On the Acceleration of Our Universe and the Effects of Inhomogeneities

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Introduction

Distant SNe-Ia (at $z \sim 0.5$) appear to be *fainter* than *expected* in *Einstein-de Sitter model*

- The concordance model

Geometry: FLRW Symmetry = Isotropic & *Homogeneous*
Main constituents: Dark Matter and Dark Energy

Still does NOT have any basis in fundamental physics

The issues of why so small and why now

We might be misinterpreting the cosmological data

- Alternative model?

Geometry: In-homogeneous
Main constituents: Dark Matter (No Dark Energy)

“ Which is more absurd,

Dark Energy or Inhomogeneous models? ”

Iguchi – Nakamura - Nakao 2002

Purpose of this talk:

- Introduce recent attempts to account for acceleration of the universe by the effects of inhomogeneities

- Point out some serious flaws in these attempts from theoretical -- relativisitc -- viewpoints

Outline

- Newtonianly perturbed FLRW universe

VS

- *Super*-horizon scale perturbations
- *Sub*-horizon perturbations & averaging
- Anti-Copernican inhomogeneous universe

Newtonianly perturbed FLRW universe

FLRW metric + scalar perturbations

$$ds^2 = -(1+2\Psi)dt^2 + a(t)^2(1-2\Phi)\gamma_{ij}dx^i dx^j$$

γ_{ij} homogeneous-isotropic 3-space

Newtonian perturbation $\Psi = \Phi$

$$|\Psi| \ll 1\,,$$

$$\left|\frac{\partial \Psi}{\partial t}\right|^2 \ll \frac{1}{a^2}(D^i\Psi)D_i\Psi\,,$$

$$(D^i\Psi D_i\Psi)^2 \ll (D^iD^j\Psi)D_iD_j\Psi$$

Stress-tensor

Smoothly distributed component

$$T_{ab}^{(s)} \approx \rho^{(s)}(t)dt^2 + P^{(s)}(t)a^2(t)\gamma_{ij}dx^i dx^j$$

e.g., Dark Energy component

Inhomogeneously distributed component

$$T_{ab}^{(m)} \approx \rho^{(m)}(t, x^i)dt^2$$

Einstein equations

$$3\left(\frac{\dot{a}}{a}\right)^2 = \kappa^2\left(\rho^{(s)} + \bar{\rho}^{(m)}\right) - 3\frac{K}{a^2}$$

$$3\frac{\ddot{a}}{a} = -\frac{\kappa^2}{2}\left(\rho^{(s)} + \bar{\rho}^{(m)} + 3P^{(s)}\right)$$

$$\frac{1}{a^2}\Delta_{(3)}\Psi = \frac{\kappa^2}{2}\delta\rho \qquad \left(\delta\rho = \rho^{(m)} - \bar{\rho}^{(m)} \right)$$

Large - scale ⇒ FLRW dynamics

Small - scale ⇒ Newtonian gravity

- It is commonly stated that when

$$\frac{\delta\rho}{\rho} \gg 1$$

we enter a non-linear regime

This is not the case

Solar system, Galaxies, Clusters of Galaxies

$$\delta\rho/\rho \approx 10^{30}, \approx 10^{5}, \approx 10^{2} \gg 1$$

$$\Psi \approx 10^{-6} \sim 10^{-5} \ll 1$$

Newtonianly perturbed FLRW metric appears to very accurately describe our universe

on all scales

(except immediate vicinity of BHs and NSs)

If this assertion is correct

higher order corrections to this metric from inhomogeneities would be negligible

… but we cannot preclude the possibility that other models could also fit all observations

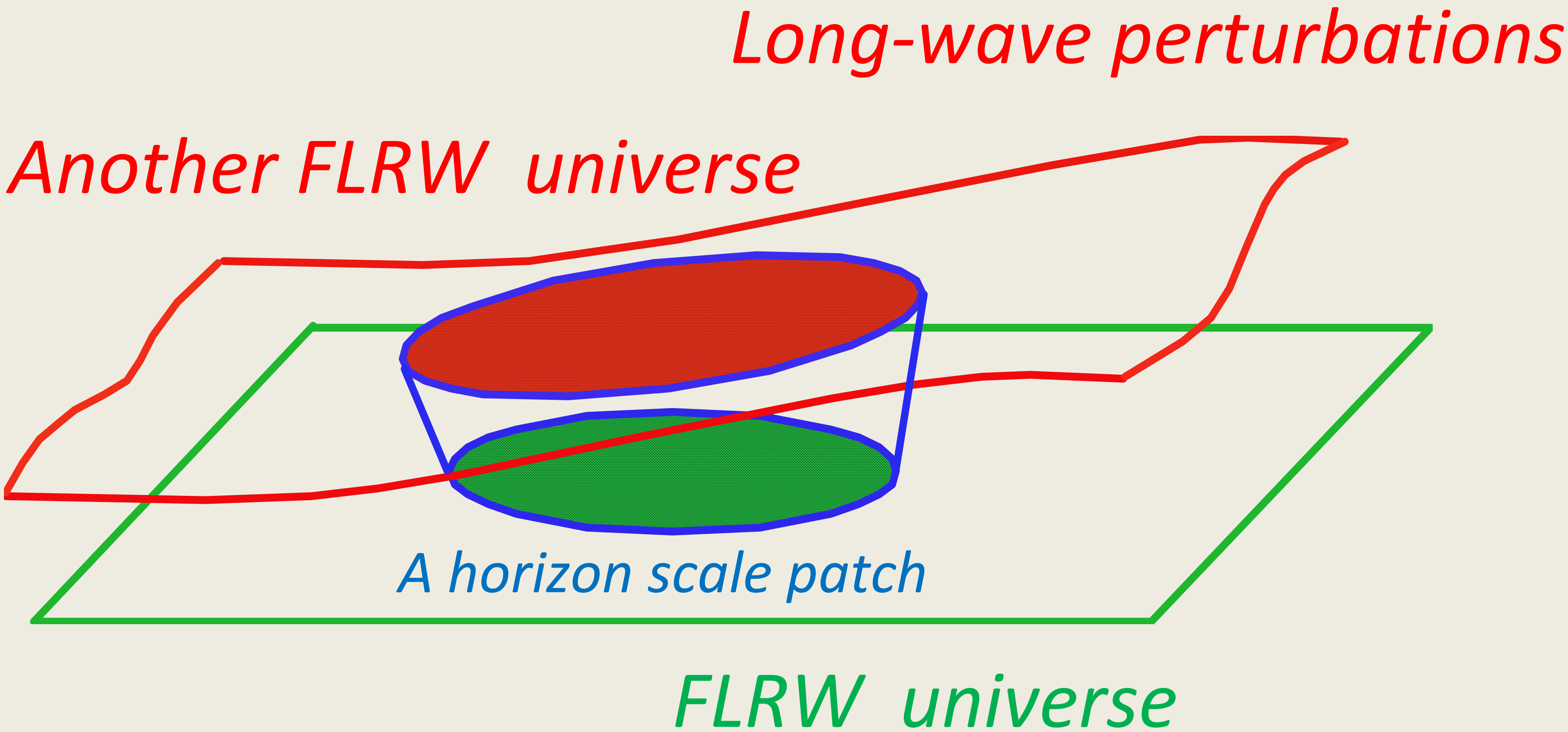
Our points

- If one wishes to propose an alternative model then it is necessary to show that all of the predictions of the proposed model are compatible with observations.

It does *not suffice* to show that some quantity (type of scale factor) behaves in a desired way

Backreaction from

Super-horizon perturbations



Long-wave perturbations
Another FLRW universe
A horizon scale patch
FLRW universe

2^{nd}-order effective stress-tensor approach

$$g(\alpha) = g_{ab}^{(0)} + \alpha g_{ab}^{(1)} + \alpha^2 g_{ab}^{(2)} + \cdots$$

0th: $G_{ab}[g^{(0)}] = 0$

For vacuum case

1st: $G_{ab}^{(1)}[g^{(1)}] = 0$

2nd: $G_{ab}^{(1)}[g^{(2)}] = -G_{ab}^{(2)}[g^{(1)}]$

View $8\pi G^{(eff)} T_{ab} := -G_{ab}^{(2)}[g^{(1)}]$

and equate as $G_{ab}[g] = 8\pi G^{(eff)} T_{ab}$

g : backreacted metric

Brandenberger et al 1997 - 2005

If the effective stress-tensor takes the form

$$^{(eff)}T_{ab} \propto -\Lambda g_{ab}$$

and has the appropriate magnitude

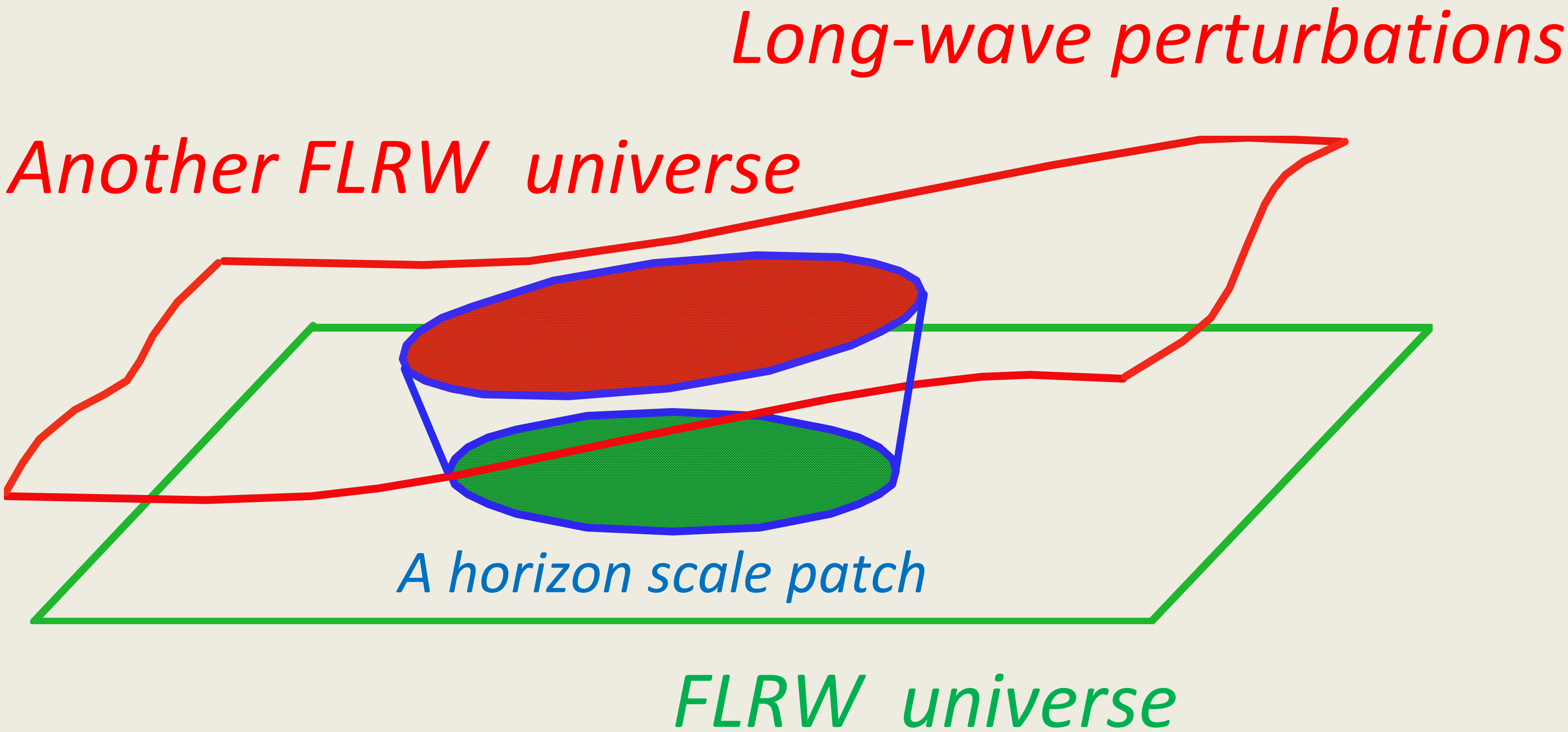
we are done … !?

Martineau – Brandenberger 2005

Some flaws in this approach

AI & Wald 2006

- “Backreaction equation” is NOT consistently constructed from “perturbation theory”
- 2^{nd}-order effective stress-tensor is gauge-dependent
- If 2^{nd}-order stress tensor has large effects, one can NOT reliably compute backreaction in 2^{nd}-order theory
- Long-wavelength limit corresponds to “other FLRW universe” (e.g., with different initial data)


Long-wave perturbations
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Backreaction from

Sub-horizon perturbations

&

spatial averaging

Inhomogeneous metric

$$ds^2 = -\alpha dt^2 + 2\beta_i dt dx^i + q_{ij} dx^i dx^j$$

Raychaudhuri equation: θ : expansion

$$\frac{d}{dt}\theta = -\frac{1}{3}\theta^2 - \sigma^2 - 4\pi G\rho + \omega^2$$

Deceleration unless one has large "*vorticity*" $\omega^2 \neq 0$

"Accelerated" expansion → need some new mechanism

For simplicity and definiteness we hereafter focus on an inhomogeneous universe with irrotational dust.

Then in the comoving synchronous gauge

$$ds^2 = -dt^2 + q_{ij}(t, x^m)dx^i dx^j$$

Spatial-Averaging

Buchert et al

Definition over Domain : $$\langle \phi \rangle_D \equiv \frac{1}{V_D} \int_D \phi d\Sigma$$

Depend on the choice of domain

Averaged scale factor: $$a_D \equiv (V_D)^{1/3}$$

Smoothing out inhomogeneities

→ Effective FLRW universe

Equations for “averaged quantities”

Buchert 2000

$$3\frac{\ddot{a}_D}{a_D} = -\frac{\kappa^2}{2}\langle\rho\rangle_D + Q_D$$

$$3\left(\frac{\dot{a}_D}{a_D}\right)^2 = \kappa^2\langle\rho\rangle_D - \frac{1}{2}\langle\mathcal{R}\rangle_D - \frac{1}{2}Q_D$$

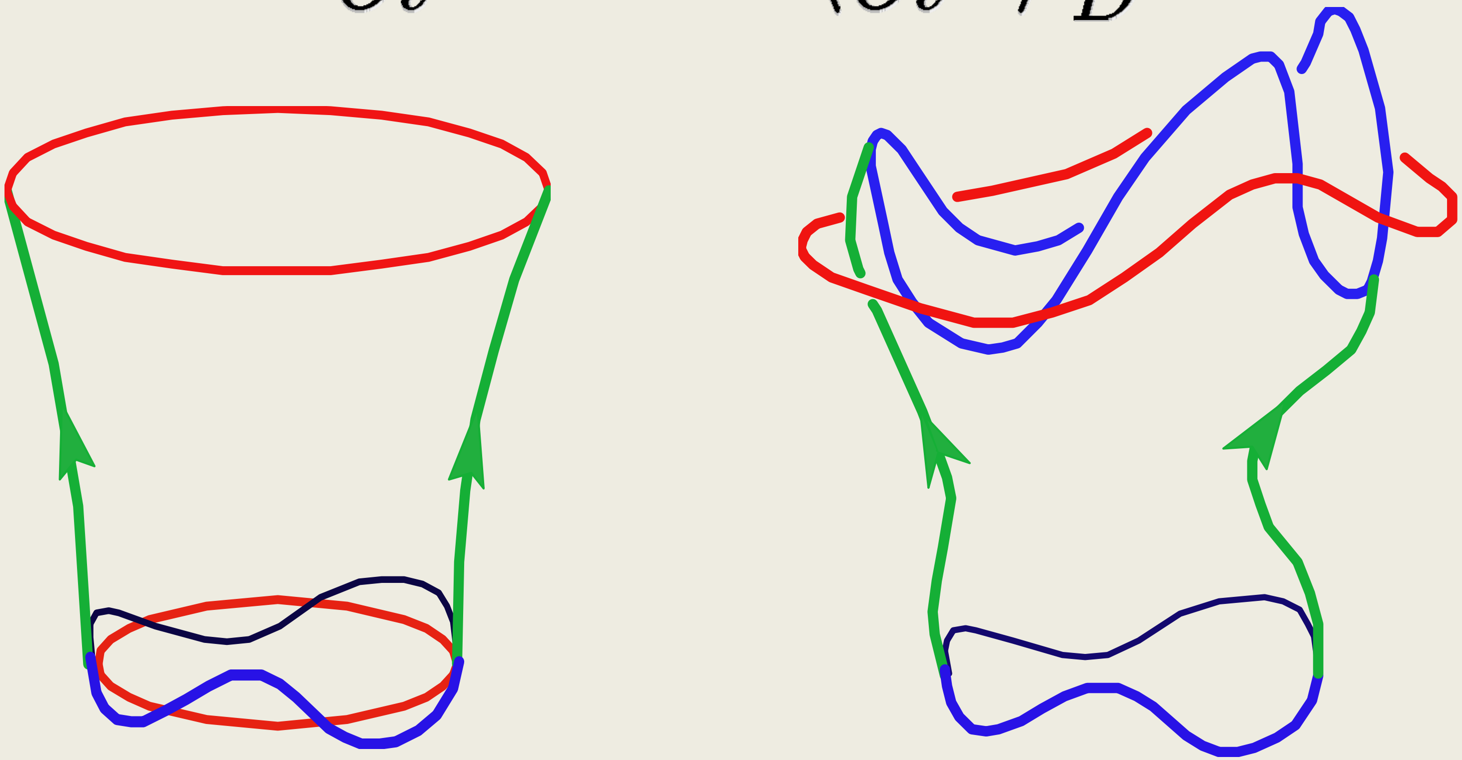
Integrability condition: $\left(a_D^6 Q_D\right)^{\cdot} + a_D^4\left(a_D^2\langle\mathcal{R}\rangle_D\right)^{\cdot} = 0$

Kinematical backreaction: $Q_D \equiv \frac{2}{3}\left(\langle\theta^2\rangle_D - \langle\theta\rangle_D^2\right) - \langle\sigma_{ij}\sigma^{ij}\rangle_D$

If $Q_D > \frac{\kappa^2}{2}\langle\rho\rangle_D$ ⇒ $\ddot{a}_D > 0$ Acceleration

Spatial averaging and time evolution do NOT commute

$$\frac{\partial}{\partial t}\langle\phi\rangle_D \neq \left\langle\frac{\partial}{\partial t}\phi\right\rangle_D$$



The same initial data

Contributions from non-linear sub-horizon perturbations to Q_D and the apparent acceleration of the volume-averaged scale factor have been studied by using *gradient expansion* method

⇒ Perturbation series appear to diverge

Kolb – Matarrese-Riotto 2005

-- The results seem to depend on the definition (e.g. choice of the domain) of the spatial averaging

-- unclear the relations btwn averaged quantities and physical observables

-- seemingly they have used the perturbation method beyond its regime of validity

An example of averaged acceleration

Averaging a portion of *expanding open* FLRW universe and a portion of *collapsing closed* FLRW universe exhibits "acceleration" in the averaged scale factor

Nambu & Tanimoto 2005

Even if $\ddot{a}_1 < 0 \quad \ddot{a}_2 < 0$

$$a_D^2 \ddot{a}_D = a_1^2 \ddot{a}_1 + a_2^2 \ddot{a}_2 + \frac{2}{a_D^3} a_1^3 a_2^3 \left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2} \right)^2$$

can be positive

This does NOT mean that we can obtain physically observable acceleration by spatial averaging

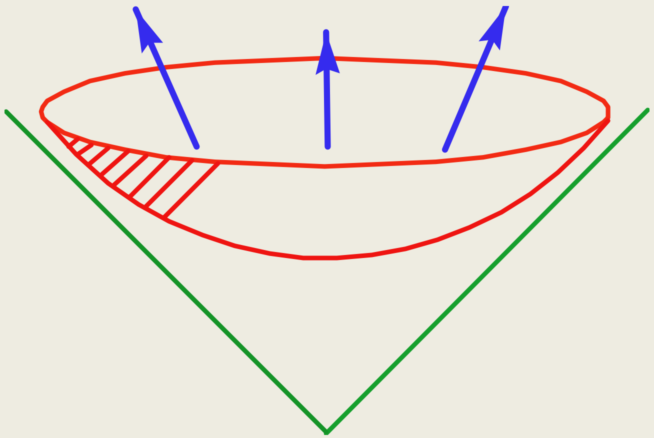
-- rather implies "spurious acceleration"

AI & Wald 2006

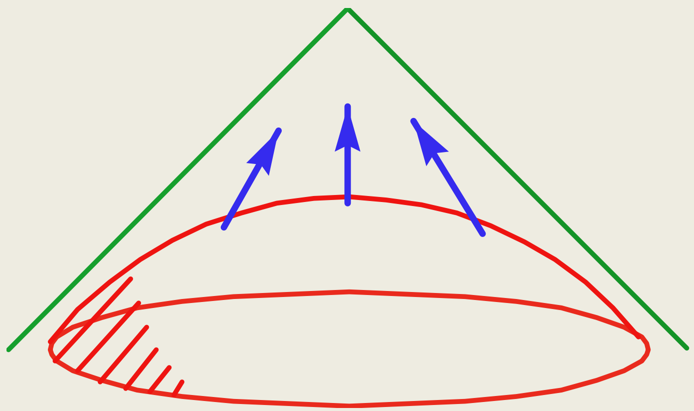
An example of spurious Acceleration

in Minkowski spacetime

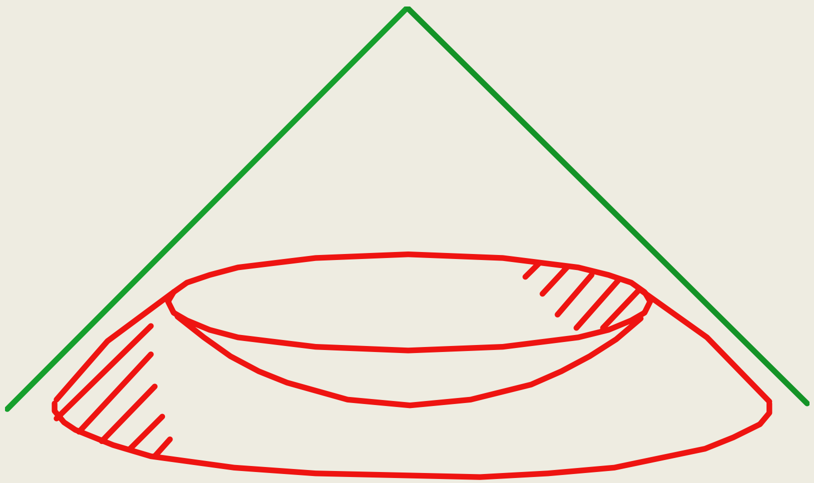
Expanding Hyperboloid



Contracting Hyperboloid



Always possible to take two (portions of) hyperboloids so that



$$\frac{\ddot{a}_D}{a_D} = -\frac{1}{3}\langle \text{Curvature of hyperboloid} \rangle_D = \frac{2}{a_D^2} > 0$$

Lessons

Gauge artifacts: The averaged scale factor displays “acceleration” without there being any physically observable consequence

No reason to believe that the averaged quantities correspond to any physical effects

Small inhomogeneities generate negligible effects

2nd-order analysis Kasai– Asada – Futamase 2006

Vanderveld et al 2007

Behrend et al 2008

Need large inhomogeneities to get large backreaction

But then averaging procedure has large ambiguities in the choice of Time-slice and Domain

Anti – Copernican universe

Inhomogeneous (non-perturvative) models

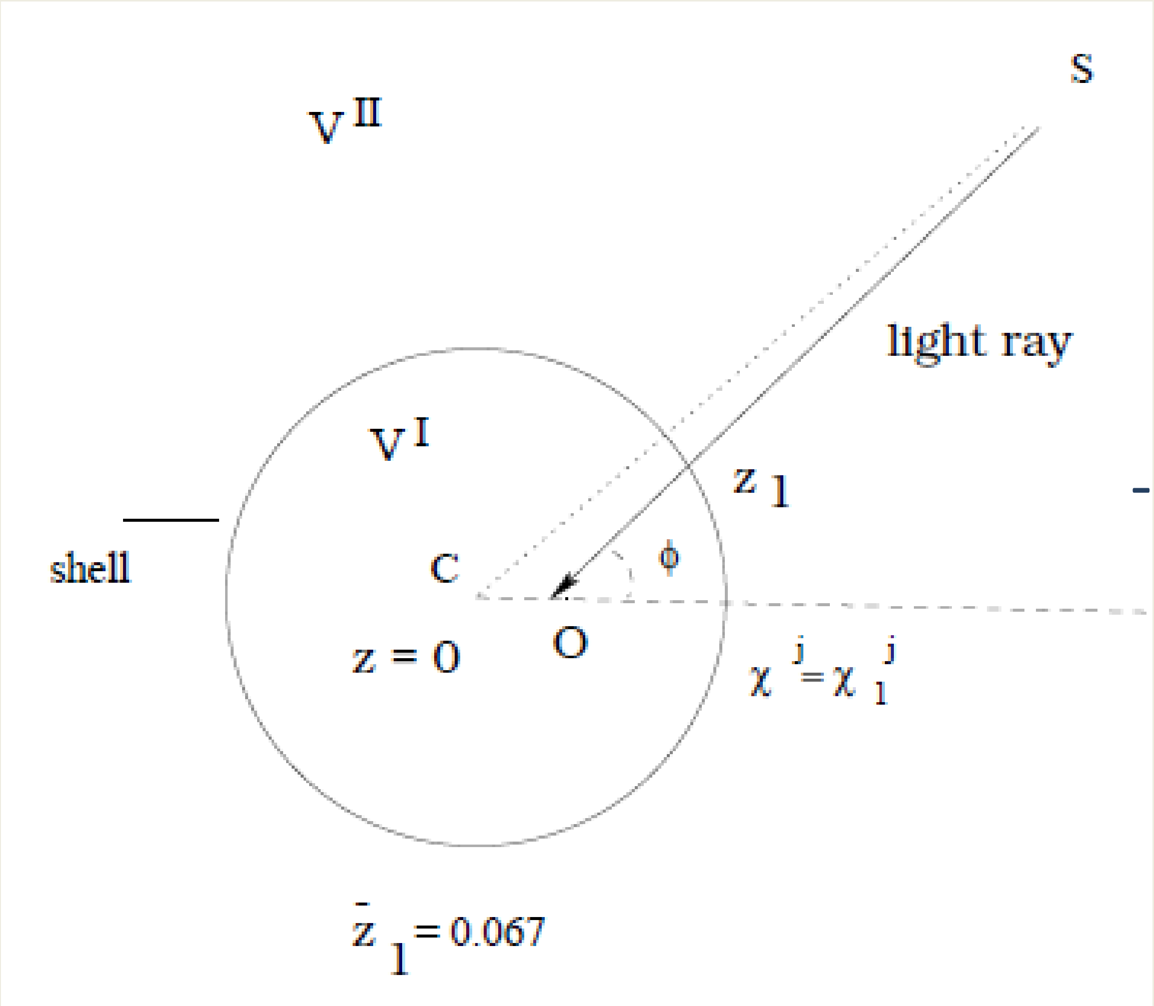
Geometry: *Spherically* Symmetric
Main constituents: Dark Matter

We are living in the center of the world

e.g. Local void of a few hundred Mpc: Tomita 2000

Local void of a few Gpc : Garcia-Bellido & Haugboelle 2008

A local void model   Tomita   2000



V^I :   low-dense region

V^{II} : high-dense outer

-- can be positioned 50Mpc away from the center

The mismatch between the local and global expansion can explain the observed dimming of SN-Ia luminosity

$$H_0^I > H_0^{II}$$

Simplest model: Lemaitre-Tolman-Bondi (LTB) metric

$$ds^2 = -dt^2 + \frac{R'(r,t)^2}{1+2E(r)}dr^2 + R(r,t)^2 d\Omega^2$$

$R(t,r) = ra(t) \quad 2E(r) = -Kr^2$ ⇒ FLRW metric

$$\dot{R}^2 = 2E + \frac{F(r)}{R}, \qquad \rho = \frac{F'}{8\pi G R' R}$$

Two arbitrary functions $E(r) \quad F(r)$

Null vector $l_a = dt_a + \frac{R'}{\sqrt{1+2E}}dr_a \quad k^a = (\partial/\partial\lambda)^a = -\omega l^a$

$$1 + z = \omega$$

Luminosity-distance: $d_L = (1+z)^2 R$

The LTB model can fit well the redshift-luminosity relation

Iguchi – Nakamura – Nakao 2002

Garfinkle 2006

$$m = M_B + 5\log(H_0 d_L)$$

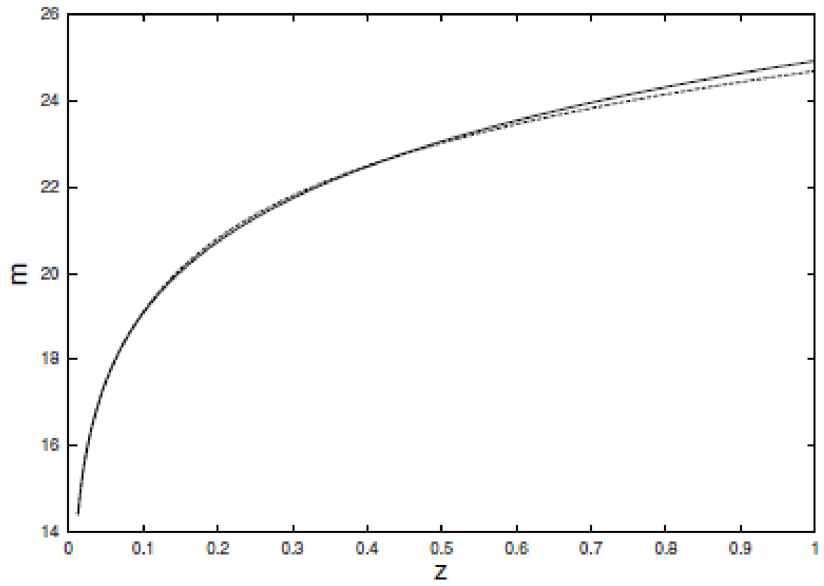


Figure 2. Plot of effective magnitude versus redshift for the standard ΛCDM model (solid and the $\Omega_M = 0.3$ LTB model (dashed curve).

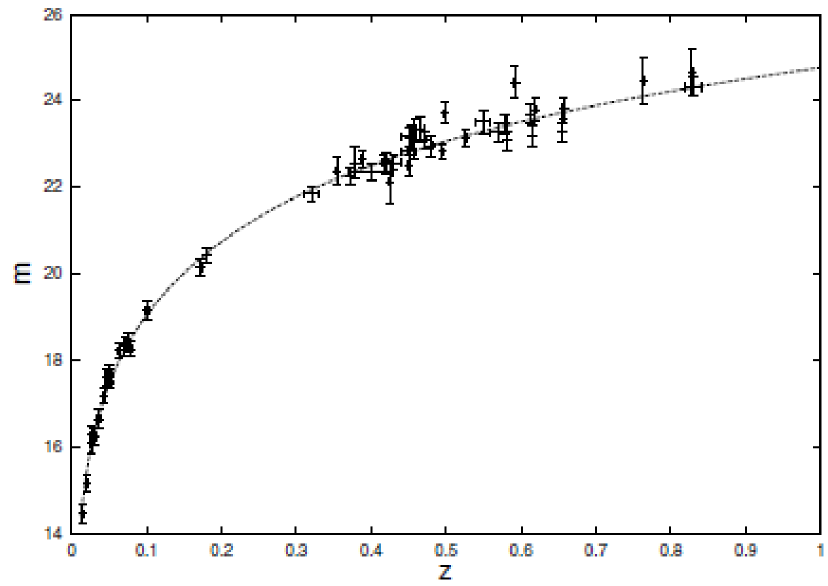


Figure 3. Plot of effective magnitude versus redshift for the $\Omega_M = 0.2$ LTB model (curve) and the supernova data.

However …

- Many LTB models contain a weak singularity at the center

  Vanderveld-Flanagan-Wesserman 2006

- We have more cosmological data than SN-Ia

- How to reconcile large scale structure formation
  without Dark Energy?

  If no Dark Energy, density perturbations would have grown too much

- How to confront with CMB spectrum?

  1st-peak of CMB power spectrum can be made to match WMAP observations

  e.g. Alnes-Amarzguioui-Groen 2006

  Garcia-Bellido – Haugboelle 2008

Conclusion

- Inhomogeneous models *can* mimic an “accelerated expansion” *without Dark Energy*
- Backreaction scenarios from perturbations and/or spatial averaging suffer from serious gauge ambiguities

- Anti-copernican models have attracted more attention

- Seems unlikely that all cosmological data can be explained by inhomogeneous models
- But the issue has not yet been settled

… noy yet definitively ruled out