Inflation, Dark Energy

&

Extra Dimensions

w/Daniel Wesley (Cambridge)

This work complements but is NOT directly related to:

no-go theorems based on SuSY, SuGRA
or based on ε or η problems
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more closely related to:

constraints on static deS

G. Gibbons (1985)
J. Maldacena & C. Nunez (2001)
S. Giddings, S. Kachru and J. Polchinski (2002)
S. Giddings and A. Maharana (2005)
Carroll, Geddes, Hoffman, Wald (2002)

For concreteness, let's consider models satisfying four conditions:

1) GR Condition: Einstein GR in 4d and higher-d

2) Flatness Condition: 4d effective theory is spatially flat

3) Boundedness Condition: extra dimension are compact

4) Metric Condition:

$$ds^2 = e^{2\Omega}(-dt^2 + a^2(t)\, dx^2) + e^{-2\Omega} g_{ab} dy^a dy^b$$

$$where \;\; R(g_{ab}) = 0$$

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5) PLUS the Null Energy Condition (NEC):

$$T_{MN}\, n^M n^N \geq 0 \;\; for\ every\ null\ n^M$$

(or $\rho + p \geq 0$)

Can prove some surprising no-go theorems … with the right bag of tricks:

Do all calculations in Einstein frame so interpretation is unambiguous

Treat $T_{\mu\nu}$ space-space components as block diagonal

$$T_{\mu\nu} = \begin{pmatrix} \rho & & & & & & \\ & -p_3 & 0 & 0 & & & \\ & 0 & -p_3 & 0 & & & \\ & 0 & 0 & -p_3 & & & \\ & & & & -p_k & 0 & 0 \\ & & & & 0 & \ldots & 0 \\ & & & & 0 & 0 & -p_k \end{pmatrix}$$

$$where \quad p_3 = \tfrac{1}{3} Tr\, T_{ij} \quad and \quad p_k = \tfrac{1}{k} Tr\, T_{IJ}$$

A-averaging

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$$\left(\int T_{MN}\, n^M n^N \boxed{e^{A\Omega}}\; e^{2\Omega}\sqrt{g}\; d^k y\right) \Big/ \left(\boxed{e^{A\Omega}}\; e^{2\Omega}\sqrt{g}\; d^k y\right) \geq 0$$

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N.B. If A-averaged NEC violated then NEC also violated (but not the converse) *consider k space – time dep. extra dimensions :*

$$\gamma_{ab}(t,y) = e^{-2\Omega}\, g_{ab}$$

$$\frac{d}{dt}\gamma_{ab} = \frac{2}{k}\,\xi\,\gamma_{ab} + \sigma_{ab}$$

take clever linear combinations of G_{00} and G_{ij} :

$$e^{-\phi}\langle e^{2\Omega}(\rho+p_3)\rangle_A = (\rho_{4d}+p_{4d}) - \frac{k+2}{2k}\langle\xi\rangle_A^2 - \frac{k+2}{2k}\langle(\xi-\langle\xi\rangle_A)^2\rangle_A - \langle\sigma^2\rangle_A$$

$$\begin{aligned} e^{-\phi}\langle e^{2\Omega}(\rho+p_k)\rangle_A = &\ \frac{1}{2}(\rho_{4d}+3p_{4d}) + 2\left(\frac{A}{4}-1\right)\frac{k+2}{2k}\langle(\xi-\langle\xi\rangle_A)^2\rangle_A \\ &- \frac{k+2}{2k}\langle\xi\rangle_A^2 - \langle\sigma^2\rangle_A \\ &+ \left[-5+\frac{10}{k}+k+A\left(-3+\frac{6}{k}\right)\right]\langle e^{2\Omega}(\partial\Omega)^2\rangle_A \\ &+ \frac{k+2}{2k}\frac{1}{a^3}\frac{d}{dt}\left(a^3\langle\xi\rangle_A\right) \end{aligned}$$

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$$\left\langle e^{2\Omega}(\rho + p_3)\right\rangle_A \propto \underbrace{(\rho^{4d} + p^{4d})}_{1+w_{total}\geq 0} - \underbrace{\frac{k+2}{2k}\left(\langle\xi\rangle_A\right)^2}_{\leq 0} + \underbrace{\text{non-positive for all A}}_{\leq 0}$$

$$\left\langle e^{2\Omega}(\rho + p_k)\right\rangle_A \propto \underbrace{\tfrac{1}{2}(\rho^{4d} + 3p^{4d})}_{\boxed{1+3\,w_{total} < 0}} + \frac{k+2}{2k}\frac{1}{a^3}\frac{d}{dt}\left(a^3\langle\xi\rangle_A\right) + \underbrace{\text{non-positive for some A}}_{\leq 0}$$

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$$\boxed{1 + 3\,w_{total} < 0}$$

only hope
is if this is non-zero

$$\leq 0$$

Illustrative example: pure cosmological constant (w_{total} = -1)

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$1 + 3\,w_{total} = -2$ *only hope* ≤ 0

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Trouble !

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$$1 + 3\,w_{total} = -2 \qquad \textit{only hope} \qquad \le 0$$

therefore, must have w_{total} > -1

Curious corollary:

Not only rules out pure Λ universe,

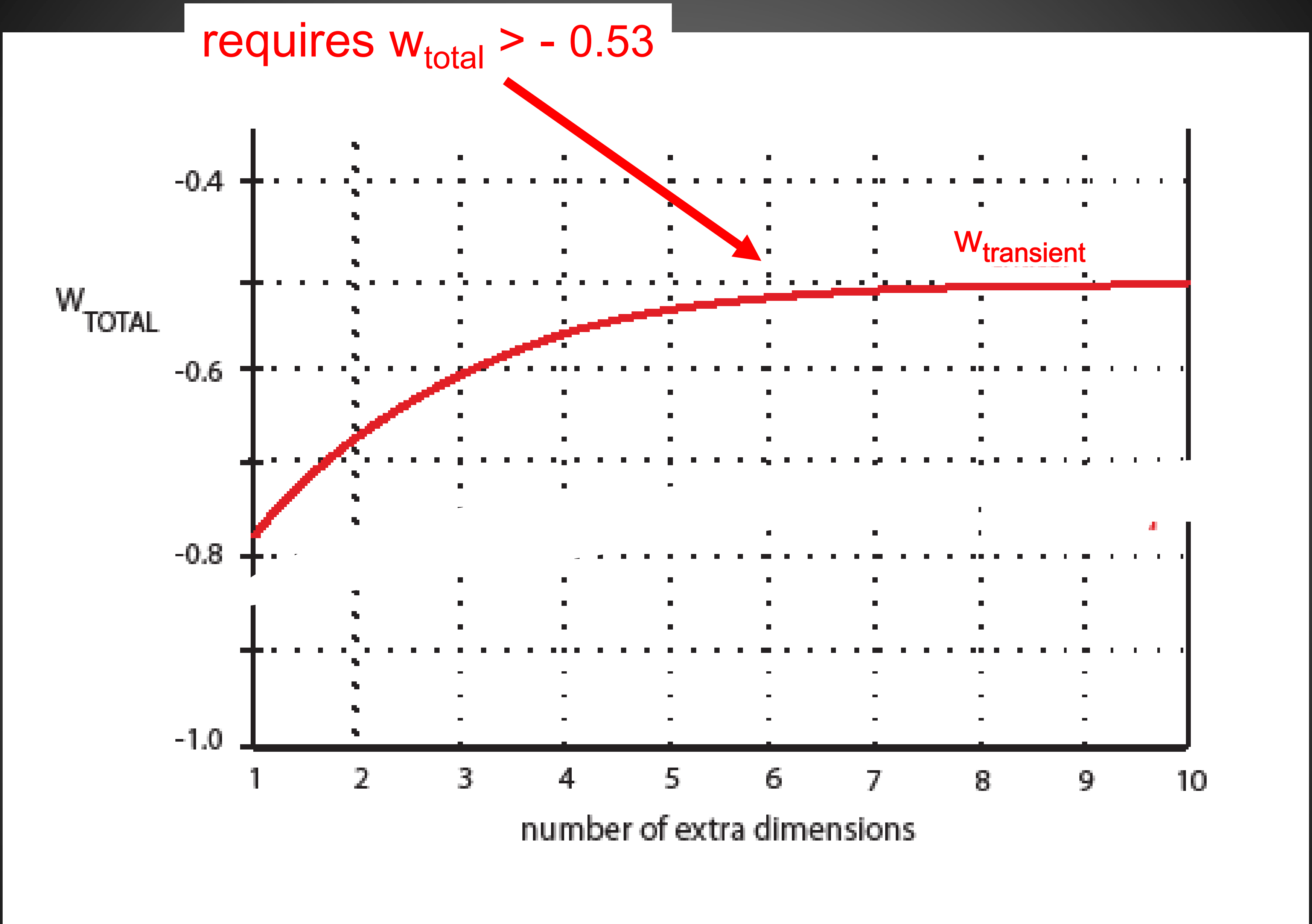
but also ΛCDM

What about $w > -1$?

Can have $w < -1/3$ if G_N is time varying…

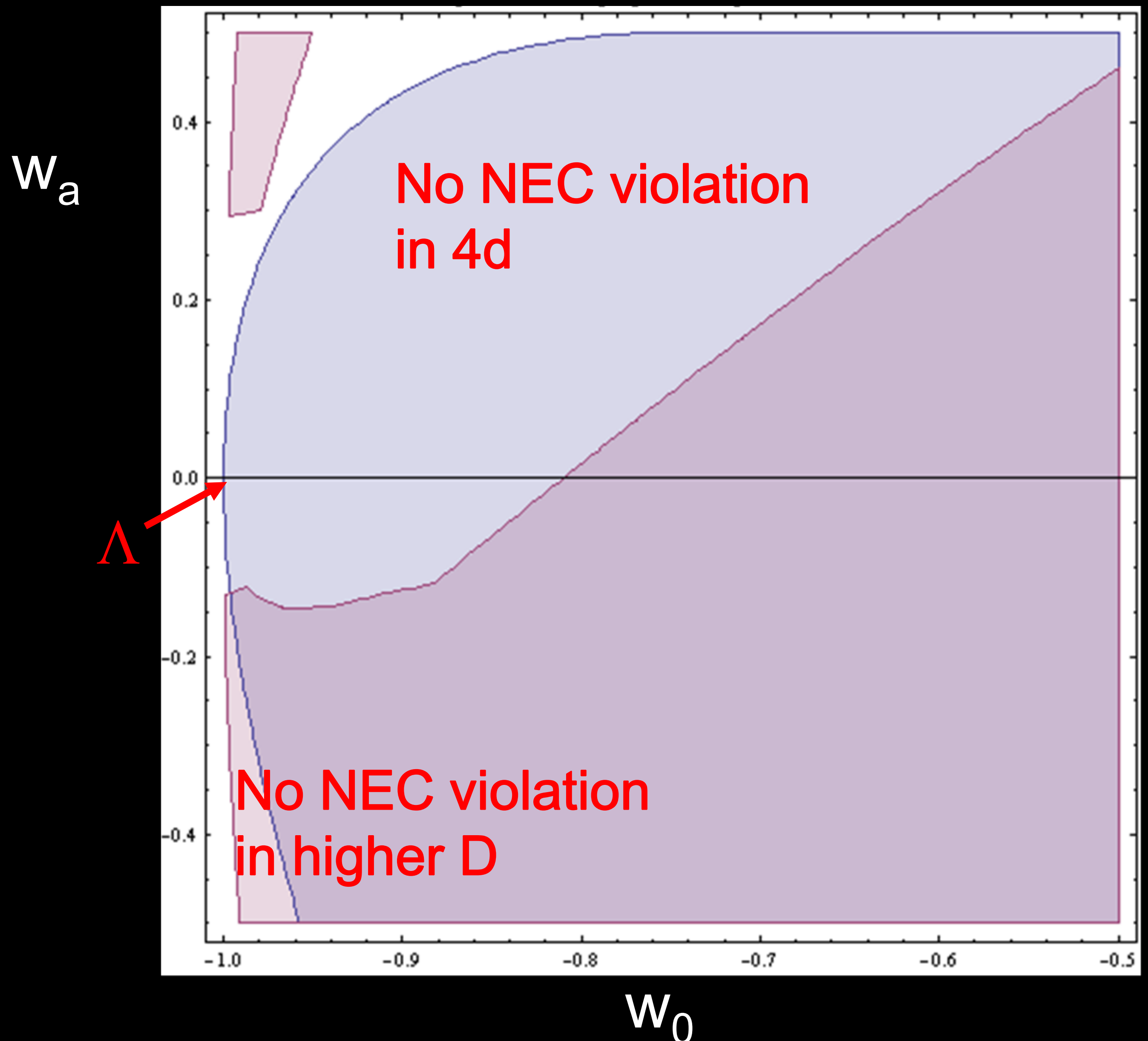
but if $w_{transient} > w > -1$

can only maintain for only a brief period;



requires $w_{total} > -0.53$
$w_{transient}$
W_{TOTAL}
-0.4
-0.6
-0.8
-1.0
1
2
3
4
5
6
7
8
9
10
number of extra dimensions

Models that satisfy constraints on $G_N(t)$ and NEC



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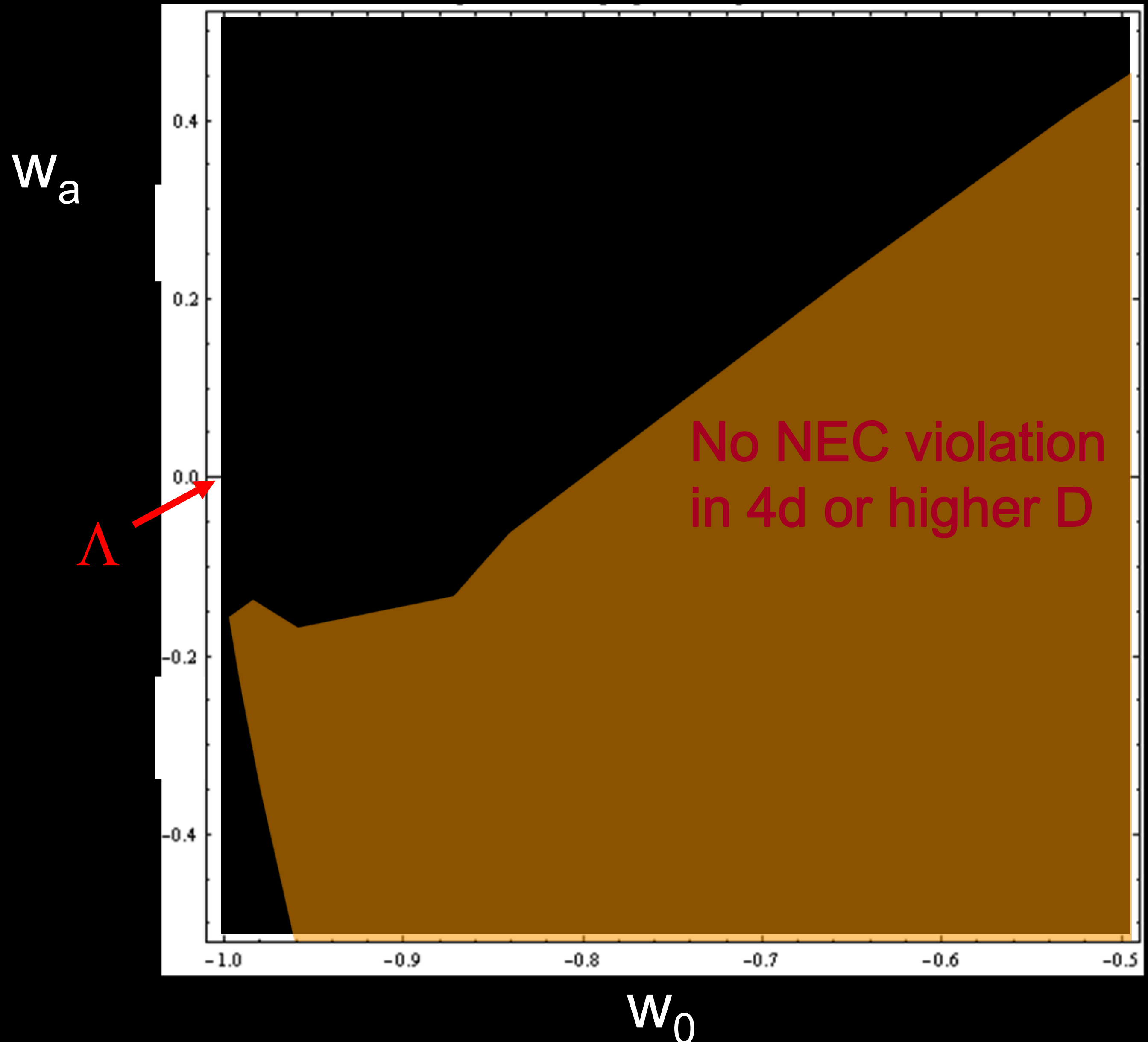

preliminary

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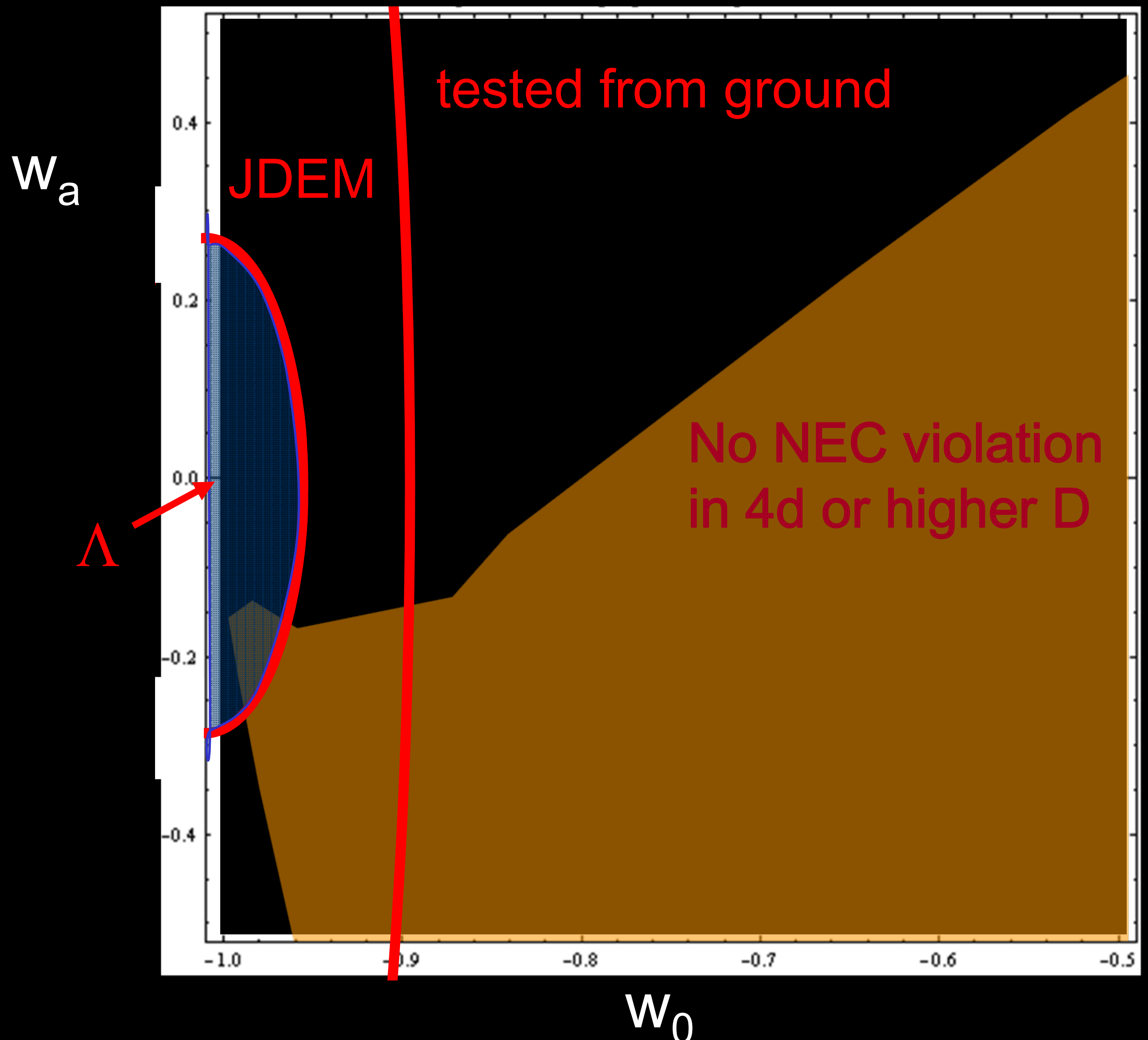

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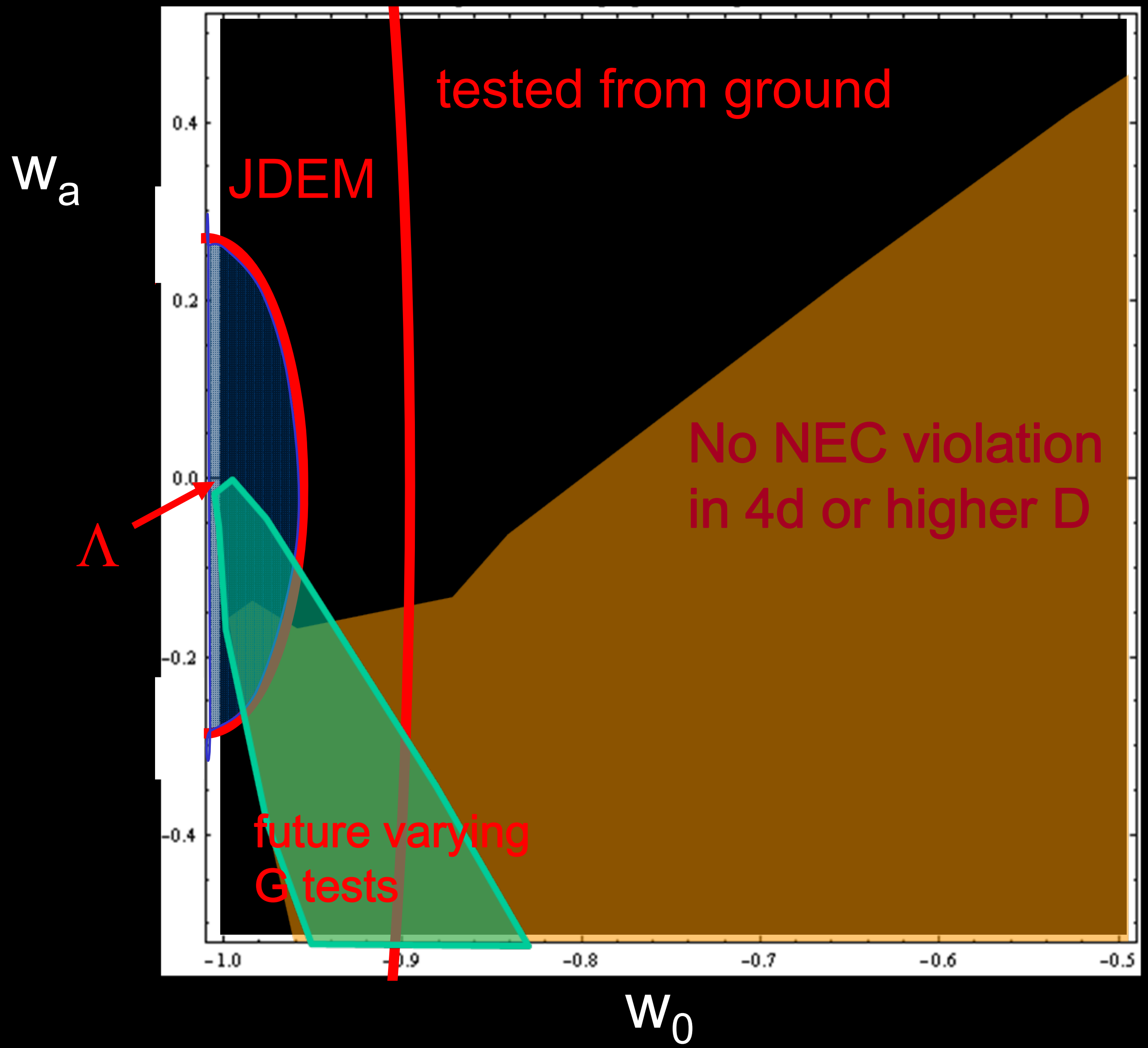

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Curious corollary:

Dark energy is barely compatible w/o NEC violation . . .

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Dark energy is barely compatible w/o NEC violation . . .

and inflation w/o NEC violation is absolutely impossible!

So, let's trade:

No G_N variation . . . but allow NEC violation

Now there are new constraints . . .

$$\left\langle e^{2\Omega}(\rho + p_3)\right\rangle_A \propto (\rho^{4d} + p^{4d}) - \cancel{\frac{k+2}{2k}\left(\langle\xi\rangle_A\right)^2} + \text{ non-positive for all A}$$

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measure of
NEC violation

choose A=A* so that
last term is zero

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measure of *NEC violation*

$\sim 1 + 3\,w$

choose A=A* so that last term is zero

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$$\left\langle e^{2\Omega}(\rho+p_3)\right\rangle_A \propto (\rho^{4d}+p^{4d}) - \cancel{\frac{k+2}{2k}\left(\langle\xi\rangle_A\right)^2} + \text{ neg. semi-def. for all A}$$

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$\sim 1+3w$

measure of *NEC violation*

choose A=A* so that last term is zero

NEC violation must be time-dependent *and proportional to ρ_{4d}*

Inflation problematic

$$\left\langle e^{2\Omega}(\rho + p_3)\right\rangle_A \propto (\rho^{4d} + p^{4d}) - \frac{k+2}{2k}\left(\left\langle \xi \right\rangle_A\right)^2 + \text{ neg. semi-def.}$$

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violation of NEC
10^{100} x DE

→ source of NEC different from DE

& must be able to annihilate it

Comment on models that violate the metric or GR conditions

$$\left\langle e^{2\Omega}(\rho + p_3)\right\rangle_A \propto (\rho^{4d} + p^{4d}) - \frac{k+2}{2k}\left(\left\langle \xi \right\rangle_A\right)^2 + \text{ non-positive for all A}$$

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No-go Theorem Summary

If four assumptions & NEC obeyed:

Inflation impossible

DE barely possible,
… but only if G and w vary with time
… can be ruled out with near further data

If four assumptions & fixed moduli /NEC violated:

NEC must be violation in compact dimensions
… must be inhomogeneous in compact dimensions
… must vary with time in sync with w4d
… violation must be substantial for inflation

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