COSMOLOGY IN HIGHER DIMENSIONS

1. Introduction
2. Overview of Higher Dimensional Cosmology
3. Cosmology in Higher Dimensions
4. String Frame
5. Summary

Kei-ichi MAEDA
Waseda University

1. INTRODUCTION

WHY HIGHER-DIMENSIONS ?

1. Difficulties (or Mysteries) in Ordinary 4D cosmology

Inflation

Initial Singularity

Creation of the Universe

Dark Energy

2. Fundamental Unified Theory predicts higher-dimensions

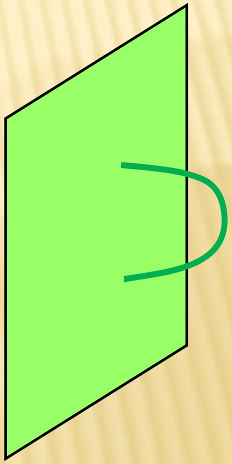
Supergravity
Superstring/M-theory

10D or 11D

How to find our present 4D universe ?

KEY 1 A brane: an interesting object in string theory

D3 brane : could be our universe



Some interesting cosmological senarios

Brane world

Ekpyrotic (or cyclic) universe

Brane inflation (Dvali-Tye , Rolling Tachyon , KKLMMT, ・・・)

•
•
•

KEY 2 Higher-order curvature corrections

$$S = \int d^D x \sqrt{-g} \left[\frac{1}{2\kappa^2} R + c_1 \alpha' e^{-2\phi} L_2 + c_2 \alpha'^2 e^{-4\phi} L_3 + c_3 \alpha'^3 e^{-6\phi} L_4 + \cdots \right]$$

$$L_2 = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2 \ (\text{Gauss} - \text{Bonnet term})$$

$$L_m = (\text{Lovelock})_m + (\text{higher derivative terms}) \ (m = 3, 4, \cdots)$$

- Inflation ?
- singularity avoidance ?
- new effects ?

theories	c_1	c_2	c_3
bosonic string	$\frac{1}{4}$	$\frac{1}{48}$	$\frac{1}{8}$
heterotic string	$\frac{1}{8}$	0	$\frac{1}{8}$
type II string	0	0	$\frac{1}{8}$

2. OVERVIEW OF HIGHER DIMENSIONAL COSMOLOGY

KALUZA-KLEIN COSMOLOGY First Stage

- Cosmological dimensional reduction

A. Chodos & S. Detweiler (1980)

5D Kasner solution

$$ds_5^2 = -dt^2 + a^2(t)d\mathrm{x}^2 + b^2(t)dy^2$$

$$a(t) \propto t^{1/2} \quad b(t) \propto t^{-1/2}$$

3 space : expanding, 5th space : contracting

→ **dynamically explain the large 3 space**

- supergravity (11D; N=1,10D) P.G.O. Freund (1982)

$\exists\ A_{\mu\nu\rho}$ $\quad a(t) \propto t \quad b(t) \propto t^{1/7}$ $\quad k_3 < 0, \quad k_7 = 0$

$a(t) \propto \cos\alpha t \quad b(t) = \text{constant}$ $\quad k_3 < 0, \quad k_7 > 0$

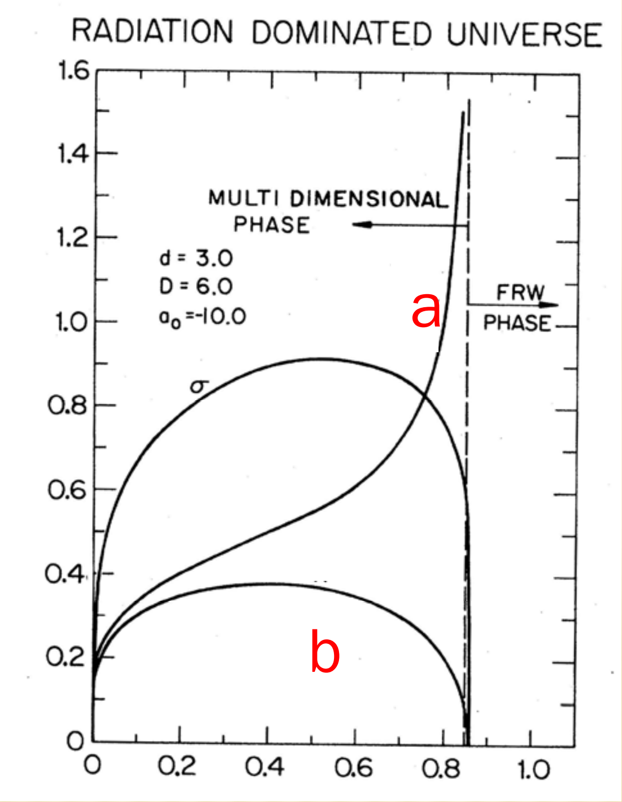
(AdS [anti de Sitter])

Kaluza-Klein inflation

D. Sahdev (1983)

perfect fluid in D-dimensions $P = w\rho$

$$k_3 = 0, \quad k_7 > 0$$



pole inflation

$$a \to \infty \quad b \to 0$$

at a finite time

However, this point is a singularity

How to exit from inflation and go beyond

b (volume modulus) : **time dependent** ⟹ ***time dependent G_N***

observational constraint

$$\left|\frac{\dot{G}_N}{G_N}\right| \leq (0.2 \pm 0.4) \times 10^{-11} \text{years}^{-1}$$ Viking Project (1983)

$$\leq (-0.06 \pm 0.2) \times 10^{-11} \text{years}^{-1}$$ binary pulsar (1996)

⟹ ***Stabilization of volume modulus***

N=2, D=6 Kaluza-Klein supergravity

KM & Nishino 1985

compactification $M_4 \times S^2$ ← internal space

our world

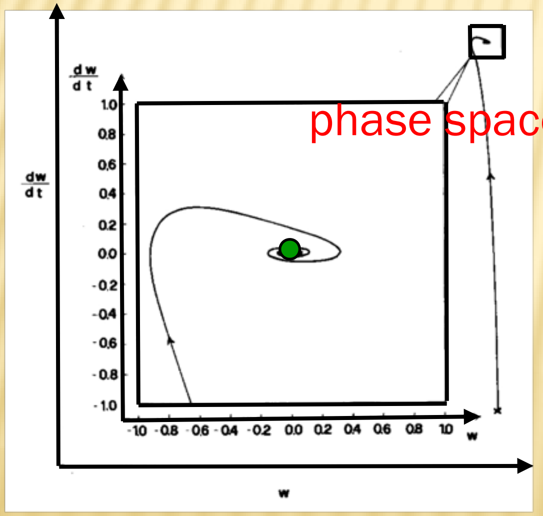
Size b(t) : small & "stabilize"

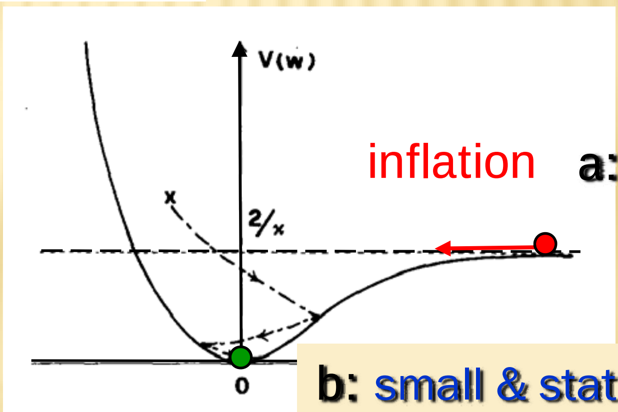
scale factor: a(t) large & inflation

scalar field in 4D spacetime $\phi = \ln b$

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right]$$

effective potential





$\Lambda=0$

transient inflation to standard Big Bang

Our universe is obtained as an attractor !

The similar analysis possible for many KK type universes

K. M., Class. Quant. Grav. 3(1986)233;651

K. M., Phys. Lett. B 166(1986) 59

4D effective equations

Using the effective potential,
we can analyze stability of our present universe.

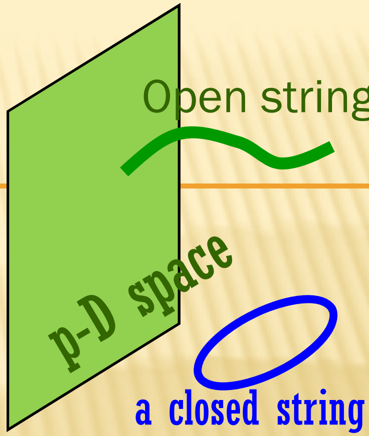
10D Einstein + dilaton + Gauss-Bonnet + Form field

↓ Calabi-Yau compactification

4D FRW universe

BRANE WORLD

Next Stage



gravity

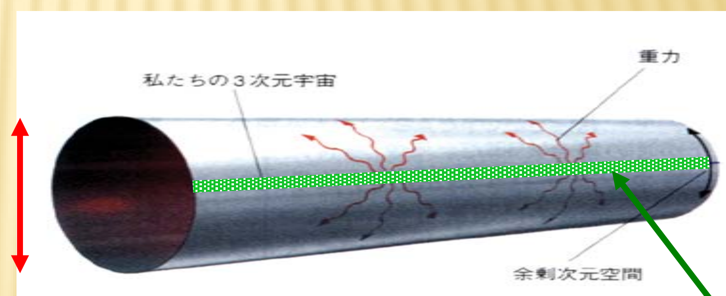
Dp brane p-dimensional (soliton like) object

Polchinsky (95)

Matter field (gauge field) is confined on Dp brane.

Large Extra Dimensions N. Arkani-Hamed, S. Dimopoulos, G. Dvali (98)

$R < 0.1\text{mm}$

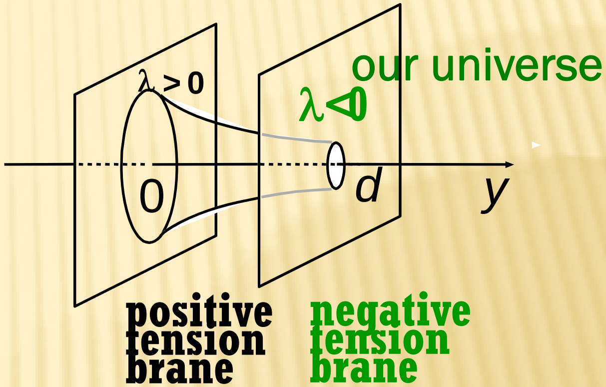


extra dimensions could be large $d < 10^{-17}$ cm

Gravity: Kaluza-Klein type

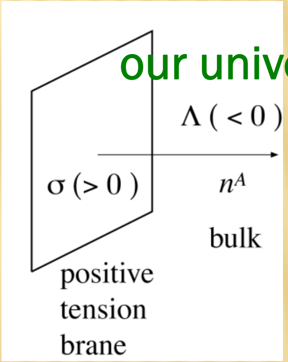
Simple toy models [5D Einstein gravity $+\Lambda(<0)$]

Randall-Sundrum model I



two-brane model

hierarchy problem

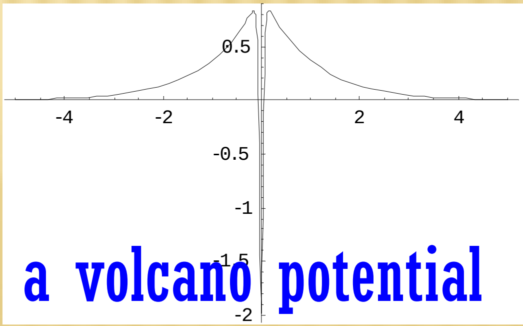


Randall-Sundrum model II

one-brane model

non-compact compactification

massless gravitons are confined in a brane



4D gravity is modified

BRANE COSMOLOGY

5D spacetime (codimension one)

- **FIVE-DIMENSIONAL APPROACH**

P. Binetruy et al (00), C. Csaki et al (99)
J. Clineet al (99), E. Flanaganet al (00)

5D Einstein eqs. with Israel's junction condition

- **EFFECTIVE FOUR-DIMENSIONAL APPROACH**

T.Shiromizu-KM-M.Sasaki (00)

4D effective Einstein eqs. By use of Gauss-Codacci eqs.

- **DOMAIN WALL APPROACH**

A domain wall motion in 5D Schwarzschild-AdS

P. Kraus

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{\kappa^2}{3}\rho + \frac{\kappa_5^4}{36}\rho^2 + \frac{\mu}{a^4}$$

- dark radiation (μ/a^4)
- brane quintessence
- inflation
- dark energy
- singularity avoidance   **U(1) or Non-abelian field**
- creation of the universe
- density perturbation

◆ codimension two (or higher)

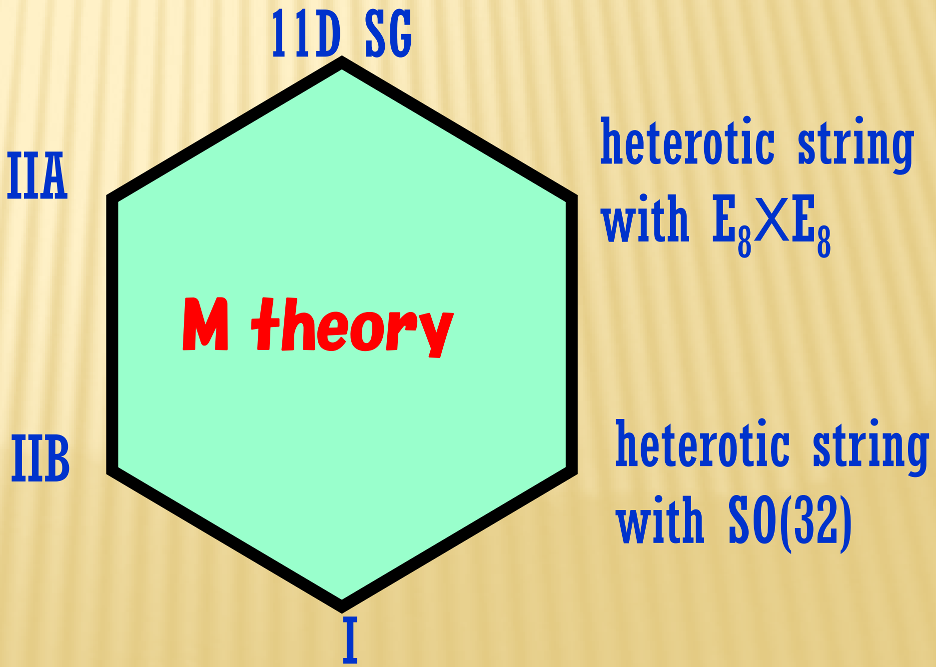
◆ induced gravity on the brane (GDP)

⇨ Cosmology based on fundamental physics

five string theories —duality→ **M theory**

10D spacetime → 11D

	string	gauge sym	SUSY
Type I	open+closed	SO(32)	N=1
Type IIA	closed	-	N=2
Type IIB	closed	-	N=2
Heterotic string (SO(32))	closed	SO(32)	N=1
Heterotic string ($E_8 \times E_8$)	closed	($E_8 \times E_8$)	N=1

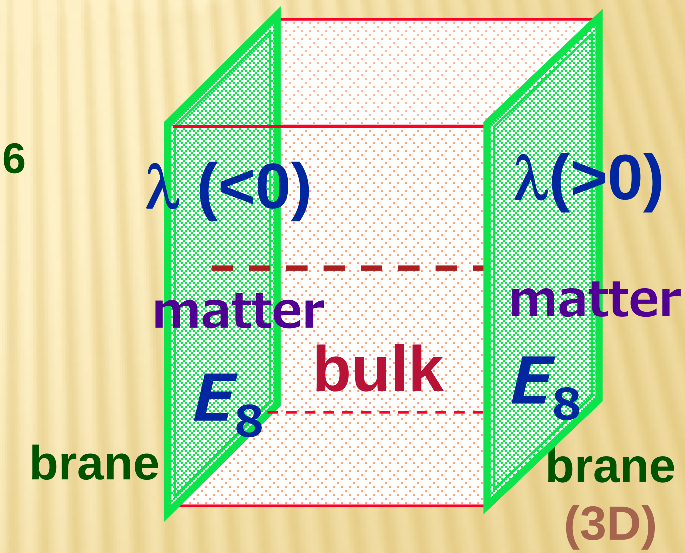


MORE "REALISTIC" MODELS

☞ **HORAVA-WITTEN** (1996)

11-D M theory compactified on S^1/Z_2 ⟷ id. 10-D $E_8 X E_8$ **heterotic string**

⇒ $M^4 X\ S^1/Z_2 X(\text{Calabi-Yau})^6$



HW model -> effective 5D theory

A. Lukas, B. Ovrut, K. Stelle, D. Waldram (99)

Effective 5D theory

$$S_5 = S_g + S_{hyper} + S_B$$

Bulk action

$$S_g = \frac{1}{2\kappa_5^2}\int_{M_5}\sqrt{-g}\left[R + \frac{3}{2}\mathcal{F}_{\alpha\beta}\mathcal{F}^{\alpha\beta} + \frac{1}{\sqrt{2}}\epsilon^{\alpha\beta\gamma\delta\epsilon}\mathcal{A}_\alpha\mathcal{F}_{\beta\gamma}\mathcal{F}_{\delta\epsilon}\right]$$

$$S_{hyper} = \frac{1}{2\kappa_5^2}\int_{M_5}\sqrt{-g}\left[\frac{1}{2V^2}\partial_\alpha V\partial^\alpha V + \frac{2}{V}\partial_\alpha\xi\partial^\alpha\bar{\xi} + \frac{1}{3V^2}\alpha^2\right.$$

$$\left. + \frac{V^2}{24}G_{\alpha\beta\gamma\delta}G^{\alpha\beta\gamma\delta} + \frac{\sqrt{2}}{24}\epsilon^{\alpha\beta\gamma\delta\epsilon}G_{\alpha\beta\gamma\delta}\left[i(\xi\partial_\epsilon\bar{\xi} - \bar{\xi}\partial_\epsilon\xi) + 2\alpha\epsilon(x^{11})\mathcal{A}_\epsilon\right]\right]$$

Brane action

$$S_B = \frac{\sqrt{2}}{\kappa_5^2}\int_{M_4^{(1)}}\sqrt{-g}V^{-1}\alpha - \frac{\sqrt{2}}{\kappa_5^2}\int_{M_4^{(2)}}\sqrt{-g}V^{-1}\alpha$$

$$-\frac{1}{16\pi\alpha_{\mathrm{GUT}}}\sum_{i=1}^{2}\int_{M_4^{(i)}}\sqrt{-g}V\,tr\left(F_{\alpha\beta}^{(i)}\right)^2$$

Cosmological solution

$$ds_5^2 = b^{-1}(t)H^{1/2}(y)\left(-dt^2 + a^2(t)d\mathbf{x}^2\right) + b^2(t)H^2(y)dy^2$$

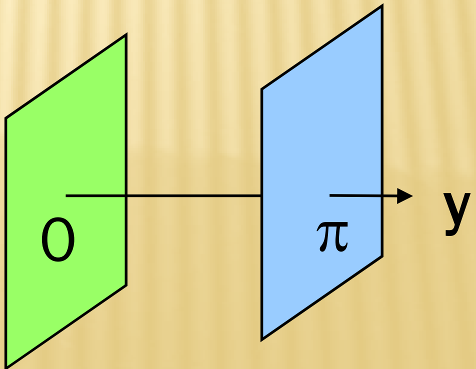
4D Einstein frame

$$H = \frac{\sqrt{2}}{3}\alpha|y| + h_0$$

$$a \propto t^p \qquad b \propto t^q \qquad \phi \propto \frac{1}{6}\ln V \sim \frac{q}{6}\ln t$$

$$p = p_\pm \equiv \frac{3}{11}\left[1 \pm \frac{4\sqrt{3}}{9}\right] \qquad q = q_\pm \equiv \frac{2}{11}\left[1 \mp 2\sqrt{3}\right]$$

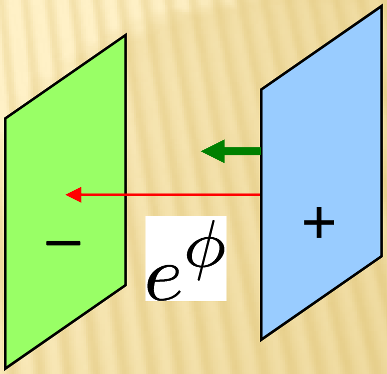
(0.48, 0.06) (–0.45, 0.81)



New idea: A brane collision

- **Ekpyrotic or cyclic universe** J. Khoury, P.Steinhardt, N.Turok

The alternative to inflation ?



- **collision of D brane & $\overline{\text{D}}$ brane**

Brane inflation

Dvali-Tye , Rolling Tachyon , KKLMMT, ・・・

+ test branes

KKLT : stable Calabi-Yau space by flux

Effective 4D theory

distance (~distance) : dilaton ⇨ expansion of the Universe

4D Effective Theories with Warped Compactification

IIB, HW model

H. Kodama, K. Uzawa (06)

Some solutions are not allowed in Higher dimensions

10D $ds_{10}^2 = h^{-1/2}(x,y)ds_4^2(x) + h^{1/2}(x,y)ds_6^2(y)$

$$h(x,y) = h_0(x) + h_1(y)$$

$$R_{\mu\nu}(x) = 0 \qquad R_{ab}(y) = \lambda g_{ab}(y)$$

$$D_\mu D_\nu h_0 = \lambda g_{\mu\nu}(x) \qquad \Delta_y h_1 = -\frac{g_s}{2}(G_3 \cdot \bar{G}_3)_y$$

4D effective theory

$$R_{\mu\nu}(x) = H^{-1}\left[D_\mu D_\nu H - \lambda g_{\mu\nu}(x)\right]$$

$$\Delta_x H = 4\lambda \qquad H = h_0(x) + V_6^{-1}\int_{Y_6} d\Omega_6 h_1(y)$$

⇨ **Careful analysis when extra dimension is time dependent**

3. COSMOLOGY IN HIGHER DIMENSIONS (1)

KK type

Higher curvature terms

Type II (or M) quartic correction terms

$$S = \frac{1}{2\kappa_{11}^2} \int \sqrt{-g}\, [R + \alpha_4 E_8 + \gamma J_0]$$

$$E_8 = -\frac{1}{2^5 \times 3} \epsilon^{\alpha_1\alpha_2\alpha_3\rho_1\sigma_1\cdots\rho_4\sigma_4} \epsilon_{\alpha_1\alpha_2\alpha_3\mu_1\nu_1\cdots\mu_4\nu_4} \times R^{\mu_1\nu_1}{}_{\rho_1\sigma_1} R^{\mu_2\nu_2}{}_{\rho_2\sigma_2} R^{\mu_3\nu_3}{}_{\rho_3\sigma_3} R^{\mu_4\nu_4}{}_{\rho_4\sigma_4}$$

$$J_0 = C^{\lambda\mu\nu\kappa} C_{\alpha\mu\nu\beta} C_\lambda{}^{\alpha\rho\sigma} C^\beta{}_{\rho\sigma\kappa} + \frac{1}{2} C^{\lambda\kappa\mu\nu} C_{\alpha\beta\mu\nu} C_\lambda{}^{\rho\sigma\alpha} C^\beta{}_{\rho\sigma\kappa}$$

$$\alpha_4 = \frac{\kappa_{11}^2 T_2}{3^2 \times 2^9 \times (2\pi)^4} \qquad \gamma = \frac{\kappa_{11}^2 T_2}{3 \times 2^4 \times (2\pi)^4}$$

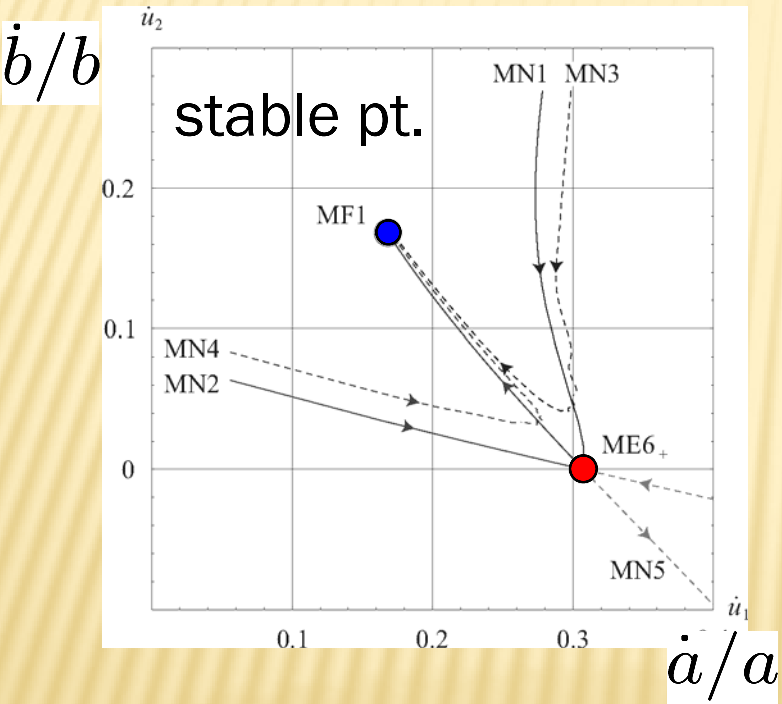
$$T_2 = \left(2\pi^2/\kappa_{11}^2\right)^{1/3}$$ membrane tension

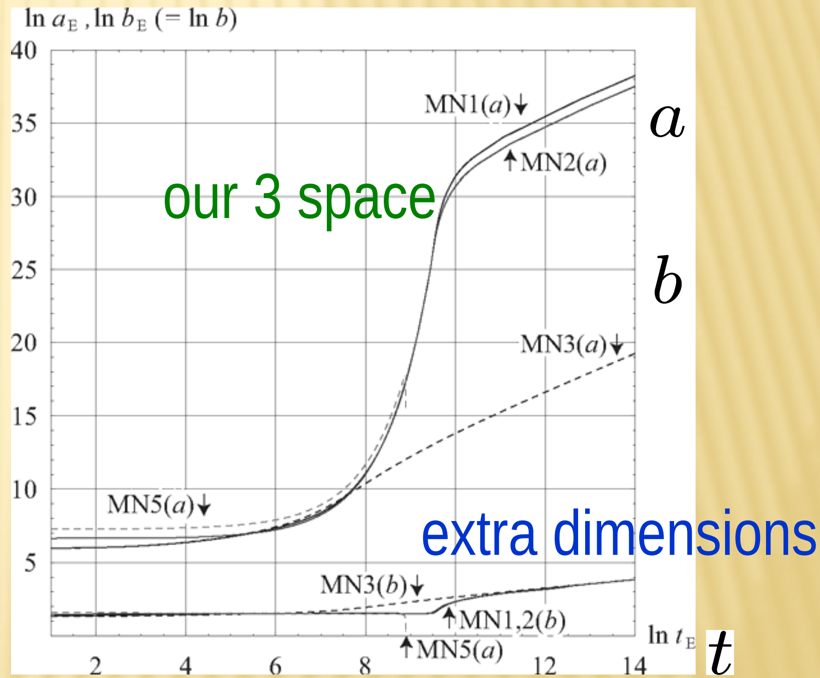
Cosmology with higher curvature

KM, N. Ohta, PLB (04), PRD (05)
K. Akune, KM, N. Ohta, PRD (06)

$$ds^2 = -dt^2 + a^2 \sum_{i=1}^{3} \left(dx^i\right)^2 + b^2 \sum_{\alpha=5}^{11} \left(dy^\alpha\right)^2$$

de Sitter : trangent attractor





inflationary phase

Heterotic type

Einstein-Gauss-Bonnet + dilaton K. Bamba, Z.-K. Guo, N. Ohta (07)

KK type inflation (pole inflation) : attractor

A singularity appears at a finite time

---

common problems

- graceful exit ?
- reheating ?
- density perturbations ?

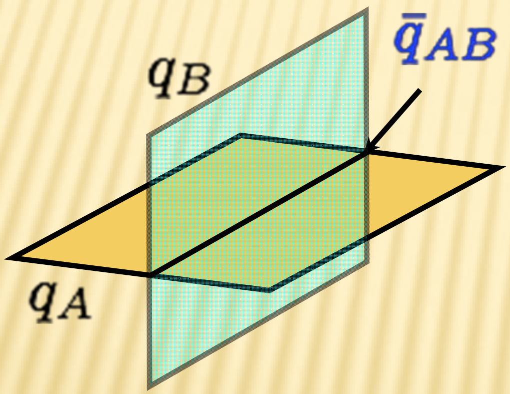
COSMOLOGY IN HIGHER DIMENSIONS (2)

Higher dimensional cosmology with branes

- microscopic description of BH by branes

S. R. Das ('96),
M. Cvetic and C. M. Hull ('88)

branes in some dimensions ⟶ gravitational sources



BHs (Black objects) in 4 or 5 dim

\# branes ~ charges

⇕

area of horizon (BH entropy)

⟹ **cosmology ?**

D-dimensional effective action

P. Binetruy, M. Sasaki , K. Uzawa (07)
KM, N. Ohta, K. Uzawa (in preparation)

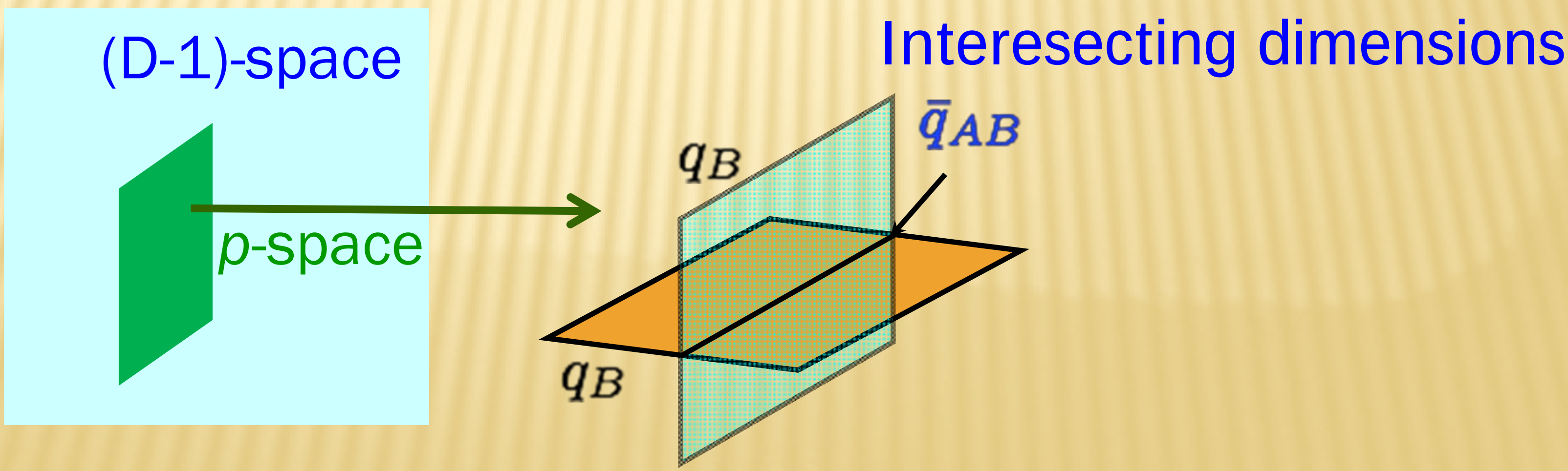
$$S = \frac{1}{2\kappa^2}\int d^D X \sqrt{-g}\left[R - \frac{1}{2}(\nabla\phi)^2 - \sum_A \frac{1}{2\cdot n_A!} e^{a_A \phi} F_{n_A}^2\right]$$

ϕ : **dilaton** F_{n_A} ***: n_A* form fields**

A: type of branes (2-brane, 5-brane etc)

Ansatz:

Source: Several types of branes in *p*-dim space



Mtheory
(D=11, supergravity) $M2 \perp M5$

4-form $q_2=2$ M2 brane dual: 7-form $q_5=5$ M5 brane

intersection rule

$$M2 \cap M2 \to q_{22} = 0\,, \quad M2 \cap M5 \to q_{25} = 1\,, \quad M5 \cap M5 \to q_{55} = 3$$

Example: 5 dimensional black hole

y_1	y_2	y_3	y_4	y_5	y_6
M2 M5 W	M5	M5	M5	M5	M2

BMPV BH J.C. Breckenridge, R.C. Myers, A.W. Peet and C. Vafa(93) time dependence

branes

$$ds^2 = -\prod_A h_A^{-\frac{D-q_A-3}{D-2}}(t,y)dt^2 + \sum_{\mu=1}^{p}\prod_A h_A^{\frac{\delta_A^\mu}{D-2}}(t,y)(dx^\mu)^2(X)$$

$$+\prod_A h_A^{\frac{q_A+1}{D-2}}(t,y)u_{ij}(Y)dy^i dy^j$$

Forms

$$F_{(q_A+2)} = d(h_A^{-1}) \wedge \Omega(X_A)$$

We classify all possible configuration

To find our 4D universe, we need compactification

However, a consistent compactification is very difficult

because $\exists$ branes → inhomogeneous

→ **We may need brane world approach**

Construct a 5D inhomogeneous spacetime

Put our 3 space as a boundary

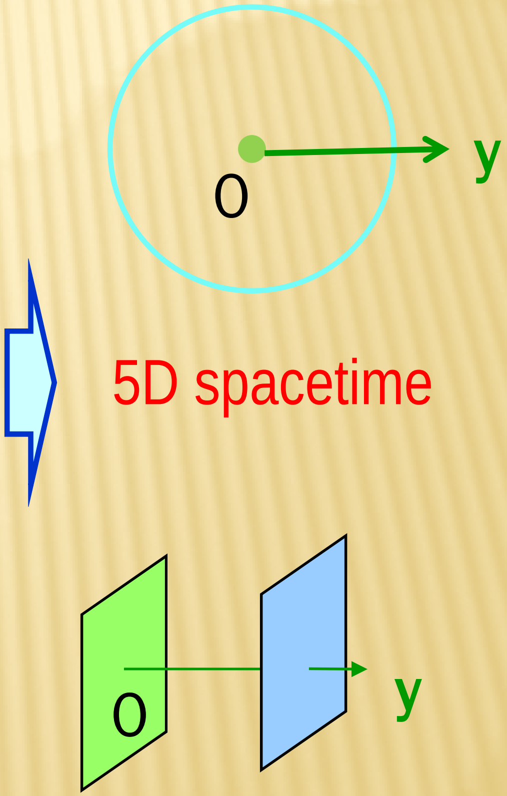
Find a motion of the boundary ⇒ Expansion of the universe

	0	1	2	3	4	5	6	7	8	9	10
M2	○	○	○								
M5	○	○		○	○	○	○				

commpactification (1–6) y (7–10)

	0	1	2	3	4	5	6	7	8	9	10
M5	○	○	○	○	○	○					
M5	○	○	○	○			○	○			
M5	○	○	○	○					○	○	

x (1–3) commpactification (4–9) y (10)



work in progress

4. STRING FRAME

Attractor Universe in Scalar-Tensor Theory

Y. Fujii, KM (in preparation)

MODEL

$$S_J = \int d^4x\sqrt{-g}\left[\frac{\xi}{2}\phi^2 R(g) - \frac{\epsilon}{2}(\nabla\phi)^2 - V_0\right] + \int d^4x\sqrt{-g}L_{\rm m}(\psi, g)$$

Non-minimal coupling (ξ) + cosmological constant (V_0)

BD parameter $\omega = 1/4\xi$

conformal transformation

$g \to g \exp(2\zeta\kappa\sigma)$ $\zeta = \sqrt{\xi/(\varepsilon+6\xi)}$

String theory $\zeta = \sqrt{1/2}$

Einstein theory (g) + scalar field σ $V = V_0 \exp(-4\zeta\kappa\sigma)$

Dynamics without matter is well-known

But, coupling with matter is important

$$H^2 + \frac{k}{a^2} = \frac{\kappa^2}{3}\left[\frac{1}{2}\left(\frac{d\sigma}{dt}\right)^2 + V + \rho\right]$$

$$\frac{d^2\sigma}{dt^2} + 3H\frac{d\sigma}{dt} + \frac{\partial V}{\partial \sigma} = \zeta\kappa(\rho - 3P)$$

$$\frac{d\rho}{dt} + 3\gamma H\rho = -\zeta\kappa(4-3\gamma)\frac{d\sigma}{dt}\rho$$

$$P = (\gamma - 1)\rho$$

Two fixed points

FP1 Scalar field dominant

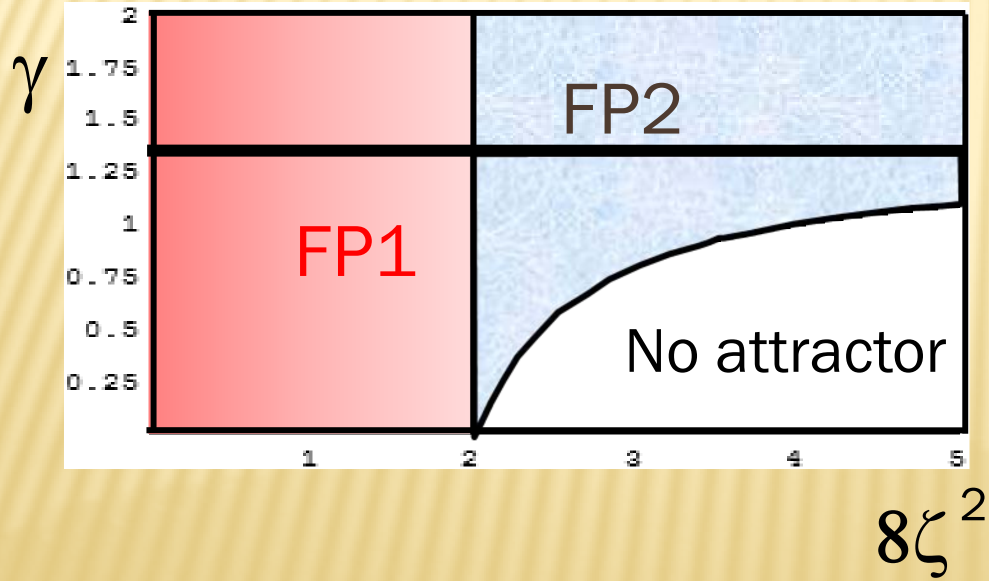
$$a \propto t^{\frac{1}{8\zeta^2}} \qquad \kappa\sigma = \frac{1}{2\zeta}\ln t + \text{const}$$

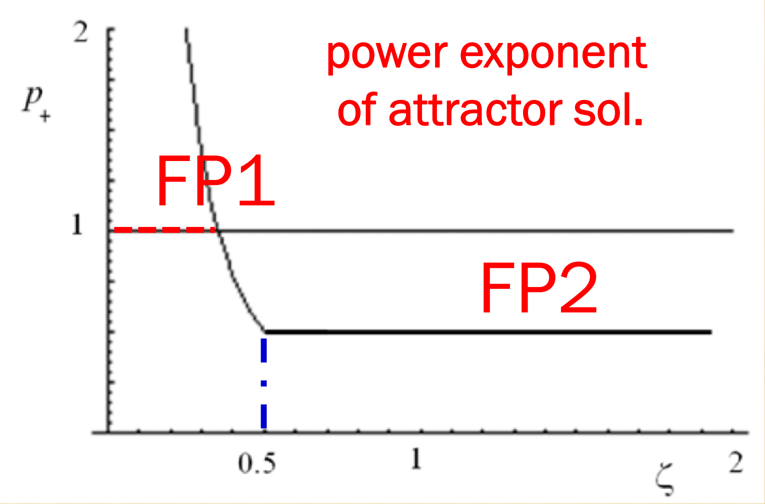
FP2 Scaling solution

$$\left(\frac{\rho}{V}\right)_2 = \frac{2(4\zeta^2-1)}{2-\gamma-2(4-3\gamma)\zeta^2} \quad \text{const}$$

$$a \propto t^{\frac{1}{2}} \qquad \kappa\sigma = \frac{1}{2\zeta}\ln t + \text{const}$$

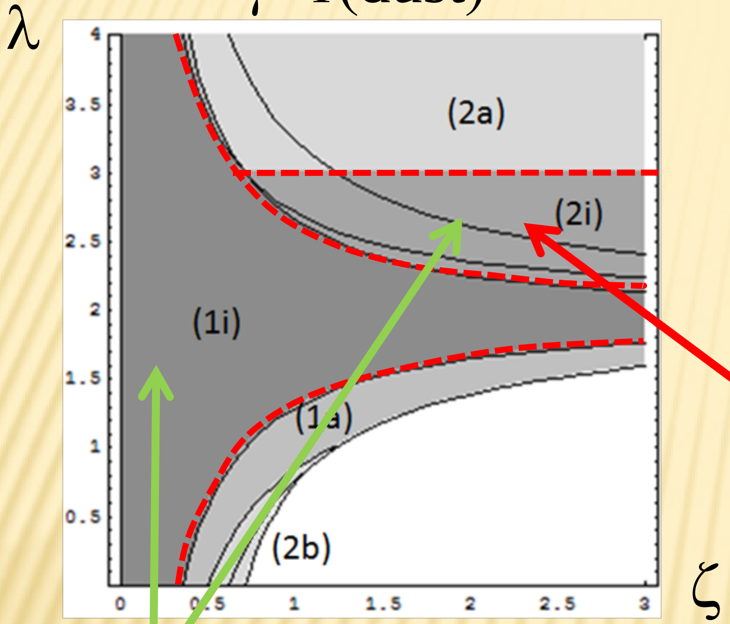
Minkowski in Jordan frame



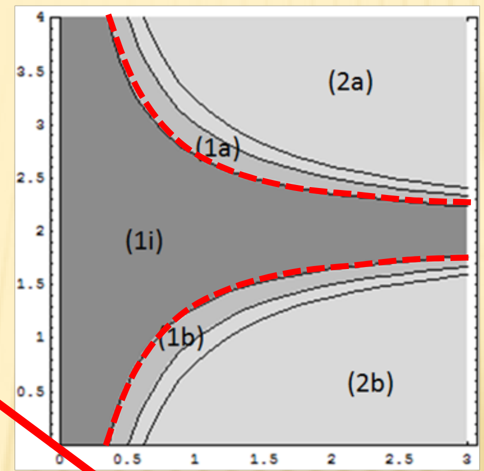


power-law potential $V_0 \to (\kappa^2\phi^2)^\lambda V_0$

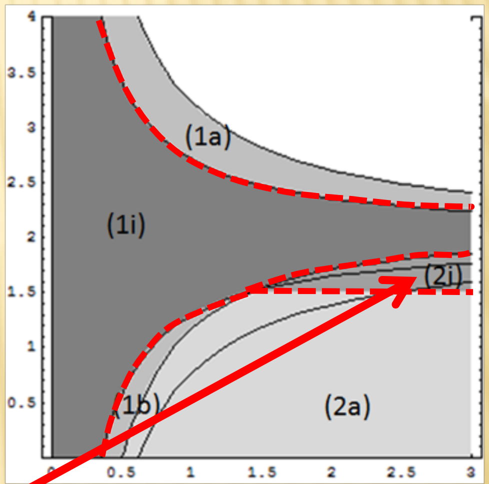
$\gamma=1$(dust)



$\gamma=4/3$(radiation)



$\gamma=2$ (stiff)



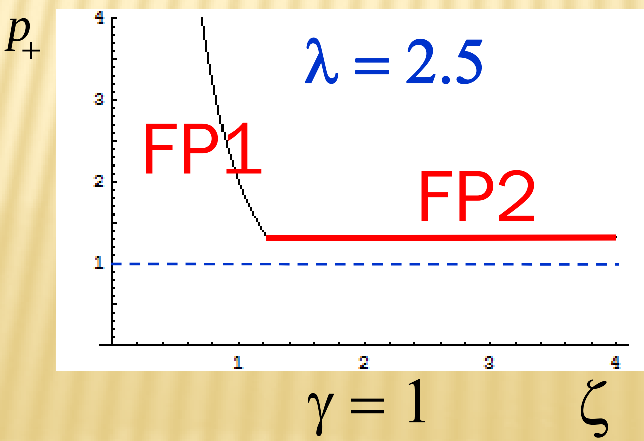
New type

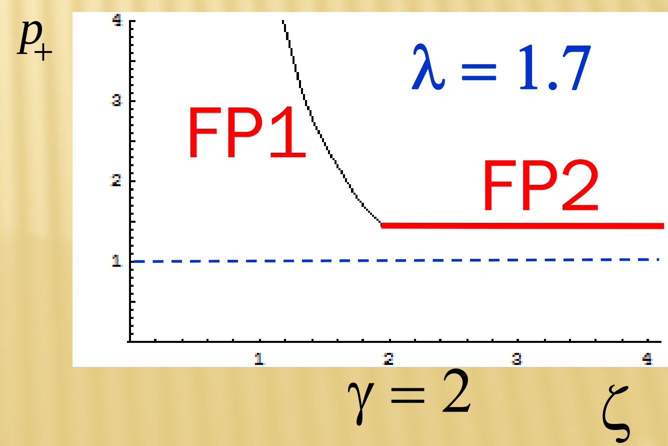
Inflation with a steep potential

Power-law inflation

power exponent of attractor sol.

$a \propto t^p$





Importance of analysis in string frame

String frame

$$S = \frac{1}{2\kappa_D^2}\int d^D x\sqrt{-\hat{g}}e^{-2\phi}\left[R(\hat{g}) + 4(\hat{\nabla}\phi)^2 + \alpha_2 R_{GB}^2(\hat{g})\right]$$

$$\hat{g}_{\mu\nu} = e^{\frac{4\phi}{D-2}} g_{\mu\nu}$$

Einstein frame

$$S = \frac{1}{2\kappa_D^2}\int d^D x\sqrt{-g}\left[R(g) - \frac{1}{2}(\nabla\phi)^2 + \alpha_2 e^{-\gamma\phi} R_{GB}^2(g)\right.$$

$$\left. + \mathcal{F}(\nabla\phi, R)\right]$$

- usually ignored
- could be important

5. SUMMARY

- We overview higher-dimensional cosmology.
- We study cosmology with higher curvature corrections.
- We study a time-dependent spacetime
  with intersecting branes in M/superstring theory.
- We point out some importance of study in string frame.