Neutrino Interactions in Dense Matter

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Messages:

- Rates of neutrino processes important for stellar collapse calculations
- Improved estimates of rates. Include mean-field effects and collisions
- Preliminary results. Rates of processes involving energy transfer reduced by up to one order of magnitude

Supernova 1987A, Hubble Space Telscope

General considerations and history

- Neutrinos seen from SN 1987A
- Early calculations. Energy transport by neutrinos.
  Colgate and White (1966)
- 1970s. Discovery of weak neutral currents ($\nu + N \rightarrow \nu + N$), in addition to charged currents ($p + e^- \rightarrow n + \nu_e$).
- Improved equations of state.
- Failure of direct explosion mechanism.
- Shock revival by neutrino heating (Bethe, Wilson).
- Still no agreed mechanism for generating explosion.
  (Convection, sound waves, rotation, magnetic fields)
- *Need improved estimates of neutrino processes*

Neutrino processes

Free nucleons

- **Scattering** from nucleons $N + \nu \rightarrow N + \nu$

Effects of N-N interactions

- Scattering modified. Initial and final state interactions
  $N + N + \nu \rightarrow N + N + \nu$
- **Bremsstrahlung** of neutrino pairs $N + N \rightarrow N + N + \nu + \bar{\nu}$
- **Pair annihilation** $\nu + \bar{\nu} + N + N \rightarrow N + N$
- Similar processes in neutron star cooling (also charged currents)

Complication. Inhomogeneity of matter (coherent scattering)

Interactions have two effects

- Mean-field effects
  - Screening of matrix elements
  - Phase space altered only quantitatively
- Real collisions
  - Allows extra processes
  - Landau-Pomeranchuk-Migdal effect

Previous calculations

- Mean-field effects
  - Iwamoto (1982) (Kagawa U.)
  - Burrows, Sawyer; Reddy, Lattimer, Prakash, Pons (1998-9)
- Real collisions
  - Allows extra processes
  - Landau–Pomeranchuk–Migdal effect
  - Raffelt, Seckel, Sigl, Hannestad (1995-8). Zero q

Challenge: To include both effects

Basic formalism

- Treat weak interactions using Golden Rule.

$$\text{Rates} \propto G_{\text{F}}^2 \int d\omega d^3 q \ldots S(q,\omega)$$

- $S(q,\omega)$ – density-density (vector) or spin-spin (axial vector) dynamical structure factors (nucleons non-relativistic)

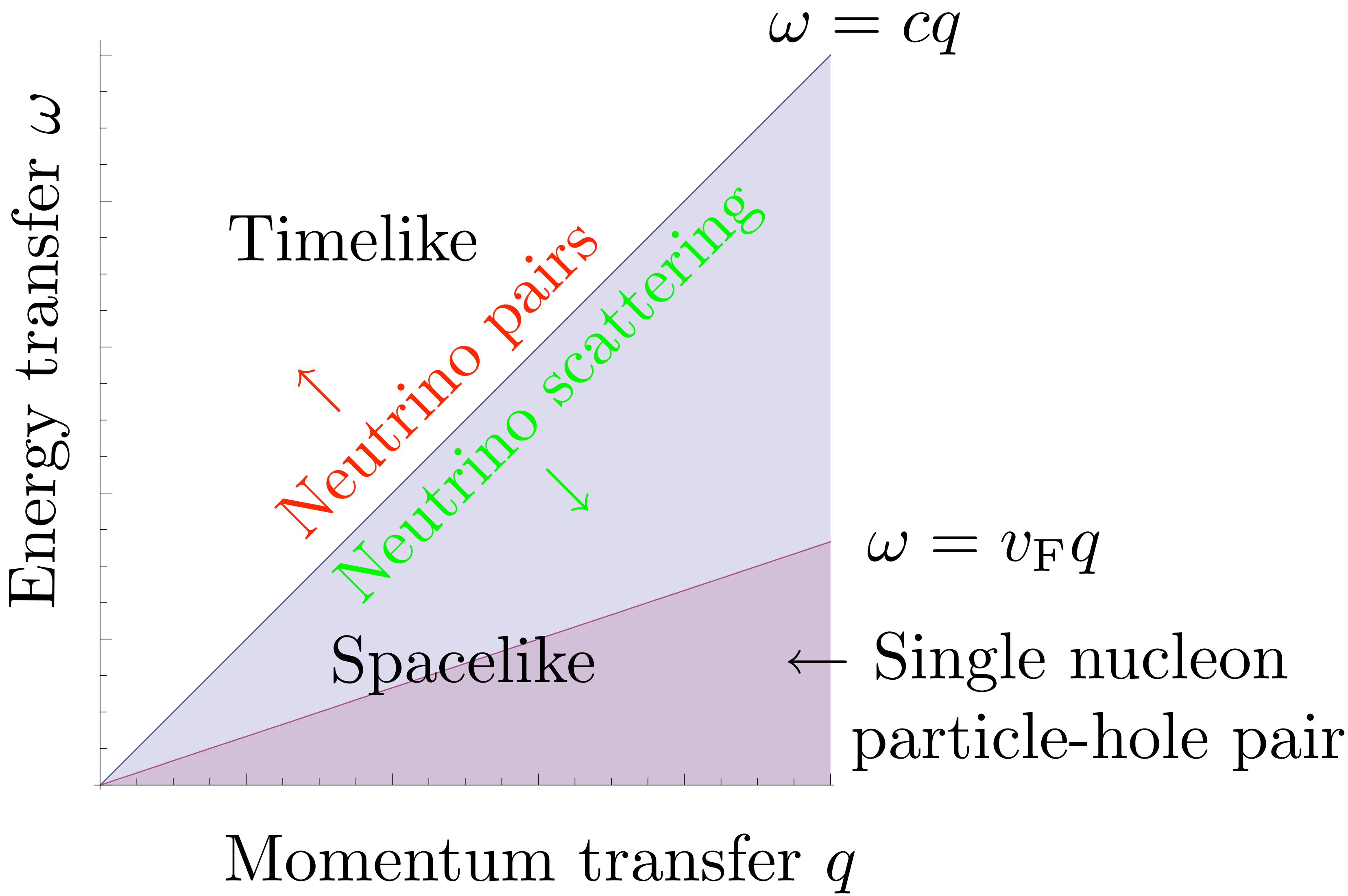
- Related to the corresponding correlation functions:

$$S(q,\omega) = \frac{1}{\pi n} \frac{1}{1 - \mathrm{e}^{-\omega/T}} \mathrm{Im}\chi(\omega, \mathbf{q})$$

- Problem is to calculate structure factors for nucleonic matter.

- Crucial densities seem to be somewhat below nuclear matter density. (High densities – neutrinos trap. Low densities – few interactions)

Phase Space



$\omega = cq$
Timelike
Neutrino pairs
Neutrino scattering
$\omega = v_{\mathrm{F}} q$
Spacelike
Single nucleon particle-hole pair
Energy transfer ω
Momentum transfer q

Phase Space

- Neutrino scattering. $(\omega, \mathbf{q})$ spacelike.
- Neutrino pair production or annihilation. $(\omega, \mathbf{q})$ timelike.
- Single nucleon particle-hole pair. $|\omega| \leq v_{\mathrm{F}} q$, i.e. spacelike.
- Need collisions between nucleons for the latter processes.
  Two or more particle-hole pair creation.
  Both timelike and spacelike.
- Also affects scattering.
- Landau-Pomeranchuk-Migdal effect. (Raffelt and coworkers.)

Interactions

- Vector and axial vector terms.
- Nucleons nonrelativistic.
  - Vector current $\propto$ number density of nucleon.
  - Axial current $\propto$ spin density of nucleon.
- Need spin-density–spin-density and density–density correlation functions.
- Wavenumbers usually small compared with k_{F}.
- Temperatures $\sim$ Fermi temperature or less.
- Start with Landau Fermi-liquid theory.
- Central interactions. N. Iwamoto, thesis (1981).

Fermi liquid theory with tensor interactions

- System on nucleons, either pure neutrons or neutron-proton mixture.
- Generally axial vector interaction most important. Factor 3.
  Vector important for coherent scattering from nuclei.
- Density response. Particle number conserved.

$$\omega_{m0}(\rho_{\mathbf{q}})_{m0} = -\mathbf{q}\cdot(\mathbf{j}_{\mathbf{q}})_{m0}, \qquad (\rho_{\mathbf{q}})_{m0} = -\frac{\mathbf{q}\cdot(\mathbf{j}_{\mathbf{q}})_{m0}}{\omega_{m0}}$$

- Spin response. Total spin not conserved. (cf. liquid ^{3}He)

$$\chi_\sigma = \chi_{\text{Landau}} + \chi_{\text{Multipair}}, \qquad \chi_{\text{Landau}} = \frac{\mu^2 N(0)}{1+F}$$

  – μ has tensor components, $\propto \boldsymbol{\sigma}$ and $\boldsymbol{\sigma}\cdot\hat{\mathbf{p}}\ \ \hat{\mathbf{p}}$.
  – F has tensor components.

- Effects well known to nuclear physicists.

Spin response function

- Relaxation time approximation.
- Only isotropic Landau interaction.

$$\chi_\sigma = \frac{X^0}{1 + g_0 X^0}$$

$$X^0 = N(0)\left[1 - \frac{\omega}{2v_\mathrm{F}q}\ln\left(\frac{\omega + \mathrm{i}/\tau_\sigma + v_\mathrm{F}q}{\omega + \mathrm{i}/\tau_\sigma - v_\mathrm{F}q}\right)\right]$$

- Generalization of usual response function to allow for collisions.
- Includes effects of nonzero q and mean field.
- Long wavelengths, $q \to 0$

$$\chi_\sigma = \frac{N(0)}{1 + G_0 - \mathrm{i}\omega\tau_\sigma}, \quad S_\sigma = \frac{N(0)}{\pi n}\frac{\omega}{1 - \mathrm{e}^{-\omega/T}}\frac{\Gamma_\sigma}{\omega^2 + [(1+G_0)\Gamma_\sigma]^2}$$

Usual relaxation form. $\Gamma_\sigma = 1/\tau_\sigma$

Relaxation rates

- **Vector**
  - Vector charge does not decay. CVC
  - Amounts to particle conservation nonrelativistically
  - Distortions of distribution which are anisotropic in momentum space can decay.
- **Axial**
  - Axial vector can decay. PCAC
  - Tensor force in one component systems
  - Central (spin-exchange) interactions in mixtures
    Neutrons and protons couple with opposite sign to Z boson

Collision Rates

- Collision rate

$$\frac{1}{\tau} = C[T^2 + (\omega/2\pi)^2]$$

- Spin relaxation rate

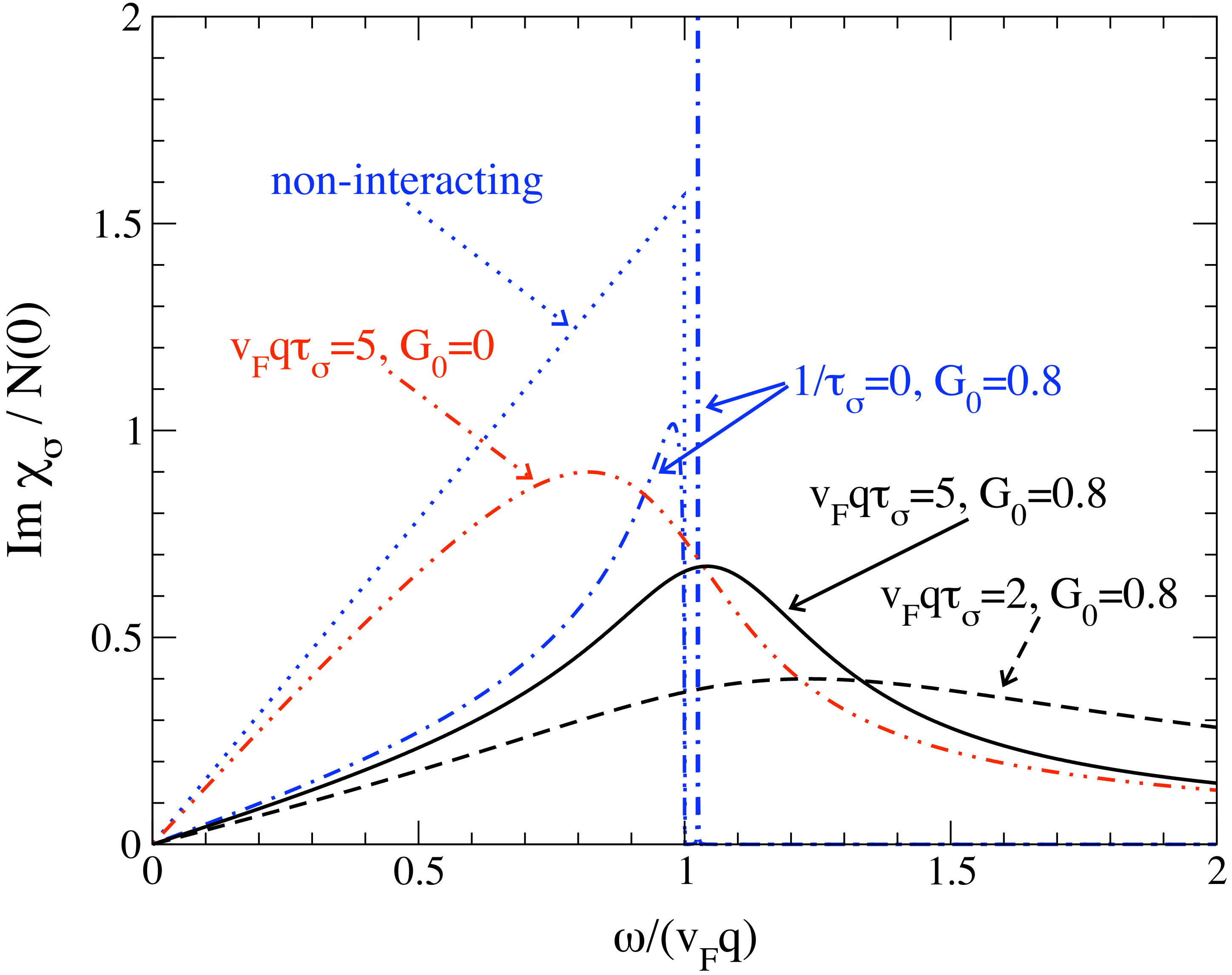
$$\frac{1}{\tau_\sigma} = C_\sigma[T^2 + (\omega/2\pi)^2]$$

Non-central, especially tensor forces essential!

Scattering rate

$$C_\sigma = \frac{\pi^3 m^*}{6k_\mathrm{F}^2}\left\langle \frac{1}{12}\sum_{j=1,2,3} \mathrm{Tr}\left\{\mathcal{A}_{\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2}(\mathbf{k},\mathbf{k}')\,\boldsymbol{\sigma}_1^j\left[(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)^j\,,\,\mathcal{A}_{\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2}(-\mathbf{k},\mathbf{k}')\right]\right\}\right\rangle_{\mathrm{FS}}$$

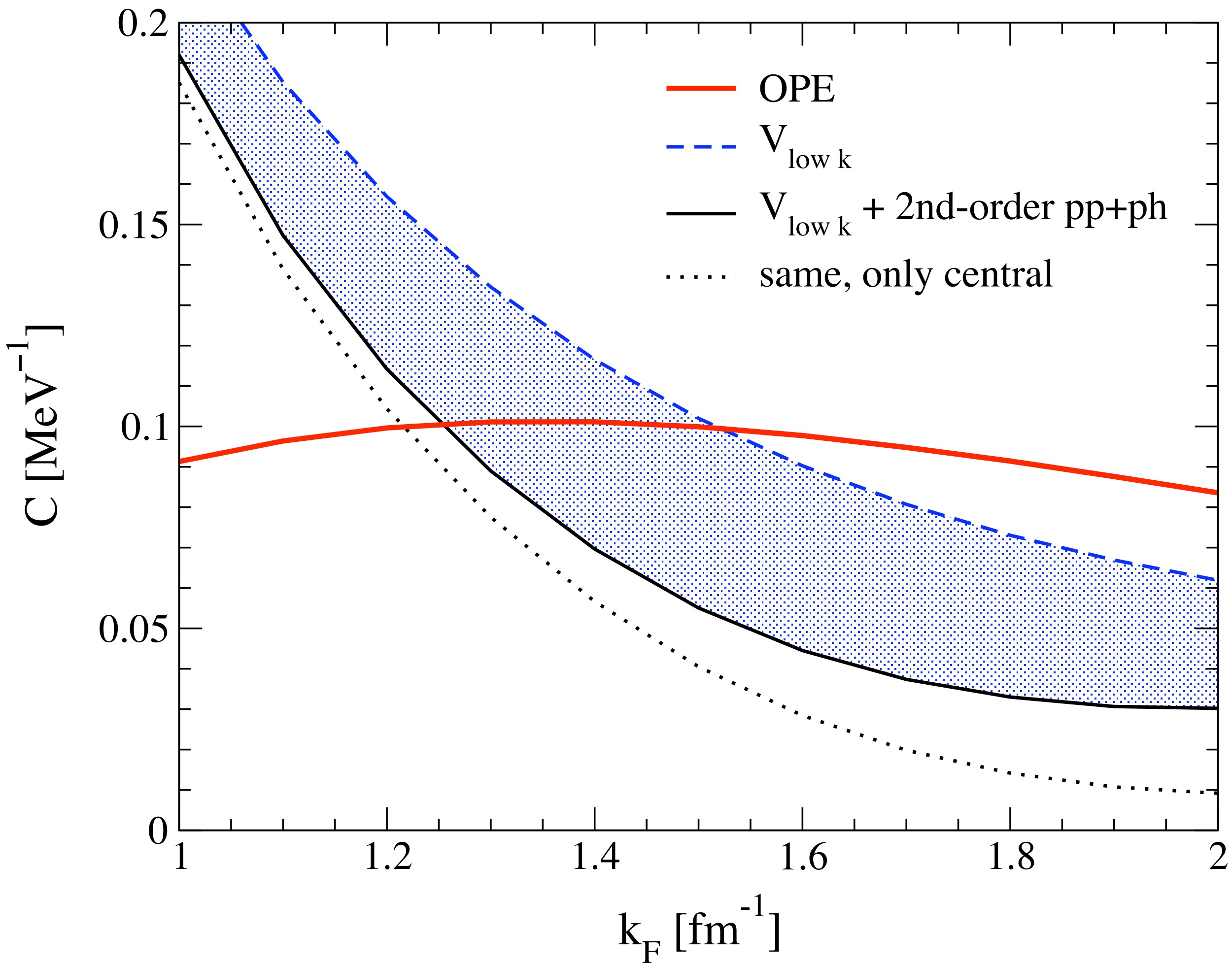
- Change of spin (more generally weak charge) in collision is important
- $\mathcal{A}$ – scattering amplitude times $N(0)$. Many different terms.
- $C_\sigma \sim 0.05$.
- Quasiparticle picture consistent for $\Gamma_\sigma \lesssim T$, or $T \lesssim 20$ MeV.



Spin dynamical structure factor

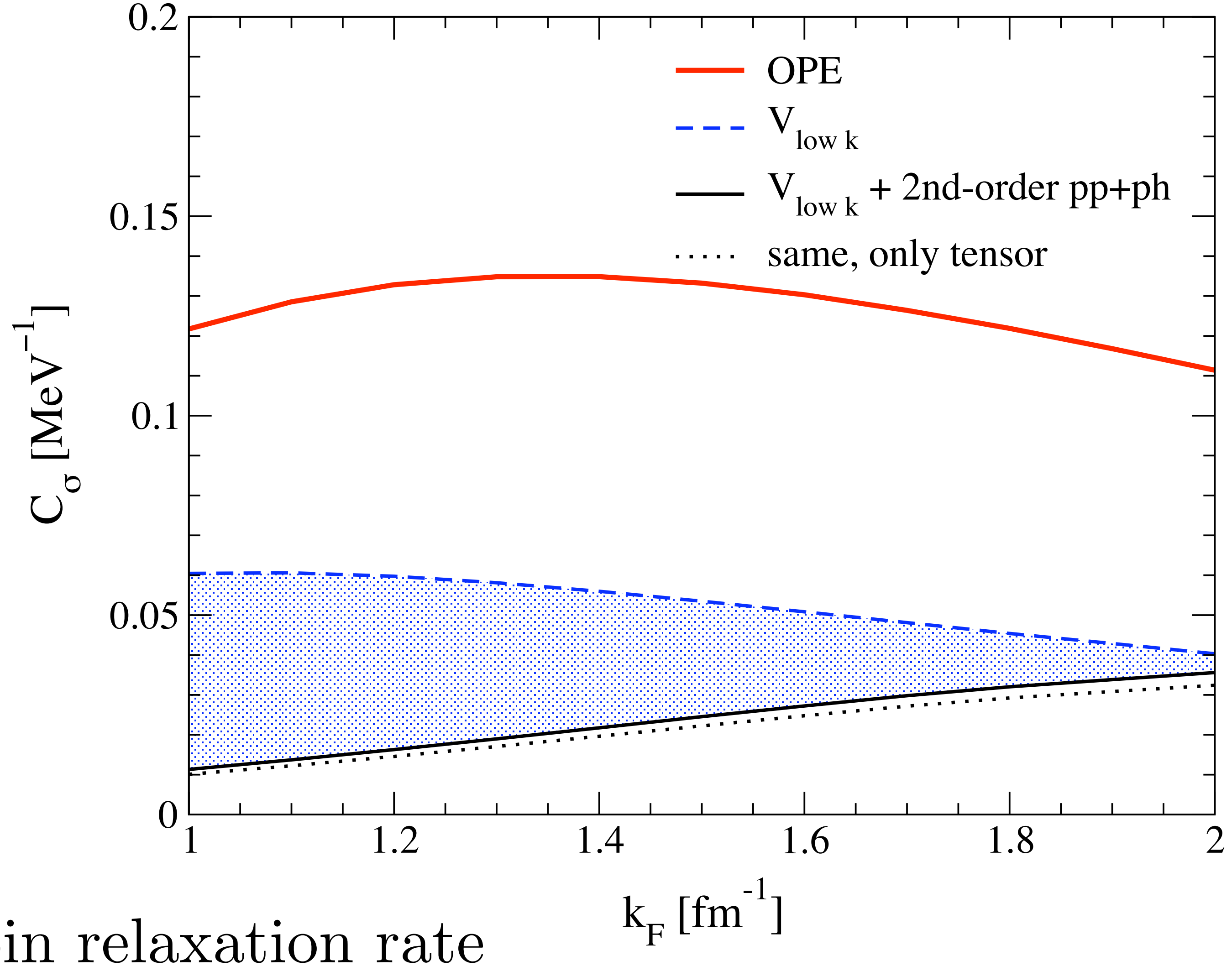
Nucleon–nucleon interactions

- Spin relaxation time depends on subtle aspects of interaction.
- One-pion exchange. $\uparrow + \uparrow \;\; \rightarrow \;\; \downarrow + \downarrow$
- Potentials fitted to scattering data (Paris, Argonne, ...)
- V_{lowk}. (Schwenk, Brown, Friman)
  - Effective interaction in reduced space.
  - Put in medium effects by perturbation theory.
- Chiral perturbation theory. Expansion in powers of momentum.



Quasiparticle relaxation rate

$$\frac{1}{\tau} = C\left[T^2 + \left(\frac{\omega}{2\pi}\right)^2\right]$$



Spin relaxation rate

$$\frac{1}{\tau_\sigma} = C_\sigma \left[T^2 + \left(\frac{\omega}{2\pi} \right)^2 \right]$$

Neutrino processes

Scattering mean free path

$$\frac{1}{l} \propto T \int_0^\infty \frac{S_\sigma(\omega, q)}{\omega} \propto \frac{m^*}{1 + G_0}$$

Total scattering rate unaffected by spin relaxation

Energy transfer sensitive

- Energy loss rate due to neutrino pair bremsstrahlung

$$Q = \frac{C_A^2 G_F^2}{20\pi^3} n \int_0^\infty d\omega \omega^6 e^{-\omega/T} S_\sigma(\omega)$$

Rate reduced by a factor of 4-10

	G_0	0	0.8	0	0.8	0	0.8
k_F [fm^{-1}]	T [MeV]	C_σ from OPE		$V_{\mathrm{low}\,k}$		$V_{\mathrm{low}\,k}$ + 2nd order	
1.0	5	1.77	1.62	0.911	0.888	0.173	0.172
	10	4.02	3.00	2.25	2.06	0.441	0.440
1.7	5	2.75	2.49	1.09	1.07	0.679	0.675
	10	6.18	4.55	2.73	2.57	1.72	1.68

TABLE II: Energy-loss rate Q of Eq. (45) due to neutrino-pair bremsstrahlung, $nn \to nn\nu\bar{\nu}$, for characteristic temperatures and Fermi momenta. Results are given without and with mean-field effects, $G_0 = 0$ and $G_0 = 0.8$ respectively, and for different spin relaxation rates $1/\tau_\sigma$ based on Fig. 1. The energy-loss rates are in units of $10^{33}\,\mathrm{erg\,cm^{-3}\,s^{-1}}$ for $T = 5\,\mathrm{MeV}$ and $10^{35}\,\mathrm{erg\,cm^{-3}\,s^{-1}}$ for $T = 10\,\mathrm{MeV}$.

• Rate of energy transfer between neutrinos and neutrons due to neutrino pair bremsstrahlung and annihilation

$$\frac{\Delta Q}{\Delta T} = \frac{C_A^2 G_F^2}{20\pi^3} \frac{n}{T^2} \int_0^\infty d\omega \omega^7 e^{-\omega/T} S_\sigma(\omega)$$

	G_0	0	0.8	0	0.8	0	0.8	
k_F [fm^{-1}]	T [MeV]	C_σ from OPE		$V_{\mathrm{low}\,k}$		$V_{\mathrm{low}\,k}$ + 2nd order		
1.0	5	2.48	2.26	1.27	1.24	0.241	0.241	$nn \leftrightarrow nn\nu\overline{\nu}$
		3.46	2.81	1.94	1.76	0.401	0.394	$\nu nn \leftrightarrow \nu nn$
	10	2.81	2.10	1.58	1.44	0.308	0.307	$nn \leftrightarrow nn\nu\overline{\nu}$
		3.41	2.24	2.20	1.79	0.502	0.485	$\nu nn \leftrightarrow \nu nn$
1.7	5	3.85	3.48	1.53	1.50	0.949	0.943	$nn \leftrightarrow nn\nu\overline{\nu}$
		5.33	4.30	2.38	2.20	1.53	1.46	$\nu nn \leftrightarrow \nu nn$
	10	4.32	3.18	1.91	1.80	1.21	1.18	$nn \leftrightarrow nn\nu\overline{\nu}$
		5.21	3.37	2.76	2.35	1.84	1.67	$\nu nn \leftrightarrow \nu nn$

TABLE III: Rate of energy transfer $\Delta Q/\Delta T$ due to neutrino-pair bremsstrahlung and absorption, $nn \leftrightarrow nn\nu\overline{\nu}$, of Eq. (46) and due to inelastic scattering, $\nu nn \leftrightarrow \nu nn$, of Eq. (47) for characteristic temperatures and Fermi momenta. Results are given without and with mean-field effects, $G_0 = 0$ and $G_0 = 0.8$ respectively, and for different spin relaxation rates $1/\tau_\sigma$ based on Fig. 1. The rates are in $10^{33}\,\mathrm{erg\,cm^{-3}\,s^{-1}\,MeV^{-1}}$ for $T = 5\,\mathrm{MeV}$ and in $10^{35}\,\mathrm{erg\,cm^{-3}\,s^{-1}\,MeV^{-1}}$ for $T = 10\,\mathrm{MeV}$.

Rates reduced by factor of up to 10

For the future

- Nonzero q (recoil).
- Mixtures of neutrons and protons.
  Central interactions can relax spins.
- Other components (Clusters – α particles, other nuclei).
- Extend to less degenerate regime.
- Prepare tables for numerical codes. Modular codes.
- Sensitivity tests. Where to put calculating effort.
- Better estimates of in-medium effects on scattering.

Dilute neutron matter – a resonant Fermi gas

- Long scattering length, $a_{\rm nn} \approx -18.5$ fm in singlet state.
- Effective range significant, $r_{\rm e} \approx 2.7$ fm.
- Like a *narrow* Feshbach resonance.
  Most resonances in ultracold gases are broad.

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