RESCEU 2008

Cosmology from Topology of Large Scale Structure of the Universe

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Why is the topology study useful?

Direct intuitive meanings

At large linear scales

Gaussianity of the primordial density field (Gott et al. 1986)

At small non-linear scales

Galaxy distribution at non-linear scales sensitive to cosmological parameters & galaxy formation mechanism

(Park, Kim & Gott 2005)

Measures of topology - Minkowski Functionals

3D

1. 3d genus (Euler characteristic)
2. mean curvature
3. contour surface area
4. volume fraction

→ 3d galaxy redshift survey data

2D

1. 2d genus (Euler characteristic)
2. contour length
3. area fraction

→ CMB temperature/polarization fluctuations, 2d galaxy surveys

1D

1. level crossings
2. length fraction

→ Lyα clouds, deep HI surveys, pencil beam galaxy surveys

The 3D Genus

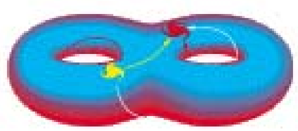
- **Definition**

G = # of holes - # of isolated regions

in iso-density contour surfaces

$= 1/4\pi \cdot \int_S \kappa \, dA$ (Gauss-Bonnet Theorem)

[ex. G(sphere)=-1, G(torus)=0, G(two tori)=+1]



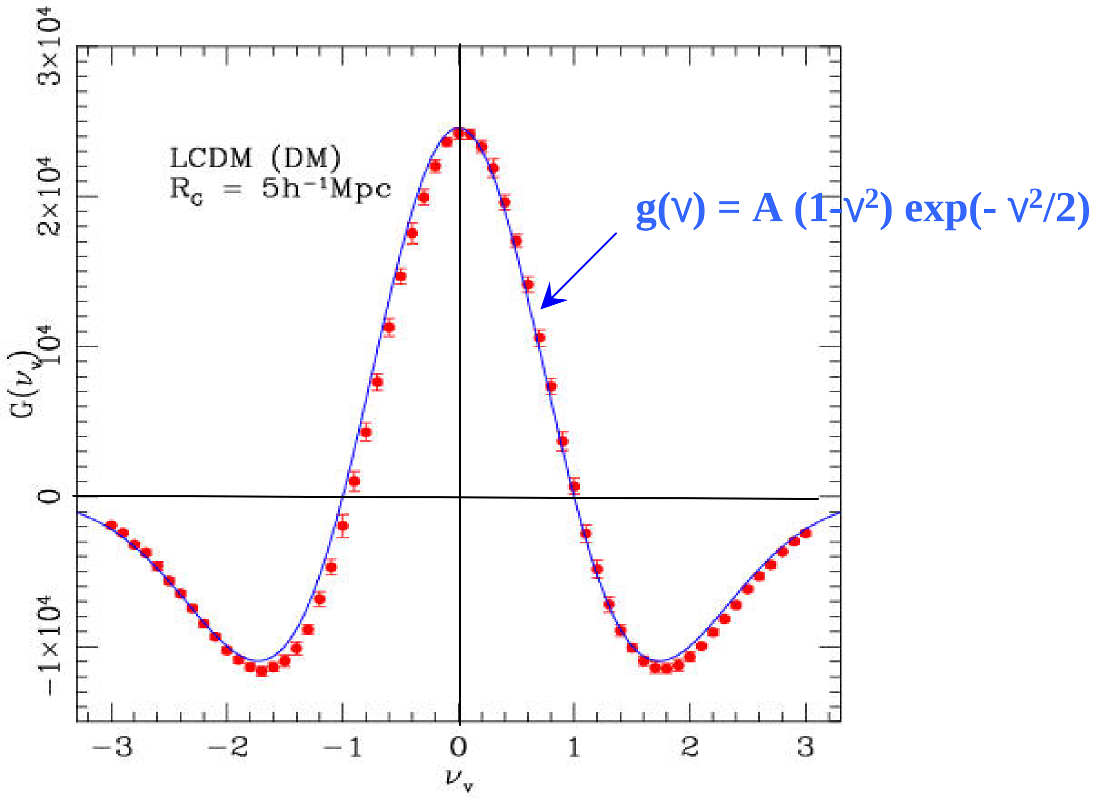
: 2 holes – 1 body = +1

Gaussian Field

Genus/unit volume $g(\nu) = A\,(1-\nu^2)\exp(-\nu^2/2)$

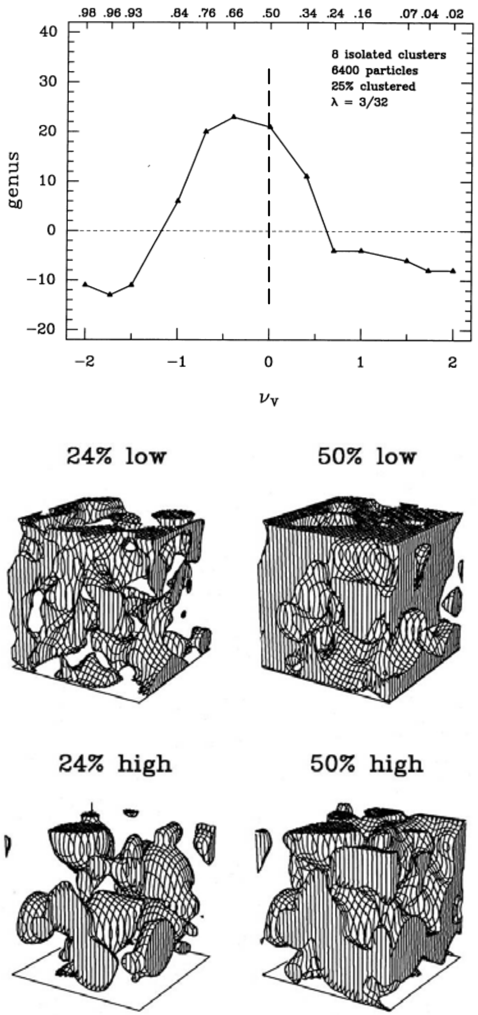
where $\nu=(\rho-\rho_b)/\rho_b\sigma$ & $A=1/(2\pi)^2 \langle k^2/3\rangle^{3/2}$

if $P(k)\sim k^n$, $A R_G^3 = [8\sqrt{2}\pi^2]^{-1} * [(n+3)/3]^{3/2}$

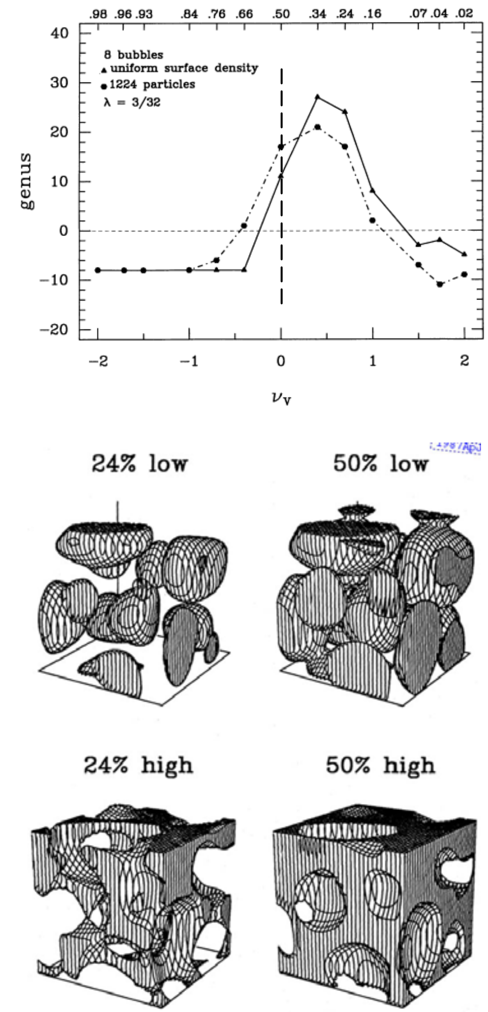


Non-Gaussian Field (Toy models)

Clusters



Bubbles



(Weinberg, Gott & Melott 1987)

History of LSS Topology Study

I. Early Works

- **1986: Hamilton, Gott, Weinberg; Gott, Melott, Dickinson**
  – smooth small-scale NL clustering to recover initial topology
- **1987-8: GWM, WGM, MWG, Gott et al.**
  – cosmological & toy models. $R_G>3r_c$ to recover initial topology
- **1989: Gott et al. – observed galaxies, dwarfs, clusters**
- **1991: Park & Gott – NL gravitational evolution& biasing effects**
- **1992: Weinberg, Cole – PS, initial skewness, biasing effects**
- **1994: Matsubara – 2nd order perturbation in weakly NL regime**
- **1996: Matsubara – redshift space distortion in L regime**
  Matsubara & Suto – NL gravitational evolution & z-space distortion
  Matsubara & Yokoyama - non-Gaussian fields

II. Recent Works

- 2000: Colley et al. – Simulation of SDSS
- 2001, 2003: Hikage, Taruya & Suto – dark halos (analytic & numerical)
- *2003: Matsubara – 2nd order perturbation theory*
- Minkowski functionals
  Gott et al. (1990) - CMB
  Mecke, Buchert & Wagner (1994); Schmalzing & Buchert (1997)
  Matsubara(2008) - perturbation theory of halo bias & redshift-space distortion

III. 3D genus analysis of observational data

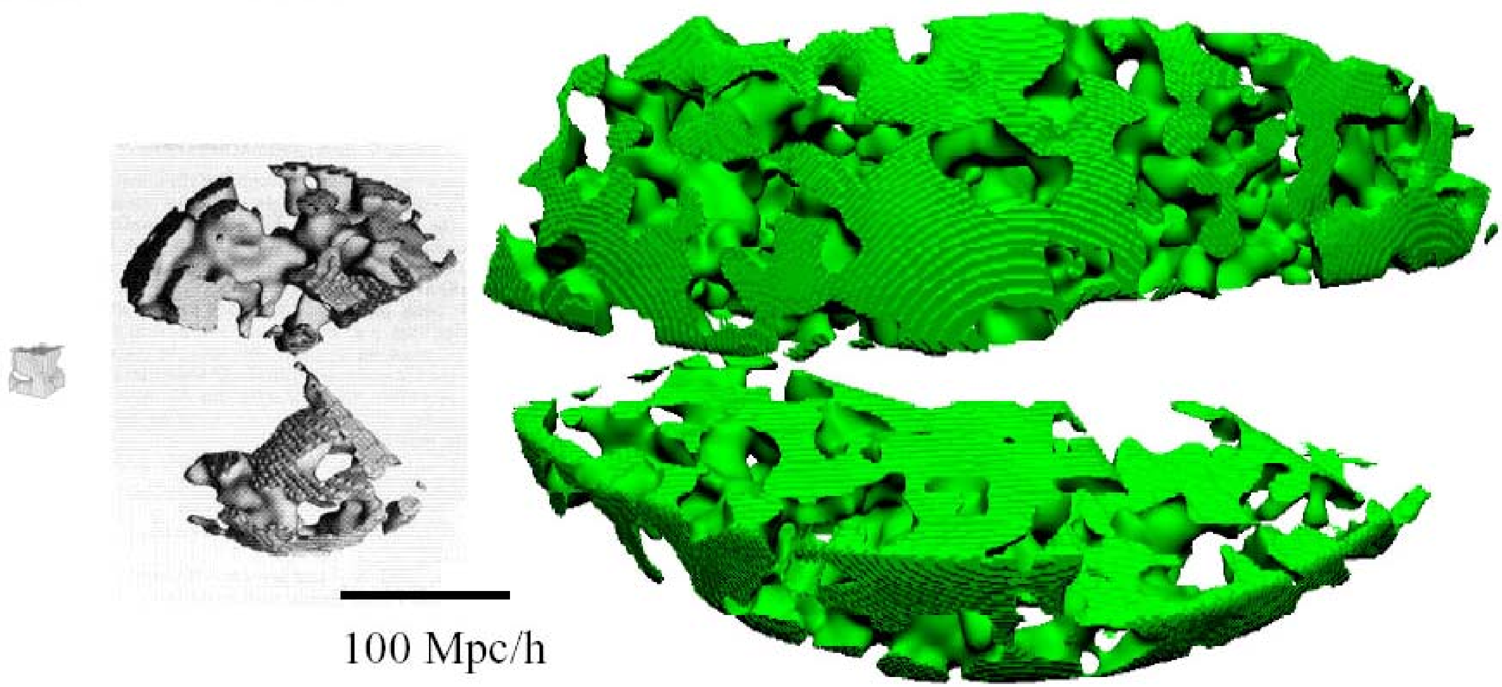
1989: Gott et al.	- CfA 1 etc.	→ consistent with Gaussian
1992: Park, Gott, & da Costa	- SSRS 1	
1992: Moore et al.	- IRAS QDOT	
1994: Rhoads et al.	- Abell Clusters	
1994: Vogeley et al.	- CfA 1+2	
1997: Protogeros & Weinbergs	- IRAS 1.2Jy	"
1998: Springel et al.	- IRAS 1.2Jy	
1998: Canavezes et al.	- IRAS PSCz	
2002: Hikage et al.	- SDSS EDR	
2003: Hikage et al.	- SDSS LSS Sample 12	
2004: Canavezes & Efstathious	- 2dFRGS	
2005: Park et al.	- SDSS LSS Sample 14	→Luminosity bias in topology

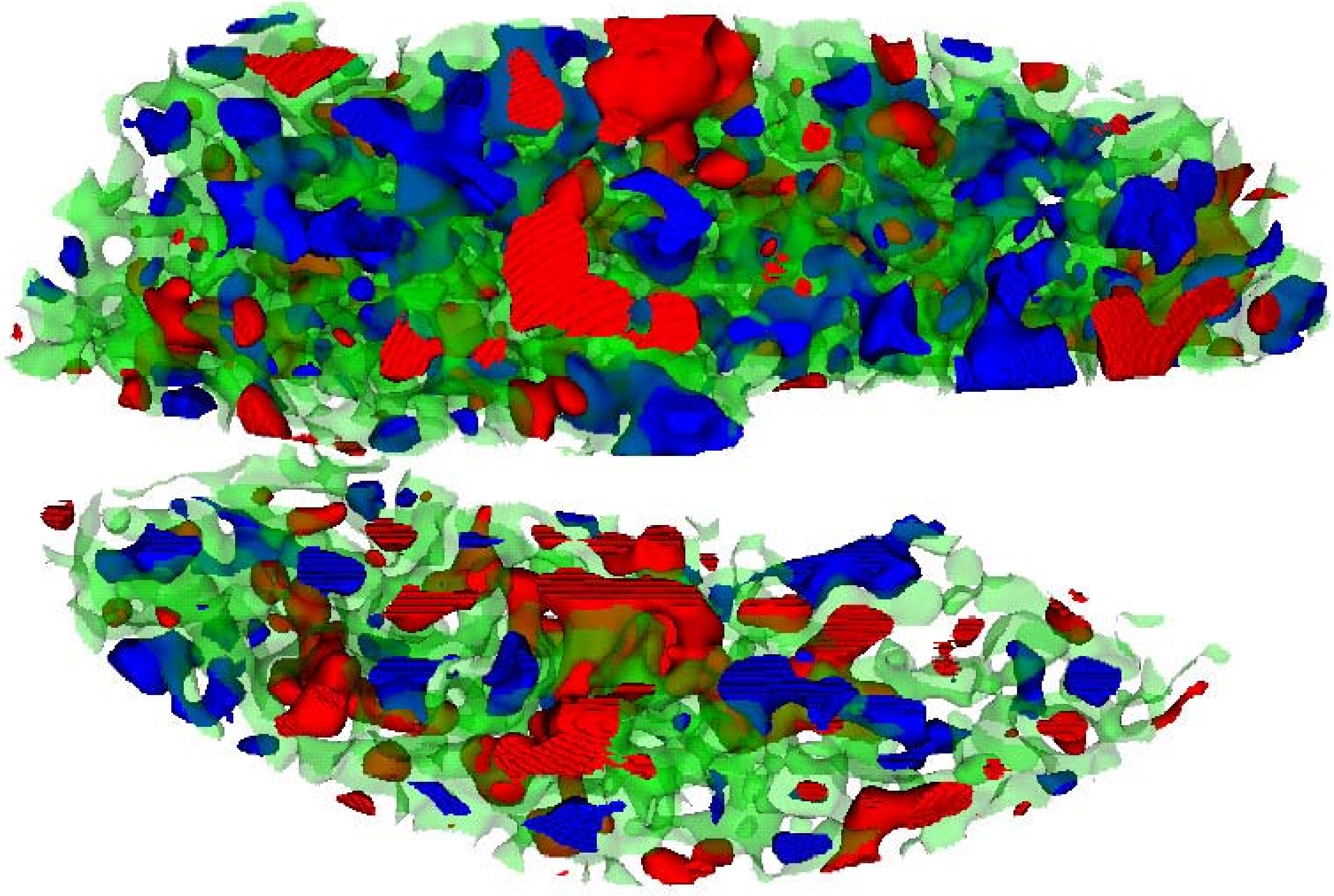
Observational sample sizes

Gott, Melott & Dickinson (1986)

Vogeley et al. (1994) : CfA2

Gott et al. (2006) : SDSS DR4plus

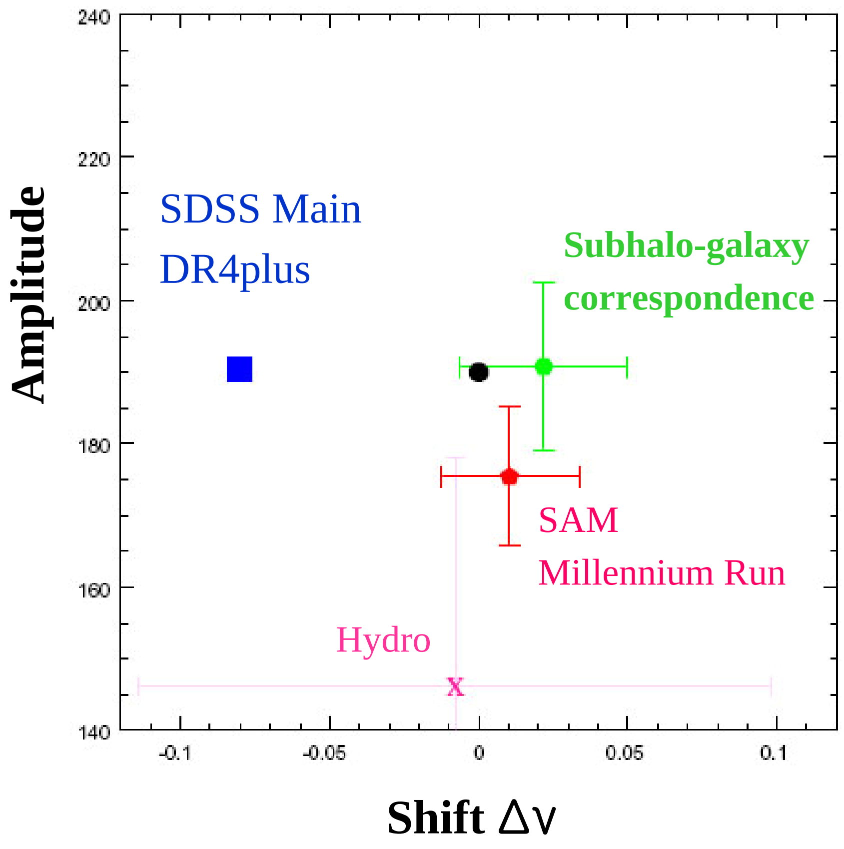


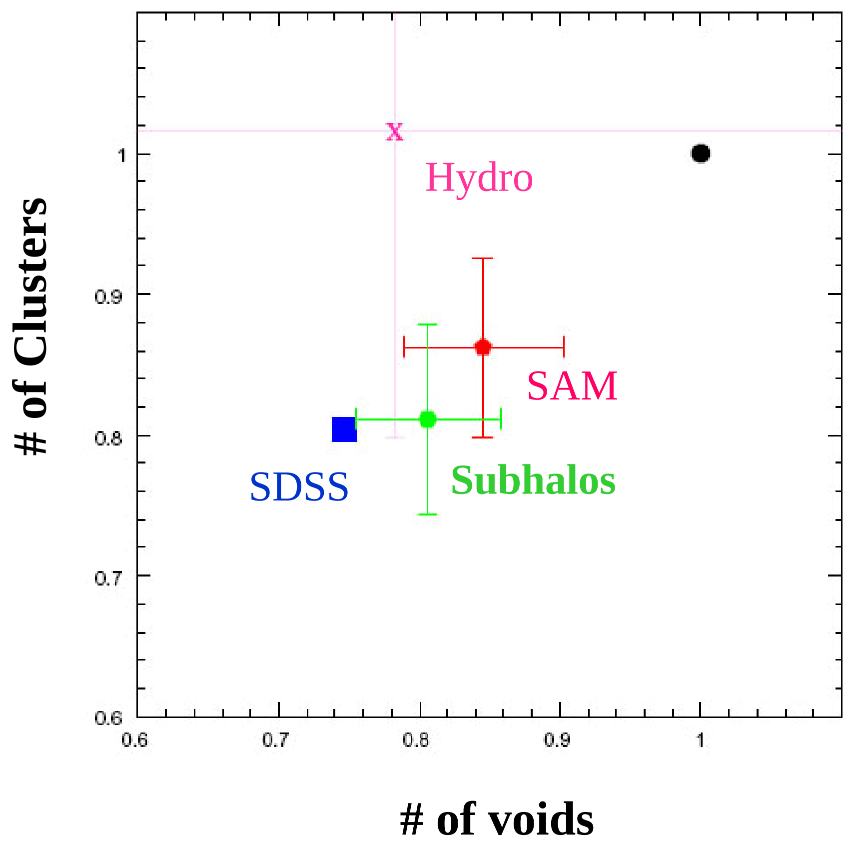


Voids (blue - 7% low), filaments/clusters (red - 7% high) in the SDSS DR4plus sample (Gott et al. 2008) => Sponge !!

SDSS DR4plus sample (Gott et al. 2008) : smoothing scale R_G=6h^{-1}Mpc

Test for galaxy formation models





Current status of LSS topology study

1. **Large scales (>> 10 h^{-1}Mpc)**

   Primordial Gaussianity → No strong constraints yet due to small sample size

   (But SDSS LRG sample & future deep redshift surveys)

2. **Small scales (< 10 h^{-1}Mpc)**

   Little study so far. Needs dense sample.

   Topology at small scales is sensitive to cosmological parameters & galaxy formation

   (gravitational evolution, galaxy biasing, internal physical properties of galaxies)

Large-scale structure

as a cosmic ruler

Large Scales

constrain the shape of power spectrum P(k) & the expansion history of space H(t)

→ cosmological parameters like $\Omega_m h$, w, etc.

Observables for cosmological parameter estimation

1. primordial fluctuations (~initial conditions)

CMB (+neutrino, gravitational wave)

=> geometry of space, matter contents, matter P(k), non-Gaussianity

2. Expansion history of the space

standard candle	$D_L(z) = (1+z)\, r(z)$	SN Ia	HST Legacy, Essence, DES, SNAP
standard ruler	$D_A(z) = (1+z)^{-1}\, r(z)$ $dV/dzd\Omega = r^2(z)/H(z)$	AP test, BAO **Topology**	redshift surveys (SDSS)

=> H(z) or $r(z) = \int_0^z \frac{dz'}{H(z')}$

Observables

3. Growth of structures

ISW	l<30 CMB CC btw CMB & LSS	CMB, LSS	WMAP-Planck * SNAP-LSST-SDSS
Population density	comoving V * # density ~> dn/dz	clusters (SZ, Xray), galaxies	SDSS, ACT, APEX, DES, SPT
Weak lensing	shear convergence	imaging, photo-z	CFHTLS, SNAP, DES, LSST

=> depends on both expansion of space H(z) & matter power spectrum P(k)

4. Properties of non-linear structures

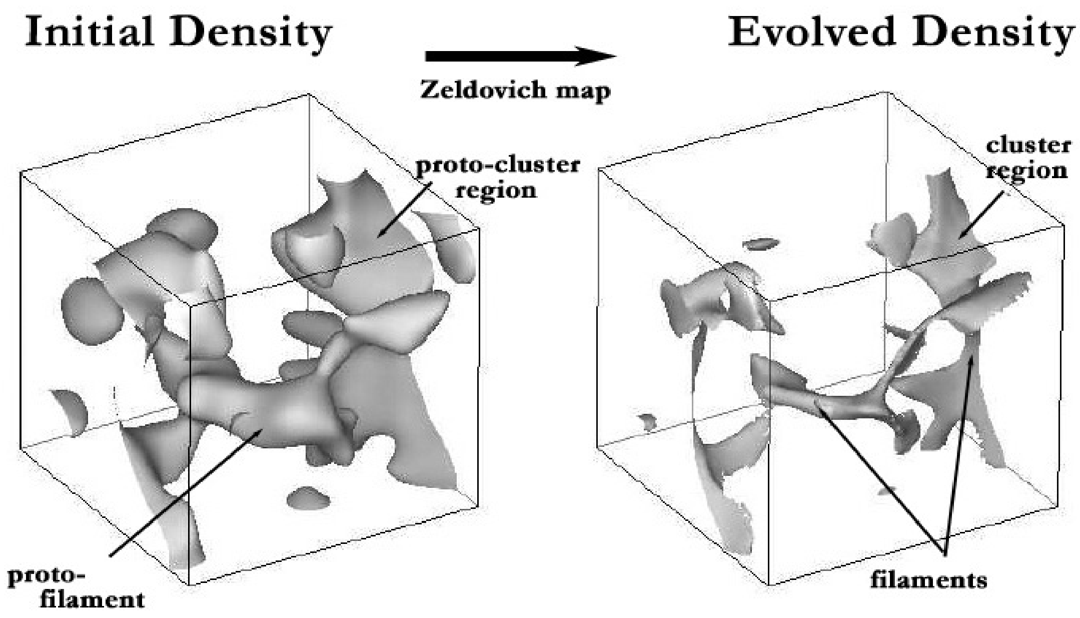
properties of galaxies, AGNs, cluster of galaxies, globular cluster

=> depends on H(z), P(k), non-linear physics

LSS as a cosmic ruler

Filament-dominated Cosmic Web

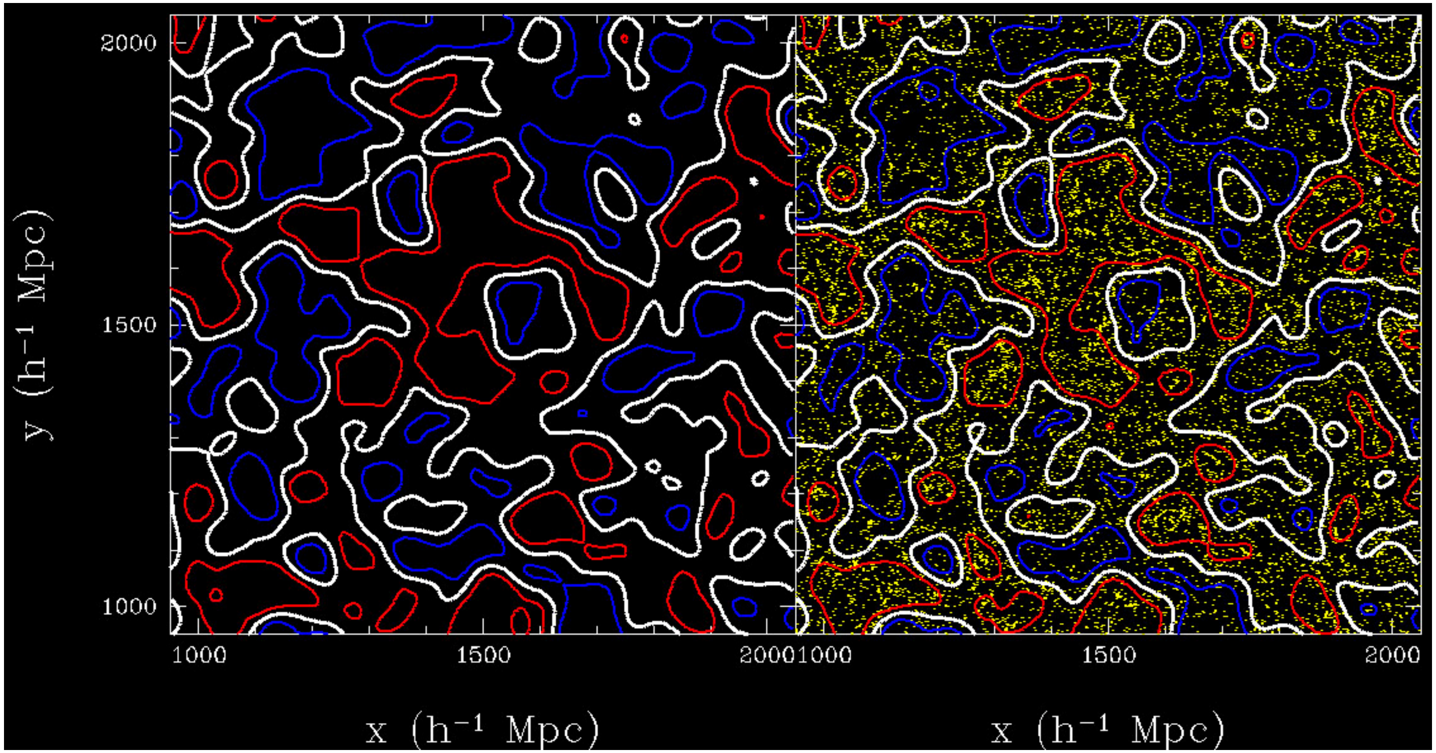
Bond et al. (1996) : Final-state web is present in embryonic form in the overdensity pattern of the initial fluctuations with NL dynamics just sharpening the image.



Cosmic Sponge Theory

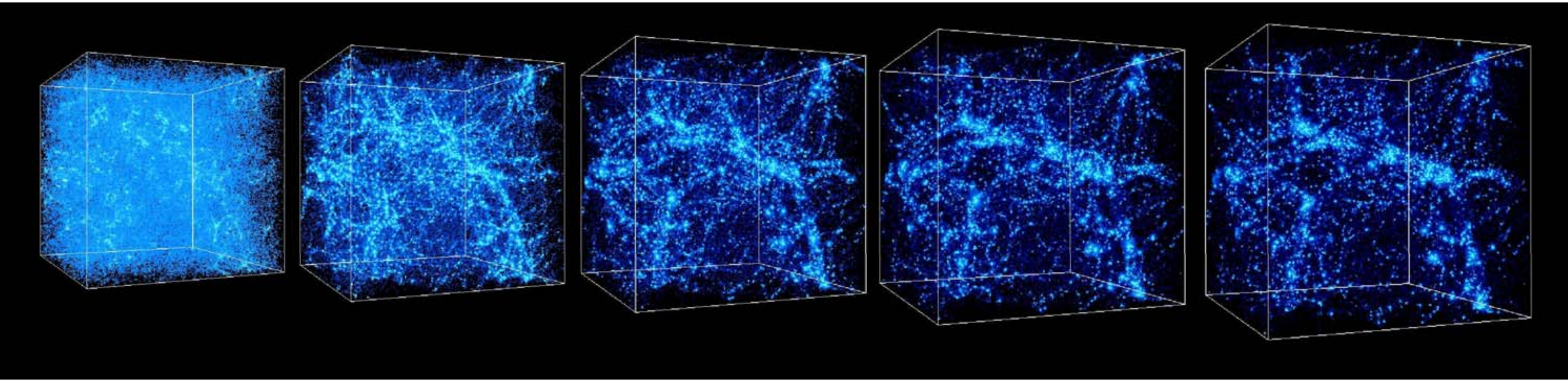
Not just overdensity patterns but all large-scale structures including voids maintain their initial topology (sponge) till the present

[Initial density field] **[Matter density field at z=0]**



flat LCDM

R_G=25h^{-1}Mpc



(courtesy: A. Kravtsov).

The LSS are in the (quasi-)linear regime,

& maintain the primordial sponge topology at all redshifts!

(= the original idea of using topology for the test for the Gaussianity of the primordial density field by Gott et al. in 1986)

Now, the LSS can be used as a cosmic ruler

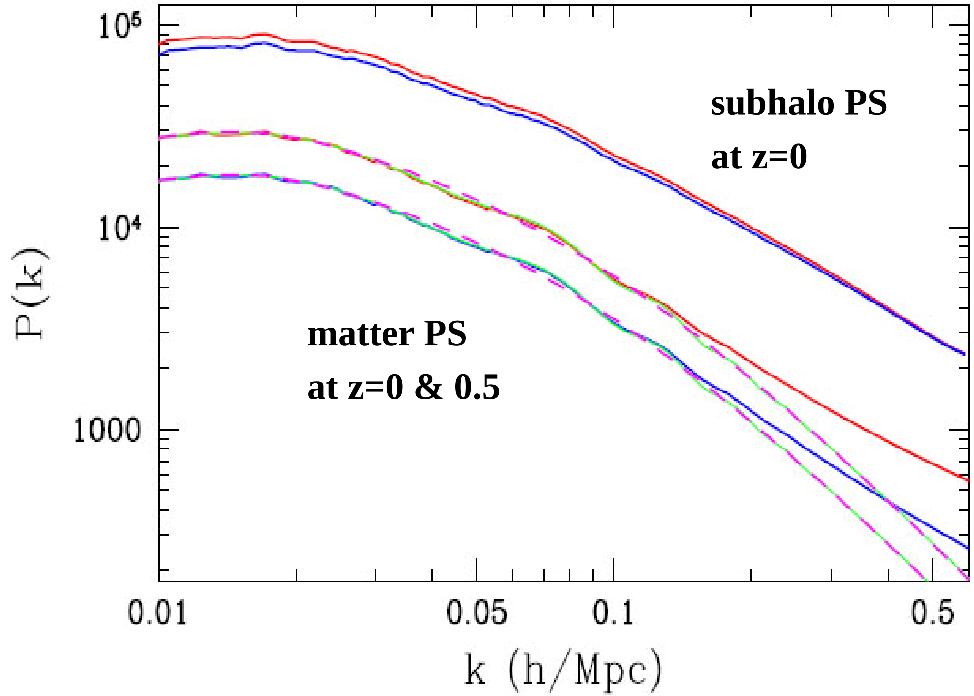
for cosmological parameter estimation!

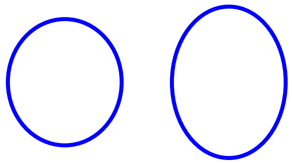
Cosmological parameter estimation from LSS topology analysis

I. Using the shape of PS

The PS of each model universe has a specific scale dependence, and one can use the whole shape of PS, not just tiny wiggles on top of smooth PS, as a cosmic ruler.

The genus amplitude depends on the shape of PS, and importantly to first order, the genus, as an intrinsic topology, is independent of all small non-linearities (gravitational evolution, biasing, redshift-space distortion)





Kim, Park & Gott (2008)

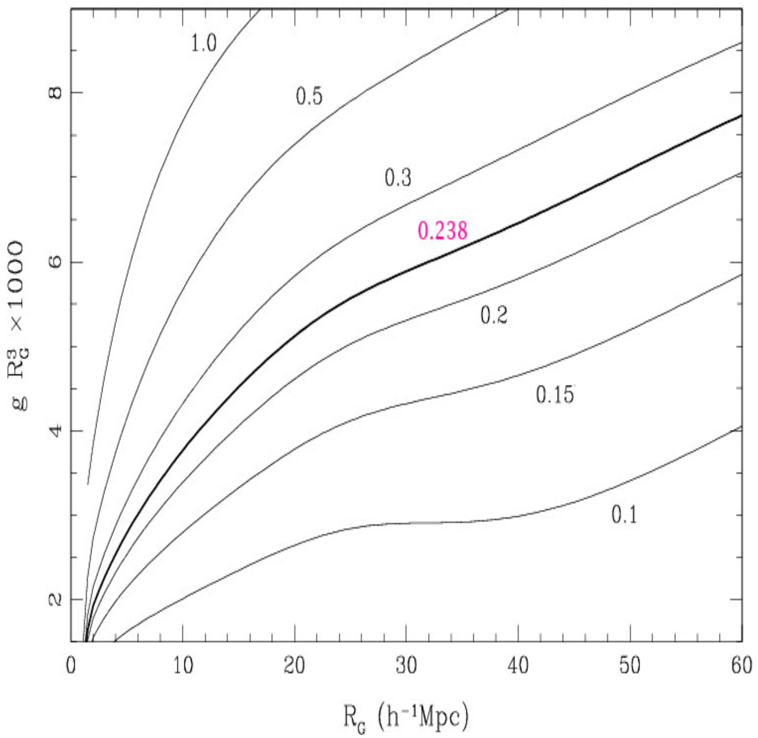
Genus amplitude for CDM PS : strong dependence on $\Omega_m h$

If we choose a wrong cosmology,

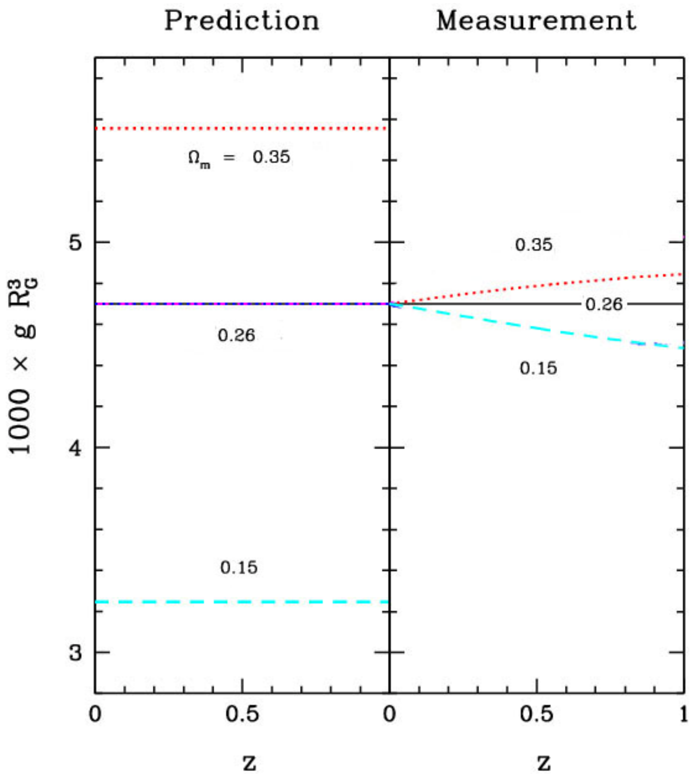
there is a difference between the predicted & measured genus.

observed z's → r(z) for a trial cosmology → compare the predicted & measured genus

Genus per smoothing volume



smoothing length



redshift

Effects of NL gravitational evolution, biasing, redshift-space distortion, discreteness, & finite pixel size

WMAP3

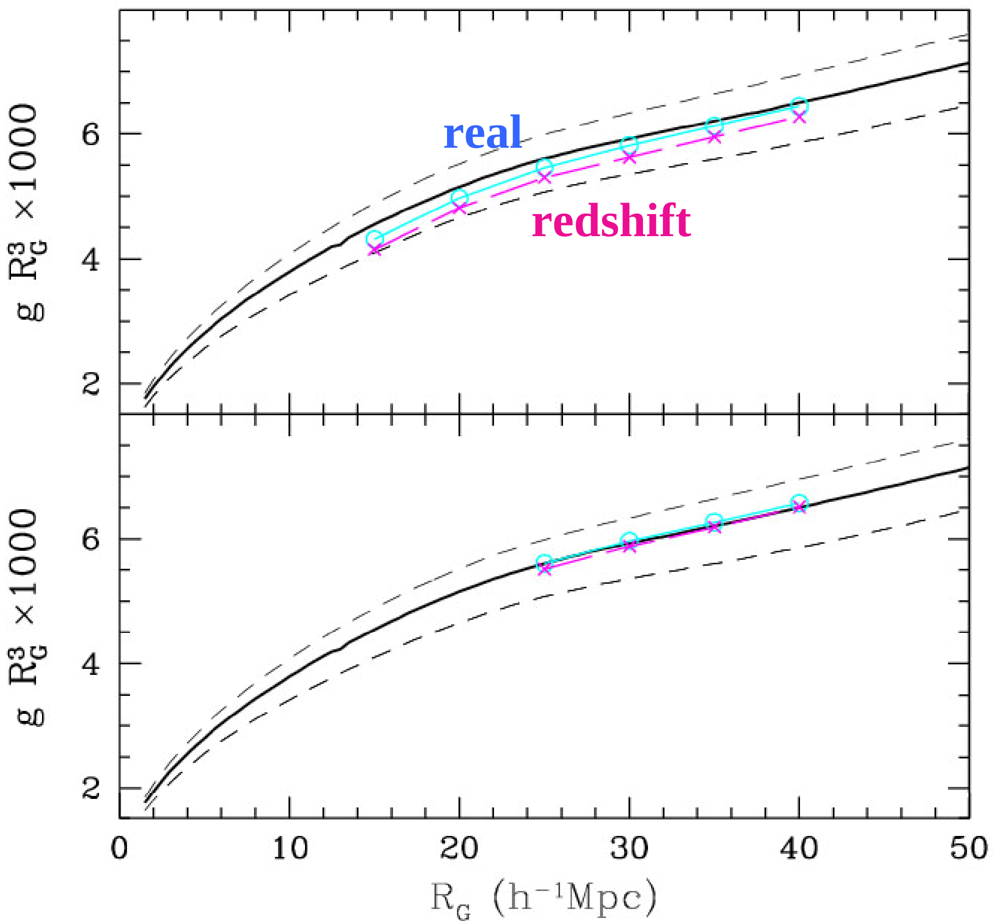
0.271

0.240

0.203

Matter in real & redshift spaces

Dark subhalos in real & redshift spaces

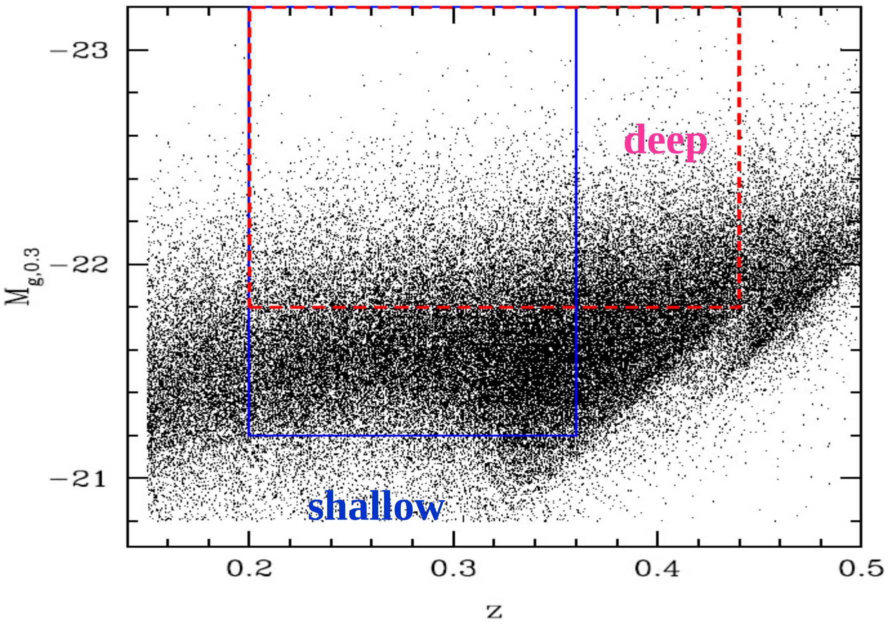


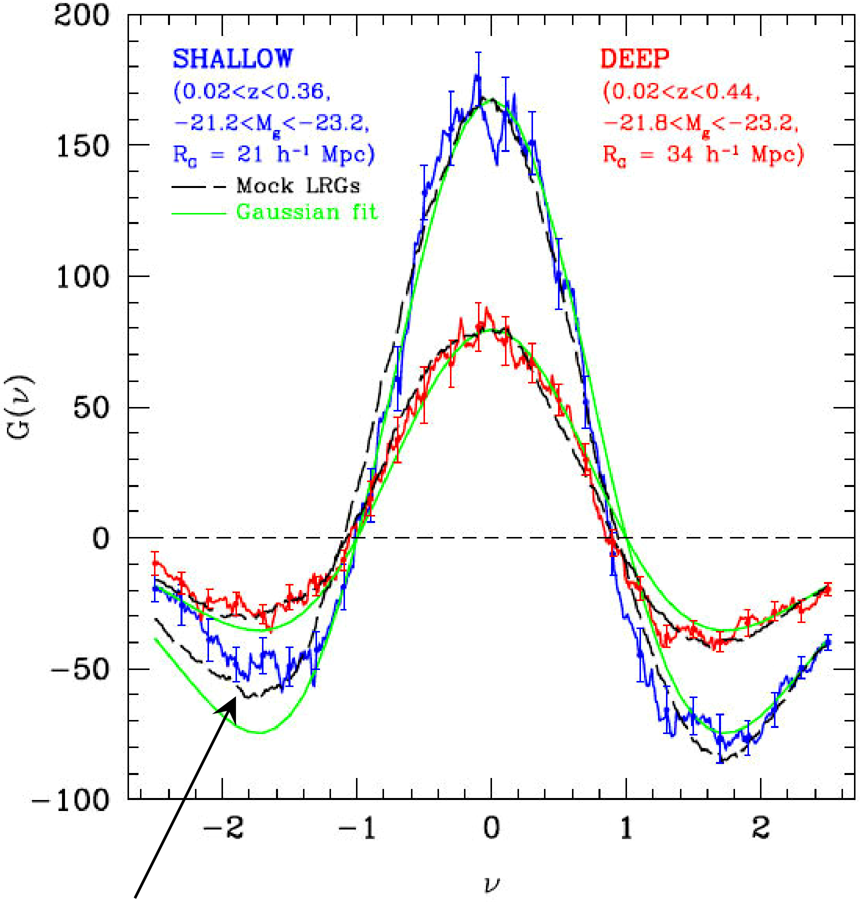
space / R_G	difference wrt linear g
real 25h^{-1}Mpc	-0.02%
redshift 25	-1.7%
real 35	+0.5%
redshift 35	-0.8%

Observational Data

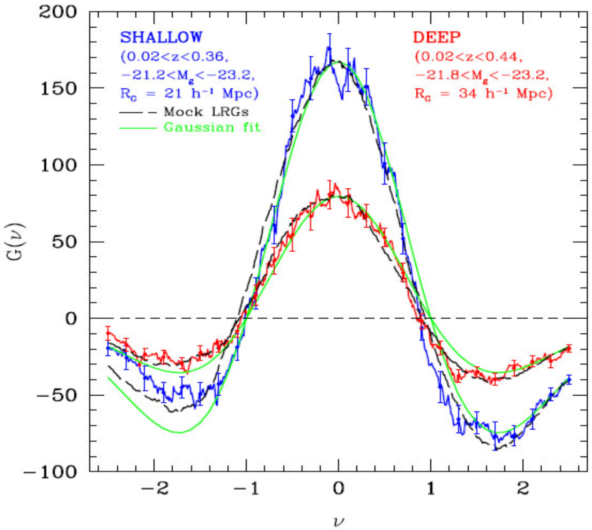
[Gott et al. 2008]

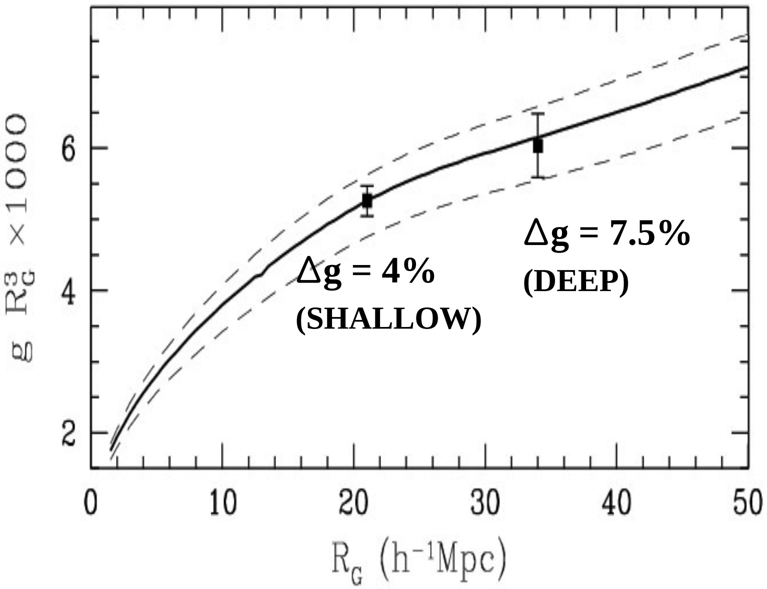
Luminous Red Galaxiess in SDSS DR4plus





LRGs in SDSS DR4plus





WMAP3

0.271

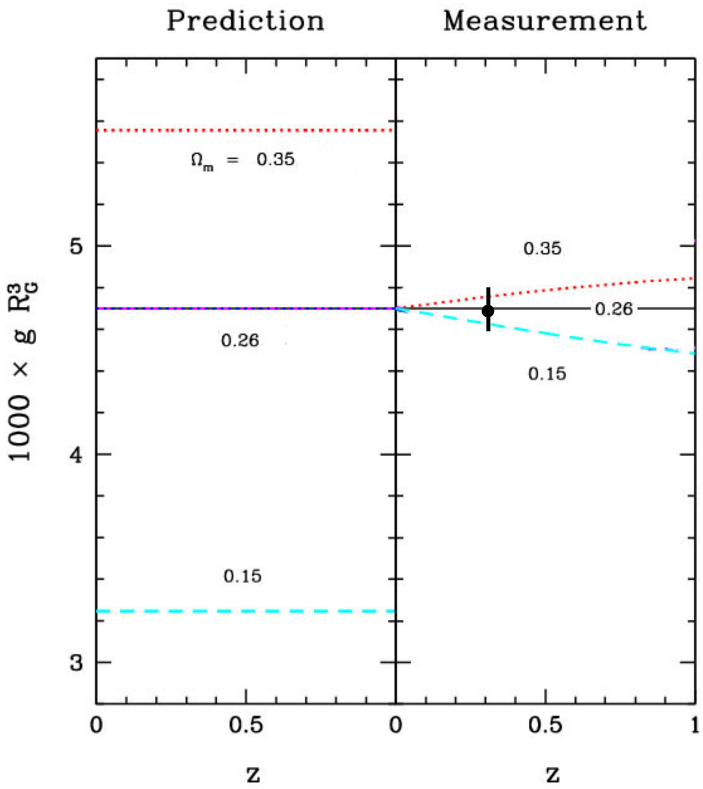
0.240

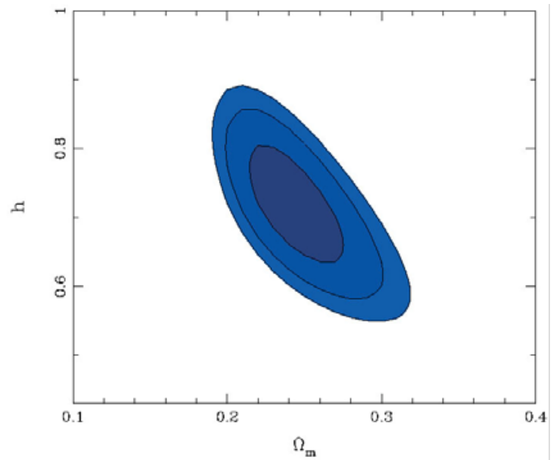
0.203

LRGs in SDSS DR4plus

: △g = 4% (shallow, R_G=21h^{-1}Mpc) & 7.5% (deep, R_G=34h^{-1}Mpc)

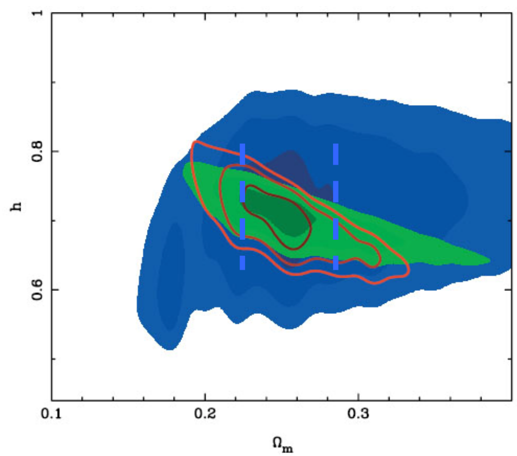
Ω_m = 0.241 ± 0.014 (if flat LCDM & h=0.72)





[Park et al. 2008]

Genus in SDSS LRG galaxies assuming flat LCDM & h=0.72 ±0.08



[Percival et al. 2007]

BAO in SDSS DR5 Main & LRG galaxies assuming flat LCDM & h=0.72 ±0.08

Future surveys

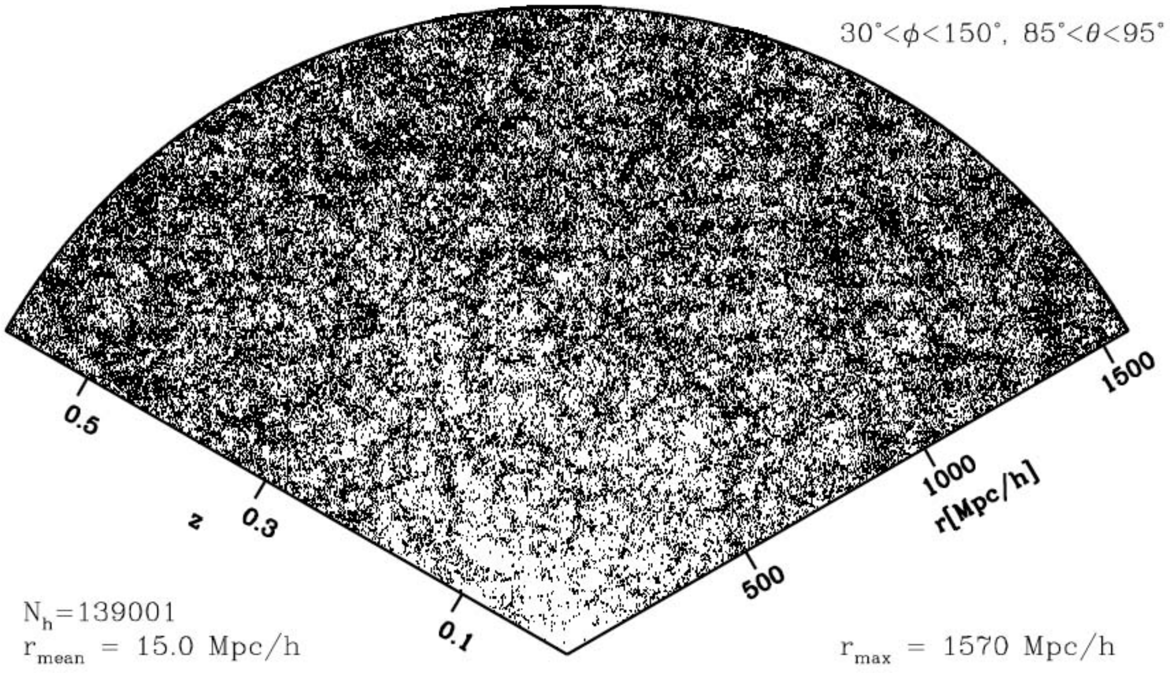
Constraint on PS shape using only the genus statistic

1. DR7 of SDSS I+II : # of LRGs ~ 100K

$\triangle g$ = ~3% & $\triangle \Omega_m$ = ~ 0.010

2. LRGs in SDSS-III :

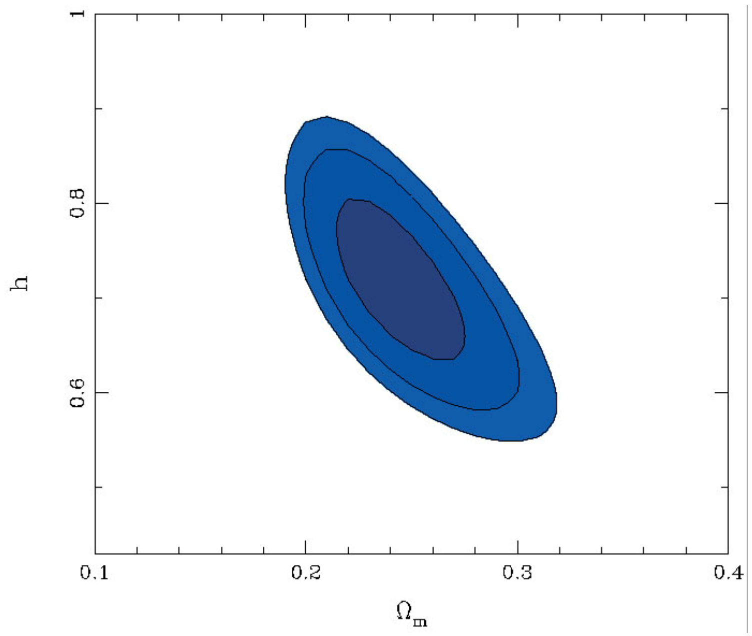
# of LRGs ~ 1.5M



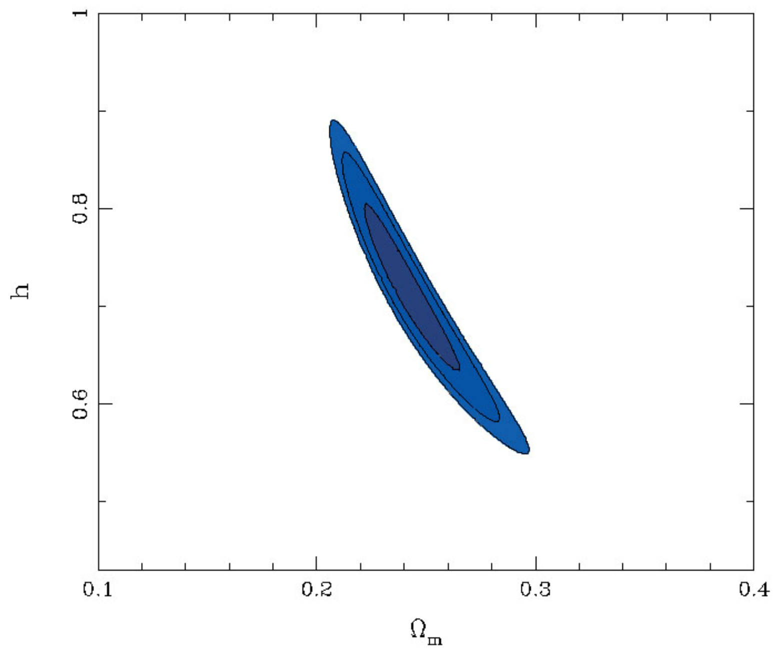
LRGs in SDSS-III : # of LRGs ~ 1.5M

$\triangle$g = ~0.8% & $\triangle\Omega_m$ ~ 0.004

Genus in SDSS DR4plus LRG galaxies assuming flat LCDM & h=0.72 ±0.08



Genus in SDSS-III LRG galaxies assuming flat LCDM & h=0.72 ±0.08



Cosmological parameter estimation from LSS topology analysis
II. Using the expansion history of the space

Focus on dark energy :

If we choose a wrong equation of state of the dark energy, there are differences between the predicted & measured genus as the redshift changes.

Strategy

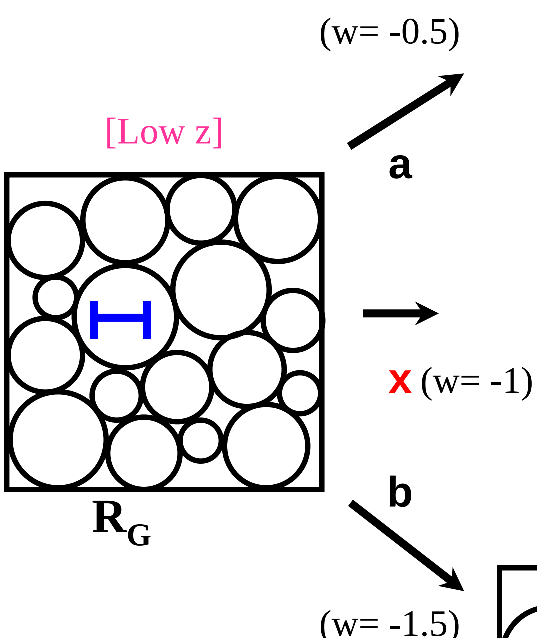
- choose a reference cosmology with a certain $w = P/\rho$
- convert z into r(z) through the reference cosmology
- calculate the genus
- compare the measured genus with the predicted genus in the reference cosmology

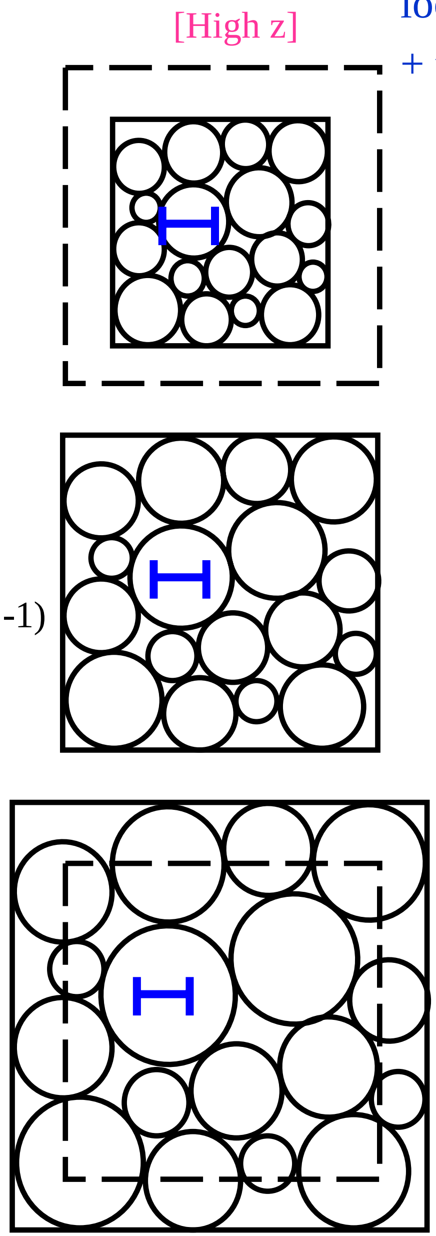
(the w-dependence originated from the different expansion history of space)

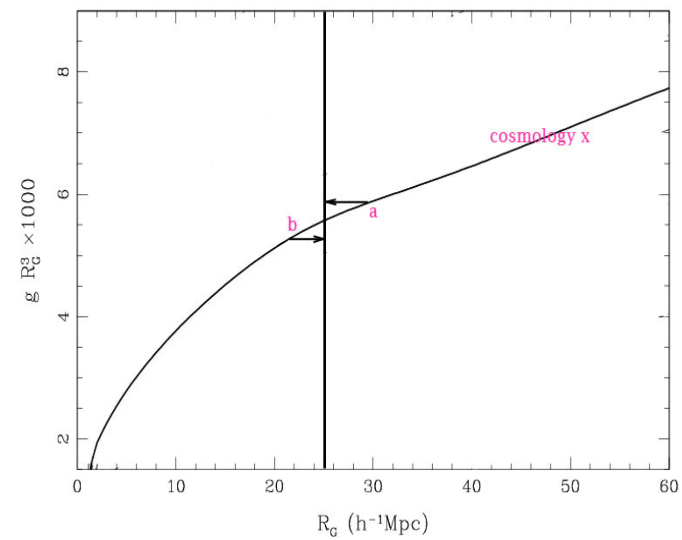
Suppose true cosmology is x

looking at a larger scale

+ taking a larger volume







Measured genus

= genus of true cosmology at scaled smoothing length

X volume factor of true cosmology

/ volume factor of wrong cosmology

Measured genus when a wrong cosmology 'a' is adopted

= genus of true cosmology at scaled R_G

X (volume factor of true cosmology / volume factor of wrong cosmology)

= g(R_G') X D_V(cosmology x) / D_V(cosmology a)

where $D_V = d_A^2/H(z)$, $R_G' = R_G$ X $[D_V(x)/D_V(a)]^{1/3}$, &

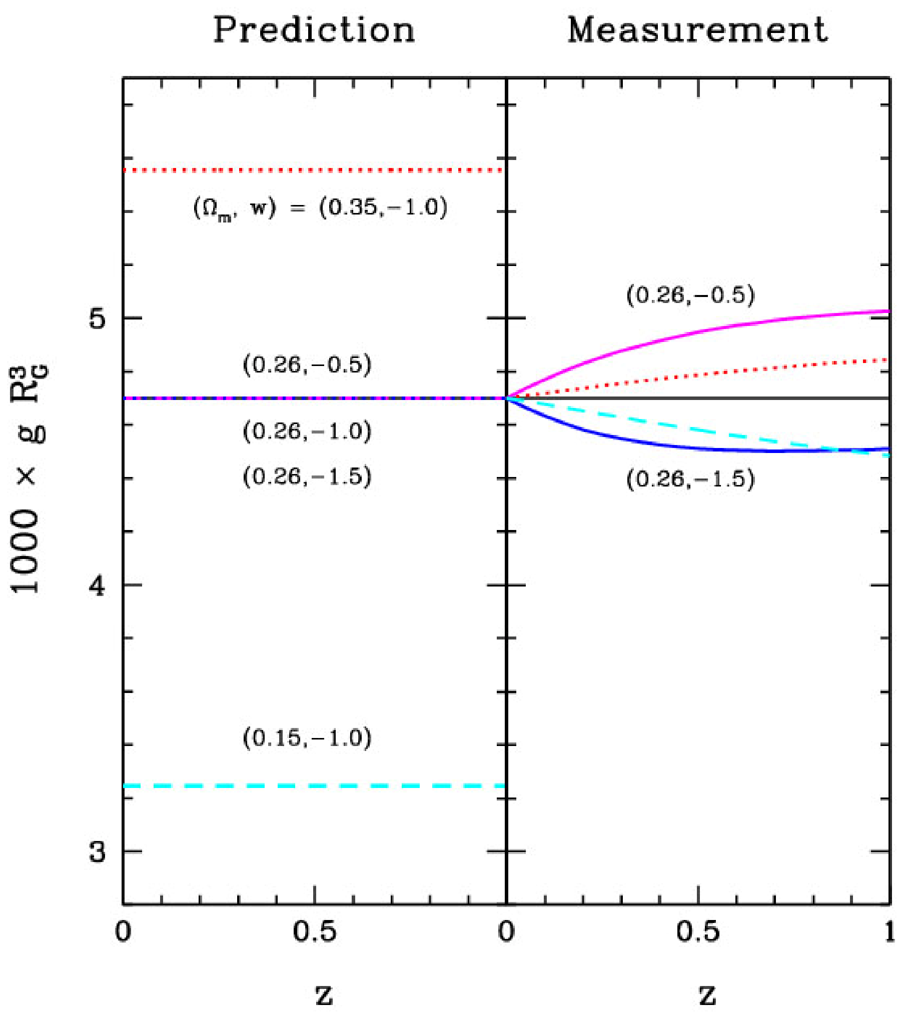
$$H(z) = H_0\sqrt{\Omega_m(1+z)^3 + \Omega_X \exp\left[3\int_0^z \frac{1+w(z)}{1+z}dz\right]}$$

Genus & Dark Energy

Suppose we live in a universe with $(\Omega_m, w) = (0.26, -1.0)$.

Let's choose a wrong w when z is converted to r(z).

Difference between the predicted and measured genus as z changes.



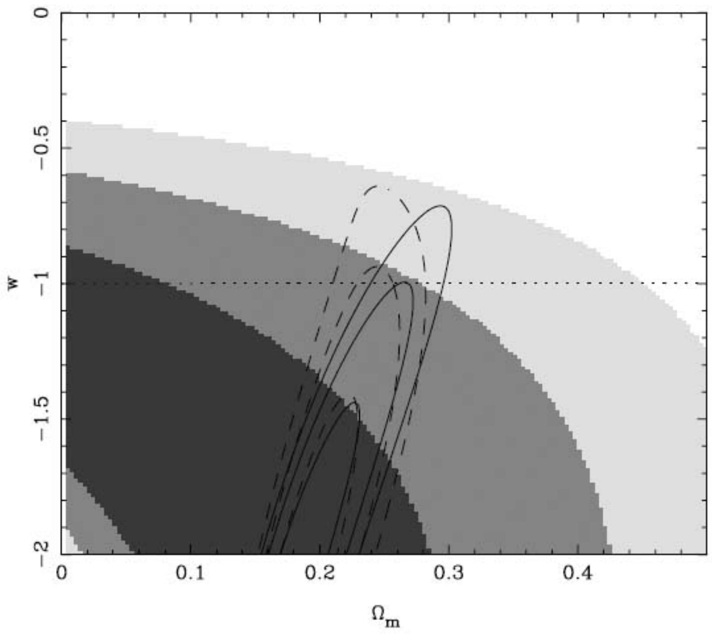
Constraint on 'w' using the genus statistic only :

LRGs in SDSS DR4plus

: △g = 4% (shallow, R_G=21h^{-1}Mpc) & 7.5% (deep, R_G=34h^{-1}Mpc)

→ Δw ~ 0.4

Likelihood contours from the BAO scale measurement for flat LCDM models with constant w. $D_V(z=0.35)/D_V(0.2)$ is used. [Percival et al. 2007]

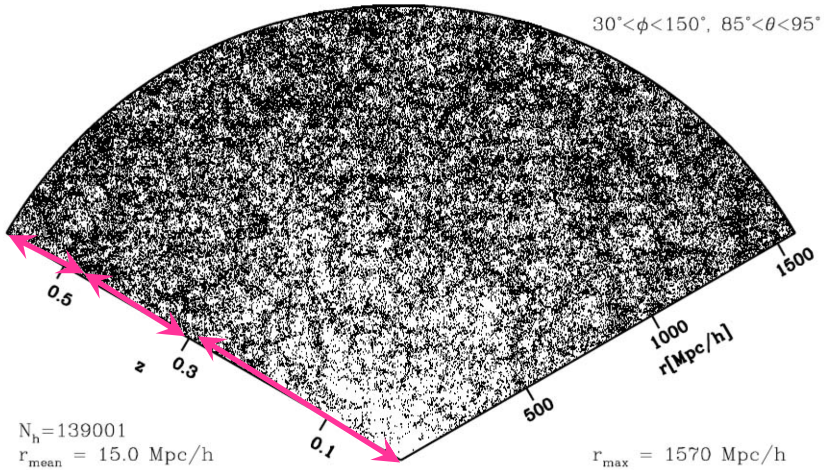


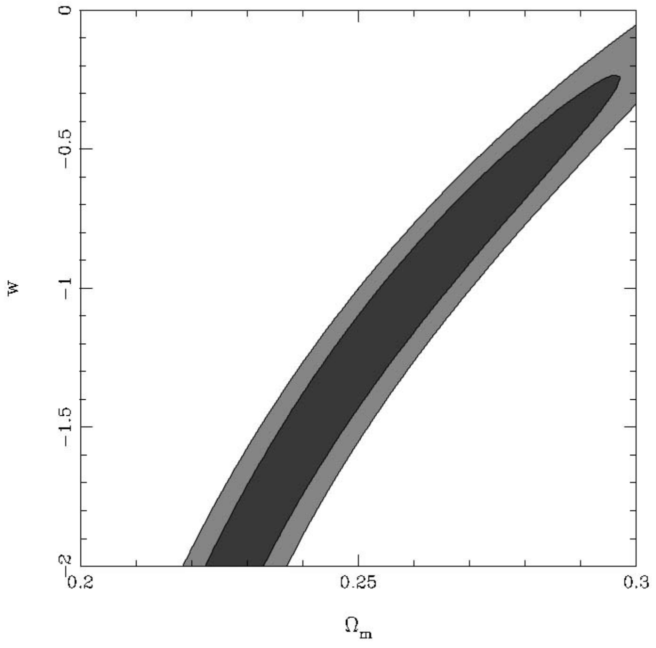
Future surveys

Constraint on 'w' using the genus statistic only :

LRGs in SDSS-III : # of LRGs ~ 1.5M

△g = ~1.5% in each of 3 z-bins → △w ~ 0.08





Summary

1. Topology of LSS has been used to examine the Gaussianity of galaxy distribution at large scales.

This was used to test for the Gaussianity of the primordial density field,

which is one of the major predictions of the simple inflationary scenarios.

2. Recently, topology of galaxy distribution at non-linear scales is being used to constrain the galaxy formation mechanisms and cosmological parameters.

3. Here we propose to use the sponge topology of LSS to

measure the shape of power spectrum P(k) & the expansion history of space

4. 2D and 1D LSS topology studies too!

Redshift slices from the deep imaging surveys - 2d topology

Line-of-sight level crossings of Ly-a forest clouds, HI gas distribution - 1d topology