QUANTUM ANOMALIES AND SUPERLUMINOUS PROPAGATION. IS IT POSSIBLE? IS IT DANGEROUS?

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Based on the work in progress with R. Akhouri and some very old papers of the previous century.

Drummond and Hathrell, 1980: radiative correction to light propagation in gravitational field (triangle graph).

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Famous diagram leading to quantum anomaly.

Similar one with one axial and two vector current creates chiral anomaly, i.e. axial current non-conservation for massless electrons, which is classicaly conserved.

Quantum correction to the coupling of two photons to graviton breaks conformal invariance:

$$T_\mu^\mu = \alpha\beta F_{\mu\nu}F^{\mu\nu}$$

Even more surprising, this diagram may break causality!

$$L_{eff} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + C R_{\alpha\beta\mu\nu} F_{\mu\nu} F^{\alpha\beta}$$

where $C = -\alpha/(360\pi\, m_e^2)$.

C **is divergent at** $m_e = 0$.

There are also $R F_{\mu\nu} F^{\mu\nu}$ **and** $R_{\alpha\beta} F^{\mu\alpha} F_{\mu}^{\beta}$ **terms, but they are absent in vacuum.**

$$D_\mu \left[F^\mu_\nu - 4C R^\mu_{\nu\alpha\beta} F^{\alpha\beta} \right] = 0$$

Characteristics of the Maxwell equations are modified so that one of light polarization propagates inside the cone, the other propagates outside the cone! Light propagates faster than light.

Quantum corrections to light propagation in external electric or magnetic field (Heisenberg-Euler): both polarizations are inside the light cone.
(S. Adler)
Heisenberg-Euler correction to refraction index vanishes at $\omega \to \infty$

DH-correction does not vanish at $\omega \to \infty$! It leads to $n(\infty) < 1$.

Group velocity is determined by $n(\omega)$ at some fixed ω and can be larger than c (anomalous dispersion).

Front velocity must be smaller than c, to this end is necessary: $n(\infty) \geq 1$

DH: the effective Lagrangian is valid only at low ω (we will see that it is not so) but dispersion relations leads to $n(\infty) < n(0)$ (we will see that it is not so too).

$$n(\omega) = n(0) + \frac{2\omega}{\pi} \int \frac{d\omega \, \mathcal{I}m \, n(\omega')}{\omega'(\omega' - \omega)}$$

Thus if $\mathcal{I}m \, n(\omega) \geq 0$, $n(\infty) < n(0)$.

However, $\mathcal{I}m\, n$ may be negative due to wave focusing in curved space-time:

$$\mathcal{I}m\, n = \lambda_s R_{1230}/2\omega^2$$

(AD, Khriplovich, 1983). In 2007 Shore made the same conclusion because of possible particle production but the effect seems to be negligible.

The DH correction to the refraction index is valid at any ω, because the $\gamma\gamma$-graviton vertex is a function of only three variables k_1^2, k_2^2 and q^2 and for on-mass-shell photons depends only on momentum transfer to gravitational field, Khriplovich, 1994.

The Heisenberg-Euler correction is 4-leg amplitude and drops with rising photon frequency, ω.

The photon-photon scattering or photon scattering in external electromagnetic field drops down with energy.

Possible higher order corrections in electromagnetic interactions : could higher powers of α be compensated by a rise of the amplitide with the photon energy, ω?

Negative, because of the known renormalizability of QED in external gravitational field.

Higher order in external gravitational field. **The amplitude F^2R^n might contain derivatives ∂F which gives some factors proportional to ω, but power counting and renormalizability demand that the amplitudes are normalized to photon energy and not to electron mass in complete analogy to photon scattering in electromagnetic field.**

Higher orders in virtual graviton exchange (quantum gravity) may contain terms of the type:

$$C_g \alpha (\partial F)(\partial F) R / m_{Pl}^2.$$

Constant C_g is small but at $\omega > m_{Pl}$ this correction can be large.
However, if there is renormalizable quantum gravity (super-strings, branes...), the corrections must be small.

Recent criticism:

1. Hollowood, Shore claimed: " ..novel non-analytical behavior due to vacuum polarization, which invalidates Kramers-Kronig dispersion relation." because of generic focusing of geodesics.

A new non-perturbative singularity in refraction index in the complex ω plane.

2. Dubovsky, Nicolis, Trincherini, Villadoro: classical theory does not admit superluminous propagation. Quantum field theory respects general principles of classical theory, if there are no quantum anomalies. "Anomalies arise due to UV effects and exhibit themselves as localized terms in position space" and cannot influence spacelike separated commutators.

AD, Zakharov, 1970: anomalies are equally infrared phenomenon and the anomalous amplitude is infrared singular, i.e. long-ranged. E.g. the vertex VVA has $1/q^2$-singularity. Imaginary part of the amplitude is $\sim m^2/q^4$ and vanishes for $m = 0$, but real part is $\sim 1/q^2 \neq 0$ if $m = 0$.

The same is known to be true for the energy-momentum tensor.

Imaginary part of the triangle diagram:

$$\mathcal{I}m A_{\alpha\beta\mu\nu} = -\frac{e^2}{8}\int d\tau_2 Tr\left\{(R_\mu\gamma_\nu + R_\nu\gamma_\mu)\right.$$
$$\left[\frac{1}{p_1k_1}(\not{p}_2 - m)\gamma_\beta(\not{p}_1 - \not{k}_1 + m)\gamma_\alpha(\not{p}_1 + m) + \right.$$
$$\left.\left.\frac{1}{p_1k_2}(\not{p}_2 - m)\gamma_\alpha(\not{p}_1 - \not{k}_2 + m)\gamma_\beta(\not{p}_1 + m)\right]\right\}$$

$$\mathcal{I}m\, C(q^2) = \frac{2\pi\alpha m^2}{q^4} \times$$

$$\left[6V - \left(3 - V^2\right) \, ln \left(\frac{1+V}{1-V} \right) \right],$$

where $V = \sqrt{1 - 4m^2/q^2}$.

$\mathcal{I}m\, C(q^2) = 0$ for $m = 0$ but

$\mathcal{R}e\, C(q^2) \neq 0$ for $m = 0$.

$$C(q^2) = \frac{1}{\pi}\int_{4m^2}^{\infty}\frac{dz}{z-q^2}\mathcal{I}m\, C(z)$$

Thus for small q^2:

$$C(0) = \frac{4\pi\alpha}{45m^2}.$$

Only acausal amplitude is purely anomalous, the other ones contain both anomalous and non-anomalous terms.

Is it possible to violate causality?

There are reference frames in which the signal, which is causal in some other frame, looks acausal

(AD, I. Novikov, 1998, disagreed by Shore).

Special relativity, ”normal” tachyons.

Tachyon is emitted at (t_1, x_1) and registered at (t_2, x_2):

$$t_2 - t_1 = (x_2 - x_1)/u > 0.$$

Here $u > 1$ is the tachyon velocity.

Causality is not violated.

Go to another frame with velocity V:

$$x' = \gamma(x - Vt), \quad t' = \gamma(t - Vx),$$

where $\gamma = 1/\sqrt{1 - V^2}$.
In the new frame:

$$t_2' - t_1' = \gamma(t_2 - t_1)(1 - Vu)$$

Thus $u' = \infty$ for $V = 1/u$,
infinite energy for $u = const > 1$??!!.
For $V > 1/u$, $t_2' - t_1' < 0$, i.e. tachyon propagates backward in time

Is it possible that in some reference frame the signal "returns" the the place of origin before it was emitted? YES! **but in more complicated gedanken experiment.**

Let us consider two tachyon sources, moving in opposite directions with velocity V. The first of them emits tachyon in direction of the other and at the moment when the second one received the signal it "shoot" back with another tachyon. If $V = 1/u$ tachyons propagate with infinite velocity and the signal returns instantly. If $V > 1/u$, the signal returns before it was emitted.

Light propagation in GR.

Metric, e.g. Schwatzschild:

$$ds^2 = a^2(r)dt^2 - b^2(r)(dx^2 + dy^2 + dz^2)$$

where $r^2 = x^2 + y^2 + z^2$, $a^2 = (1 - r_g/4r)^2/(1 + r_g/4r)^2$, and $b^2 = (1 + r_g/4r)^4$. The explicit form of a and b is not essential.

Change of coordinates: One of the spatial coordinate lines, l, is the trajectory of the superluminal photon in this metric and the coordinate running along this trajectory we denote as l. The other two, $x_\perp$, are orthogonal (in three-D sense). The metric along this trajectory, where $x_\perp = 0$, can be written in the form:

$$ds^2 = A^2(l)dt^2 - B^2(l)dl^2$$

New coordinate frame moving w.r.t. the original one with velocity V along l at large distances from the center. The corresponding coordinate transformation can be chosen as

$$t' = \gamma\,[t - Vl - V f(l)]$$
$$l' = \gamma\,[l + f(l) - Vt]$$

The function $f(l)$ is chosen in such a way so that the crossed terms $dt'dl'$ do not appear in the metric. It can be achieved if

$$f(l) = \int^{l} dl \left(\frac{B}{A} - 1\right)$$

In this moving frame the metric is:

$$ds^2 = A^2 \left(dt'^2 - dl'^2\right)$$

where A should be substituted as a function of l' and t'.

The motion of the tachyonic photon in the original frame satisfies the condition:

$$u = \frac{Bdl}{Adt} = 1 + \delta u$$

The overshoot of c:

$$\delta u = \frac{\alpha r_g \rho}{30 \pi r^4 m_e^2}$$

The effect may be large for small BHs with $r_g < 1/m_e$.

E.g. for $M_{BH} = 10^{14}$ g and $\tau \sim t_U$, $r_g \approx 1/\text{GeV}$.

Possible implications: BH evaporation and information loss?