The Equation of State for Supernovae and Neutron Stars

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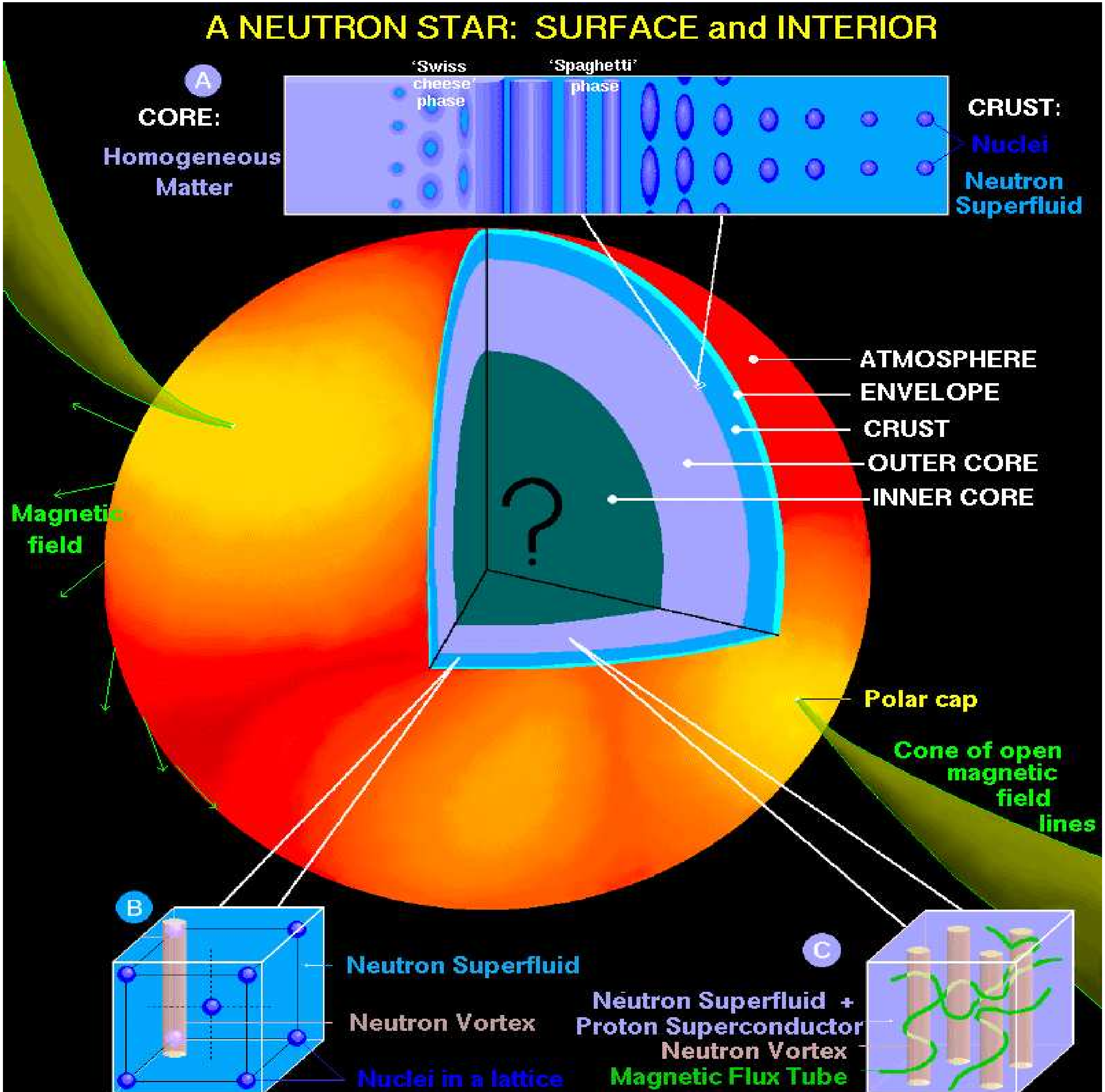
K. Sato's Early Astrophysical Contributions

- **Nuclei in Neutron Matter (Bethe, Borner & Sato 1970)**
  - Derived $E_{surf} = 2E_{Coul}$, foreshadowing the "Nuclear Virial Theorem" of Baym, Bethe & Pethick (1971)
- **Formation of Elements in Neutron Rich Ejected Matter of Supernovae (Sato, Nakazawa & Ikeuchi 1973)**
  - First discussion that matter becomes opaque to electron neutrinos
- **Supernova Explosion and Neutral Currents of Weak Interaction (Sato 1975)**
  - Neutrino trapping
  - Neutrino diffusion
  - Equation of state of neutrino-trapped matter

Outline

- **Neutron Star Structure**
  - Extreme Properties
  - Pulsar Constraints – Rotation and Mass
  - Pressure–Radius Correlation
  - Nuclear Symmetry Energy
  - Nuclear Structure Constraints
  - Inverting the TOV Equations
- **Radius Constraints**
  - Radiation Radius
  - Seismology
  - Moment of Inertia
  - Tidal Effects in Mergers
- **Neutron Star Cooling**
  - Binding Energy From Neutrinos
  - Crustal Cooling
  - Core Cooling and the URCA Process



Credit: Dany Page, UNAM

Neutron Star Structure

Tolman-Oppenheimer-Volkov equations of relativistic hydrostatic equilibrium:

$$\frac{dp}{dr} = -\frac{G}{c^2}\frac{(m+4\pi pr^3)(\epsilon+p)}{r(r-2Gm/c^2)}$$
$$\frac{dmc^2}{dr} = 4\pi\epsilon r^2$$

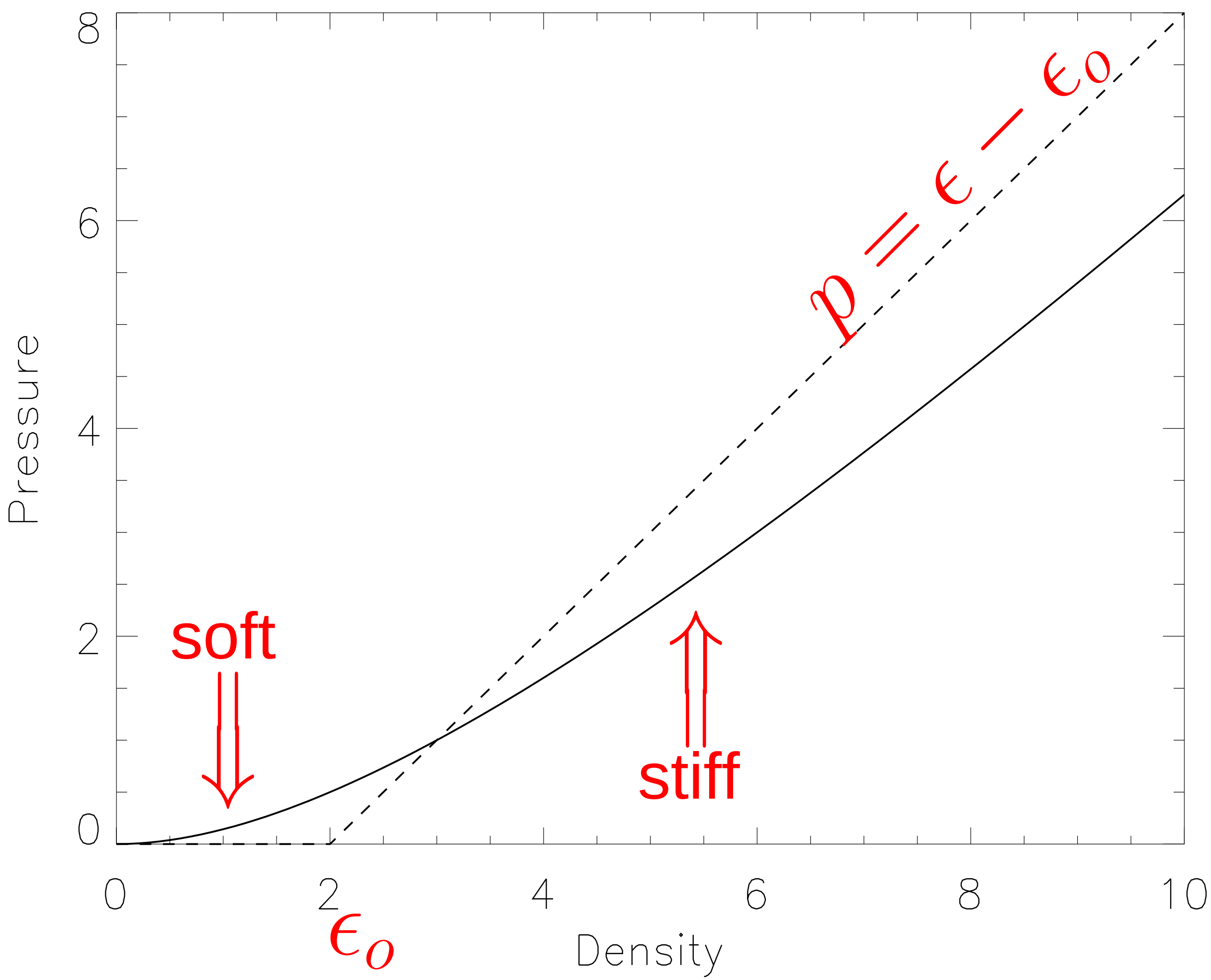
p is pressure, ϵ is mass-energy density

Useful analytic solutions exist:

- Uniform density $\epsilon =$ constant
- Tolman VII $\epsilon = \epsilon_c[1-(r/R)^2]$
- Buchdahl $\epsilon = \sqrt{pp_*} - 5p$

Extreme Properties of Neutron Stars

- The most compact configurations occur when the low-density equation of state is "soft" and the high-density equation of state is "stiff".



Maximally Compact Equation of State

Koranda, Stergioulas & Friedman (1997)

$$
\begin{aligned}
p(\epsilon) = 0, \qquad \epsilon &\leq \epsilon_o \\
p(\epsilon) = \epsilon - \epsilon_o, \qquad \epsilon &\geq \epsilon_o
\end{aligned}
$$

This EOS has a parameter ϵ_o, which corresponds to the surface energy density. The structure equations then contain only this one parameter, and can be rendered into dimensionless form using

$$y = m\epsilon_o^{1/2}, \qquad x = r\epsilon_o^{1/2}, \qquad q = p\epsilon_o^{-1}.$$

$$
\begin{aligned}
\frac{dy}{dx} &= 4\pi x^2(1+q) \\
\frac{dq}{dx} &= -\frac{(y+4\pi q x^3)(1+2q)}{x(x-2y)}
\end{aligned}
$$

The solution with the maximum central pressure and mass and the minimum radius:

$$
\begin{aligned}
q_{max} &= 2.026, \qquad y_{max} = 0.0851, \qquad x_{min}/y_{max} = 2.825 \\
p_{max} &= 307\left(\frac{\epsilon_o}{\epsilon_s}\right) \text{ MeV fm}^{-3}, \quad M_{max} = 4.2\left(\frac{\epsilon_s}{\epsilon_o}\right)^{1/2} \text{ M}_\odot, \quad R_{min} = 2.825\frac{GM_{max}}{c^2}.
\end{aligned}
$$

Moreover, the scaling extends to the axially-symmetric case, yielding

$$P_{min} \propto \left(\frac{M_{max}}{R_{min}^3}\right)^{1/2} \propto \epsilon_o^{-1/2}, \qquad P_{min} = 0.82\left(\frac{\epsilon_s}{\epsilon_o}\right)^{1/2} \text{ ms}$$

Maximum Mass, Minimum Period

Theoretical limits from GR and causality

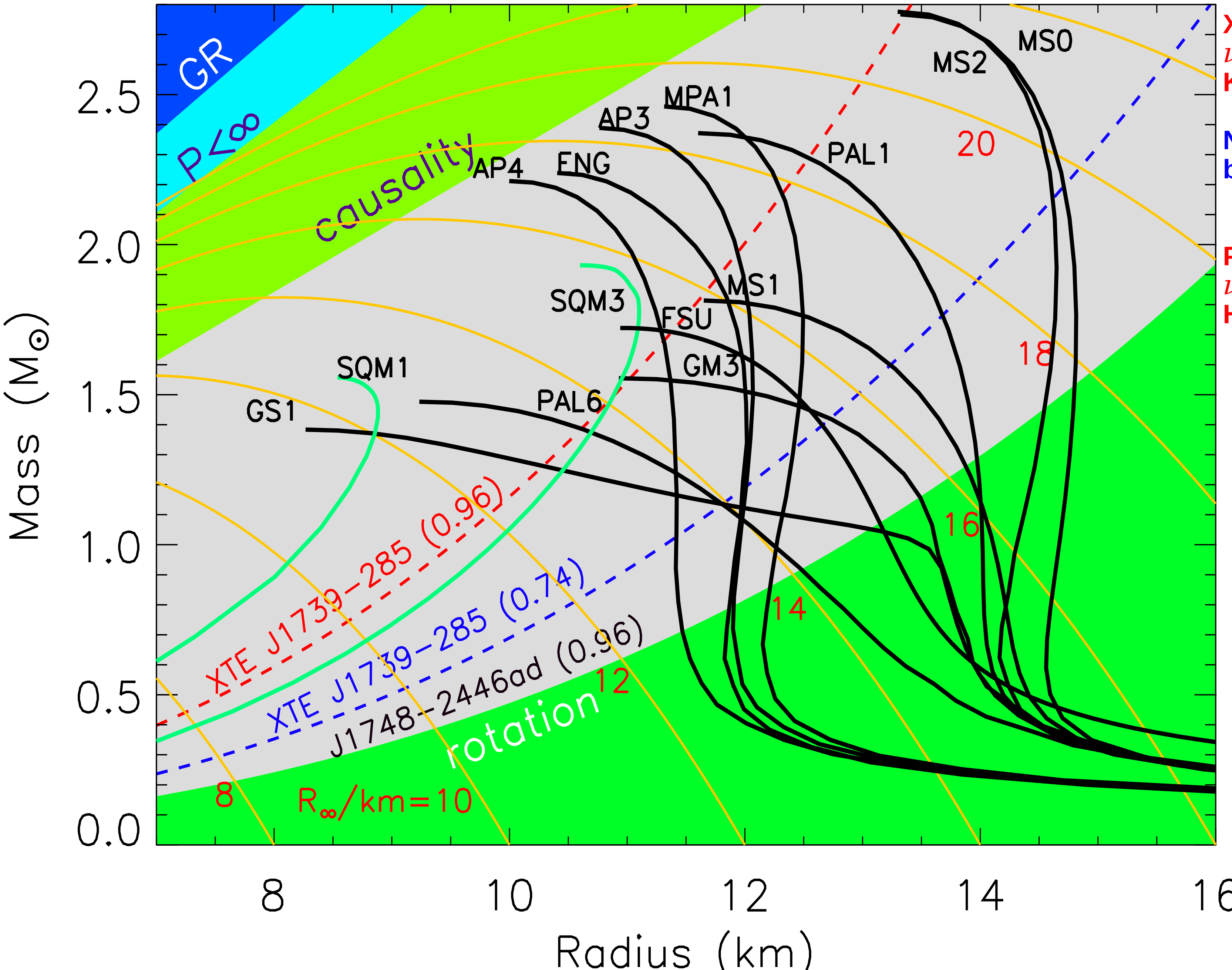
- $M_{max} = 4.2(\epsilon_s/\epsilon_f)^{1/2}\ \mathrm{M}_\odot$ **Rhoades & Ruffini (1974), Hartle (1978)**
- $R_{min} = 2.9GM/c^2 = 4.3(M/\mathrm{M}_\odot)$ km

  Lindblom (1984), Glendenning (1992), Koranda, Stergioulas & Friedman (1997)
- $\epsilon_c < 4.5 \times 10^{15}(\mathrm{M}_\odot/M_{largest})^2$ g cm^{-3} **Lattimer & Prakash (2005)**
- $P_{min} \simeq (0.74 \pm 0.03)(\mathrm{M}_\odot/M_{sph})^{1/2}(R_{sph}/10\ \mathrm{km})^{3/2}$ ms

  Koranda, Stergioulas & Friedman (1997)
- $P_{min} \simeq 0.96(\mathrm{M}_\odot/M_{sph})^{1/2}(R_{sph}/10\ \mathrm{km})^{3/2}$ ms (empirical)

  Lattimer & Prakash (2004)
- $\epsilon_c > 0.91 \times 10^{15}(1\ \mathrm{ms}/P_{min})^2$ g cm^{-3} (empirical)
- $cJ/GM^2 \lesssim 0.5$ (empirical, neutron star)

Constraints from Pulsar Spins



XTE J1739-285
$\nu = 1122$ **Hz**
Kaaret et al. 2006

Not confirmed to be a rotation rate

PSR J1748-2446ad
$\nu = 716$ **Hz**
Hessels et al. 2006

Observed Masses

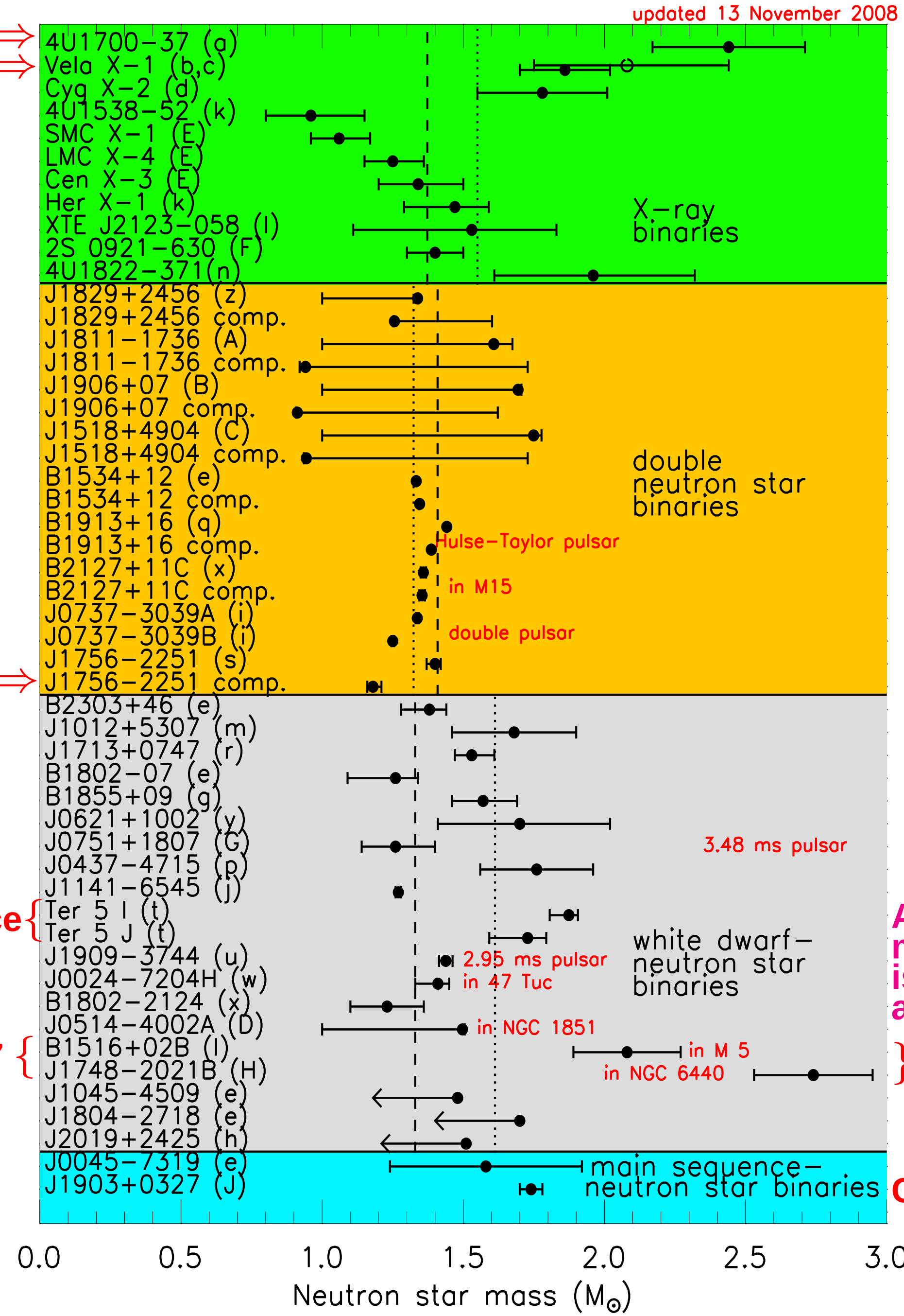
Black hole? ⇒

Firm lower mass limit? ⇒

$M \simeq 1.18$ M$_\odot$ ⇒

$M > 1.68$ M$_\odot$, **95% confidence** {

Freire et al. 2007 {



Although simple average mass of w.d. companions is 0.27 M$_\odot$ larger, weighted average is 0.015 M$_\odot$ larger

} **w.d. companion? statistics?**

Champion et al. 2008

Neutron Star Matter Pressure and the Radius

$p \simeq K\epsilon^{1+1/n}$

$n^{-1} = d\ln p/d\ln\epsilon - 1 \sim 1$

$R \propto K^{n/(3-n)} M^{(1-n)/(3-n)}$

$R \propto p_*^{1/2}\epsilon_*^{-1}M^0$

$(1 < \epsilon_*/\epsilon_0 < 2)$

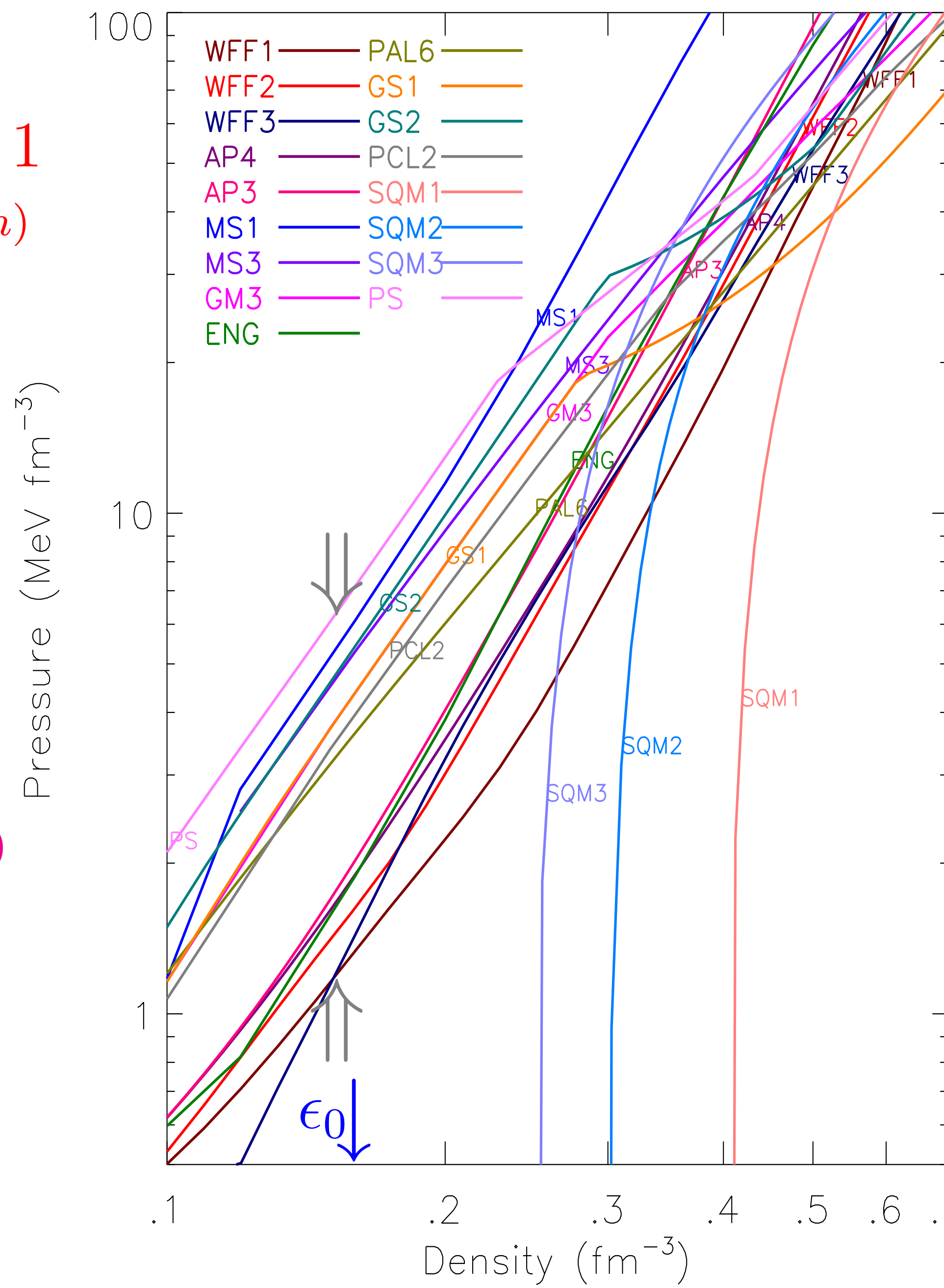
Wide variation:

$1.2 < \frac{p(\epsilon_0)}{\mathrm{MeV\ fm^{-3}}} < 7$

GR phenomenological result (Lattimer & Prakash 2001)

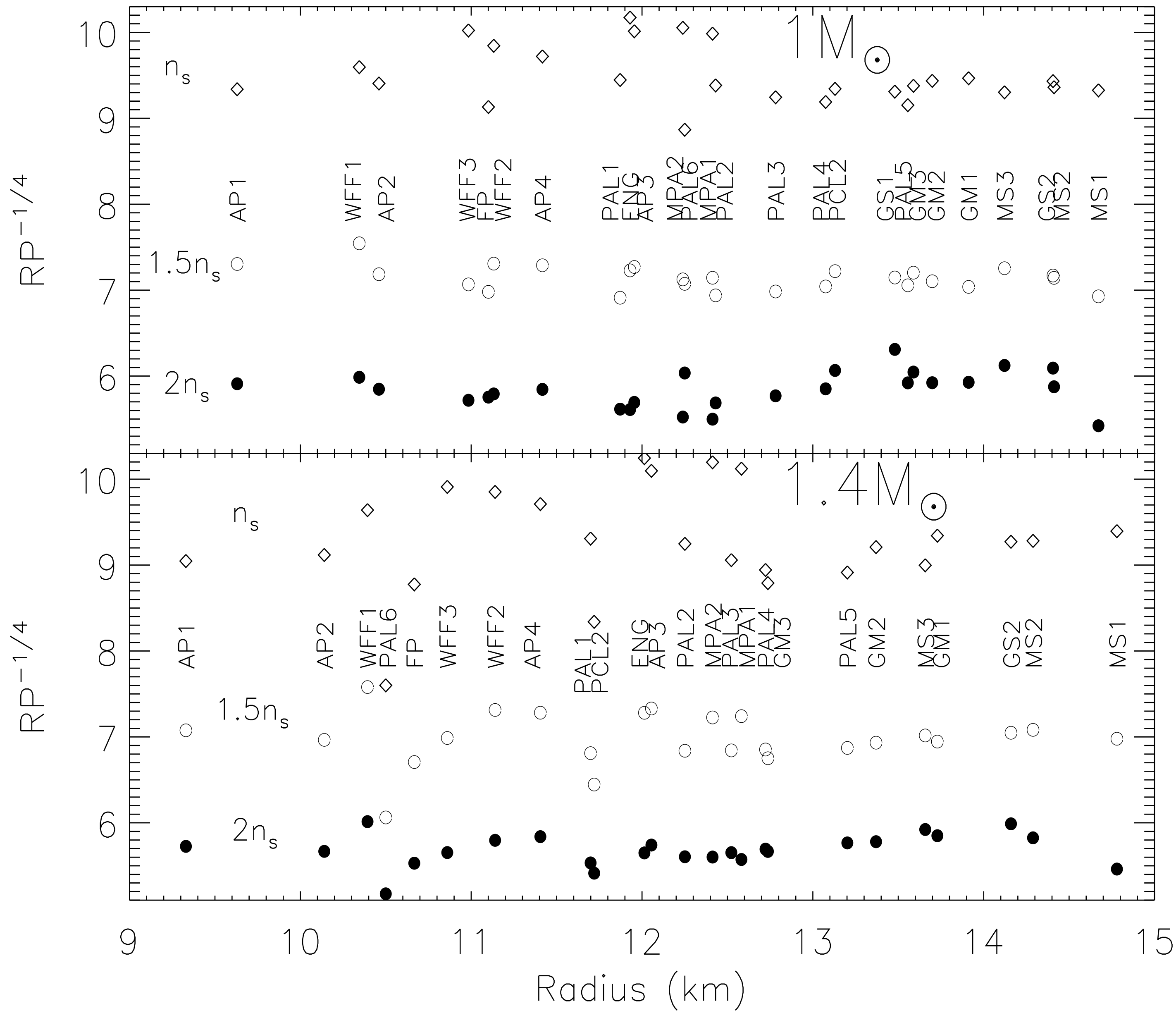
$R \propto p_*^{1/4}\epsilon_*^{-1/2}$

$p_* = n^2\frac{dE_{sym}}{dn} = \frac{n^2 L}{3n_s}$



The Radius – Pressure Correlation

$R \propto p^{1/4}$



Lattimer & Prakash (2001)

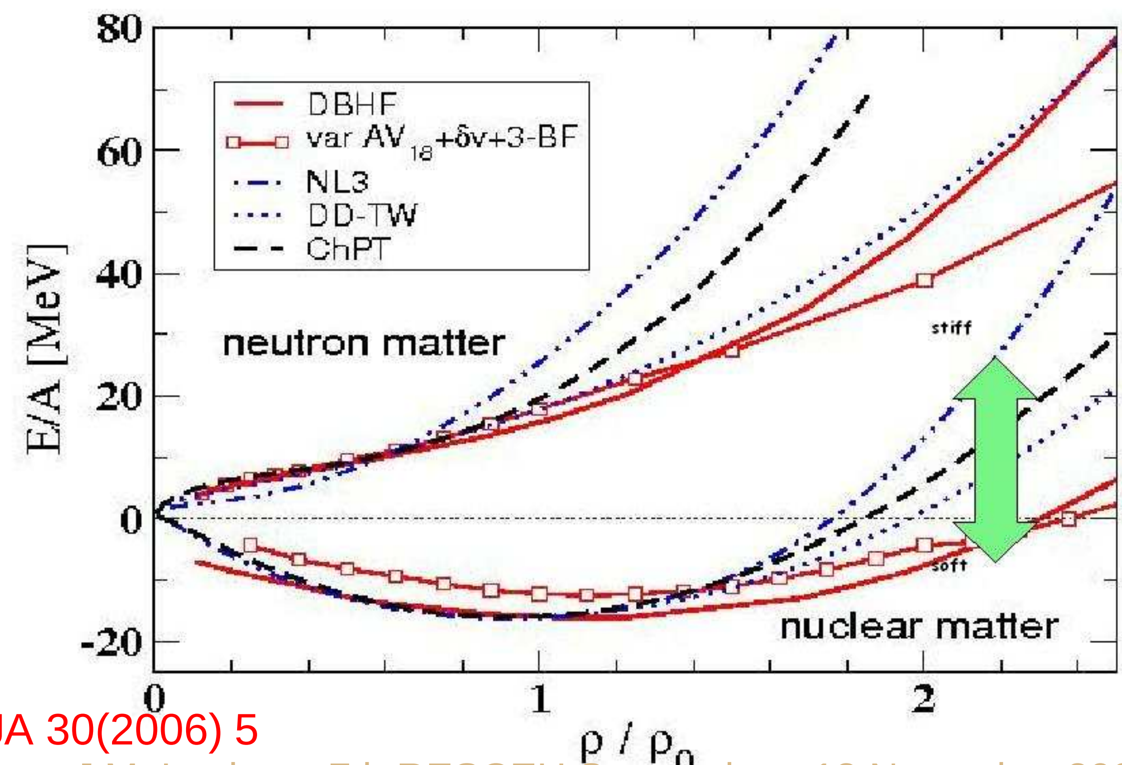
Nuclear Symmetry Energy

The density dependence of $E_{sym}(n)$ is crucial. Some information is available from nuclei (for $n < n_s$). Heavy ion collisions have potential for constraining it for $n > n_s$. It is common to expand $E_{sym}(n)$ as

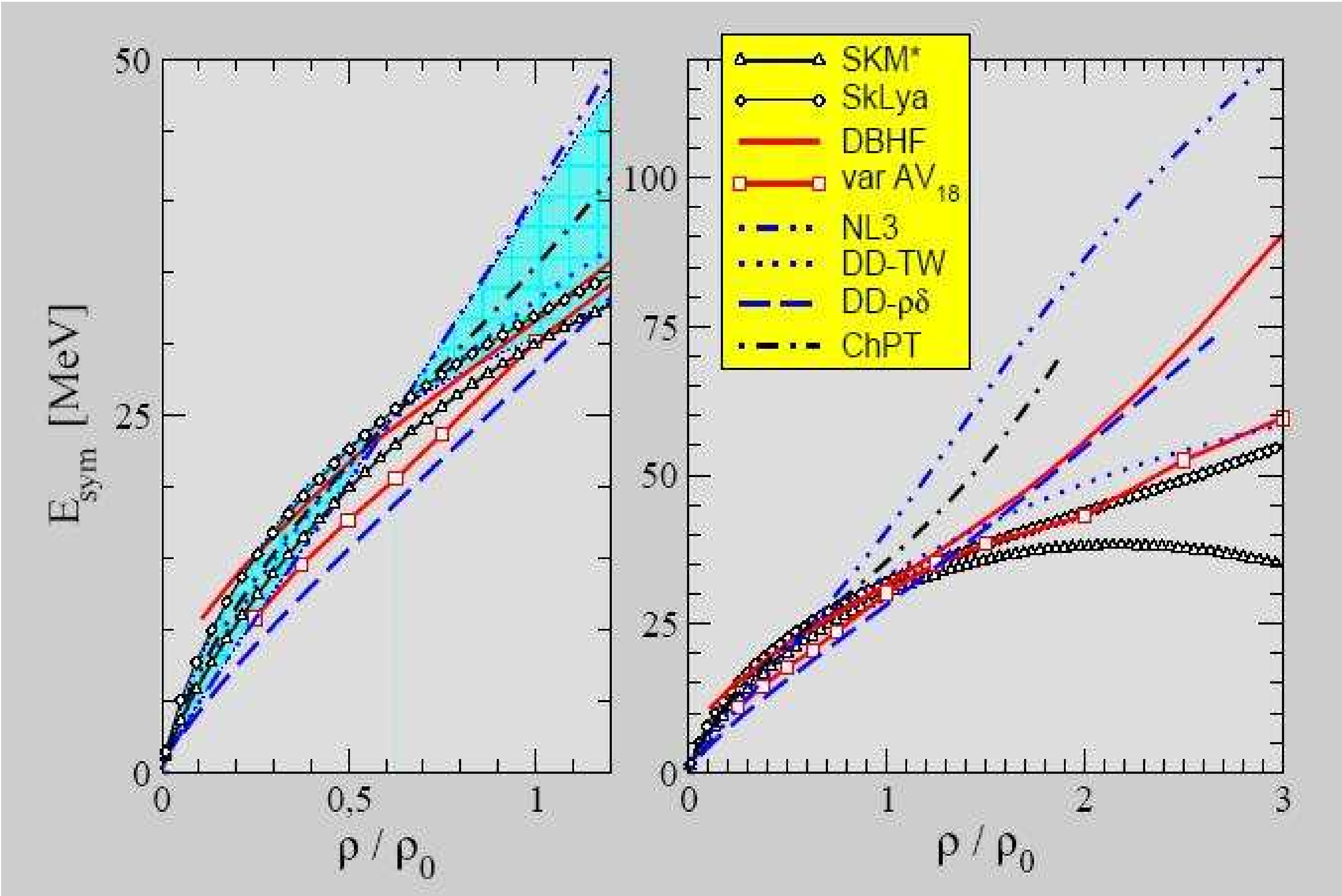
$$E_{sym}(n) \simeq J + \frac{L}{3}\left(\frac{n}{n_s} - 1\right) + \frac{K_{sym}}{18}\left(\frac{n}{n_s} - 1\right)^2 + \cdots$$

$$J = E_{sym}(n_s), \qquad L = 3n_s\left(\frac{\partial E_{sym}}{\partial n}\right)_{n_s}, \qquad K_{sym} = 9n_s^2\left(\frac{\partial^2 E_{sym}}{\partial n^2}\right)_{n_s}$$

Almost no information is available for K_{sym}.



C. Fuchs, H.H. Wolter, EPJA 30(2006) 5



C. Fuchs, H.H. Wolter, EPJA 30(2006) 5

Schematic Dependence

Nuclear Hamiltonian:

$$H = H_B + \frac{Q}{2} n'^2 \,, \quad H_B \simeq n \left[-B + \frac{K}{18} \left(1 - \frac{n}{n_s} \right)^2 \right] + E_{sym} (1-2x)^2$$

Lagrangian minimization of energy with respect to n (symmetric matter):

$$H_B - \mu_0 n = \frac{Q}{2} n'^2 = \frac{K}{18} n \left(1 - \frac{n}{n_s} \right)^2 \,, \qquad \mu_0 = -a_v$$

Liquid Droplet surface parameters: $a_s = 4\pi r_0^2 \sigma_0, \qquad S_s = 4\pi r_0^2 \sigma_\delta$

$$\sigma_0 \;=\; \int_{-\infty}^{+\infty} [H - \mu_0 n] dz = 2 \int_0^{n_s} (H_B - \mu_0 n) \frac{dn}{n'} = \frac{4}{45} \sqrt{Q K n_s^3}$$

$$t_{90-10} \;=\; \int_{0.1 n_s}^{0.9 n_s} \frac{dn}{n'} = 3 \sqrt{\frac{Q n_s}{K}} \int_{0.1}^{0.9} \frac{du}{\sqrt{u}(1-u)} \simeq 9 \sqrt{\frac{Q n_s}{K}}$$

$$\sigma_\delta \;=\; S_v \sqrt{\frac{Q}{2}} \int_0^{n_s} n \left(\frac{S_v}{E_{sym}} - 1 \right) (H_B - \mu_0 n)^{-1/2} dn$$

$$\frac{S_s}{S_v} \;=\; \frac{t_{90-10}}{r_0} \int_0^1 \frac{\sqrt{u}}{1-u} \left(\frac{S_v}{E_{sym}} - 1 \right) du, \qquad \delta R = \sqrt{\frac{3}{5}} r_0 \frac{S_s}{S_v} \frac{N-Z}{3Z}$$

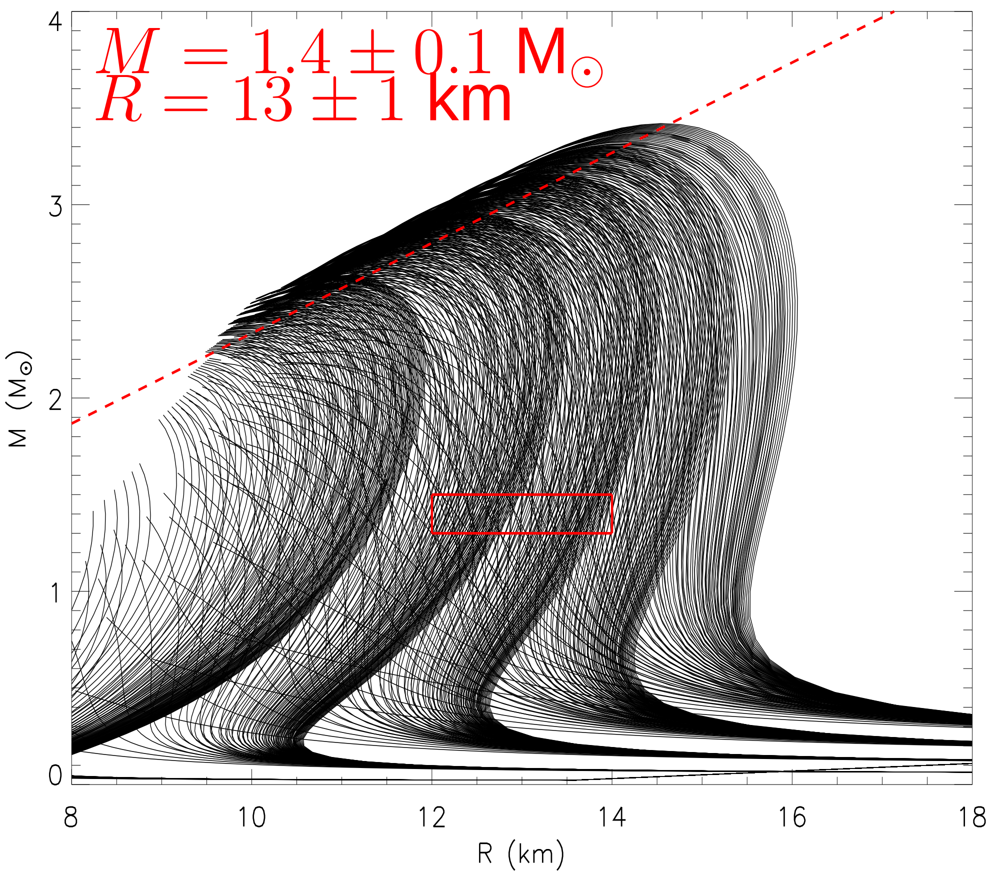
$$E_{sym} \;\simeq\; S_v \left(\frac{n}{n_s} \right)^p \Longrightarrow \int \to 0.28 \; (p = \frac{1}{2}) \,, \; 0.93 \; (p = \frac{2}{3}) \,, \; 2.0 \; (p = 1)$$

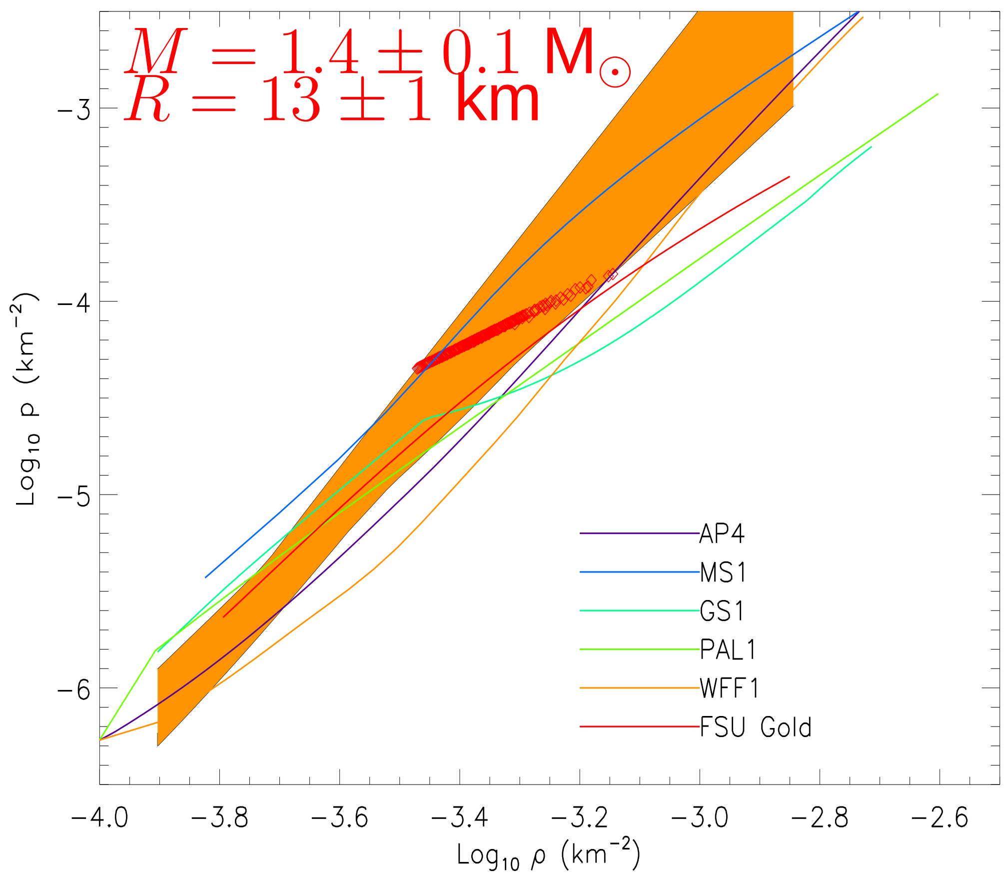
$$E_{sym} \;\simeq\; S_v + \frac{L}{3} \left(\frac{n}{n_s} - 1 \right) \Longrightarrow \int \to 2 - 2 \sqrt{\frac{3 S_v}{L} - 1} \tan^{-1} \sqrt{\left(1 + \frac{S_v}{3L} \right)^{-1}} \simeq \frac{2L}{3 S_v}$$

TOV Inversion

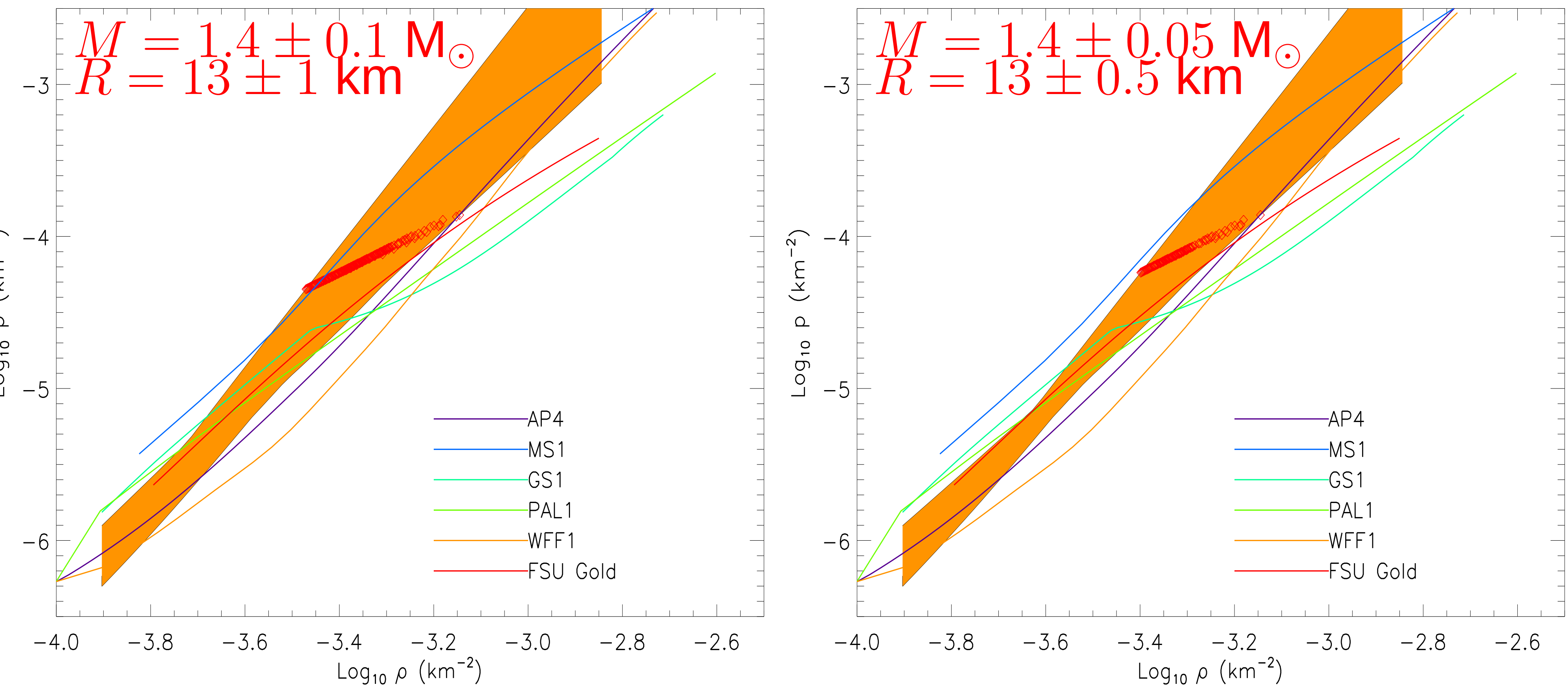
How would a simultaneous $M-R$ determination constrain the EOS? Each M-R curve specifies a unique $p-\rho$ relation.

- Generate physically reasonable $M-R$ curves and the $p-\rho$ relations that they specify.
- Generate physical $p-\rho$ relations and compute $M-R$ curves from them; select those $M-R$ curves passing within the error box.





TOV Inversion (cont.)



Observational Constraints for Neutron Stars

- Maximum and Minimum Masses (binary pulsars)
- Minimum Rotational Period*
- Radiation Radius or Redshift*
- Core Cooling Timescale (URCA or not)*
- Crustal Cooling Timescale*
- Seismology*
- Moment of Inertia*
- Proto-Neutron Star Neutrinos (Binding Energy, Opacities, Radii)*
- Pulse Shape Modulation*
- Gravitational Radiation* (Masses, Radii from tidal effects)

* Significant dependence on symmetry energy

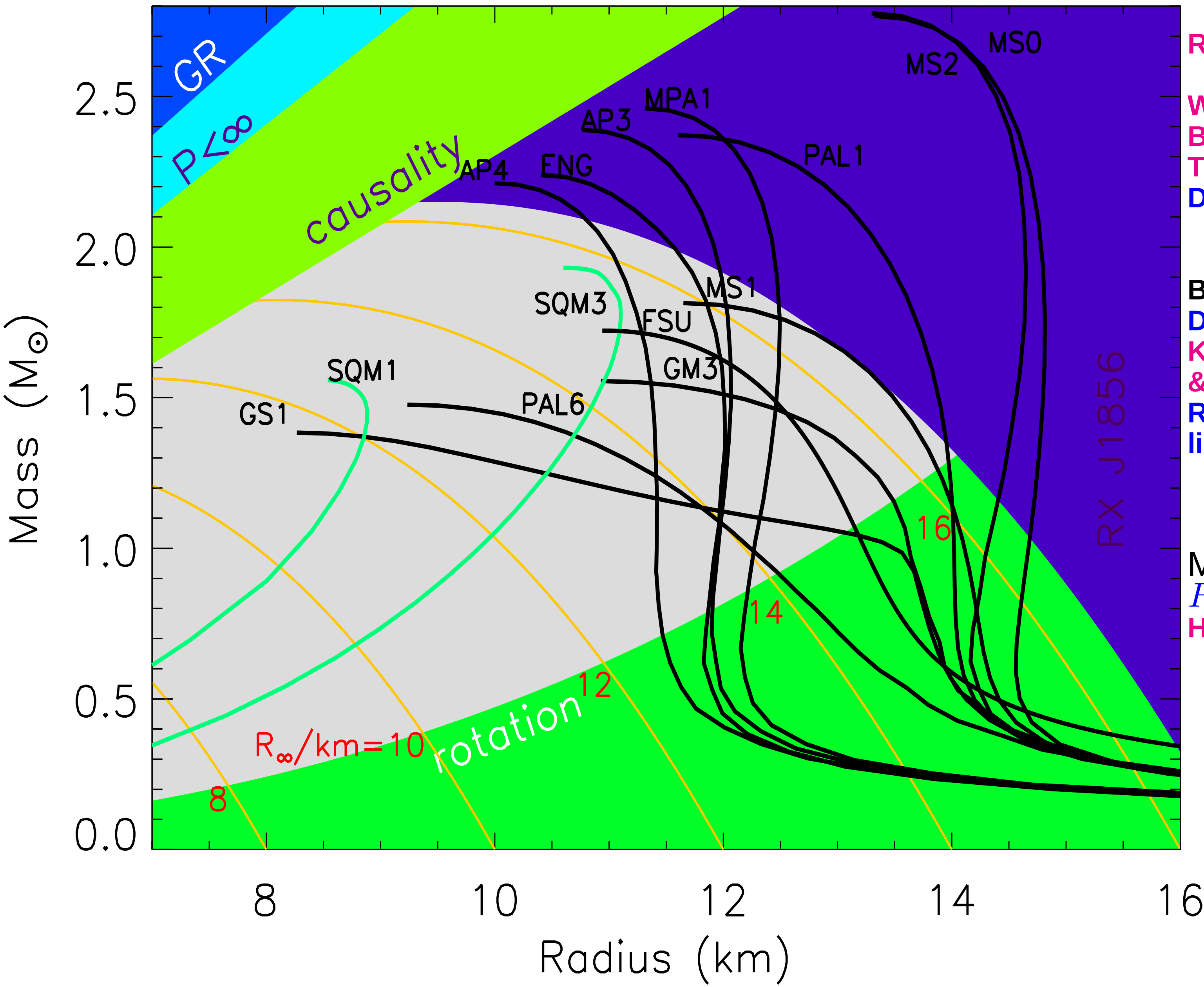
Radiation Radius

- Combination of flux and temperature measurements yields apparent angular diameter (pseudo-BB):

$$\frac{R_\infty}{d} = \frac{R}{d} \frac{1}{\sqrt{1 - 2GM/Rc^2}}$$

- Observational uncertainties include distance, interstellar H absorption (hard UV and X-rays), atmospheric composition
- Best chances for accurate radii are from
  - Nearby isolated neutron stars (parallax measurable)
  - Quiescent X-ray binaries in globular clusters (reliable distances, low B H-atmospheres)
  - X-ray pulsars in systems of known distance
    - CXOU J010043.1-721134 in the SMC: $R_\infty \geq 10.8$ km (Esposito & Mereghetti 2008)

Radiation Radius: Nearby Neutron Star



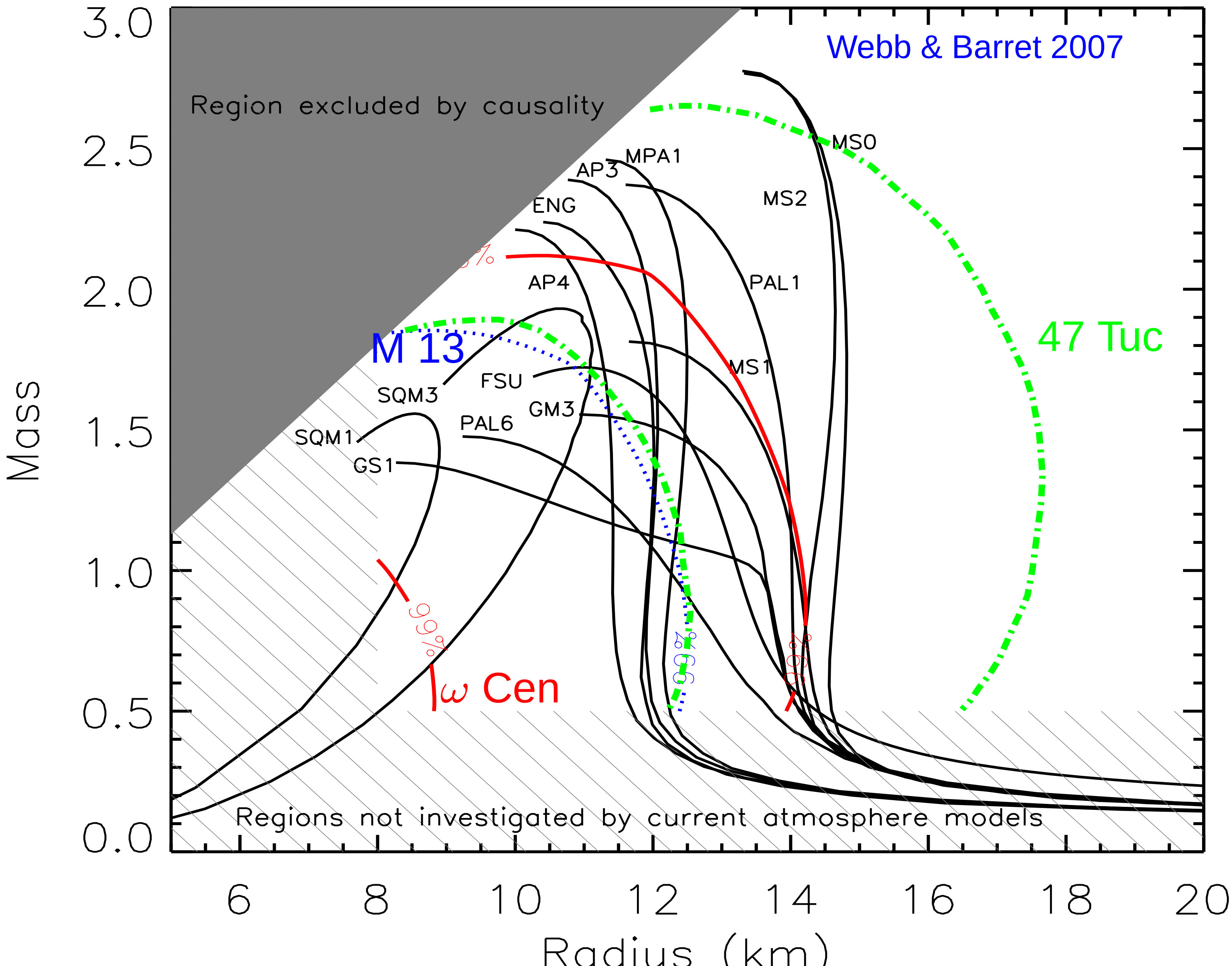
RX J1856-3754:

Walter & Lattimer 2002
Braje & Romani 2002
Truemper 2005
D=120 pc

BUT
D=140 pc
Kaplan, van Kerkwijk & Anderson 2002
Raises R_∞ limit to 19.5 km

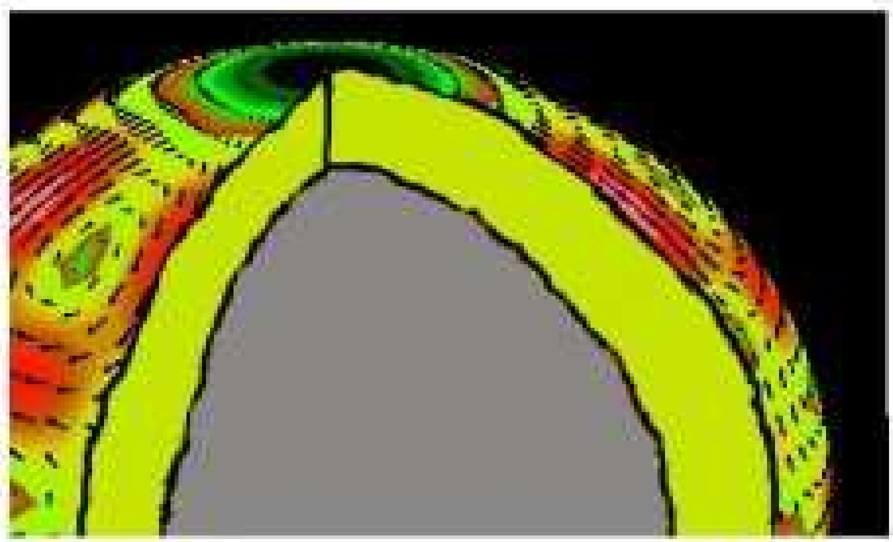
Magnetic H atmosphe
$R_\infty \approx 17$ **km**
Ho et al. 2007

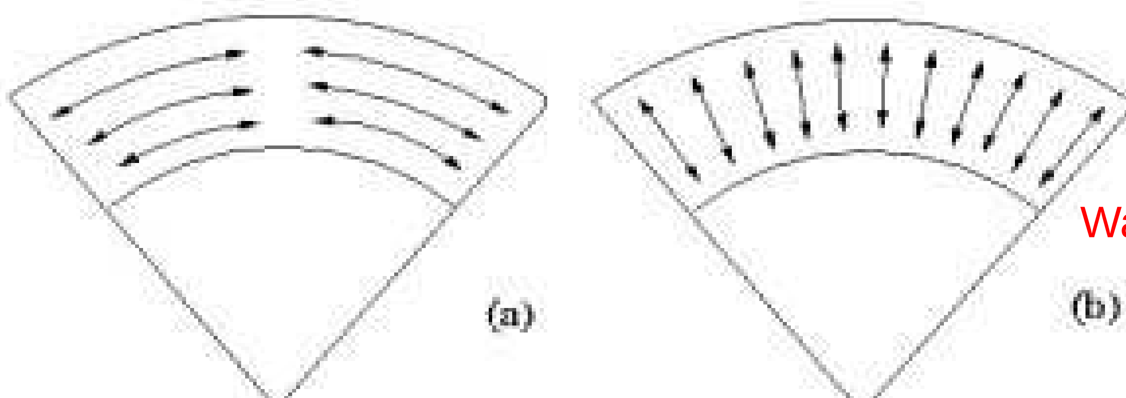
Radiation Radius: Globular Cluster Sources



Giant Flares in Soft Gamma-Ray Repeaters (SGRs)

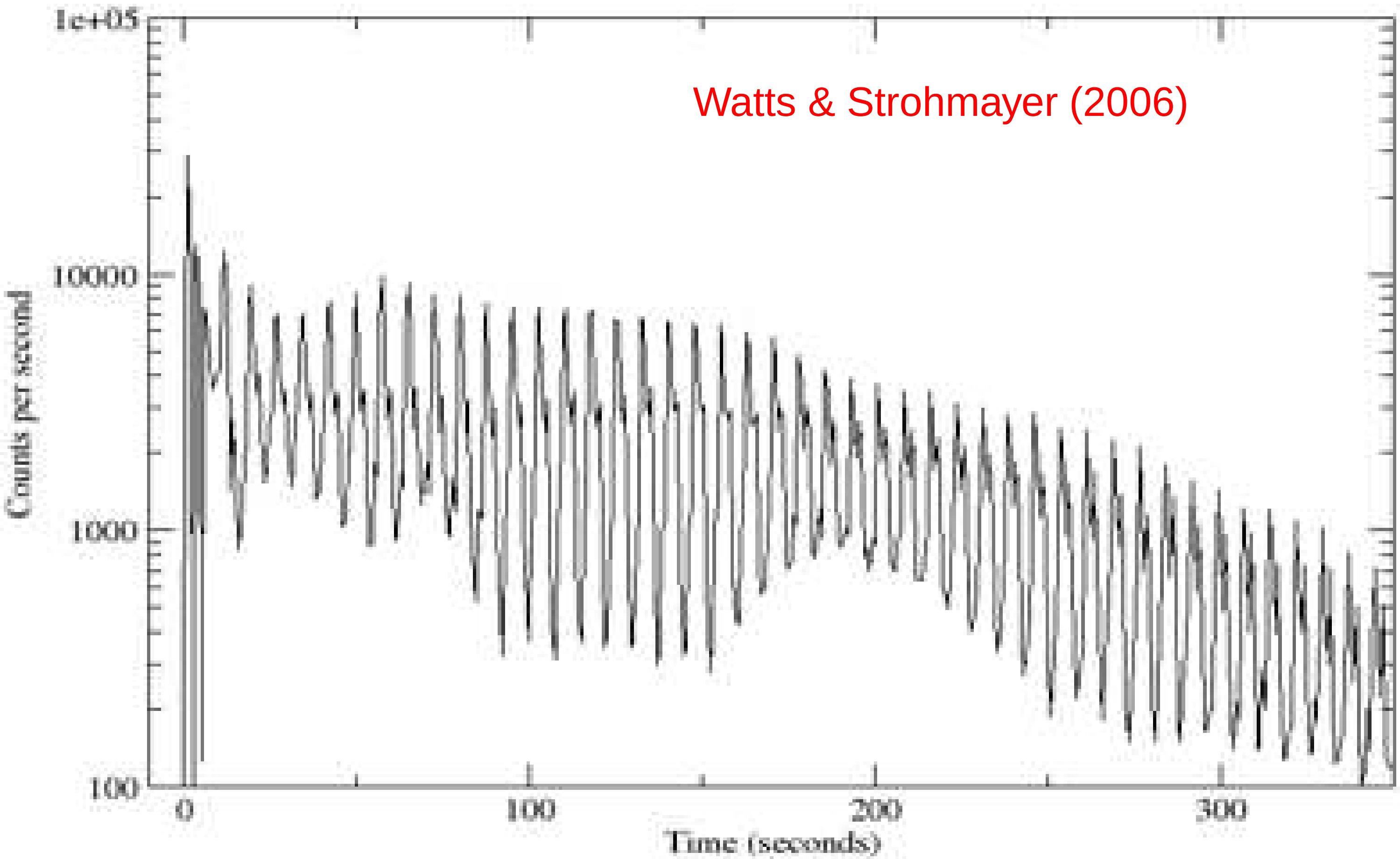
Quasi-periodic oscillations observed following giant flares in three soft gamma-ray repeaters (Israel et al. 2005; Strohmayer & Watts 2005, 6; Watts & Strohmayer 2006) which are believed to be highly magnetized neutron stars (magnetars).
Fields decay and twist, becoming periodically unstable. Eventually, the field lines snap and shift, launching starquakes and bursts of gamma-rays. Torsional shear modes are much easier to excite than radial modes.





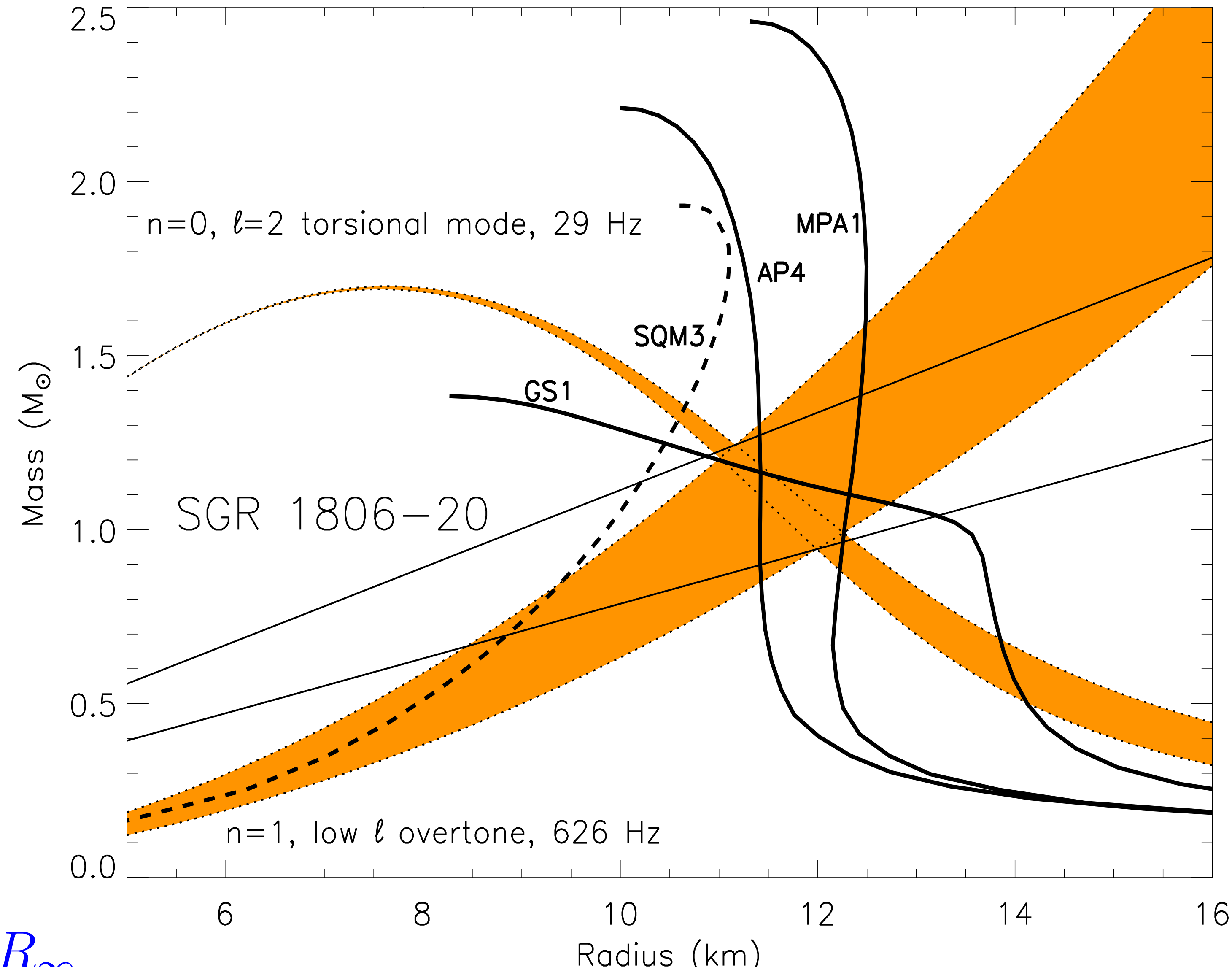
Watts & Reddy (2006)

Observations



Typical frequencies observed: 28-29 Hz, 50-150 Hz, 625 Hz
(SGRs 1806-20, 1900+14, 0526-66)
Frequencies of fundamental mode (28-29 Hz) agree well with expected torsional mode of neutron star crust (Duncan 1998)

Neutron Star Seismology



$$f_{n=0} \sim v_t/R_\infty$$
$$f_{n>0} \sim v_r(1-2\beta)/\Delta \sim (v_r M/R^2 \ln \mathcal{H})$$

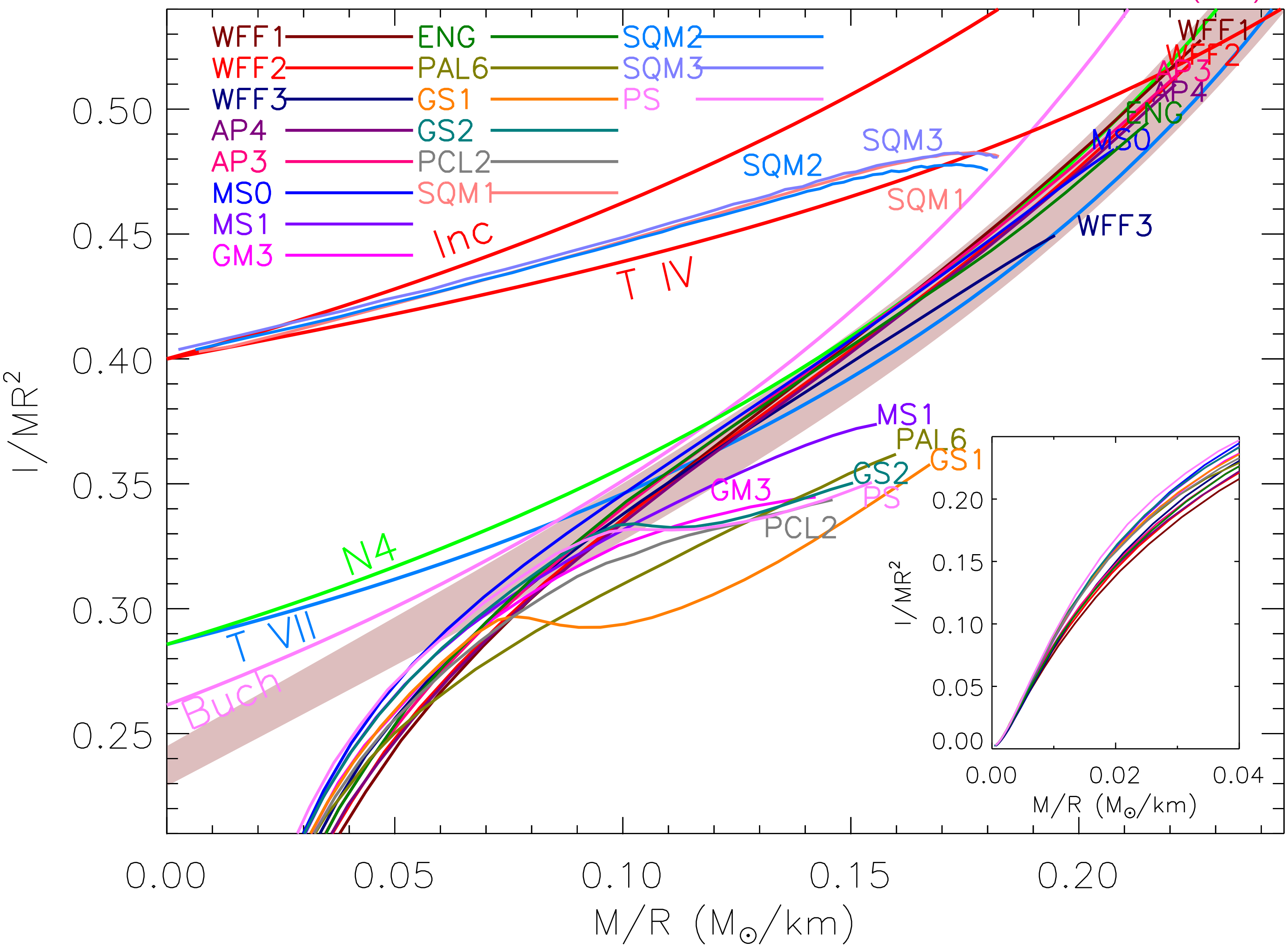
Strohmayer & Watts (2005)
Samuelsson & Andersson (2006)
Lattimer & Prakash (2006)

Moment of Inertia

- Spin-orbit coupling of same magnitude as post-post-Newtonian effects (Barker & O'Connell 1975, Damour & Schaeffer 1988)
- Precession alters inclination angle and periastron advance
- More EOS sensitive than R: $I \propto MR^2$
- Requires extremely relativistic system to extract
- Double pulsar PSR J0737-3037 is a marginal candidate
- Even more relativistic systems should be found, based on dimness and nearness of PSR J0737-3037

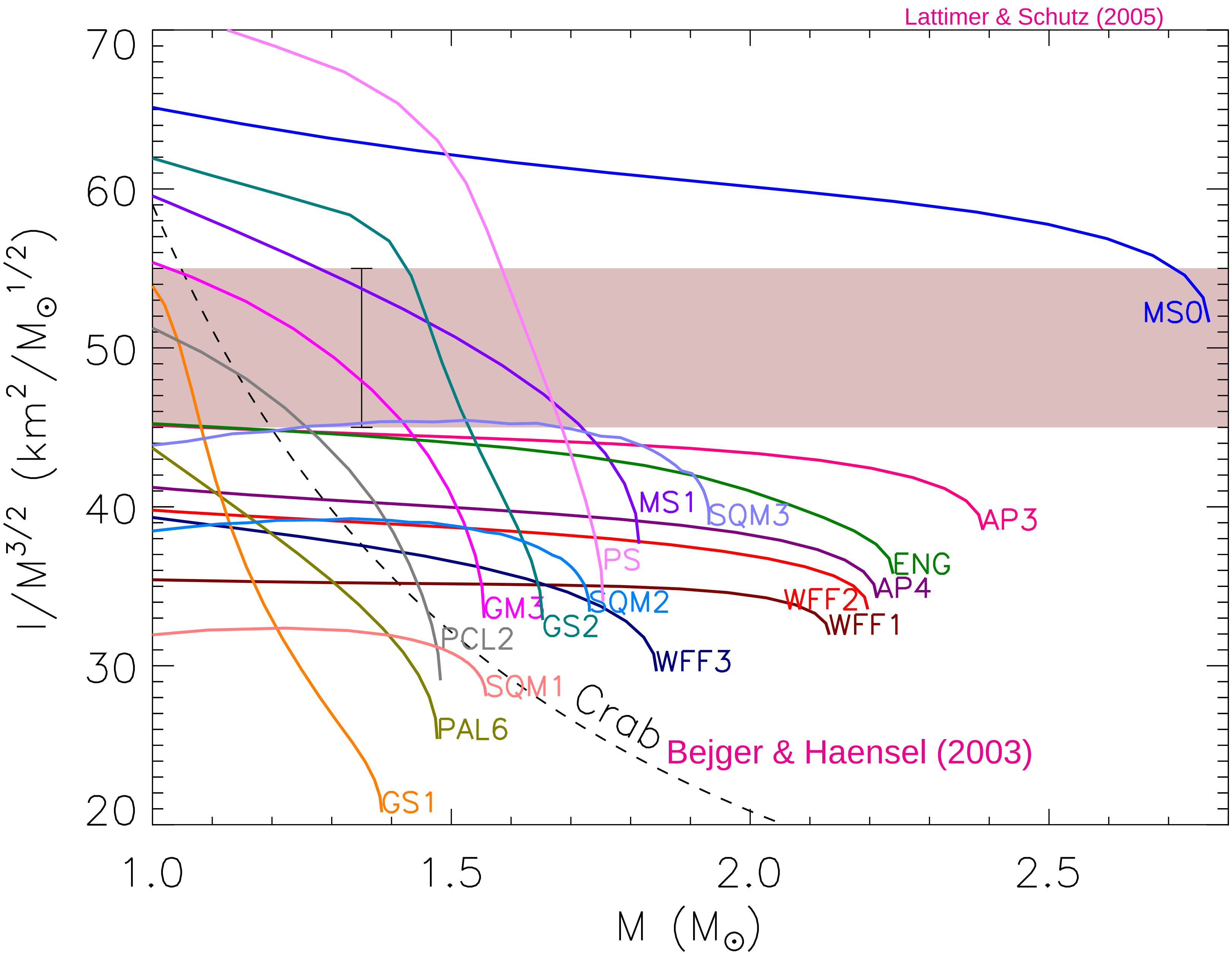
$I(M, R)$

Lattimer & Prakash (2001)

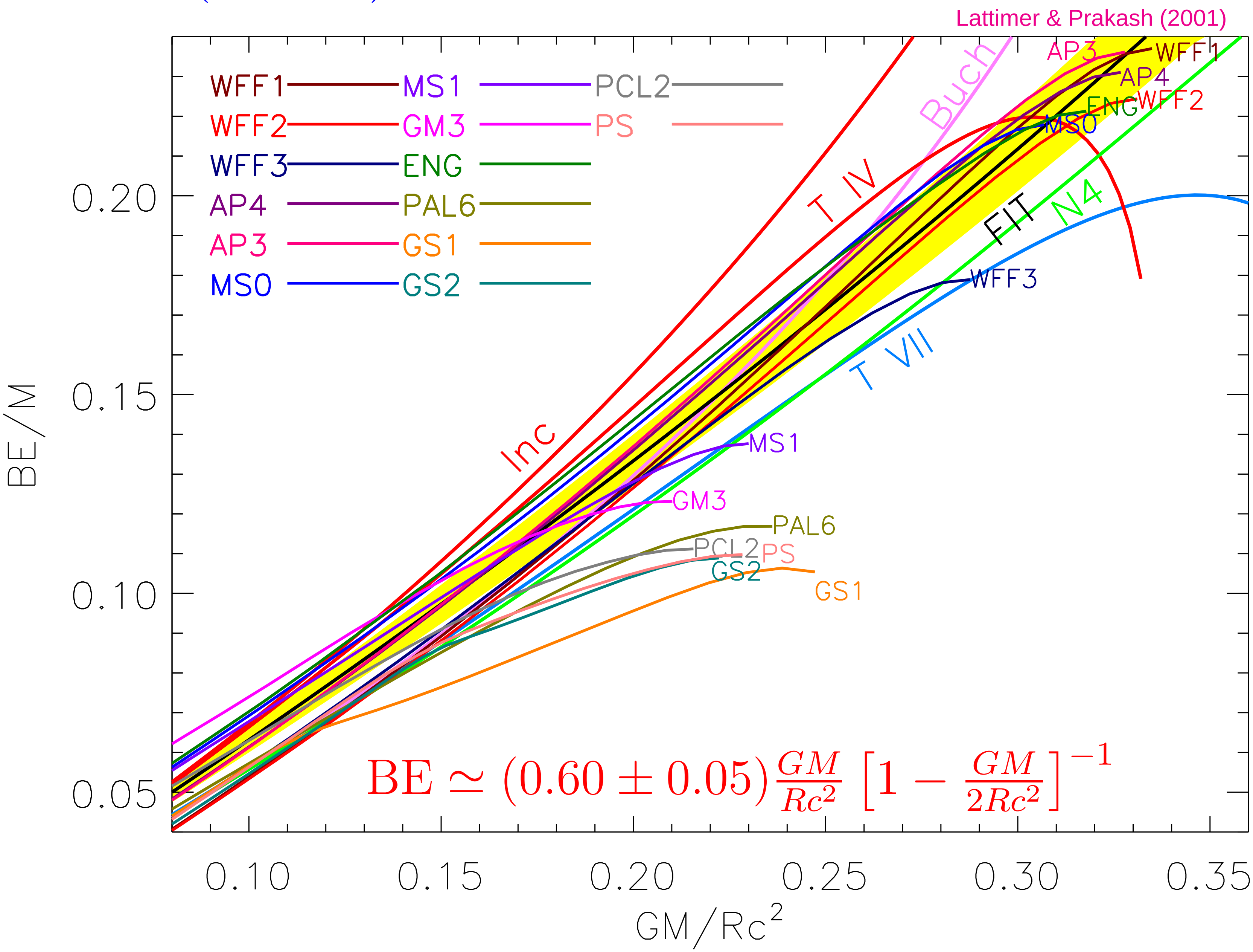


$$I \simeq (0.237 \pm 0.008) M R^2 \left[1 + 4.2 \frac{M \text{ km}}{R \text{ M}_\odot} + 90 \left(\frac{M \text{ km}}{R \text{ M}_\odot} \right)^4 \right]$$

EOS Constraint



BE(M, R)

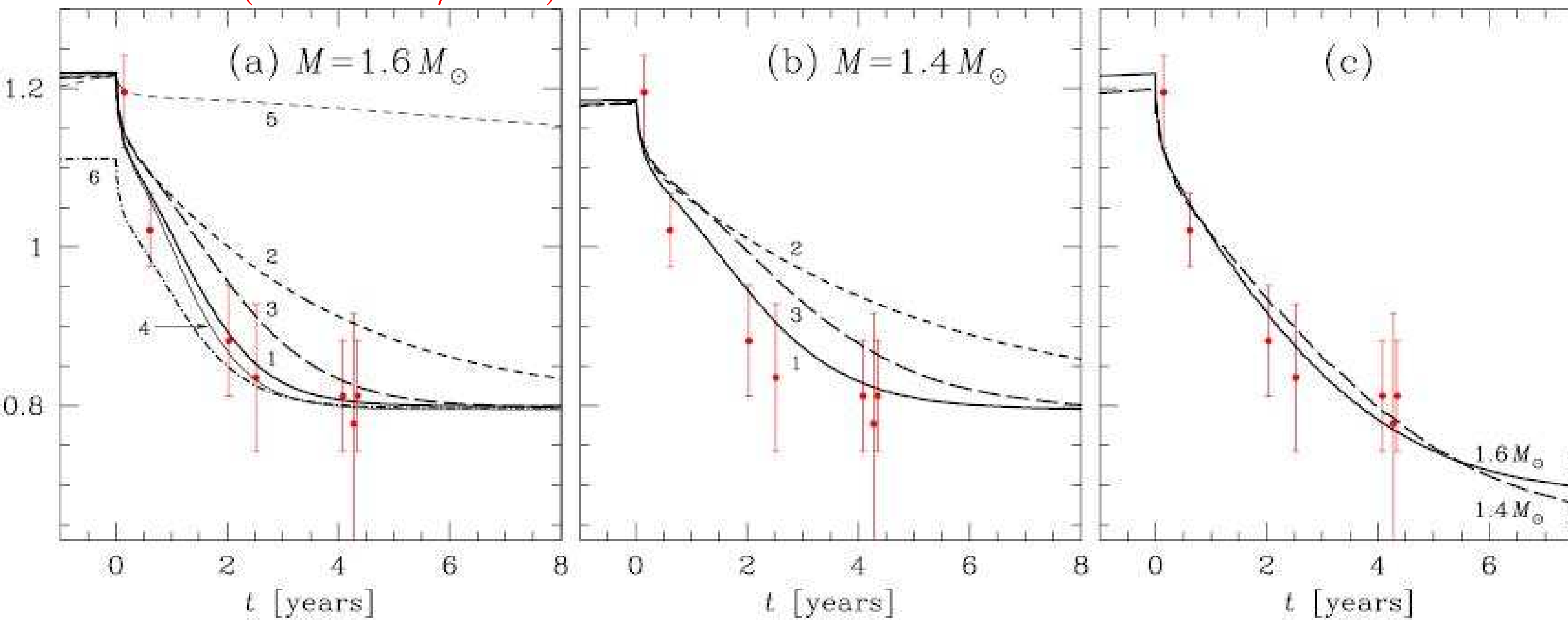


$$\mathrm{BE} \simeq (0.60 \pm 0.05)\frac{GM}{Rc^2}\left[1 - \frac{GM}{2Rc^2}\right]^{-1}$$

Crustal Heating in X-Ray Transients

Observations:
Cackett, Wijnands, Linares, Miller, Homan & Lewin (2006)

$$\tau \propto \frac{C_V \Delta^2}{\kappa(1-2GM/Rc^2)^{3/2}} \propto \frac{C_V(1-2GM/c^2)^{1/2}\ln^2 \mathcal{H} R^4}{\kappa M^2}$$

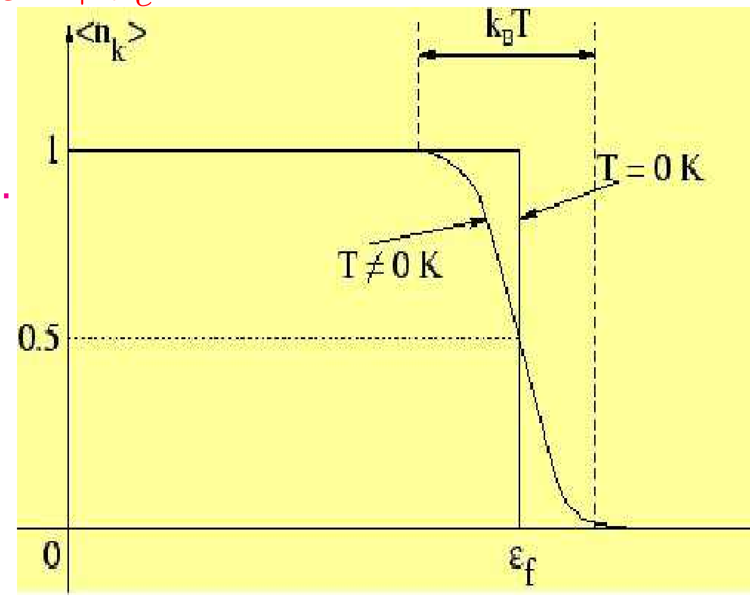


Shertnin, Yakovlev, Haensel & Potekhin (2007)

Neutron Star Cooling

Gamow & Schönberg proposed the direct Urca process: nucleons at the top of the Fermi sea beta decay.

$$n \to p + e^- + \nu_e\,, \qquad p \to n + e^+ + \bar\nu_e$$



Energy conservation guaranteed by beta equilibrium
$\mu_n - \mu_p = \mu_e$
Momentum conservation requires $|k_{Fn}| \le |k_{Fp}| + |k_{Fe}|$.
Charge neutrality requires $k_{Fp} = k_{Fe}$,
therefore $|k_{Fp}| \ge 2|k_{Fn}|$.
Degeneracy implies $n_i \propto k_{Fi}^3$, thus $x \ge x_{DU} = 1/9$.
With muons ($n > 2n_s$), $x_{DU} = \frac{2}{2+(1+2^{1/3})^3} \simeq 0.148$
If $x < x_{DU}$, bystander nucleons needed: modified Urca process is then dominant.

$$(n,p) + n \to (n,p) + p + e^- + \nu_e, \qquad (n,p) + p \to (n,p) + n + e^+ + \bar\nu_e$$

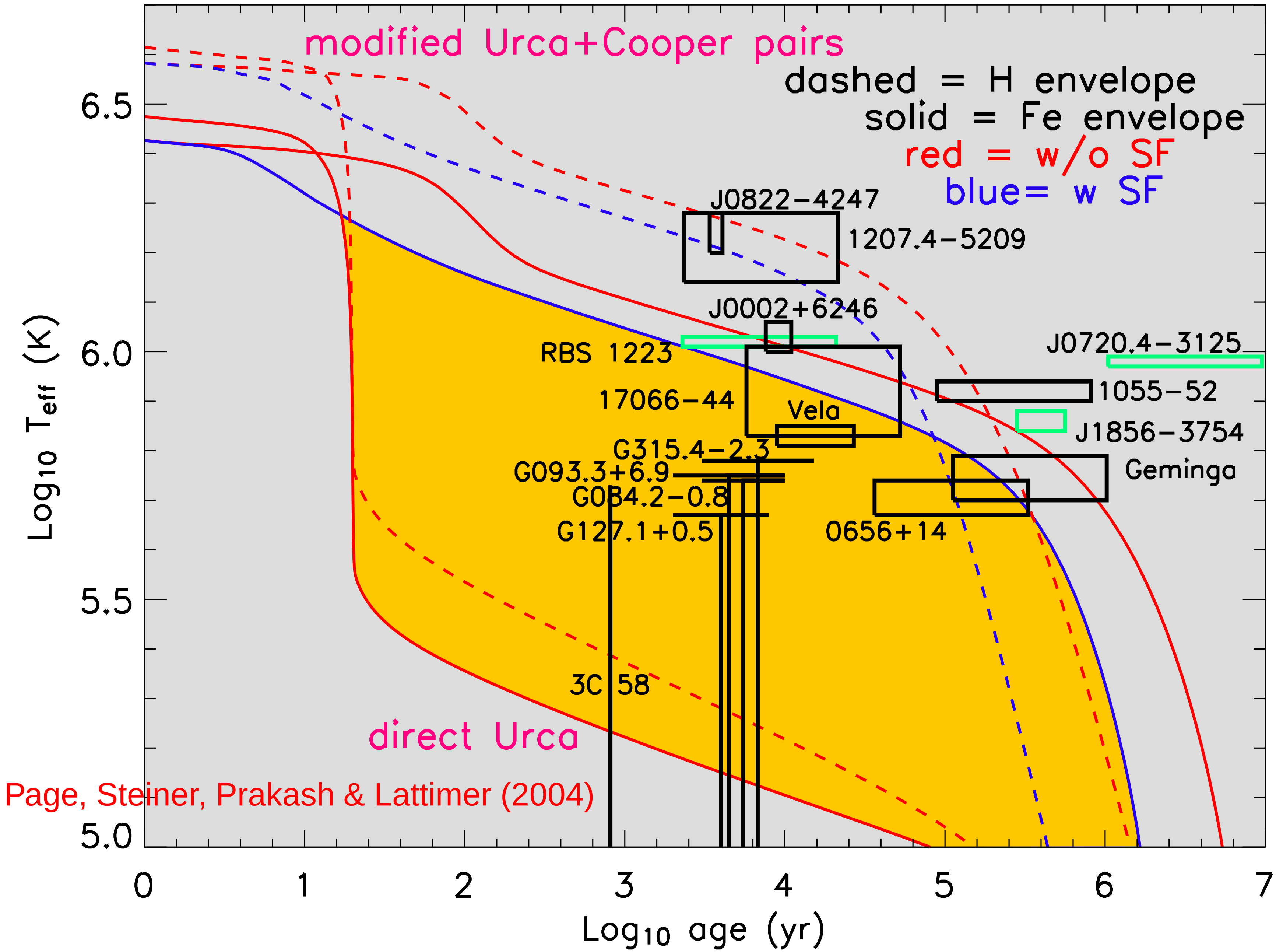
Neutrino emissivities:

$$\dot\epsilon_{MURCA} \simeq \left(\frac{T}{\mu_n}\right)^2 \dot\epsilon_{DURCA}\,.$$

Beta equilibrium composition:

$$x_\beta \simeq (3\pi^2 n)^{-1}\left(\frac{4E_{sym}}{\hbar c}\right)^3 \simeq 0.04\left(\frac{n}{n_s}\right)^{0.5-2}\,.$$

Neutron Star Cooling



Conclusions

- **We thank Katsu for his many contributions to the study of hot, dense matter in supernovae and neutron stars.**
- **Neutron stars are a powerful laboratory to constrain dense matter physics, especially the symmetry energy and composition at supranuclear densities.**
- **Increasing evidence exists for massive neutron stars ($M \gtrsim 1.7$ $M_\odot$).**
- **Many kinds of observations are becoming available to measure neutron star radii, although no definitive measures yet exist.**