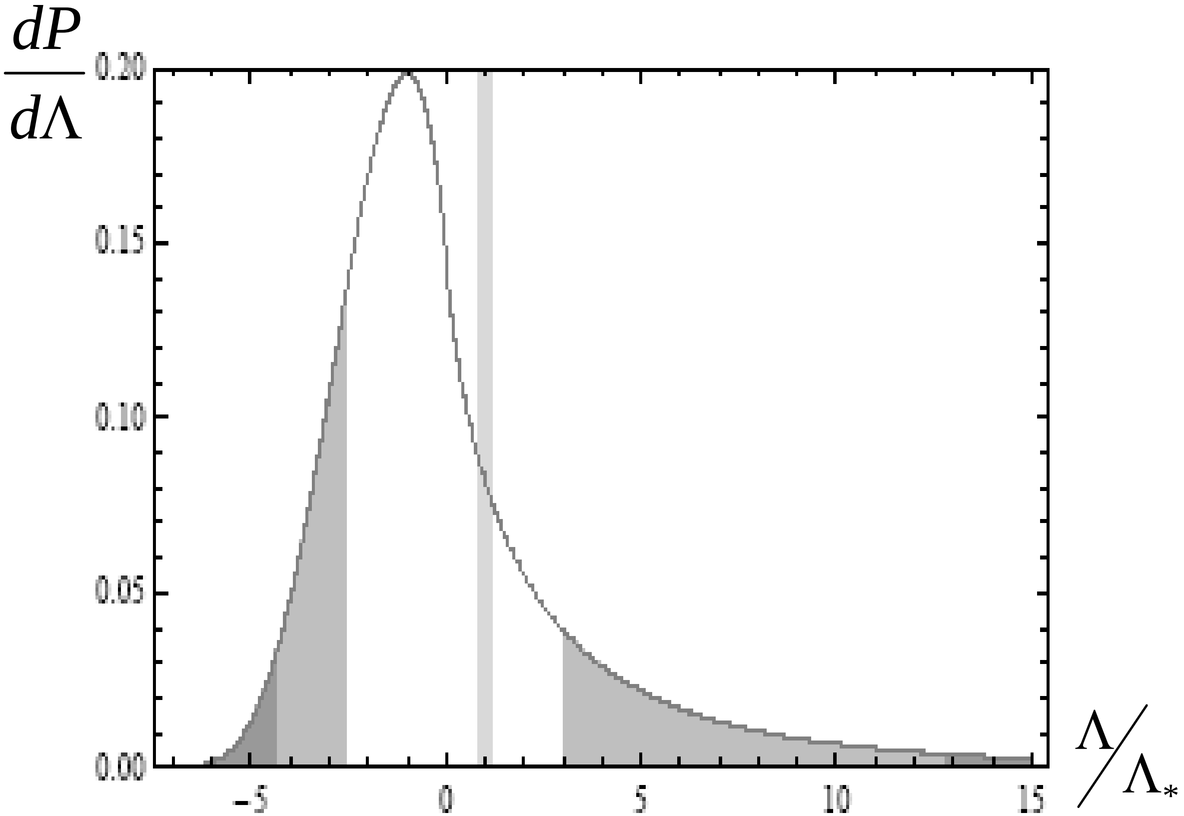
Cutoff at $a = a_c$.

Press-Schechter Warren et. al.

Proper time corresponding to scale factor change *(a_c /a).*

Probability distribution for Λ in scale factor cutoff measure

De Simone, Guth, Salem & A.V. (2008)



Can the measure be derived from fundamental theory?

Holographic measure of the multiverse

Based on work with Jaume Garriga

- The dynamics of the multiverse may be encoded in its future boundary (suitably defined).

  Inspired by holographic ideas: *Quantum dynamics of a spacetime region is describable by a boundary theory.*

- The measure can be obtained by imposing a UV cutoff in the boundary theory.
  Related to scale-factor cutoff.

First review the holographic ideas.

AdS_{D+1}/CFT_D correspondence

Maldacena (1998)
Susskind & Witten (1998)

String Theory in AdS is equivalent (dual) to a CFT on the boundary.

Euclidean AdS:

$$ds^2 = dr^2 + \sinh^2 r \, d\Omega_D^2$$

Regulate boundary theory:

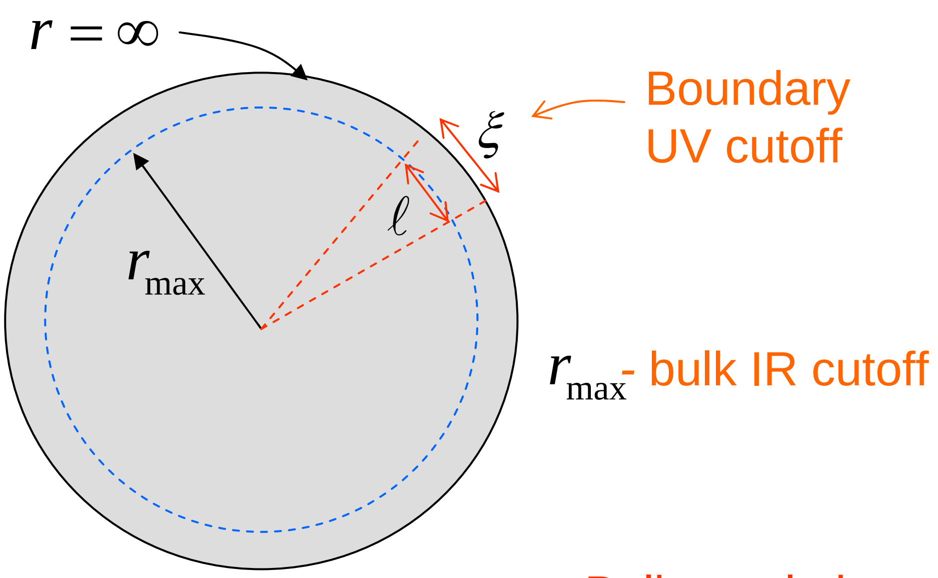
Integrate out short-wavelength modes of wavelength $\lambda_B < \xi$.

The corresponding 4D modes have minimum wavelength

$$\lambda_{\min}(r) = \xi \sinh r.$$

$$\Rightarrow \quad \sinh r_{\max} = \ell / \xi$$

$$r_{\max} \to \infty \quad \Leftrightarrow \quad \xi \to 0.$$

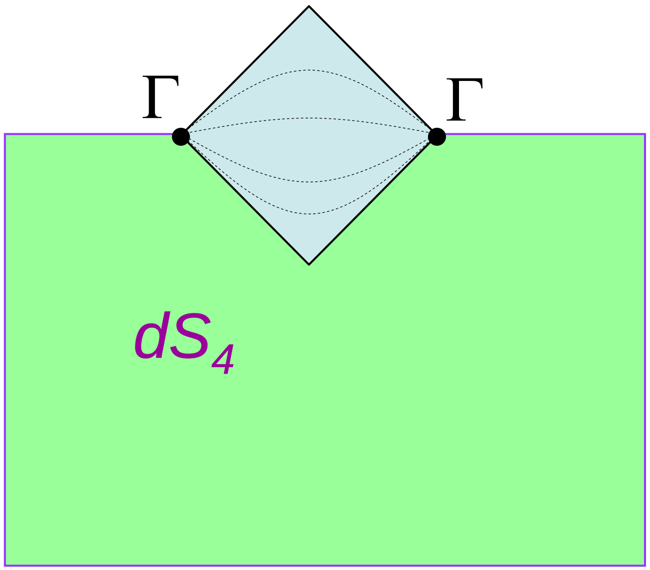


Require $\lambda_{\min} < \ell$ ← Bulk resolution scale

Variation of r_{max} $\Leftrightarrow$ RG flow in the boundary theory.

CdL/CFT correspondence

Freivogel, Sekino, Susskind & Yeh (2006) (FSSY)



Bubble interior:

$$ds^2 = -dt^2 + a^2(t)(dr^2 + \sinh^2 r \, d\Omega_2^2)$$

AdS_3

FSSY: *The 4D theory inside the bubble is equivalent to a Euclidean 2D field theory on Γ.*

The boundary theory includes a Liouville field $L(\Omega)$, which describes fluctuations of the boundary geometry:

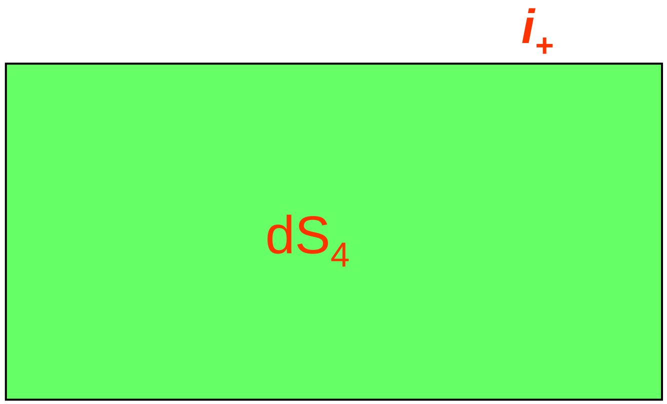
$$d\Omega_2^2 \Rightarrow e^{L(\Omega)} d\Omega_2^2.$$

This additional field plays the role of time variable ***t***, as in Wheeler-DeWitt equation, while ***r*** is recovered from RG flow.

dS/CFT correspondence

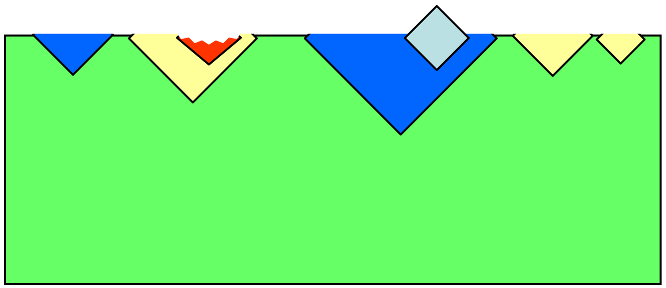
Strominger (2001)

The 4D theory describing an asymptotically de Sitter space is equivalent to a 3D Euclidean theory at the future infinity i_+.



But dS space is metastable, so there is no such thing as asymptotically dS space.

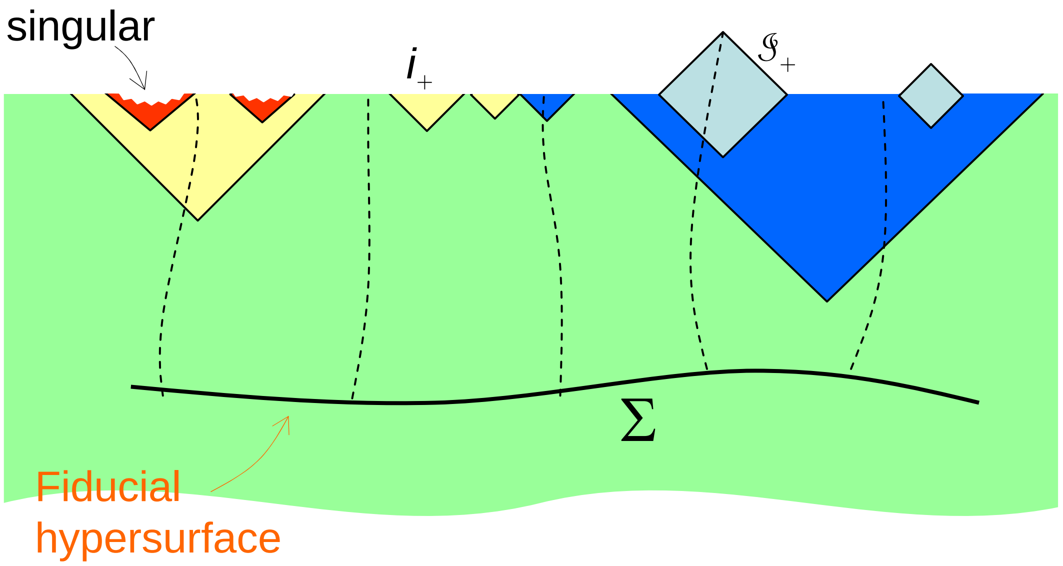
The actual future infinity looks like this:



Our conjecture:

The 4D theory describing the evolution of the multiverse is equivalent to a 3D Euclidean theory at the future infinity (suitably defined).

Defining future infinity



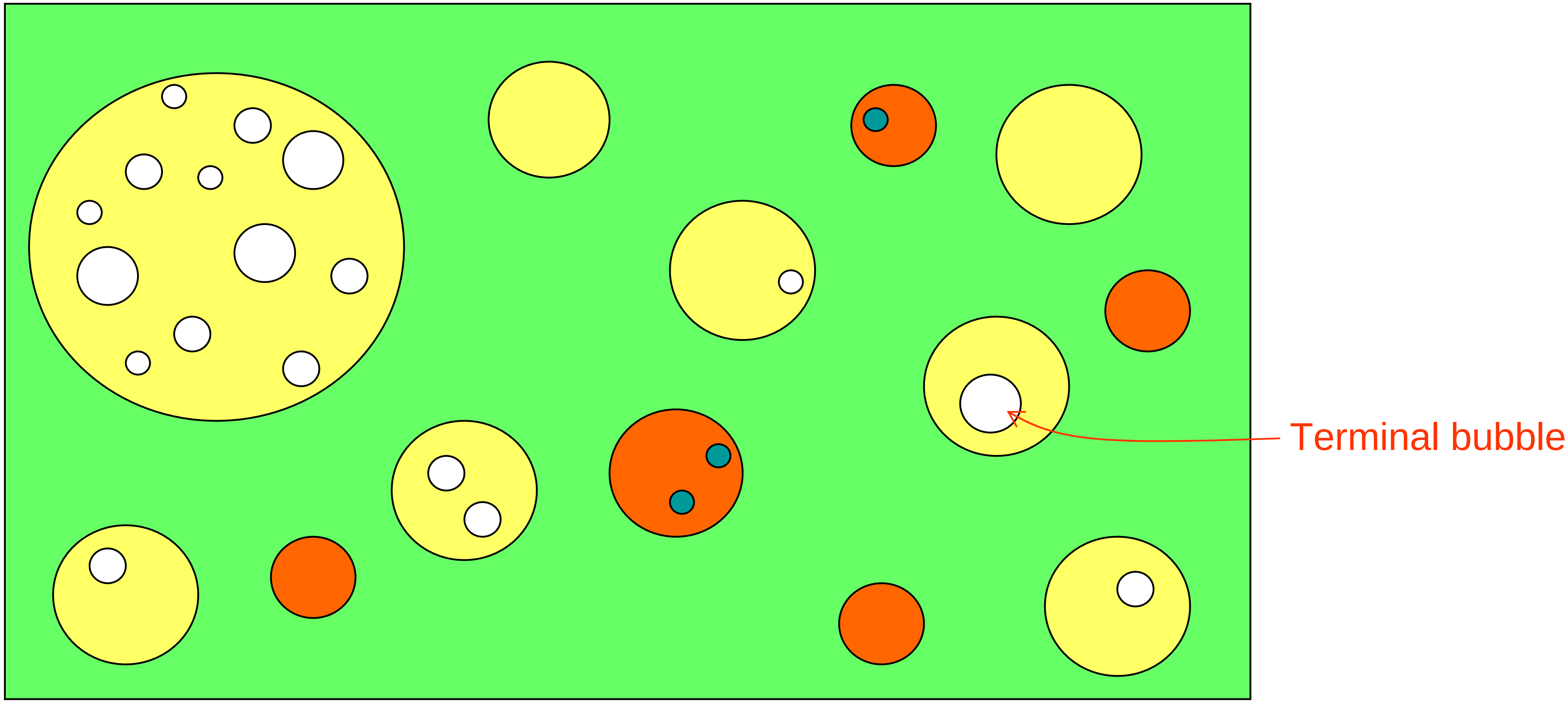
- Geodesic congruence projects bubbles onto Σ . $\Rightarrow$ Map of future infinity.
- Excise images of Minkowski bubbles. (They are described by the *2D* boundary degrees of freedom.) *(FSSY)*
- AdS bubbles can be excised in a similar way (?).

The future infinity $\mathcal{E}$ includes eternal points and the boundaries of excised terminal bubbles.

The metric $g_{ij}(\boldsymbol{x})$ on Σ defines a metric on $\mathcal{E}$.

Different choices of Σ are related by conformal transformations.

Structure of the future boundary $\mathcal{E}$



- Each bubble becomes a fractal "sponge" in the limit.
- Terminal bubbles correspond to holes (with 2D CFTs on their boundaries).
- Bubbles correspond to instantons of the 3D theory on $\mathcal{E}$.

Boundary measure

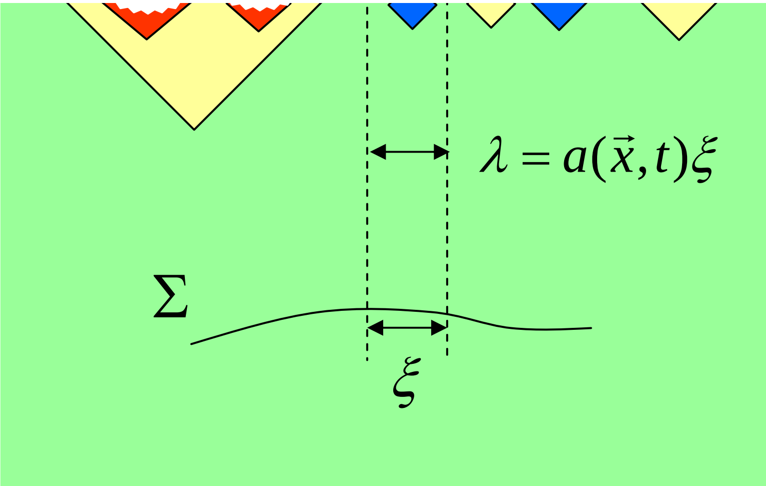
Renormalization of boundary theory:

Integrate out short-wavelength modes of wavelength $\lambda_0 < \xi$.

Boundary UV cutoff

The corresponding 4D modes have minimum wavelength

$$\lambda_{\min}(\vec{x},t) = a(\vec{x},t)\,\xi.$$



Require $\lambda_{\min} < \ell$

Bulk resolution scale

$$\Rightarrow \quad a_{\max} = \ell / \xi$$

-- scale factor cutoff

$$\xi \to 0 \quad \Rightarrow \quad a_{\max} \to \infty.$$

UV cutoff on the boundary $\Leftrightarrow$ *scale factor cutoff in 4D.*

SUMMARY

- Inflation is a never ending process, with new “bubble universes” constantly being formed.
- All possible events will happen an infinite number of times. To calculate probabilities, we need to regulate the infinities.
- The scale factor cutoff appears to be the most promising measure proposal. It does not suffer from any paradoxes and gives a prediction for Λ in good agreement with the data.
- This measure may be obtained by imposing a UV cutoff in a holographic boundary theory.