Linear Analysis of Shock Instability in Core-collapse Supernovae: Effects of fluctuations from inside

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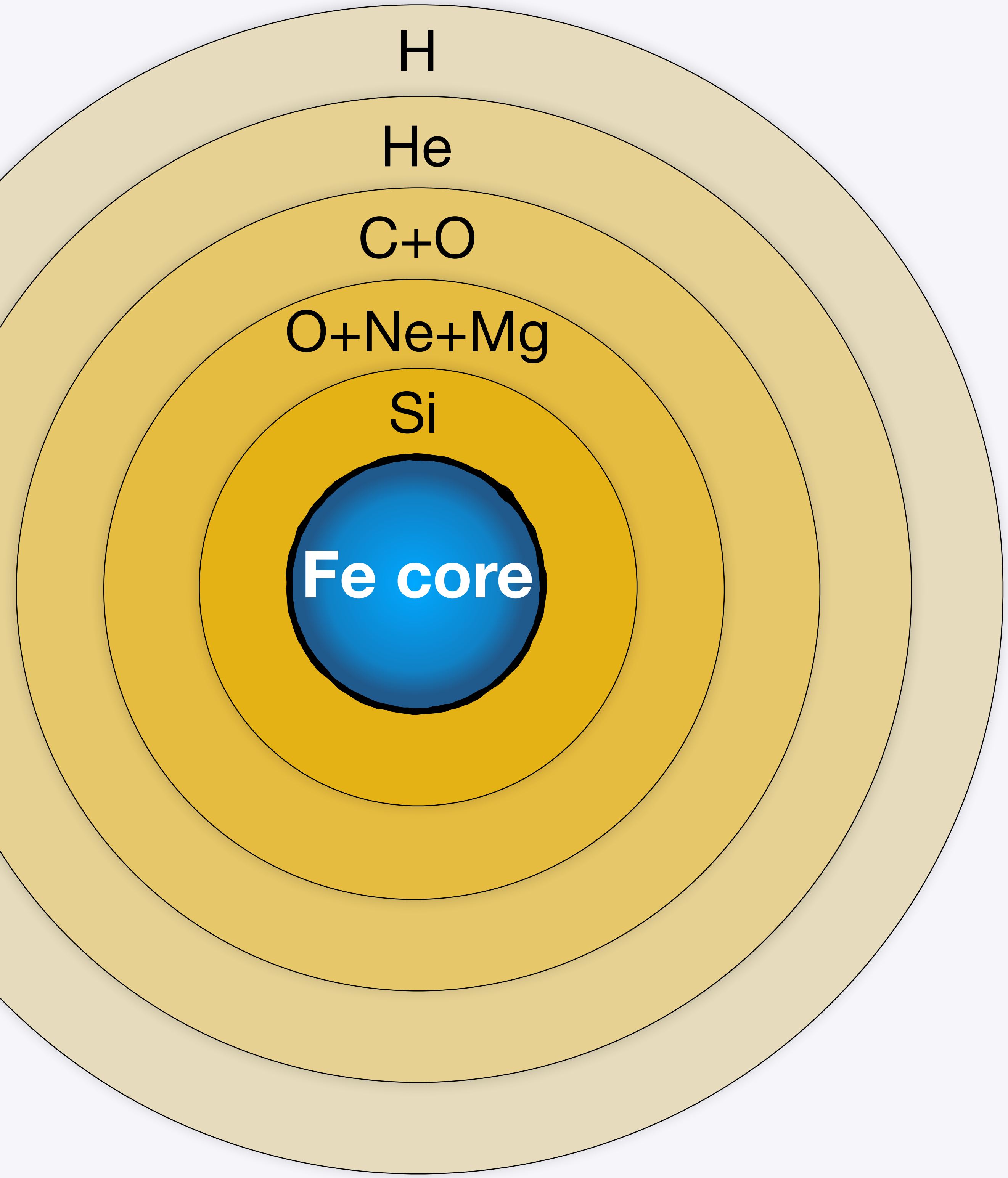
arXiv: 1903.00480

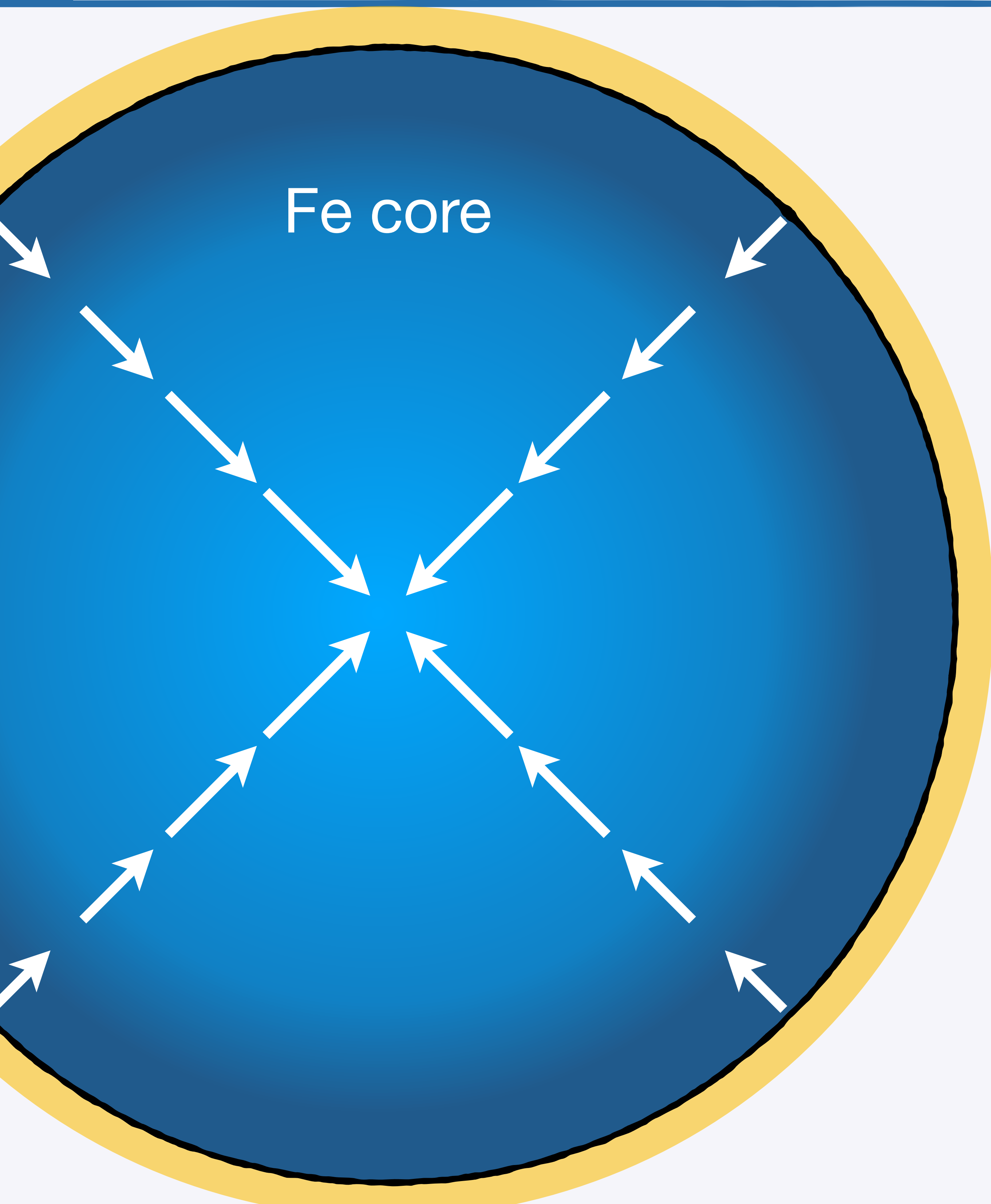
CCSNe scenario and neutrino heating mechanism



- **CCSNe scenario**
  Core-collapse of massive star
  →Core bounce + Formation of shock wave (SW)
  →Propagation of SW
- **Propagation of SW once stagnate.**
- **Neutrino heating mechanism:**
  Heating of accreting matter by emitted neutrinos from PNS

$$\nu_e + n \to p + e^-$$
$$\bar{\nu}_e + p \to n + e^+$$

CCSNe scenario and neutrino heating mechanism



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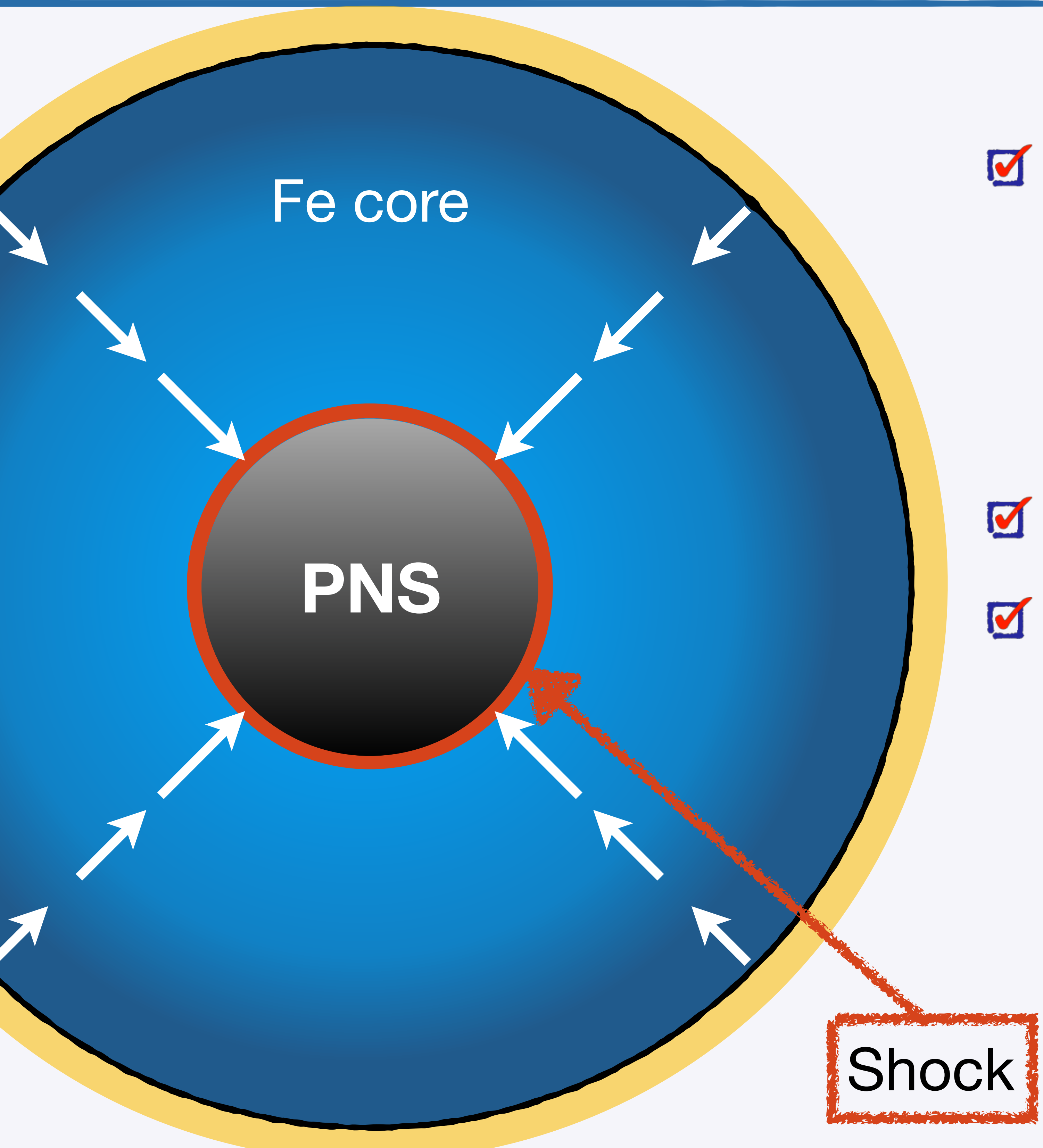

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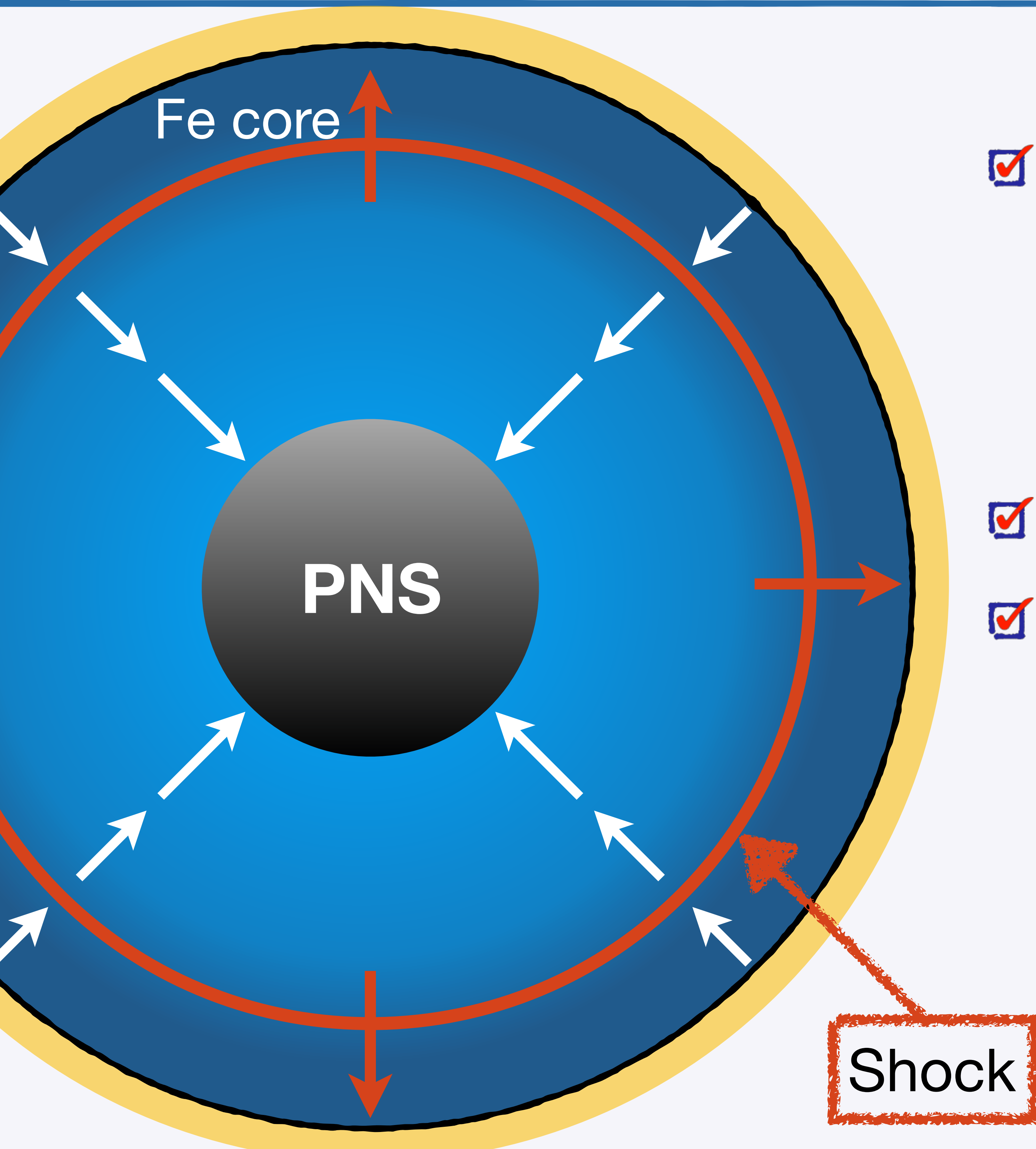

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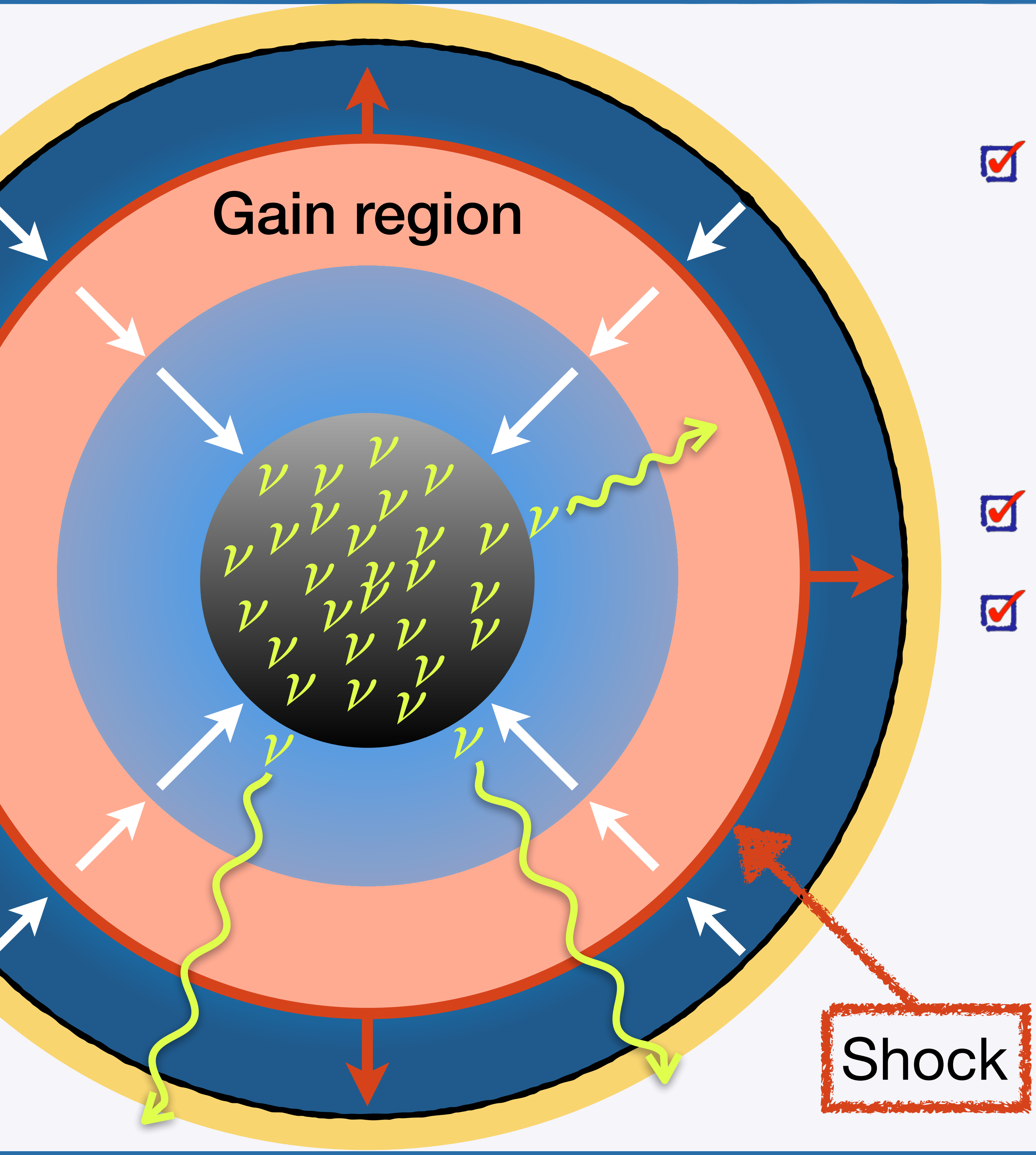

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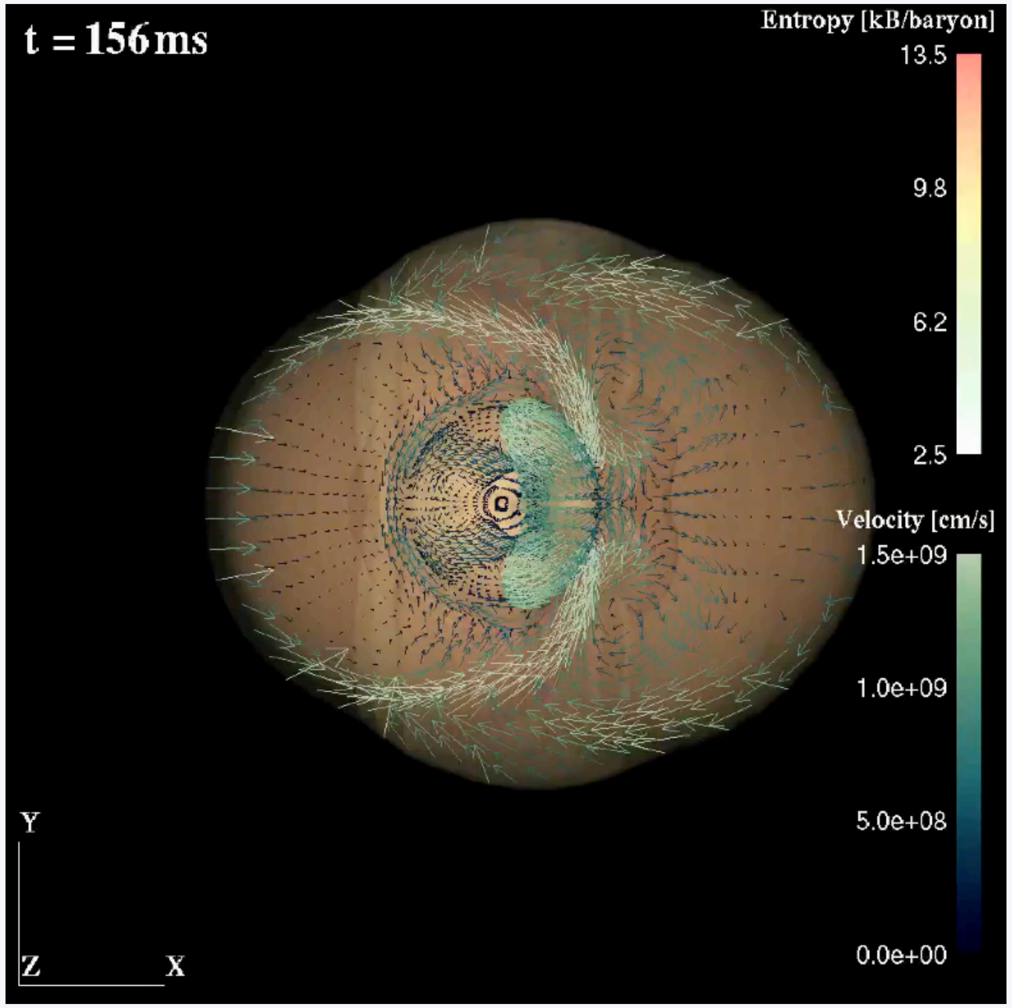
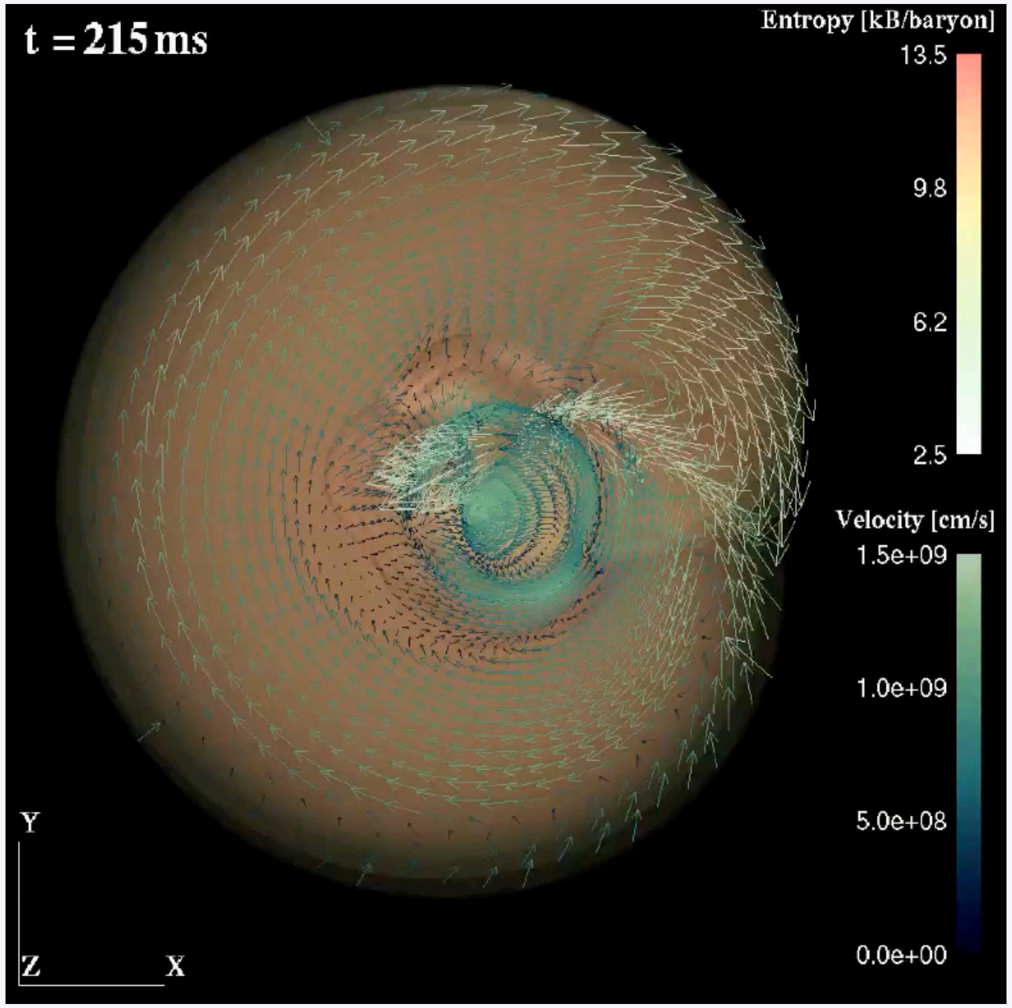
Deformation of SW:
Standing Accretion Shock Instability (SASI)

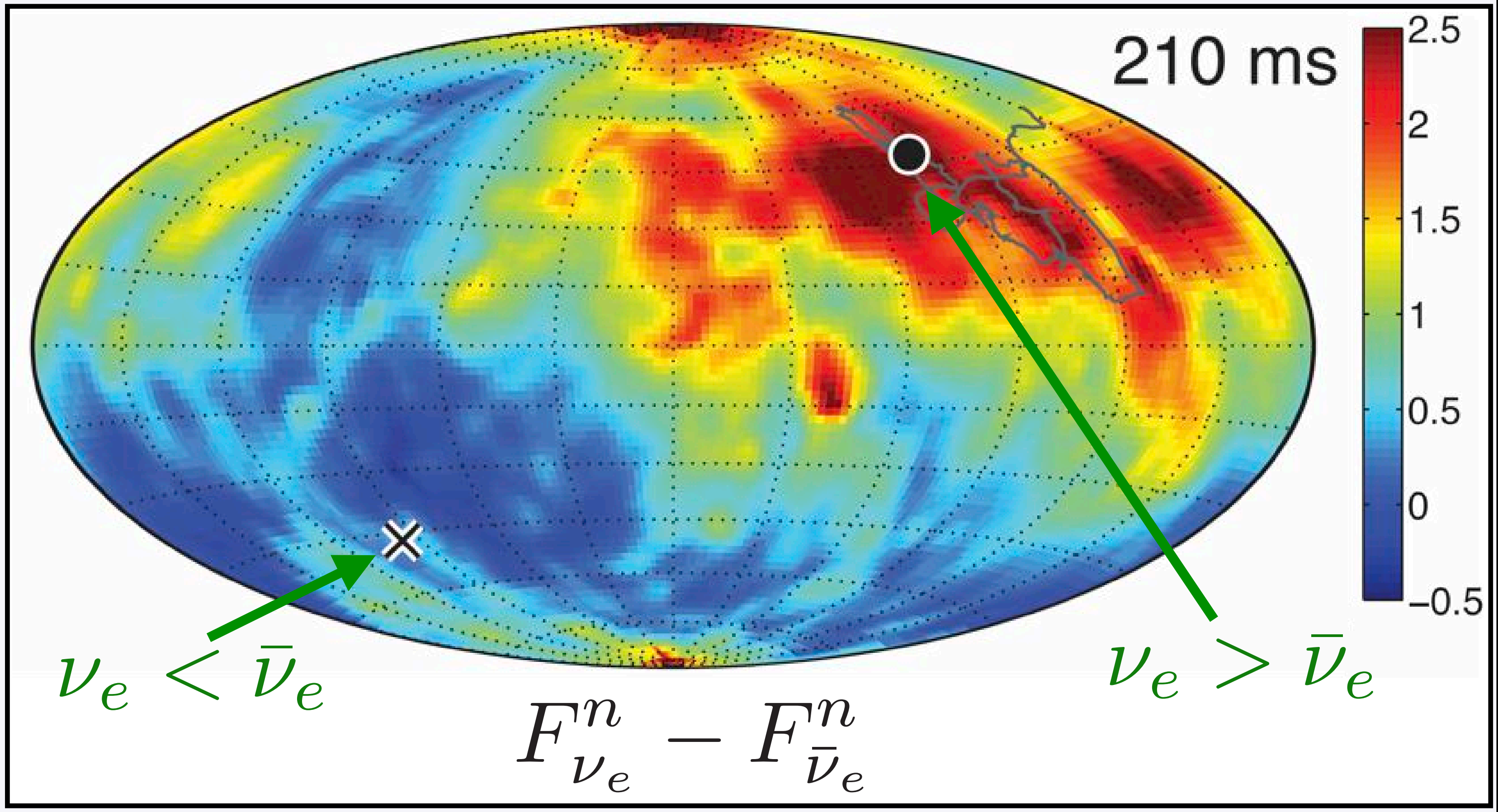




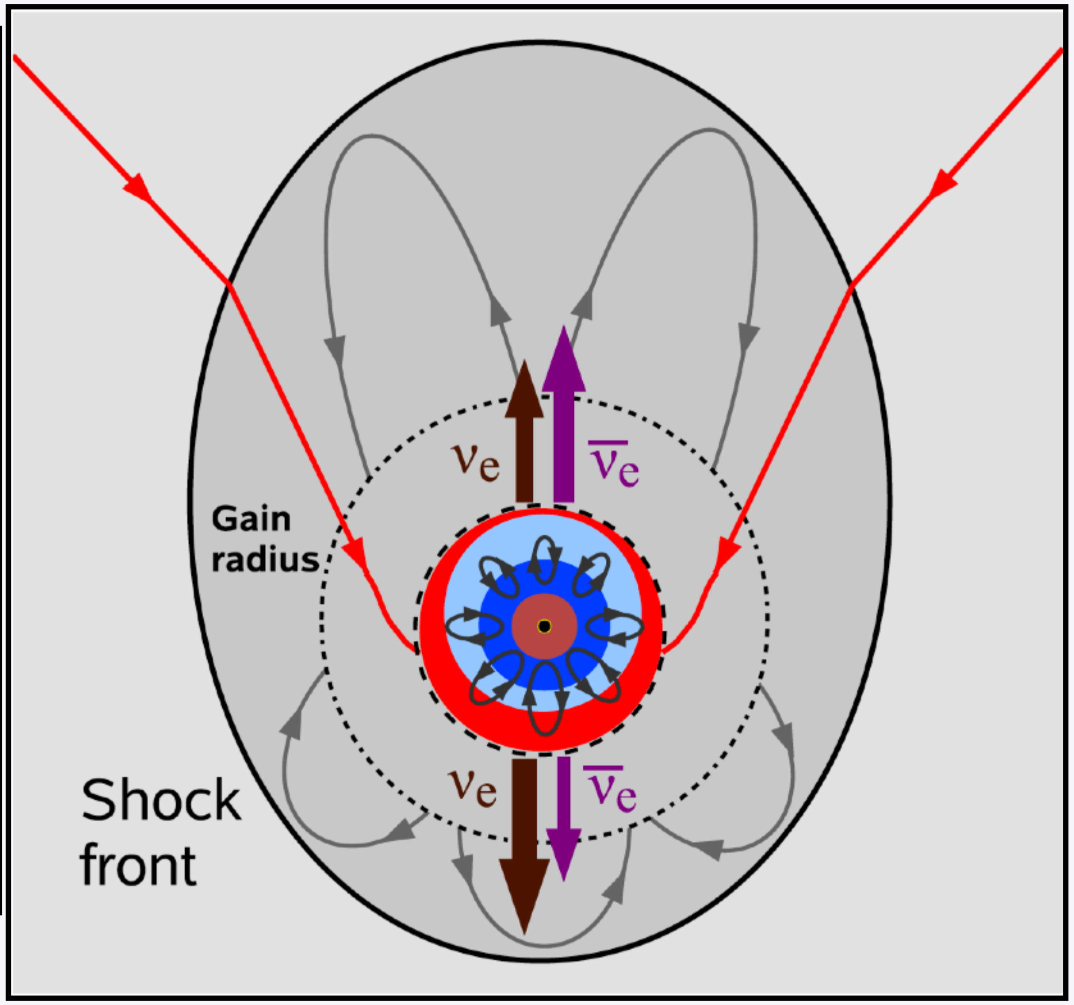
Iwakami et al. (2014)

- Instability of the spherically symmetric standing SW
- Induction of dipolar, quadrupolar deformation of the SW
- Turbulence montion are generated below the SW and turbulent pressure supports SW

Deformation of SW: LEpton-number Self-sustained Asymmetry (LESA)



Dipolar deformation of lepton-number flux



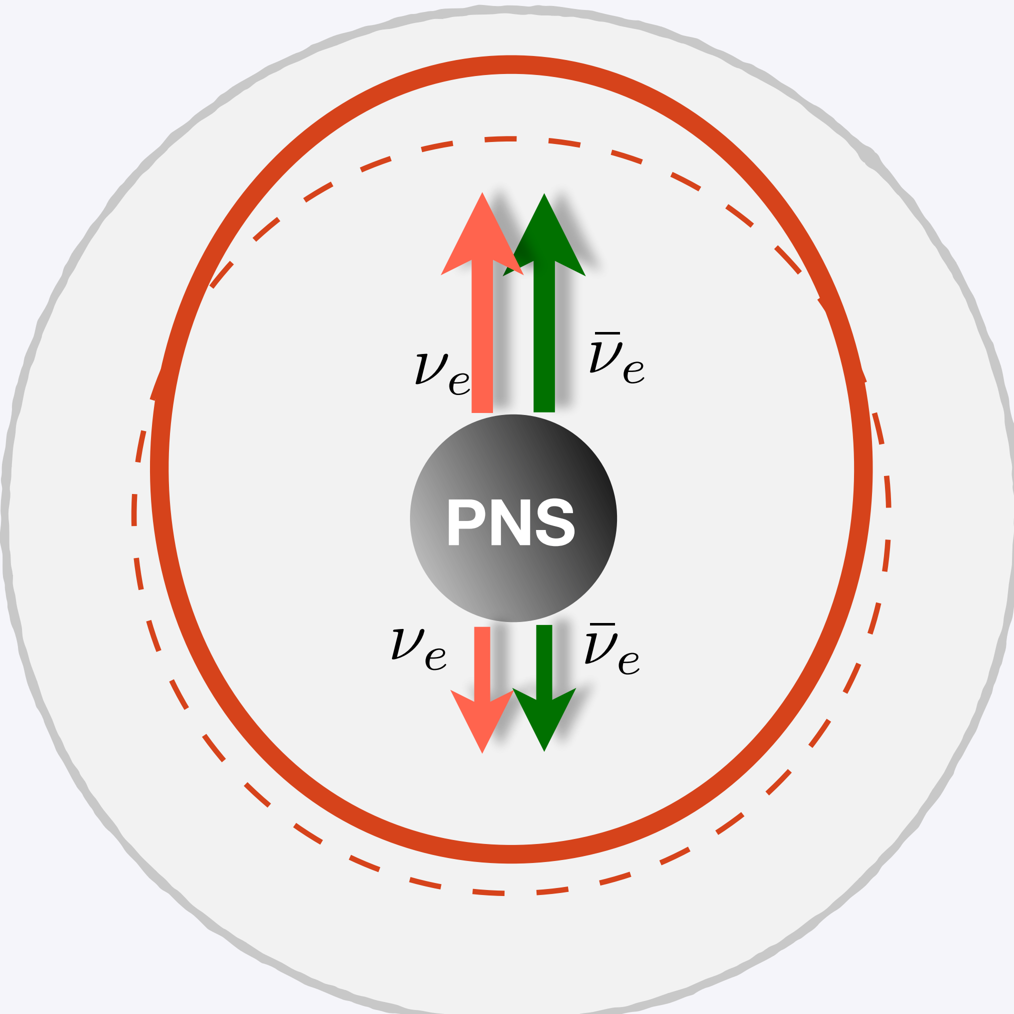
Tamborra et al. (2014)

- ☑ Deformation of SW accompanied by the dipolar deformation of the lepton-number flux distribution
- ☑ The deformation sustained for long time (~ a few hundreds ms).
- ☑ $\langle \epsilon_{\nu_e} \rangle < \langle \epsilon_{\bar{\nu}_e} \rangle \Rightarrow$ Stronger neutrino heating occur in one hemisphere.

Deformation of SW: LEpton-number Self-sustained Asymmetry (LESA)

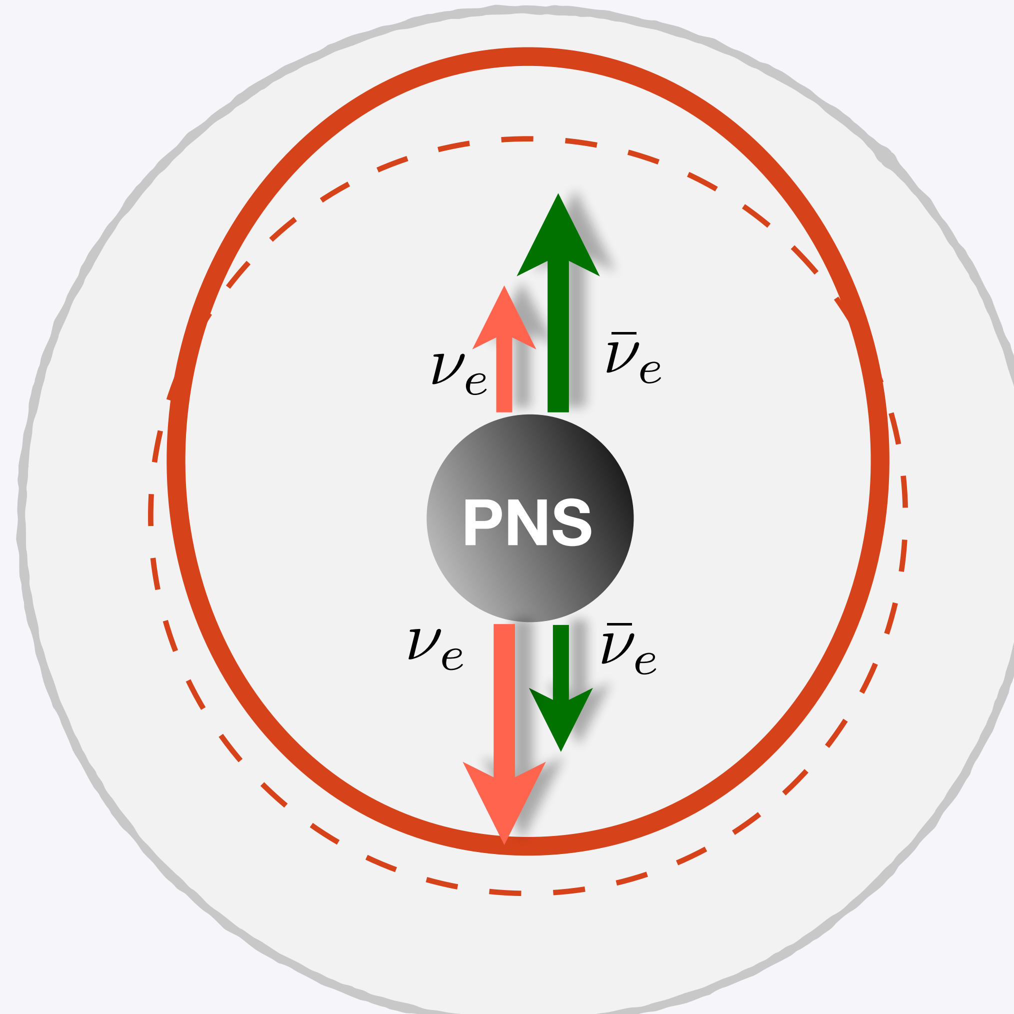
Deformation of the SW is related to …?

$$\delta r_{\rm sh} \leftrightarrow F_{\nu_e} + F_{\bar{\nu}_e}$$



Dolence et al. (2015)

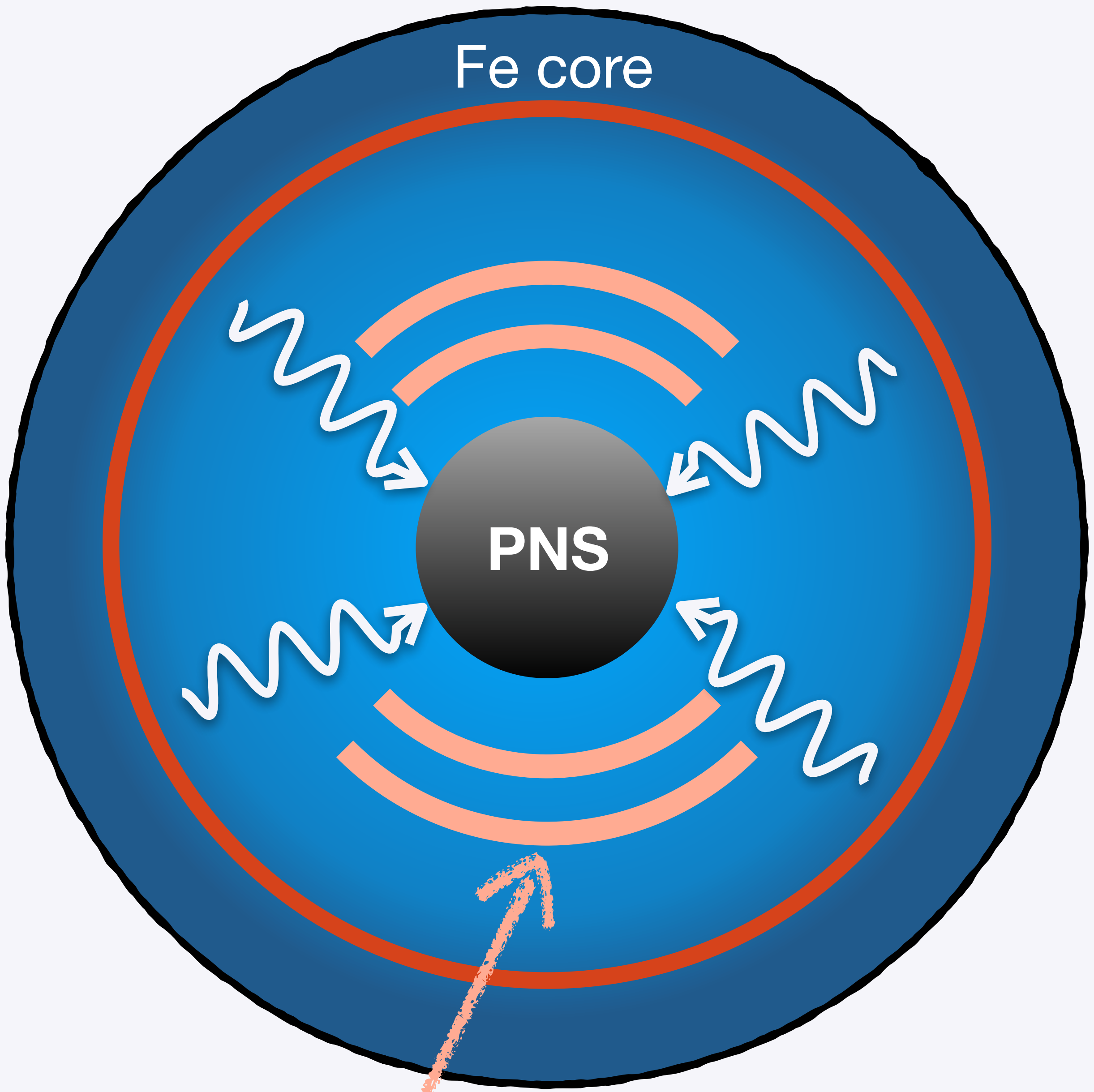
$$\delta r_{\rm sh} \leftrightarrow F^n_{\nu_e} - F^n_{\bar{\nu}_e}$$



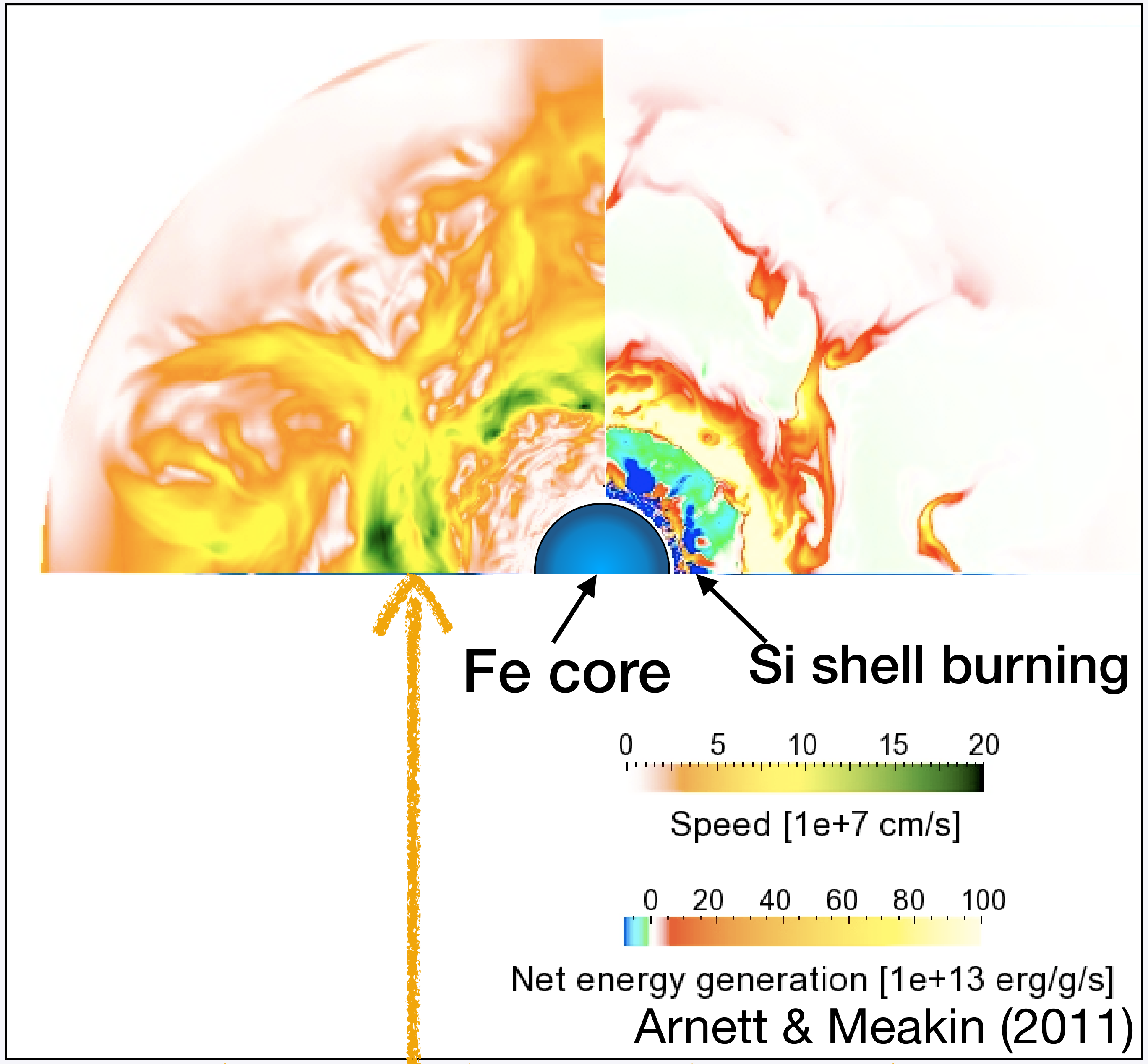
LESA: Tamborra et al. (2014)

Other multi-dimensional effects

- Acoustic injection from PNS
- Turbulence in pre-shock matter



Acoustic injection



Convection due to nuclear shell burning

Sorting of multi-dimensional effects

Dynamics of SW deformation

- ☑ SASI: Instability of SW
- ☑ LESA: Sustaining of SW deformation

Extrinsic factors of SW deformation

- ☑ Turbulence in pre-shock layer
- ☑ Acoustic injection from PNS
- ☑ Fluctuations of neutrino radiation

Sorting of multi-dimensional effects

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Linear analysis of SW deformation

Time-dependent boundary conditions

Sorting of multi-dimensional effects

Dynamics of SW deformation

- SASI: Instability of SW

Eigenmodes of deformation and its instabilities

- LESA:
  Sustaining of SW deformation

Steady solution of perturbation equation

Extrinsic factor of SW deformation

- Turbulence in pre-shock layer

Outer boundary condition

Takahashi et al. (2016)

- Acoustic injection from PNS
- Fluctuations of neutrino radiation

Inner boundary condition

Linear analysis of SW deformation

Time-dependent boundary conditions

Purpose of this study

Linear analysis of spherically symmetric steady accretion flow with standing shock

Model 1: Analysis of SASI

Investigation of influences of inner boundary conditions to the instability of standing shock

Model	Acoustic injection	Fluctuations of neutrino luminosity
A	no	no
B	yes	no
C	yes	yes

Model 2: Steady solution of perturbation equation

Are there any structures that SW deformation is sustained by fluctuation of the neutrino luminosity?

$$\delta r_{\rm sh} \leftrightarrow F_{\nu_e} + F_{\bar{\nu}_e} \text{ or } \delta r_{\rm sh} \leftrightarrow F^n_{\nu_e} - F^n_{\bar{\nu}_e} \text{ ?}$$

Method

Basic equations → Linear perturbation around background → **Linearized equation (Initial-boundary value problem)** → Laplace transform → **Linearized equation (Boundary value problem)**

Conservation of mass

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v}) = 0$$

Conservation of momentum

$$\frac{\partial}{\partial t}(\rho \boldsymbol{v}) + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v}\boldsymbol{v} + P\boldsymbol{I}) = -\rho \frac{GM}{r^2}\frac{\boldsymbol{r}}{r}$$

Conservation of energy

$$\frac{d\varepsilon}{dt} + P\frac{d}{dt}\left(\frac{1}{\rho}\right) = q(T_\nu)$$

Conservation of electron number

$$\frac{\partial}{\partial t}(nY_e) + \boldsymbol{\nabla} \cdot (nY_e \boldsymbol{v}) = \lambda(T_\nu)$$

($Y_e := n_e/n_B$: electron fraction)

Neutrino reactions

$$\nu_e + n \leftrightarrows e^- + p$$

$$\bar{\nu}_e + p \leftrightarrows e^+ + n$$

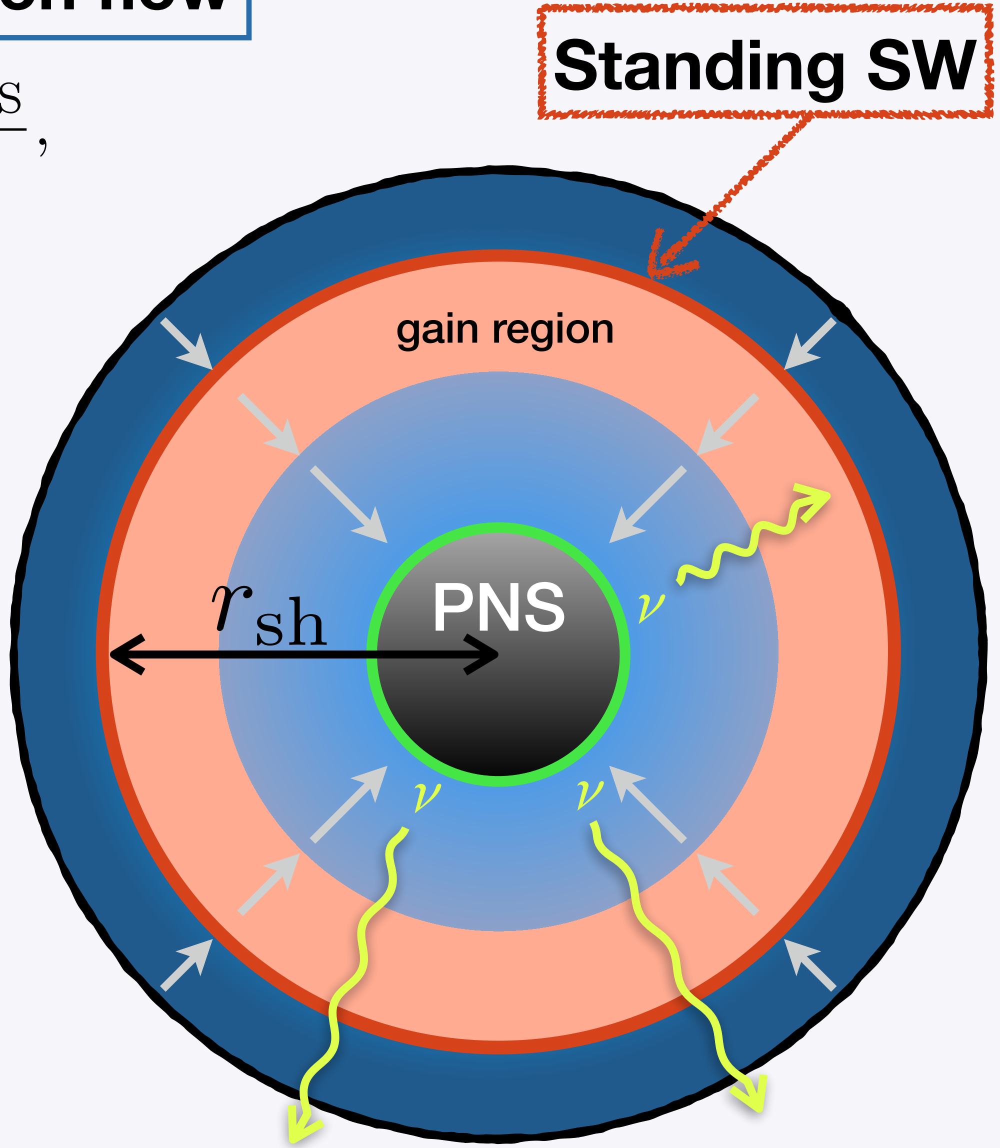
Method

Basic equations → Linear perturbation around background → **Linearized equation (Initial-boundary value problem)** → Laplace transform → **Linearized equation (Boundary value problem)**

Spherically symmetric, steady shocked accretion flow

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(\rho_0 v_{r0} r^2\right) = 0, \quad v_{r0}\frac{\mathrm{d}v_{r0}}{\mathrm{d}r} + \frac{1}{\rho_0}\frac{\mathrm{d}P_0}{\mathrm{d}r} = -\rho_0\frac{GM_{\mathrm{PNS}}}{r^2},$$

$$v_{r0}\frac{\mathrm{d}\varepsilon_0}{\mathrm{d}r} - \frac{P_0 v_{r0}}{{\rho_0}^2}\frac{\mathrm{d}\rho_0}{\mathrm{d}r} = q_0, \quad \rho_0 v_{r0}\frac{\mathrm{d}Y_{e0}}{\mathrm{d}r} = \lambda_0 m_b$$



Accreting matter: Free falling matter

$(S = 3k_B, Y_e = 0.5)$

Accretion rate $\dot{M} = 0.6 M_\odot/\mathrm{s}$

Outer b.c. (at SW): Rankine-Hugoniot condition

Inner b.c. (at PNS surface): $\rho = 10^{11}\mathrm{g/cm^3}$

$T_{\nu_e} = T_{\bar{\nu}_e} = 4.5\mathrm{MeV}$

Parameter: Neutrino luminosity $L_{\nu_e}, L_{\bar{\nu}_e}$

Method

Basic equations → Linear perturbation around background → **Linearized equation (Initial-boundary value problem)** → Laplace transform → **Linearized equation (Boundary value problem)**

Scalar variables

$$X(\boldsymbol{r},t) = X_0(r) + \delta X(\boldsymbol{r},t)$$

SW deform.

$$r_{\rm sh}(\theta,\phi,t) = r_{\rm sh} + \delta r_{\rm sh}(\theta,\phi,t)$$

Fluctuations of L_ν

$$T_\nu(\theta,\phi,t) = T_\nu + \delta T_\nu(\theta,\phi,t)$$

Velocity

$$\boldsymbol{v}(\boldsymbol{r},t) = v_{r0}\boldsymbol{e}_r + \delta\boldsymbol{v}(\boldsymbol{r},t)$$

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$$X(\boldsymbol{r},t) = X_0(r) + \delta X(\boldsymbol{r},t)$$

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Fluctuations of L_ν

$$T_\nu(\theta,\phi,t) = T_\nu + \delta T_\nu(\theta,\phi,t)$$

$$\delta X(\boldsymbol{r},t) = \sum_{l,m} \delta X^{(l,m)}(r,t) Y_{lm}(\theta,\phi)$$

$$\delta r_{\rm sh}(\theta,\phi,t) = \sum_{l,m} \delta r_{\rm sh}^{(l,m)}(t) Y_{lm}(\theta,\phi)$$

$$\delta T_\nu(\theta,\phi,t) = \sum_{l,m} \delta T_\nu^{(l,m)}(t) Y_{lm}(\theta,\phi)$$

Spherical harmonics expansion

Velocity

$$\boldsymbol{v}(\boldsymbol{r},t) = v_{r0}\boldsymbol{e}_r + \delta\boldsymbol{v}(\boldsymbol{r},t)$$

Vector spherical harmonics expansion

$$\delta\boldsymbol{v}(\boldsymbol{r},t) = \sum_{l,m} \delta v_r^{(l,m)}(r,t) Y_{lm}(\theta,\phi)\hat{\boldsymbol{r}} + \delta v_\perp^{(l,m)}(r,t)\left[\hat{\boldsymbol{\theta}}\frac{\partial Y_{lm}}{\partial\theta} + \frac{\hat{\boldsymbol{\phi}}}{\sin\theta}\frac{\partial Y_{lm}}{\partial\phi}\right] + \delta v_{rot}^{(l,m)}(r,t)\left[-\hat{\boldsymbol{\phi}}\frac{\partial Y_{lm}}{\partial\theta} + \frac{\hat{\boldsymbol{\theta}}}{\sin\theta}\frac{\partial Y_{lm}}{\partial\phi}\right]$$

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SW deform.

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$$\delta T_\nu(\theta,\phi,t) = \sum_{l,m} \delta T_\nu^{(l,m)}(t) Y_{lm}(\theta,\phi)$$

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Method

Basic equations → Linear perturbation around background → **Linearized equation (Initial-boundary value problem)** → Laplace transform → **Linearized equation (Boundary value problem)**

Linearized equations

$$A(r)\frac{\partial \boldsymbol{y}^{(l,m)}}{\partial t}(r,t) + \frac{\partial \boldsymbol{y}^{(l,m)}}{\partial r}(r,t) = B(r)\boldsymbol{y}^{(l,m)}(r,t) + \boldsymbol{u}(r)\delta T_\nu^{(l,m)}(t)$$

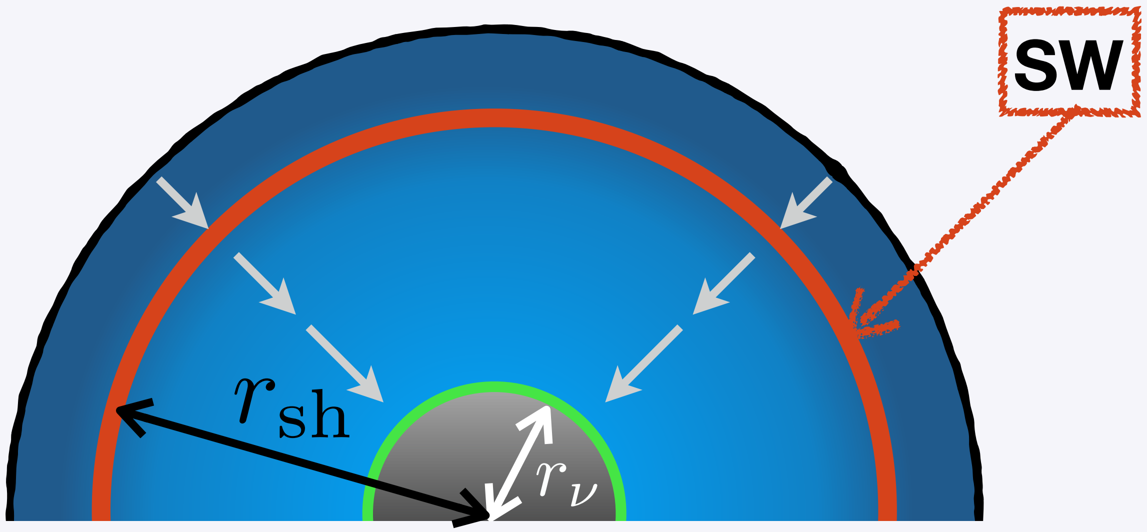
Determined by background ($A(r)$, $B(r)$, $\boldsymbol{u}(r)$)

$$\boldsymbol{y}(r,t) = \left(\frac{\delta\rho}{\rho_0}, \frac{\delta v_r}{v_{r0}}, \frac{\delta v_\perp}{v_{r0}}, \frac{\delta\varepsilon}{\varepsilon_0}, \frac{\delta Y_e}{Y_{e0}}, \frac{\delta v_{\rm rot}}{v_{r0}}\right)^T \quad \boldsymbol{u} = \left(0, 0, 0, \frac{1}{v_{r0}}\frac{\partial q}{\partial T_\nu}, \frac{m_b}{\rho_0 Y_{e0}}\frac{\partial \lambda}{\partial T_\nu}, 0\right)^T$$

Boundary conditions

Outer b.c.: SW front $(r = r_{\rm sh})$

Inner b.c.: PNS surface $(r = r_\nu)$



Method

Basic equations → Linear perturbation around background → **Linearized equation (Initial-boundary value problem)** → Laplace transform → **Linearized equation (Boundary value problem)**

Laplace transformed linearized equations

$$\frac{\partial \boldsymbol{y}^{*(l,m)}}{\partial r}(r,s) = \left(sA(r) + B(r)\right)\boldsymbol{y}^{*(l,m)}(r,s) + A(r)\boldsymbol{y}_0^{(l,m)}(r) + \boldsymbol{u}(r)\delta T_\nu^{*(l,m)}(s)$$

$$\boldsymbol{y}^*(r,s) = \left(\frac{\delta\rho^*}{\delta\rho_0}, \frac{\delta v_r^*}{\delta v_{r0}}, \frac{\delta v_\perp^*}{\delta v_{r0}}, \frac{\delta\varepsilon^*}{\delta\varepsilon_0}, \frac{\delta Y_e^*}{\delta Y_{e0}}, \frac{\delta v_{\mathrm{rot}}^*}{\delta v_{r0}}\right)^T$$

Laplace transform $f^*(s) := \int_0^\infty f(t)\mathrm{e}^{-st}dt \quad (s \in \mathbb{C})$

Boundary conditions

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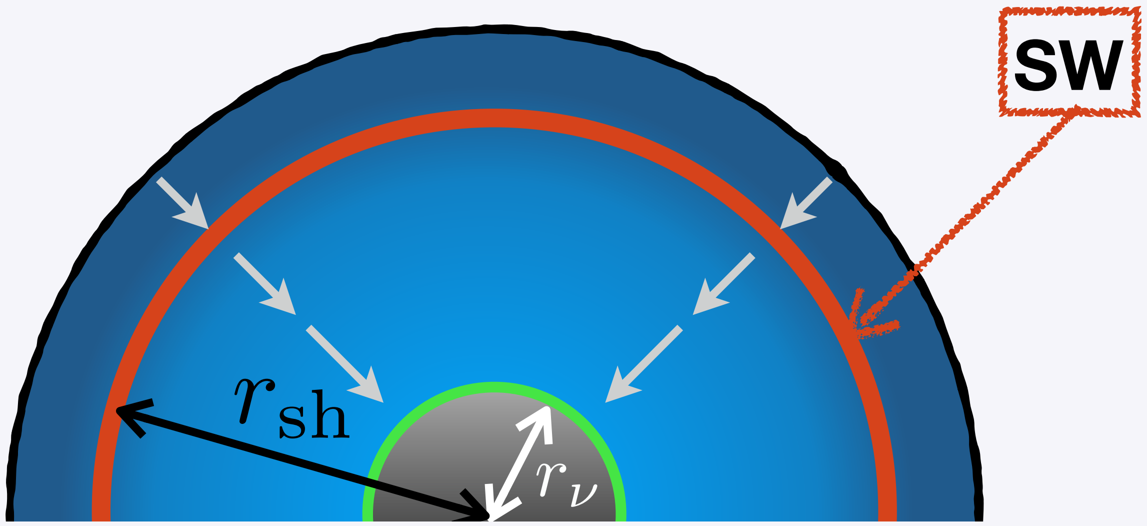

Boundary conditions

Model 1: Analysis of SASI

Outer b.c. ($r = r_{\rm sh}$) :
Linearized Rankine-Hugoniot cond.

$$\boldsymbol{y}^*(r_{sh}, s) = (s\boldsymbol{c} + \boldsymbol{d})\frac{\delta r_{sh}^*}{r_{sh}}(s) + R\boldsymbol{z}^*(r_{sh}, s)$$

Inner b.c. ($r = r_\nu$) :

☑ **Acoustic injection**

$$\frac{\delta p}{v_{r0} c_s \rho_0} + \frac{\delta v_r}{v_{r0}} = \sin\left(\omega_{\rm PNS}\, t\right)$$

Outgoing acoustic mode

☑ **Fluctuations of neutrino temp.**

$$\left(\frac{\partial P}{\partial Y_e}\right)_{\rho,T} \delta Y_e(r_{\nu_e}, t) + \left(\frac{\partial P}{\partial T}\right)_{\rho,Y_e} \delta T_\nu(t) = 0$$

Model 2: Steady sol. of perturbed eq.

Outer b.c. ($r = r_{\rm sh}$) :
Linearized Rankine-Hugoniot cond.

$$\boldsymbol{y}^*(r_{\rm sh}, s) = (s\boldsymbol{c} + \boldsymbol{d})\frac{\delta r_{\rm sh}^*}{r_{\rm sh}}(s)$$

Inner b.c. ($r = r_\nu$) :

☑ **Fluctuations of neutrino luminosity**

$$\frac{\delta L_{\nu_e}}{L_0} = 4\frac{\delta T_{\nu_e}}{T_{\nu_e 0}} + c_{Y_e}\frac{\delta Y_e}{Y_{e0}}$$

$$\frac{\delta L_{\bar{\nu}_e}}{L_0} = 4\frac{\delta T_{\bar{\nu}_e}}{T_{\bar{\nu}_e 0}} - c_{Y_e}\frac{\delta Y_e}{Y_{e0}}$$

<u>We consider 2 cases.</u>

$$\begin{cases} c_{Y_e} = 0 & (\Leftrightarrow \delta L_{\nu_e} - \delta L_{\bar{\nu}_e} = 0) \\ c_{Y_e} = 3.5 & (\Leftrightarrow \delta L_{\nu_e} - \delta L_{\bar{\nu}_e} \neq 0) \end{cases}$$

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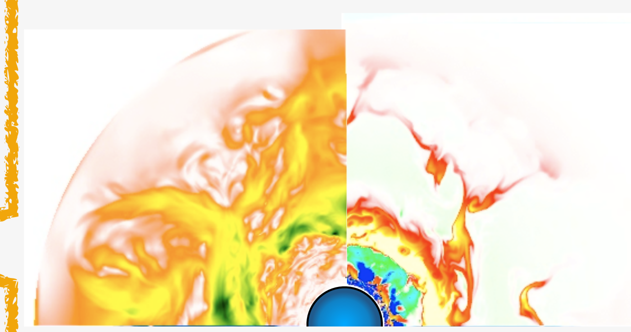
Turbulence in pre-shock layer



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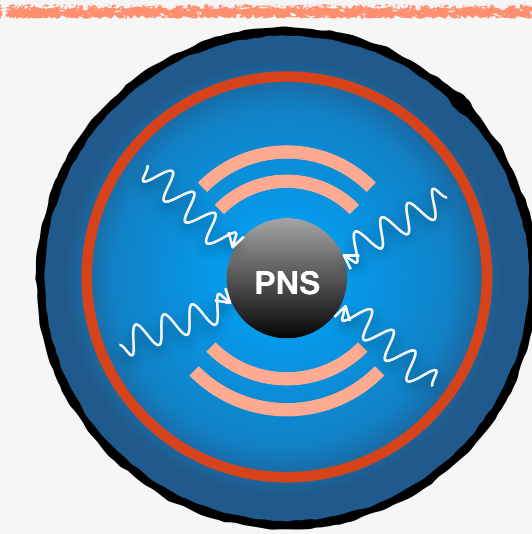
Outgoing acoustic mode



☑ **Fluctuations of neutrino temp.**

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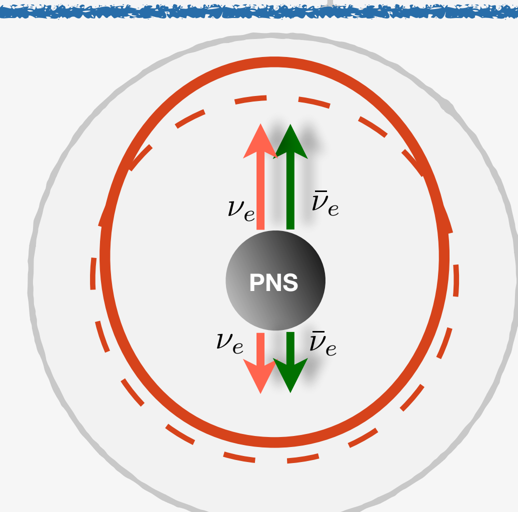


Boundary conditions

Model 1: Analysis of SASI

Outer b.c. ($r = r$ ...
Linearized Rankin...

$y^*(r_{sh}, s) = (s\boldsymbol{c} + \boldsymbol{d})\ \frac{\ldots}{\ldots_{sh}}(s) + Rz^*(r_{sh}, s)$

No turbulence
in pre-shock layer

Inner b.c. (...

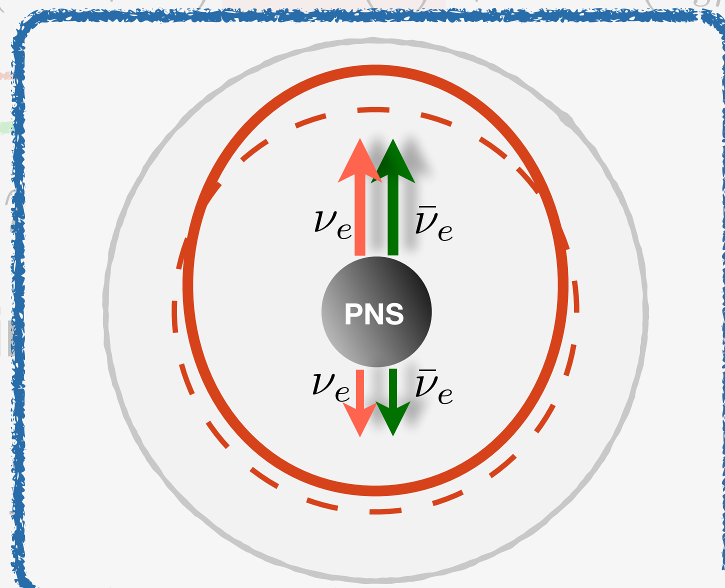
☑ Acoustic i...

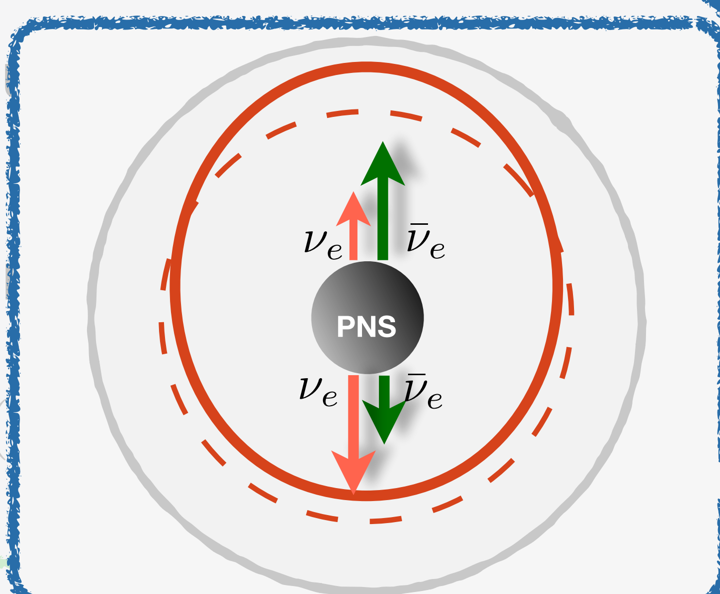
$\frac{\delta p}{v_{r0} c_s \rho_0} + \ldots$

Outgoing aco...

☑ Fluctuatio...

$\left(\frac{\partial P}{\partial Y_e}\right)_{\rho,T} \delta Y_e(\ldots \ldots = 0$





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Inner b.c. ($r = r_\nu$) :

☑ **Fluctuations of neutrino luminosity**

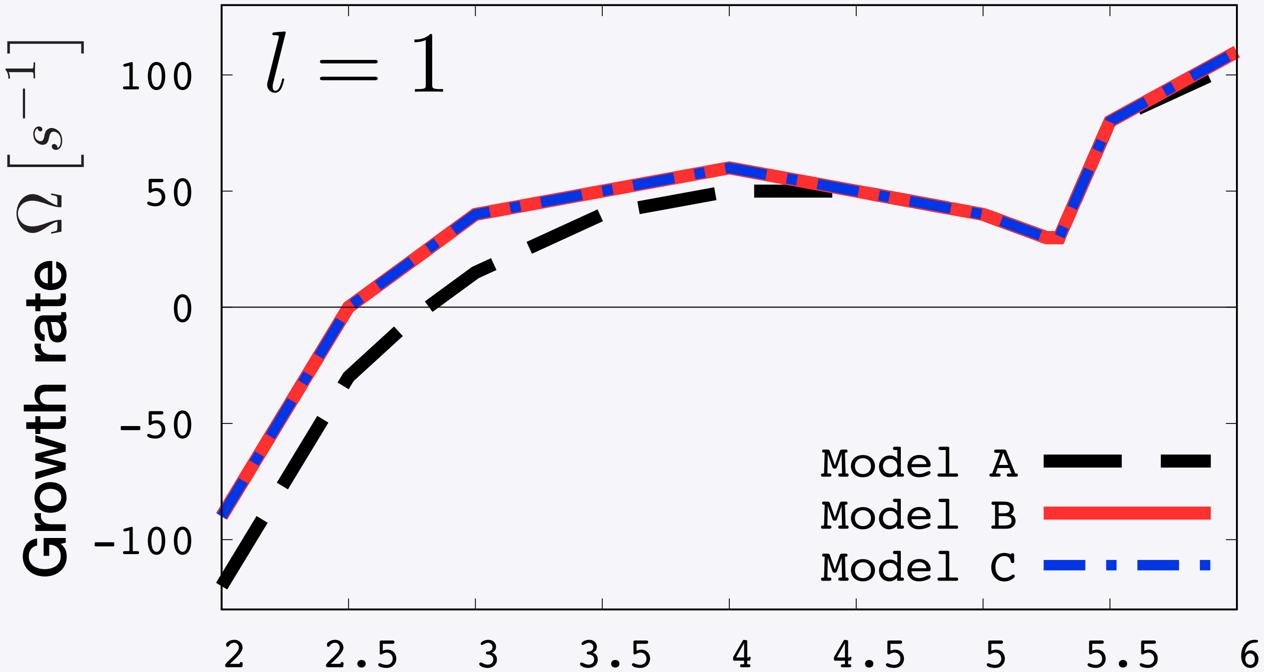
$$\frac{\delta L_{\nu_e}}{L_0} = 4\frac{\delta T_{\nu_e}}{T_{\nu_e 0}} + c_{Y_e}\frac{\delta Y_e}{Y_{e0}}$$

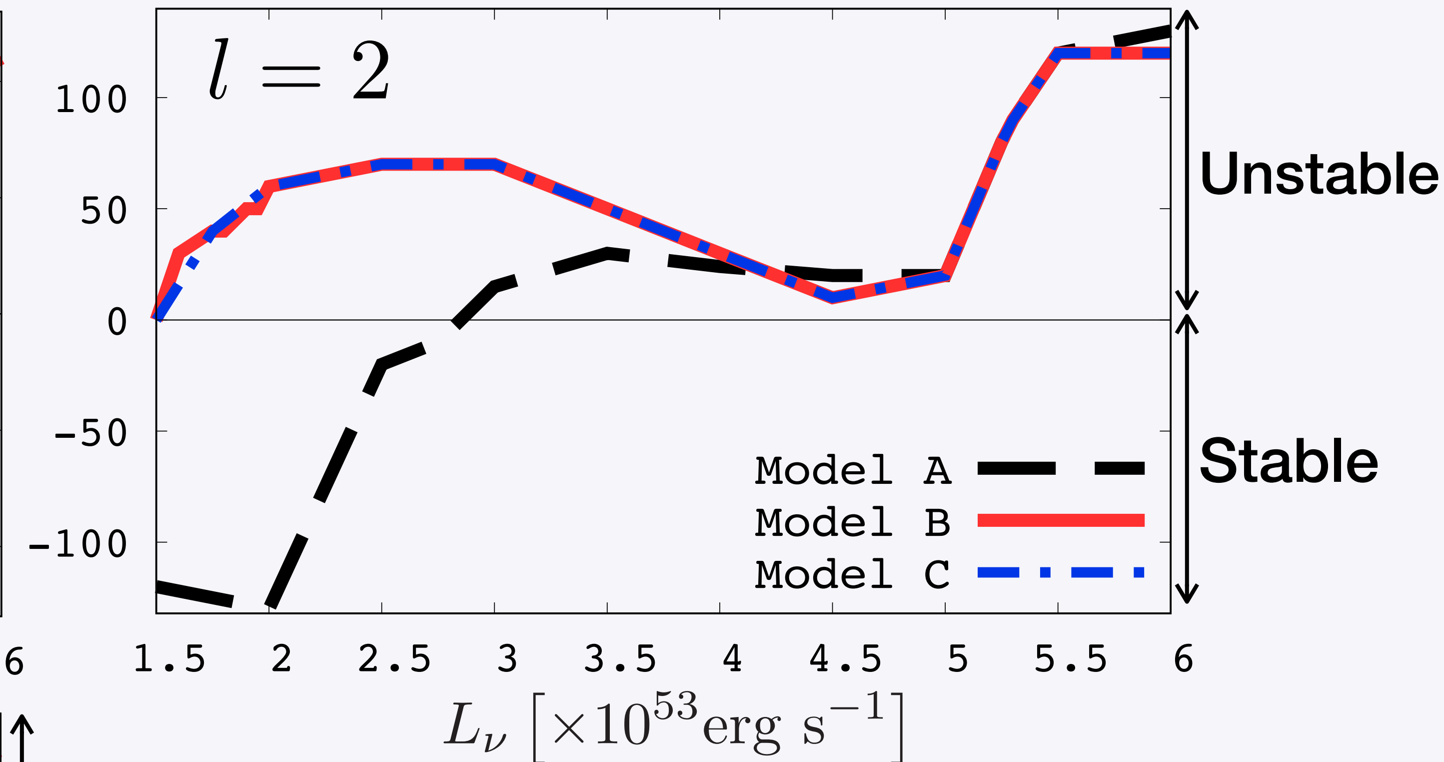
$$\frac{\delta L_{\bar{\nu}_e}}{L_0} = 4\frac{\delta T_{\bar{\nu}_e}}{T_{\bar{\nu}_e 0}} - c_{Y_e}\frac{\delta Y_e}{Y_{e0}}$$

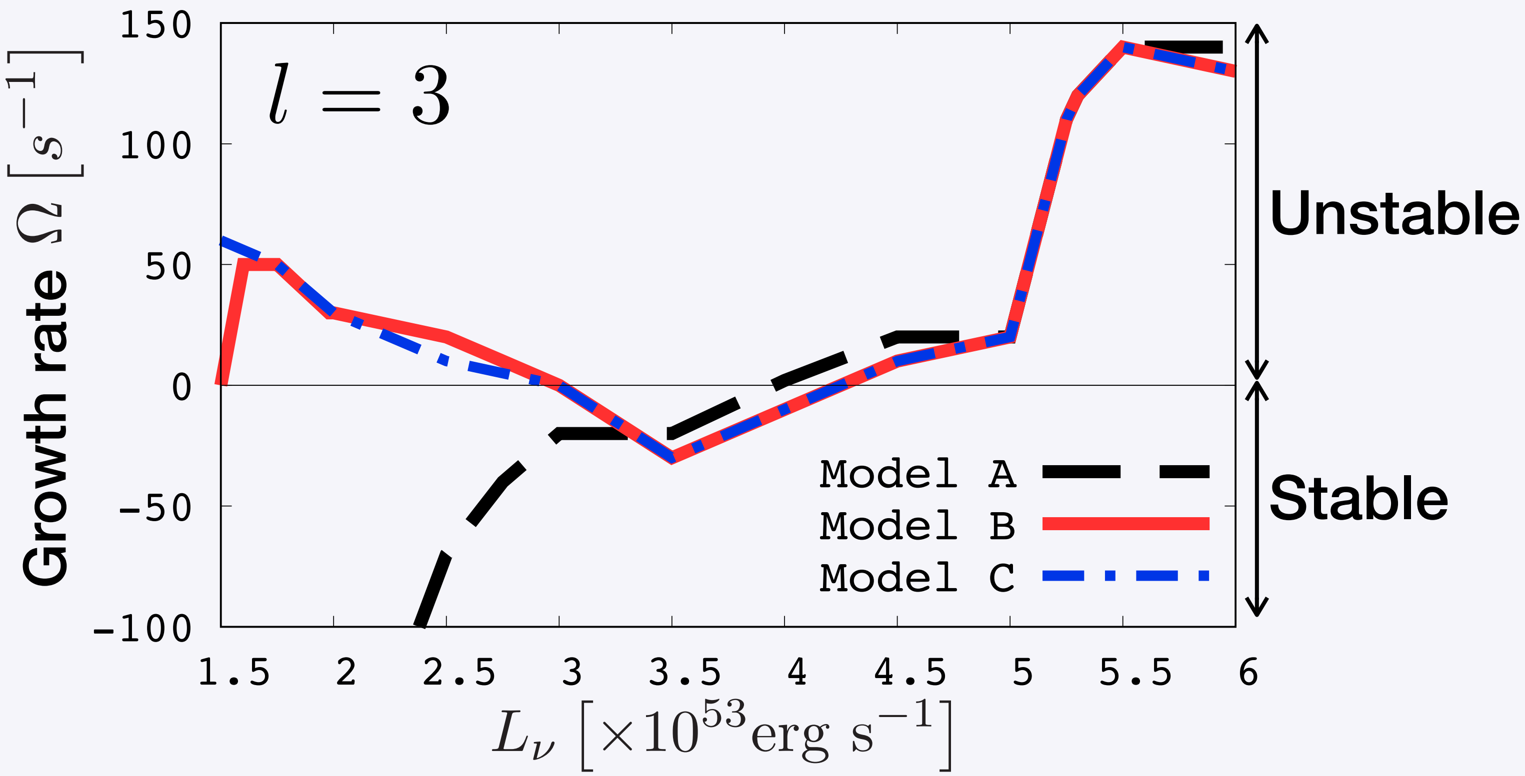
<u>We consider 2 cases.</u>

$$\begin{cases} c_{Y_e} = 0 \quad (\Leftrightarrow \delta L_{\nu_e} - \delta L_{\bar{\nu}_e} = 0) \\ c_{Y_e} = 3.5 \quad (\Leftrightarrow \delta L_{\nu_e} - \delta L_{\bar{\nu}_e} \neq 0) \end{cases}$$

Model 1: Growth rates of SW deformation





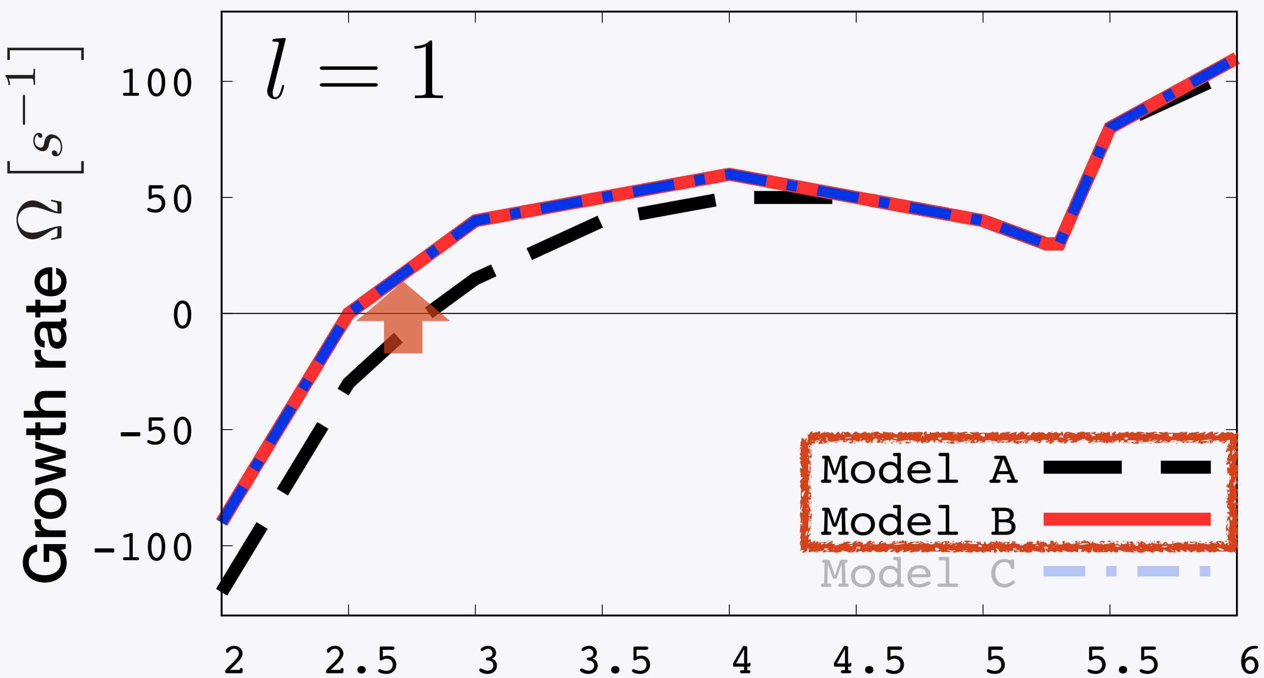


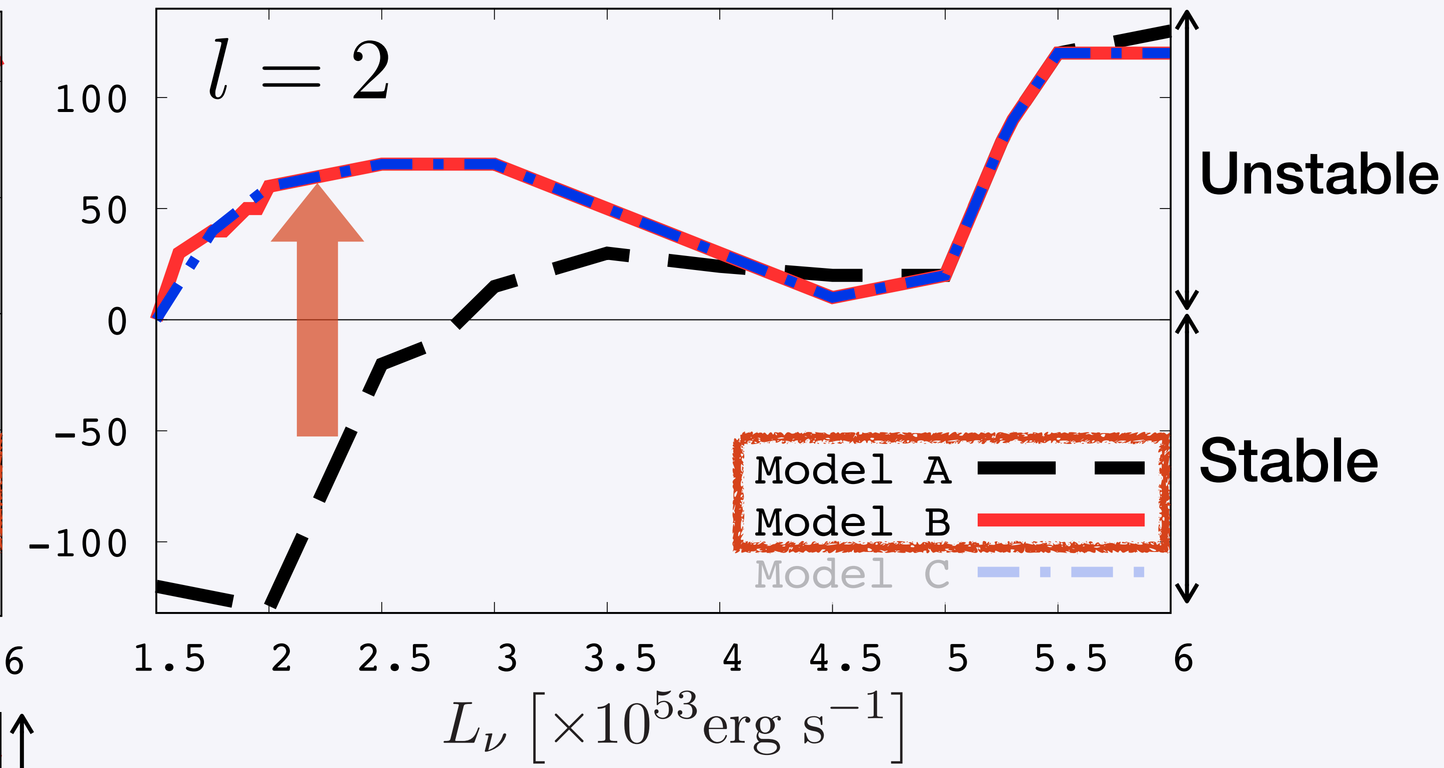
Model	Acoustic injection	Fluctuations of neutrino luminosiry
A	no	no
B	yes	no
C	yes	yes

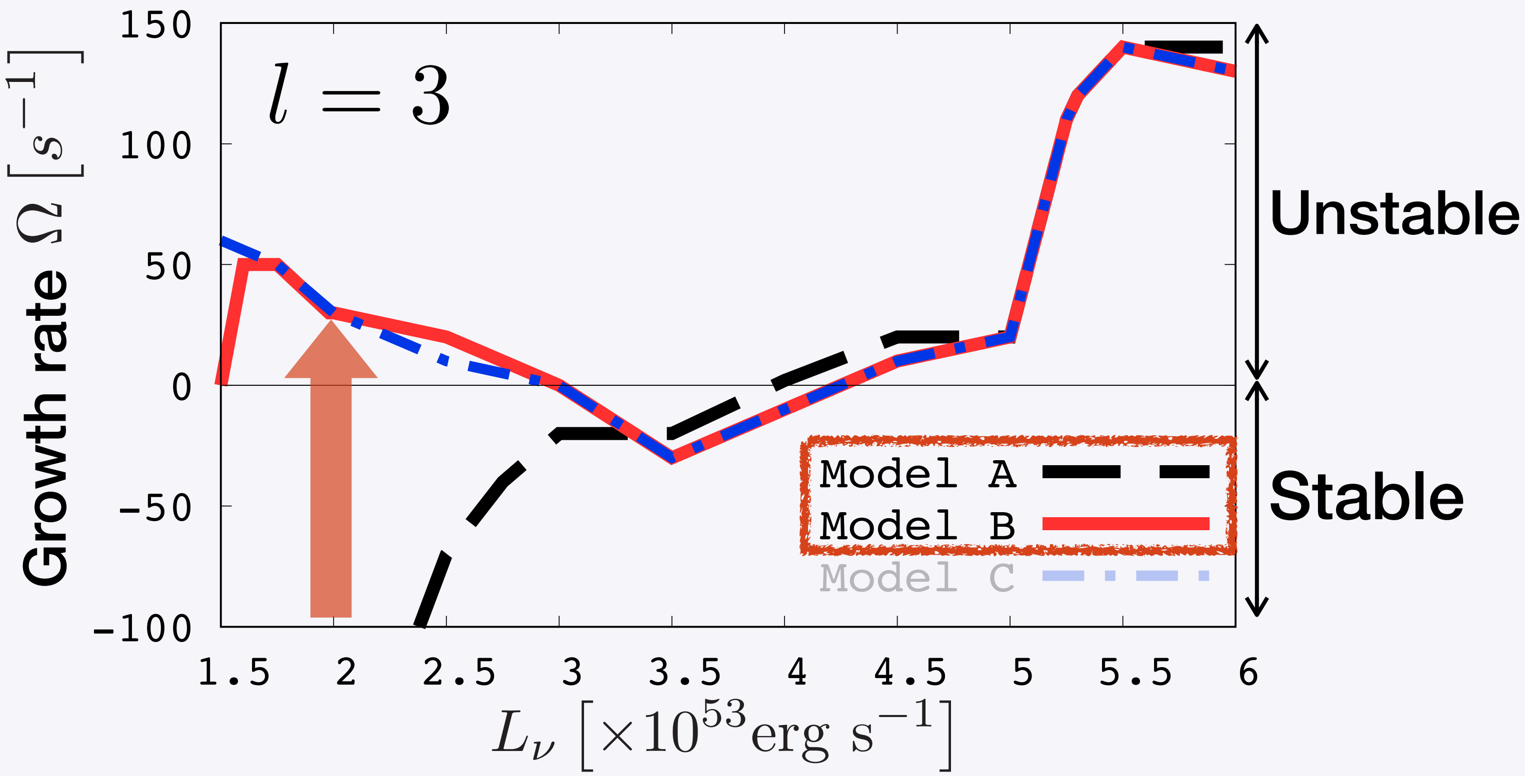
Eigenmodes expansion

$$\frac{\delta r_{\mathrm{sh}}}{r_{\mathrm{sh}}}(t) = \sum_{(l,m)} \sum_{j} a_j^{(l,m)} e^{\Omega_j^{(l,m)} t} e^{i\omega_j^{(l,m)} t} Y_{lm}(\theta, \phi)$$

Model 1: Growth rates of SW deformation







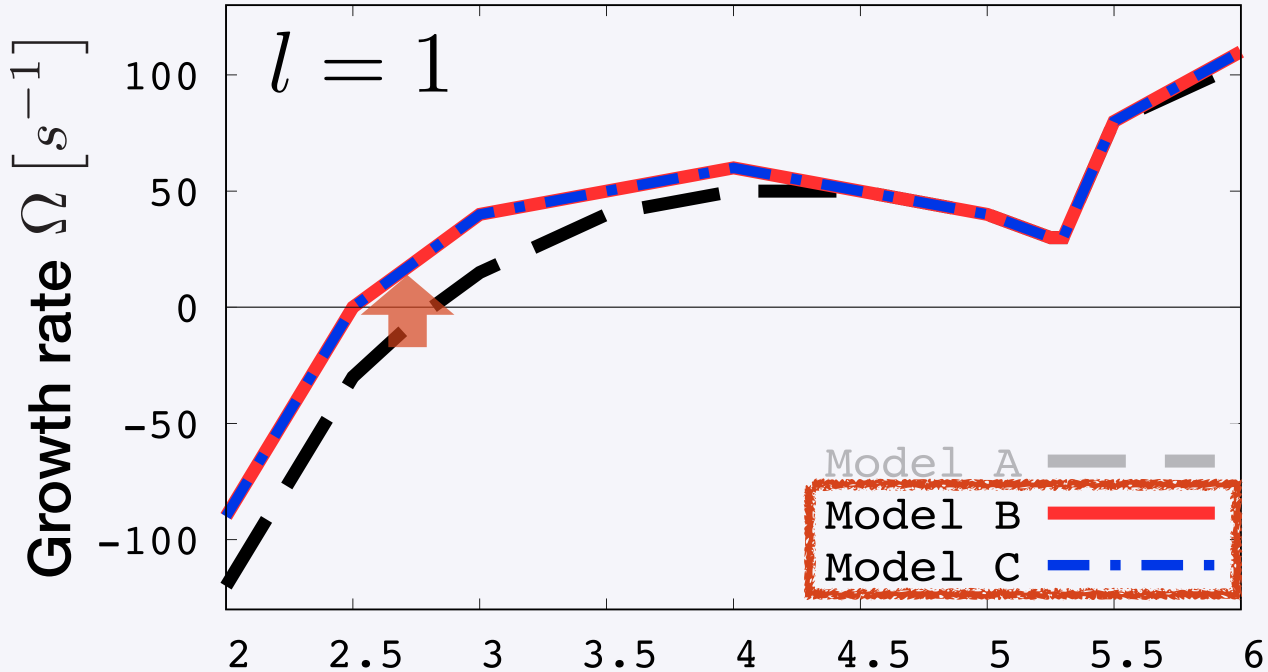
☑ **Acoustic injection enhances the growth rates.**

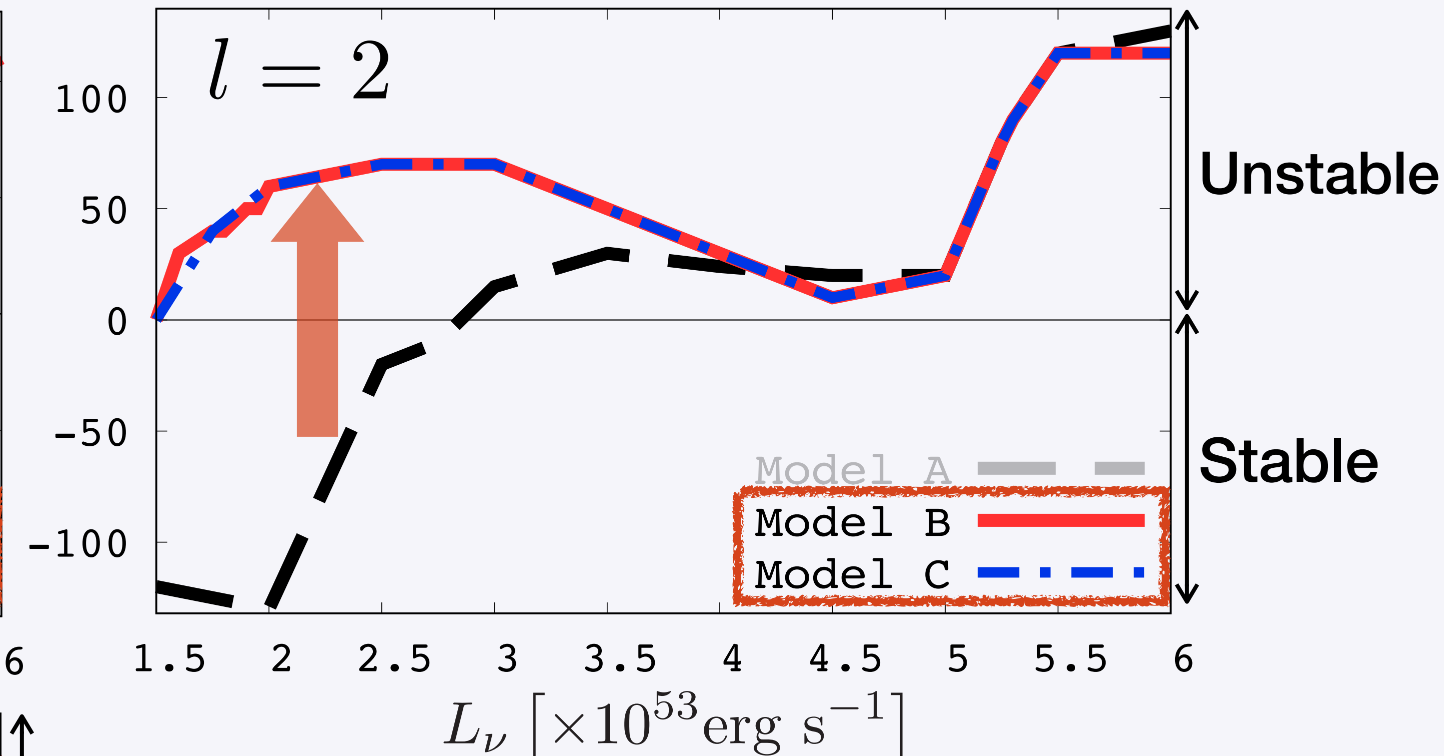
Model	Acoustic injection	Fluctuations of neutrino luminosiry
A	no	no
B	yes	no
C	yes	yes

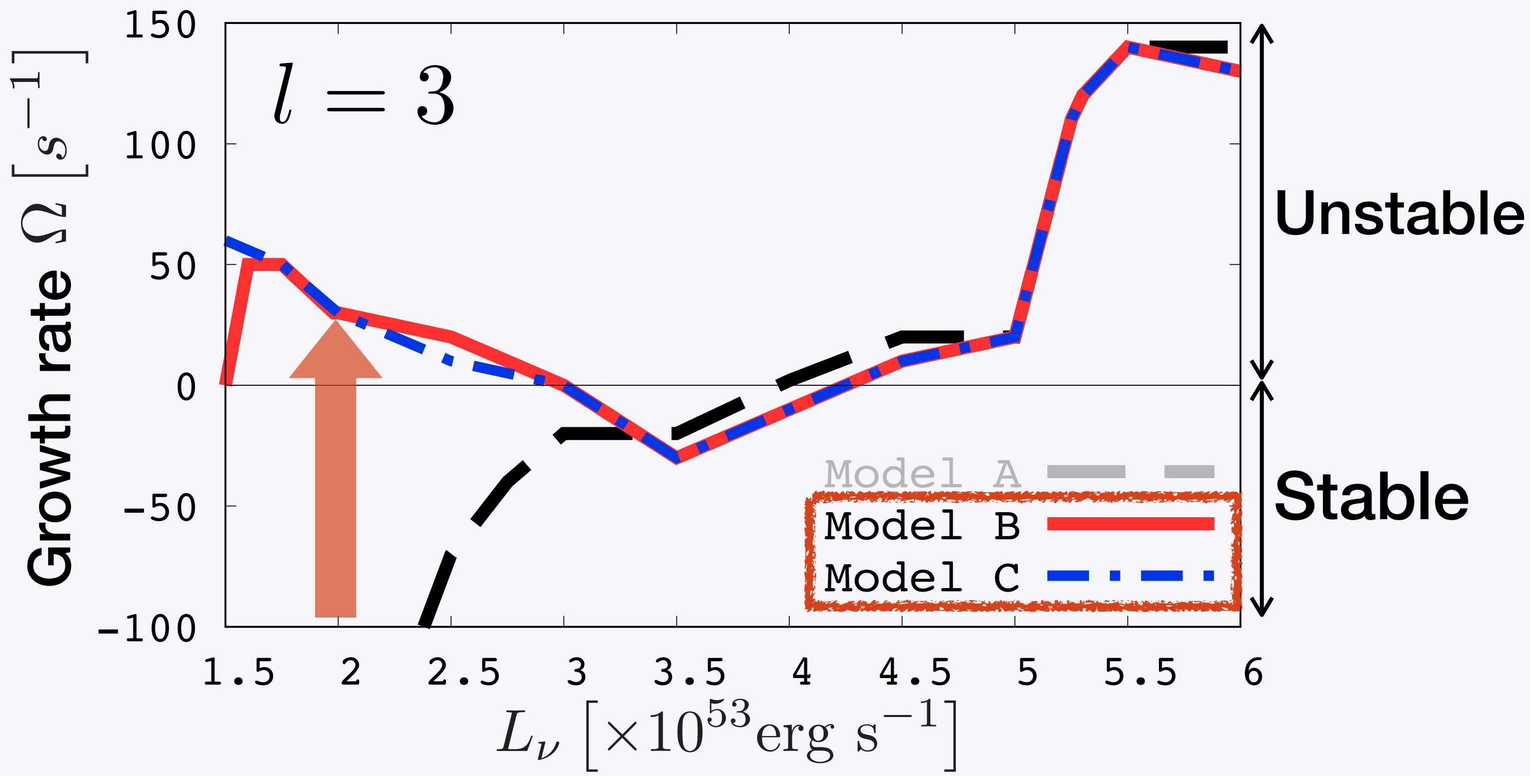
Eigenmodes expansion

$$\frac{\delta r_{\rm sh}}{r_{\rm sh}}(t) = \sum_{(l,m)} \sum_{j} a_j^{(l,m)} e^{\Omega_j^{(l,m)} t} e^{i\omega_j^{(l,m)} t} Y_{lm}(\theta, \phi)$$

Model 1: Growth rates of SW deformation







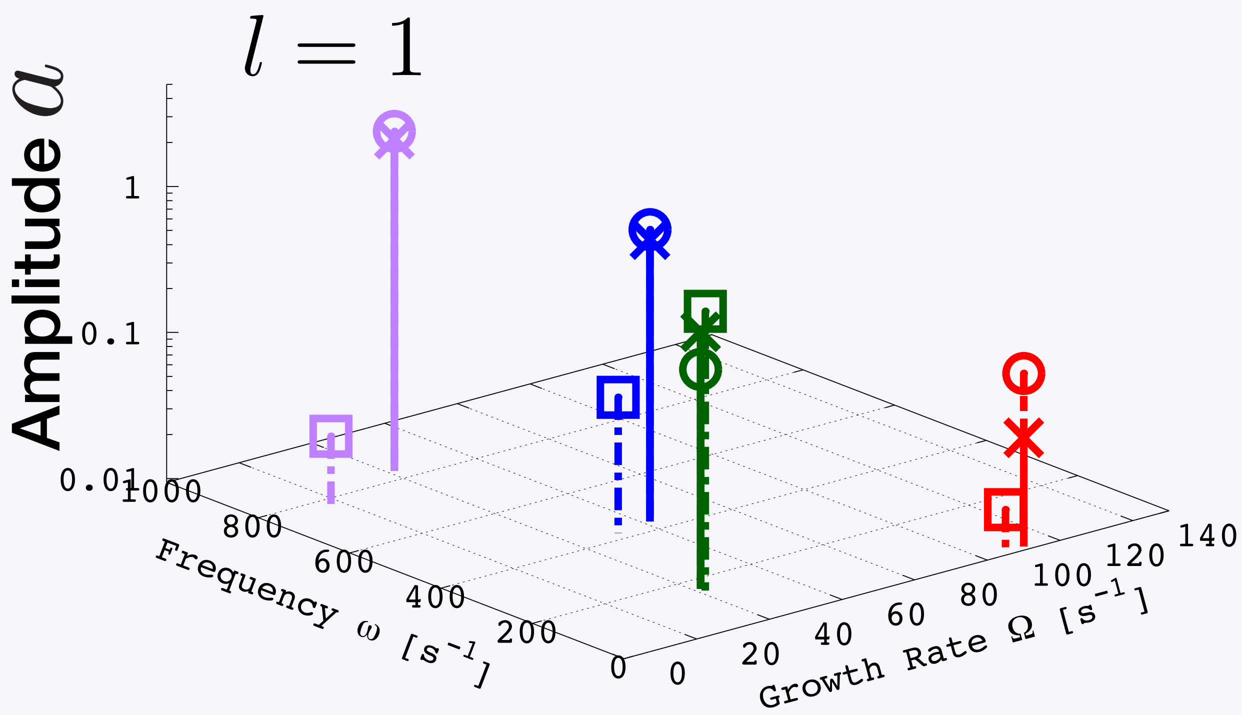
- ☑ **Acoustic injection enhances the growth rates.**
- ☑ **Fluctuations of L_ν give slight effects on the growth rates.**

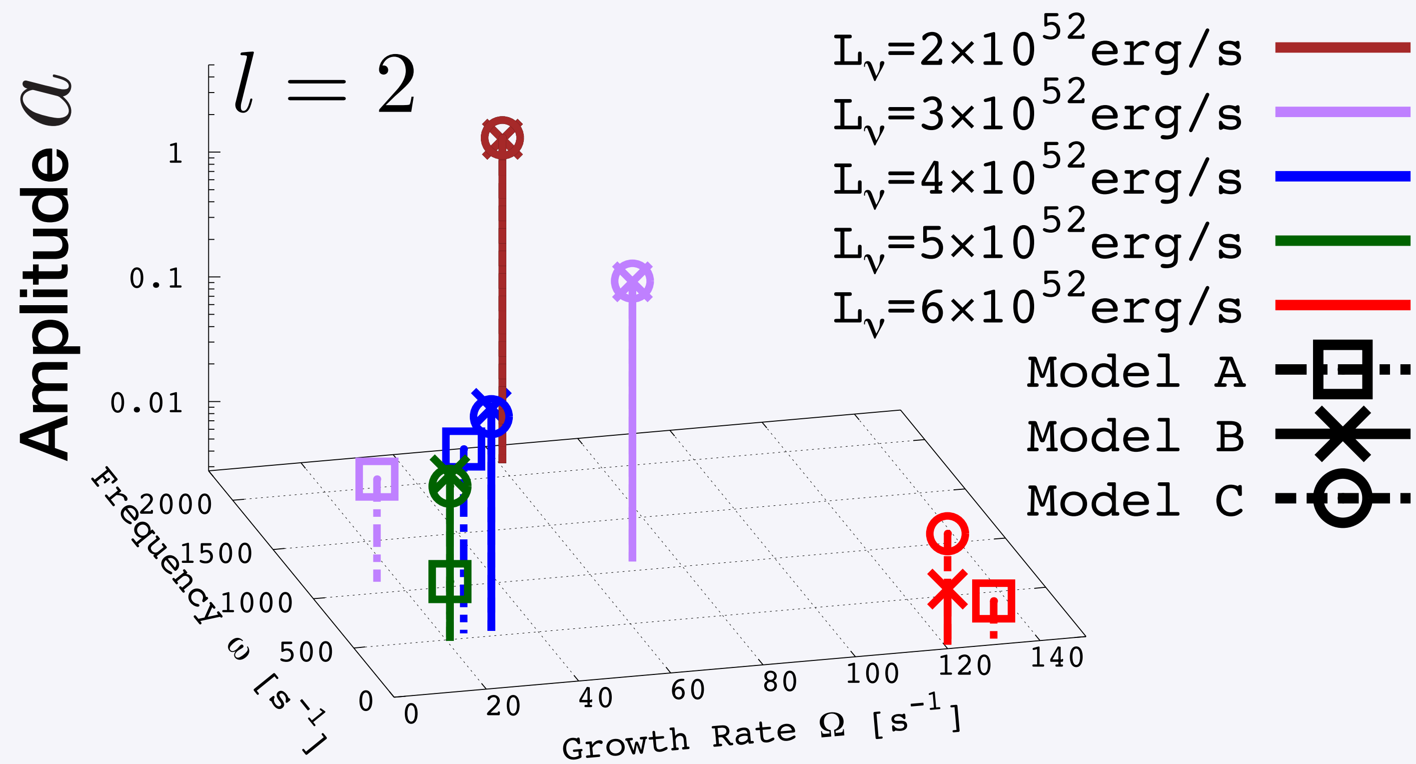
Model	Acoustic injection	Fluctuations of neutrino luminosiry
A	no	no
B	yes	no
C	yes	yes

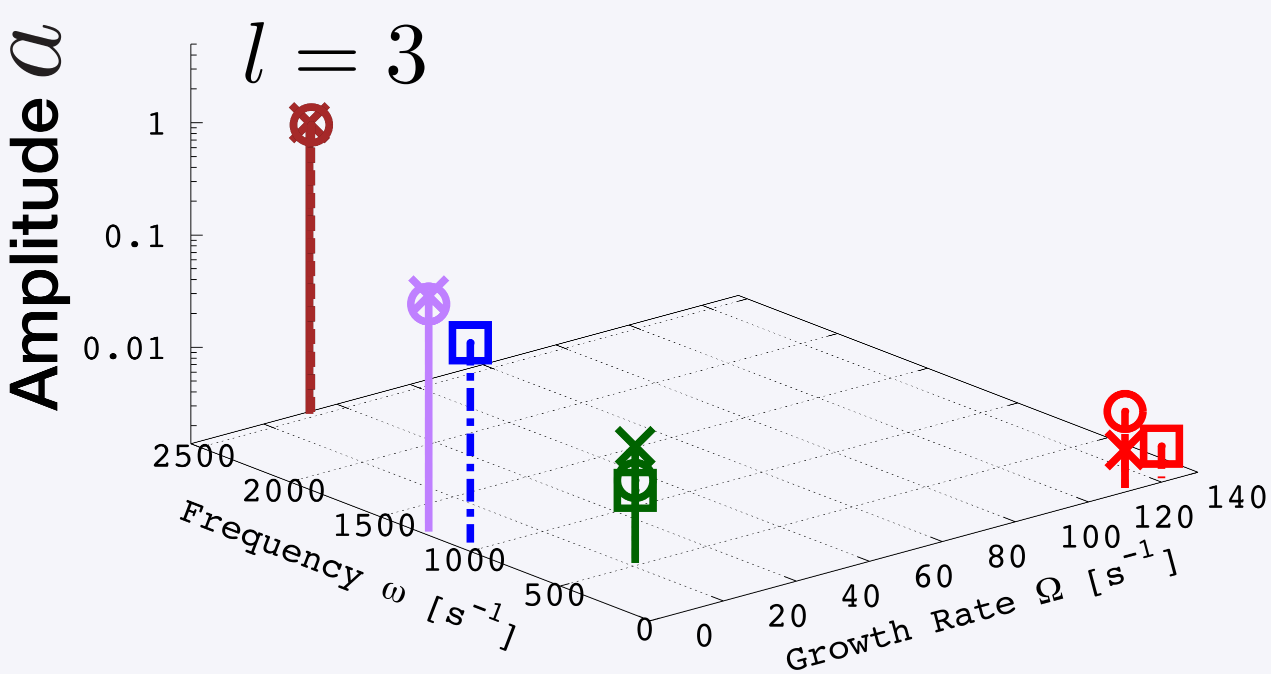
Eigenmodes expansion

$$\frac{\delta r_{\rm sh}}{r_{\rm sh}}(t) = \sum_{(l,m)} \sum_{j} a_j^{(l,m)} e^{\Omega_j^{(l,m)} t} e^{i\omega_j^{(l,m)} t} Y_{lm}(\theta, \phi)$$

Model 1: Amplitudes of shock deformation





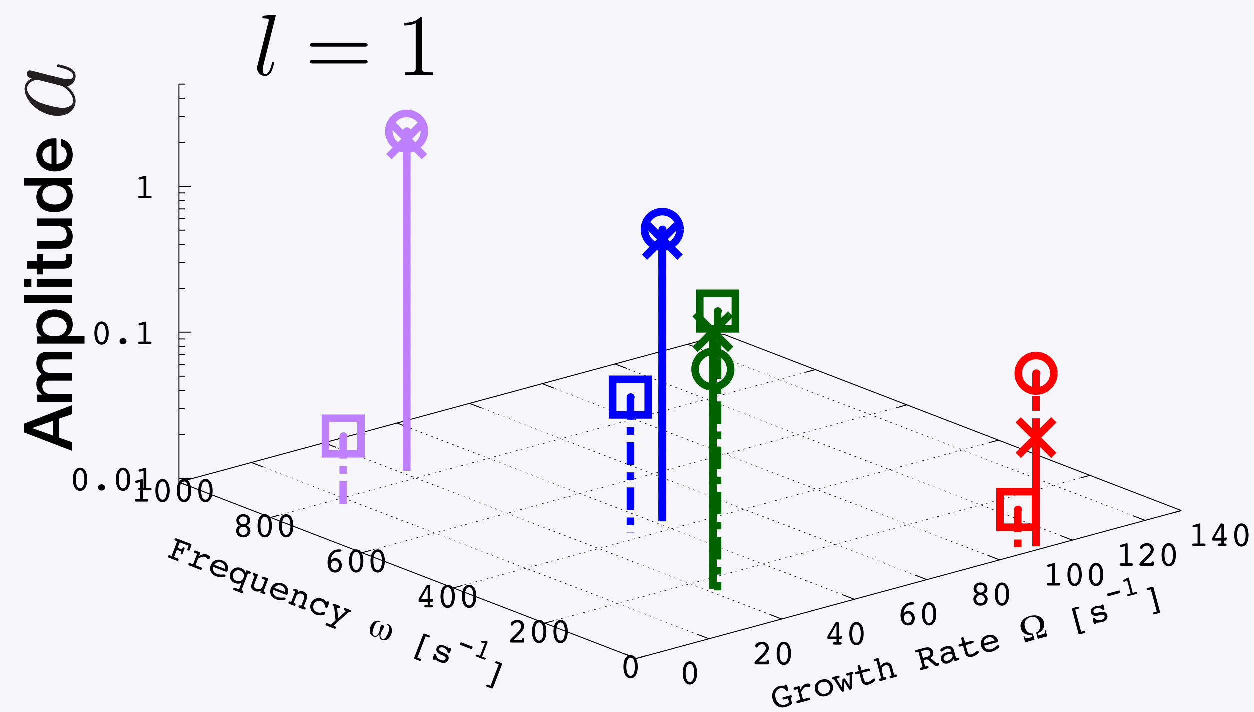


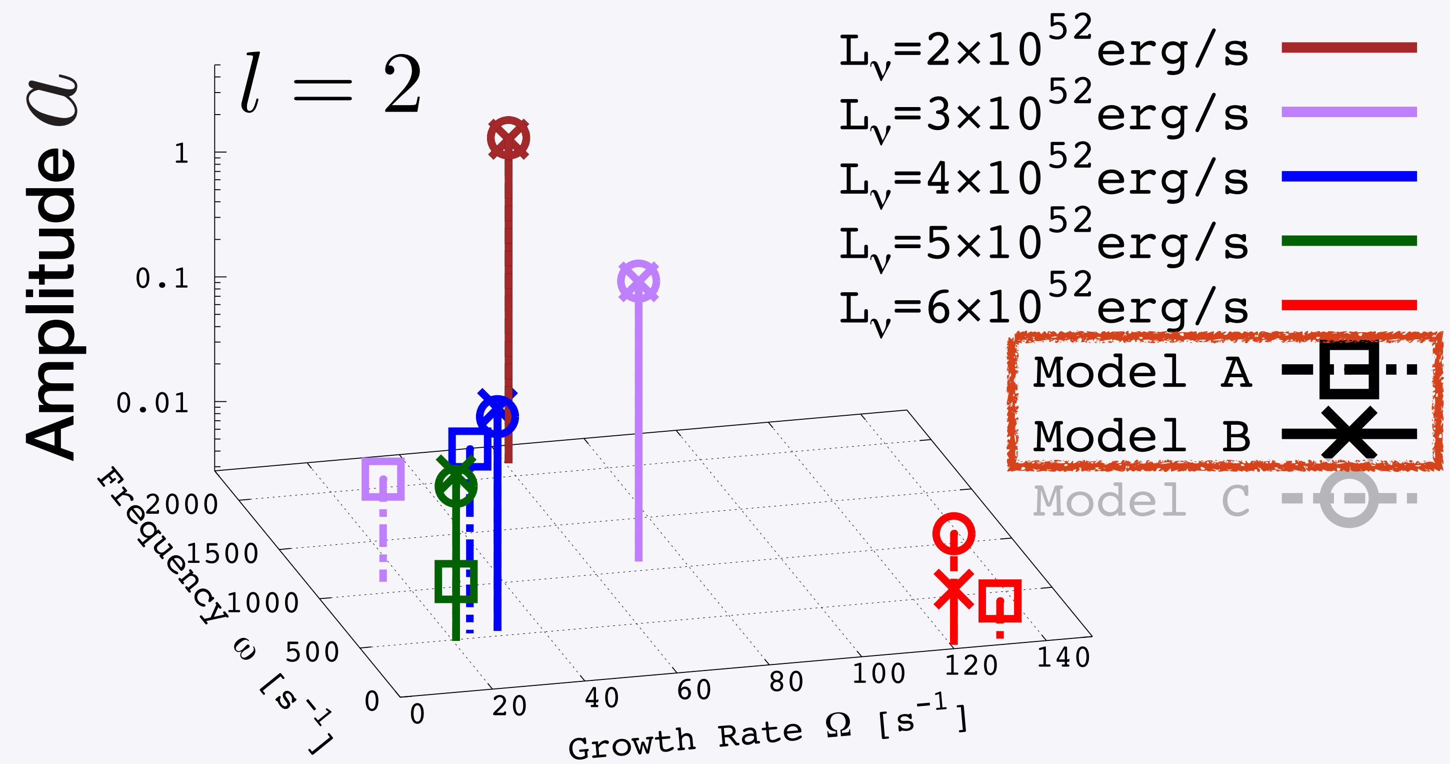
Model	Acoustic injection	Fluctuations of neutrino luminosiry
A	no	no
B	yes	no
C	yes	yes

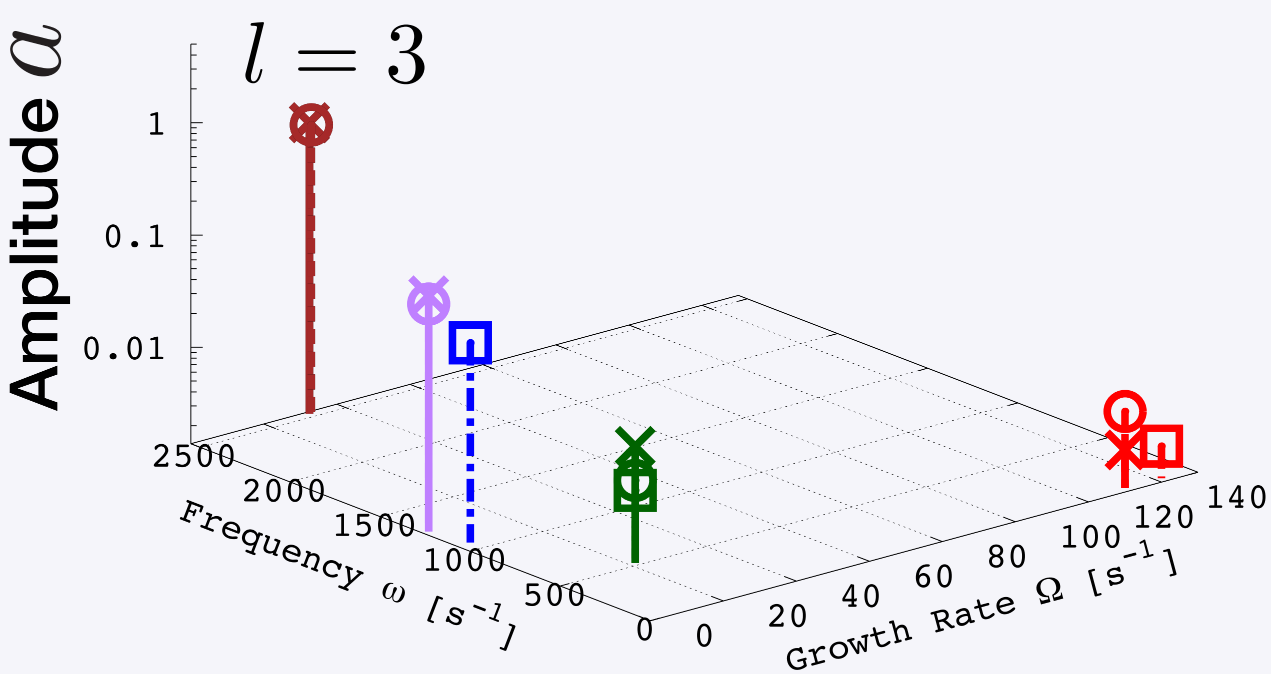
Eigenmodes expansion

$$\frac{\delta r_{\rm sh}}{r_{\rm sh}}(t) = \sum_{(l,m)} \sum_{j} a_j^{(l,m)} \mathrm{e}^{\Omega_j^{(l,m)} t} \mathrm{e}^{\mathrm{i}\omega_j^{(l,m)} t} Y_{lm}(\theta, \phi)$$

Model 1: Amplitudes of shock deformation







☑ **Excitation of eigenmode amplitude occur when the neutrino luminosity is low.**

Model	Acoustic injection	Fluctuations of neutrino luminosiry
A	no	no
B	yes	no
C	yes	yes

Eigenmodes expansion

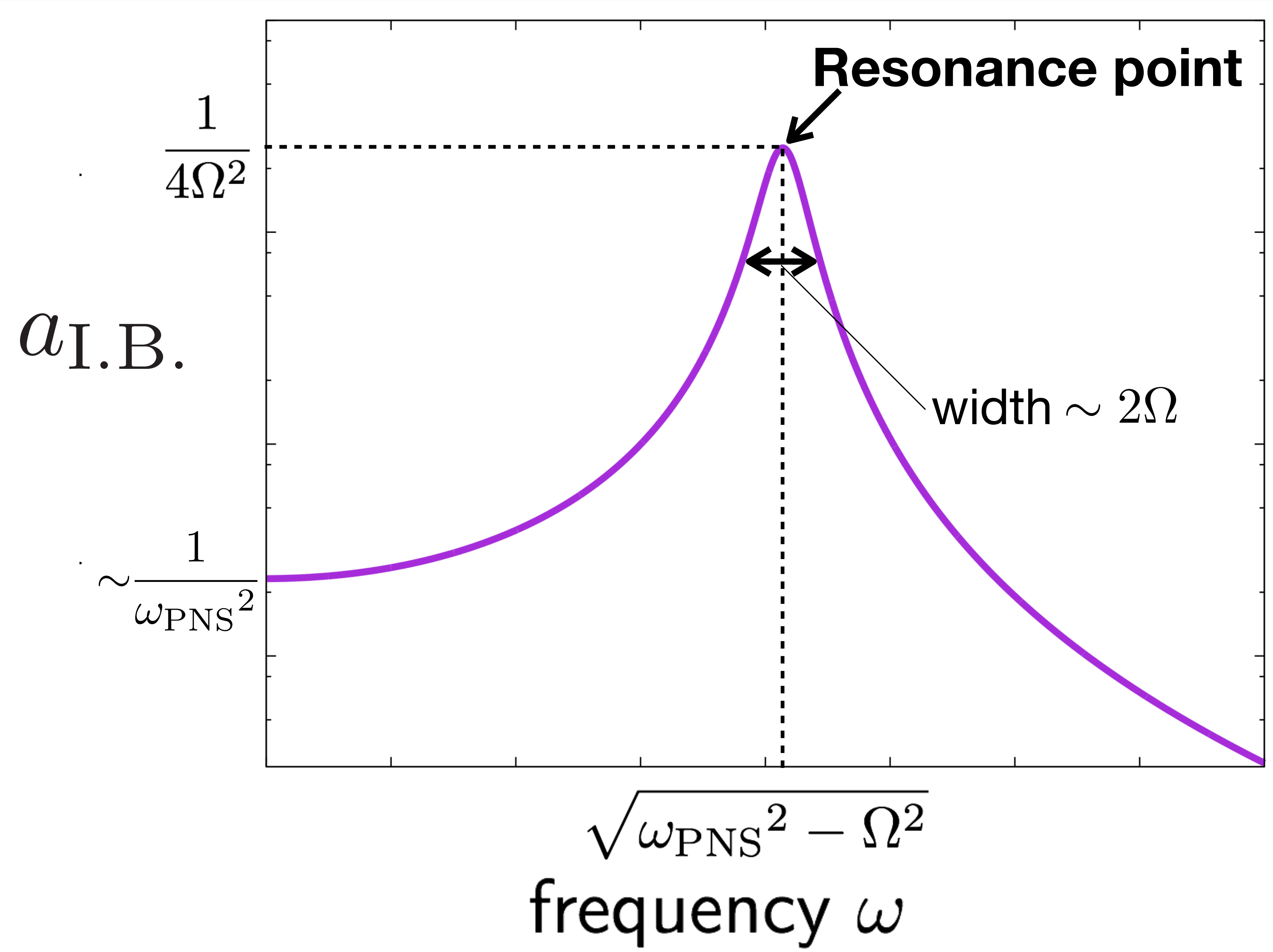
$$\frac{\delta r_{\rm sh}}{r_{\rm sh}}(t) = \sum_{(l,m)} \sum_j a_j^{(l,m)} e^{\Omega_j^{(l,m)} t} e^{i\omega_j^{(l,m)} t} Y_{lm}(\theta, \phi)$$

Resonance of acoustic wave and SASI

- **Acoustic injection** $\frac{\delta p}{v_{r0} c_s \rho_0} + \frac{\delta v_r}{v_{r0}} = \sin(\omega_{\rm PNS}\, t), \quad \omega_{\rm PNS} = 2000 \times l \; [{\rm s}^{-1}]$

- $a_i = (a_i)_{\rm upstream} + (a_i)_{\rm I.B.}, \quad (a_i)_{\rm I.B.} \propto |(\sin \omega_{\rm PNS} t)^*|$

$$= \left| \frac{\omega_{\rm PNS}}{(\Omega + {\rm i}\omega)^2 + {\omega_{\rm PNS}}^2} \right|$$



- **Resonance**

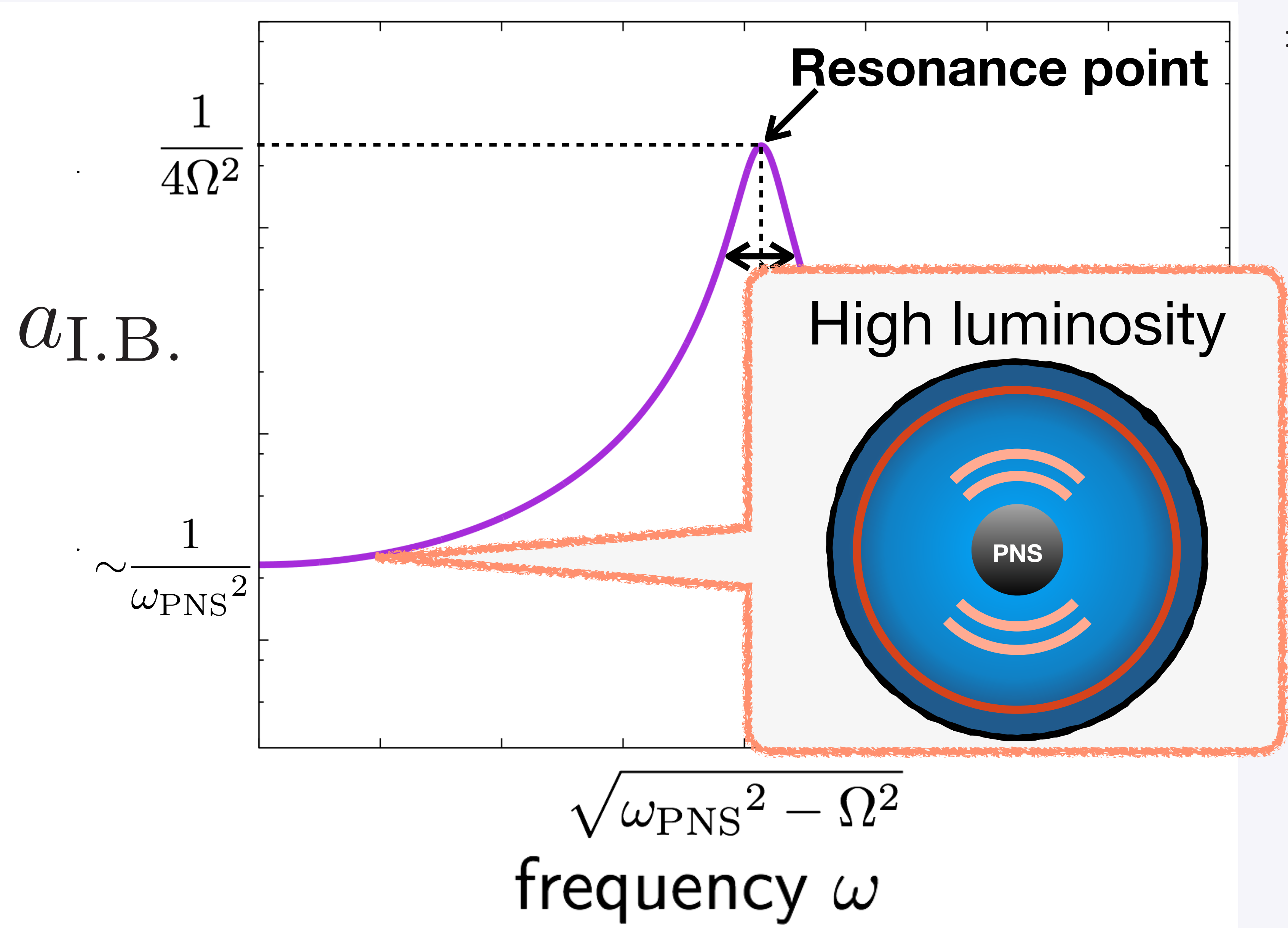
  $\Omega \sim 100\ {\rm s}^{-1} \ll \omega_{\rm PNS}$

  → Resonance occur when $\omega \sim \omega_{\rm PNS}$

Resonance of acoustic wave and SASI

- **Acoustic injection** $\frac{\delta p}{v_{r0} c_s \rho_0} + \frac{\delta v_r}{v_{r0}} = \sin\left(\omega_{\rm PNS}\, t\right), \quad \omega_{\rm PNS} = 2000 \times l\ [{\rm s}^{-1}]$
- $a_i = (a_i)_{\rm upstream} + (a_i)_{\rm I.B.}, \quad (a_i)_{\rm I.B.} \propto \left|(\sin \omega_{\rm PNS} t)^*\right|$

$$= \left| \frac{\omega_{\rm PNS}}{(\Omega + {\rm i}\omega)^2 + {\omega_{\rm PNS}}^2} \right|$$



- **Resonance**
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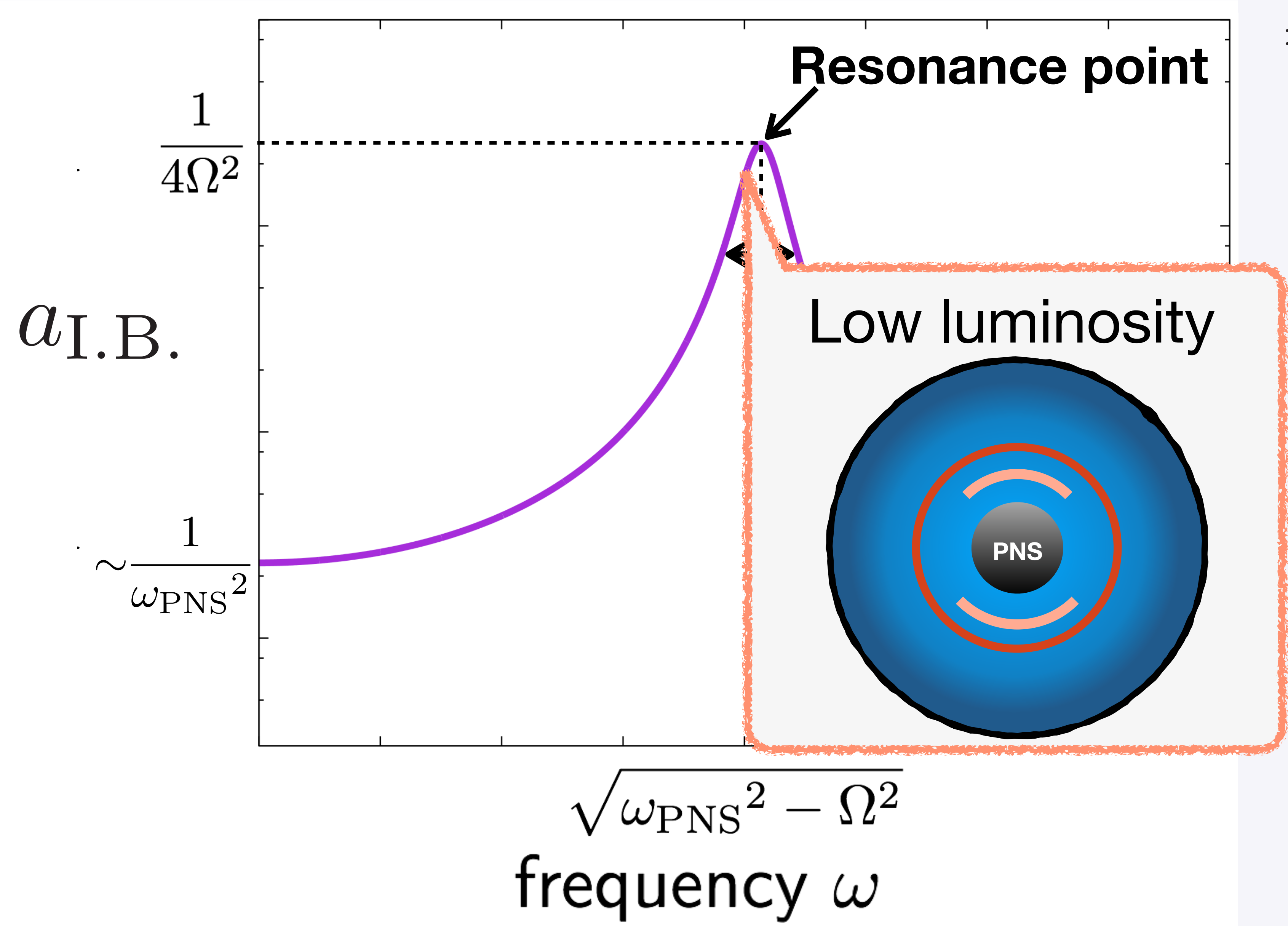
Resonance of acoustic wave and SASI

- **Acoustic injection** $\dfrac{\delta p}{v_{r0} c_s \rho_0} + \dfrac{\delta v_r}{v_{r0}} = \sin(\omega_{\rm PNS}\, t), \quad \omega_{\rm PNS} = 2000 \times l \ [{\rm s}^{-1}]$
- $a_i = (a_i)_{\rm upstream} + (a_i)_{\rm I.B.}, \quad (a_i)_{\rm I.B.} \propto |(\sin \omega_{\rm PNS} t)^*|$

$$= \left| \frac{\omega_{\rm PNS}}{(\Omega + i\omega)^2 + {\omega_{\rm PNS}}^2} \right|$$

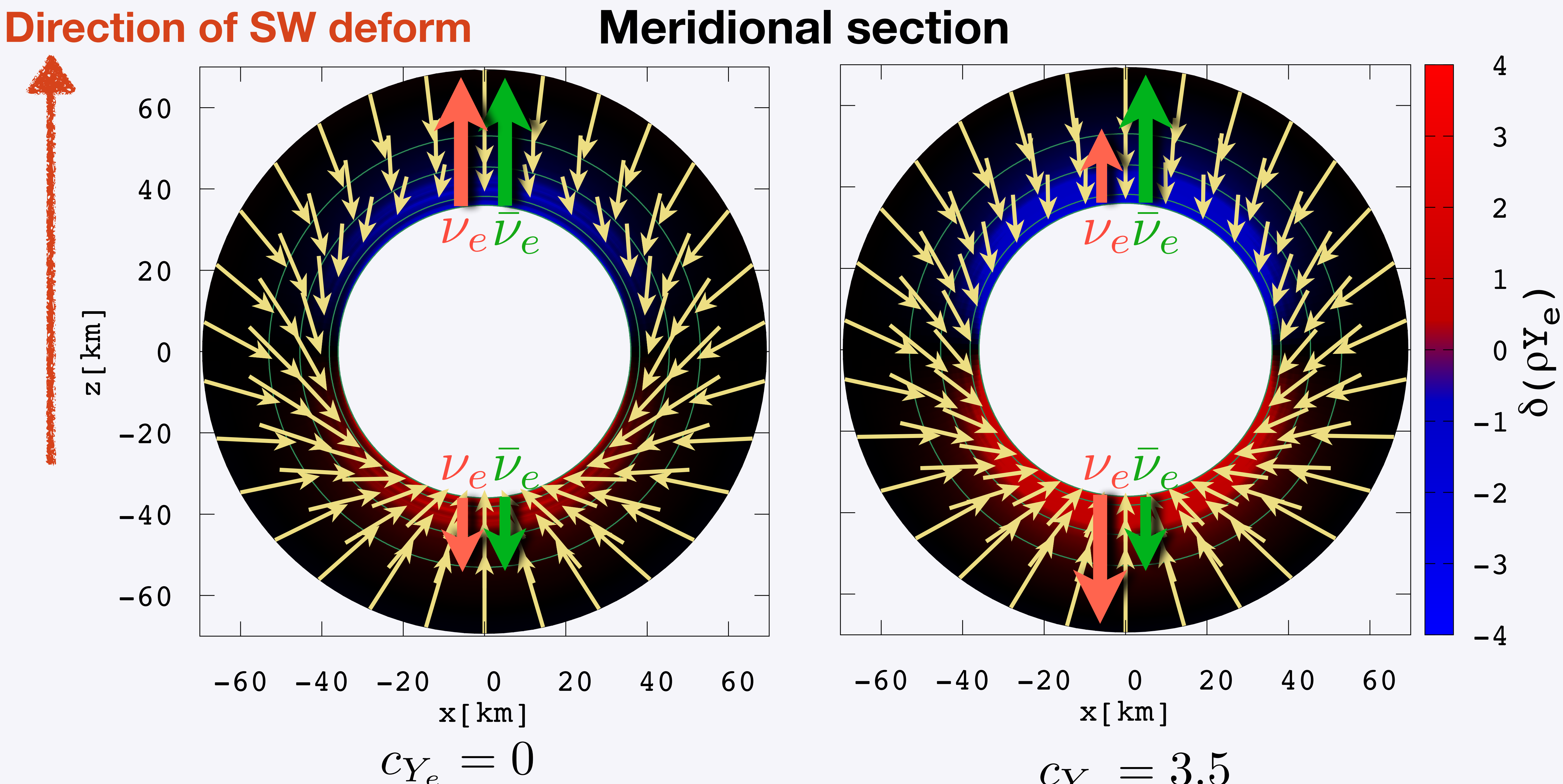


- **Resonance**

  $\Omega \sim 100\ {\rm s}^{-1} \ll \omega_{\rm PNS}$

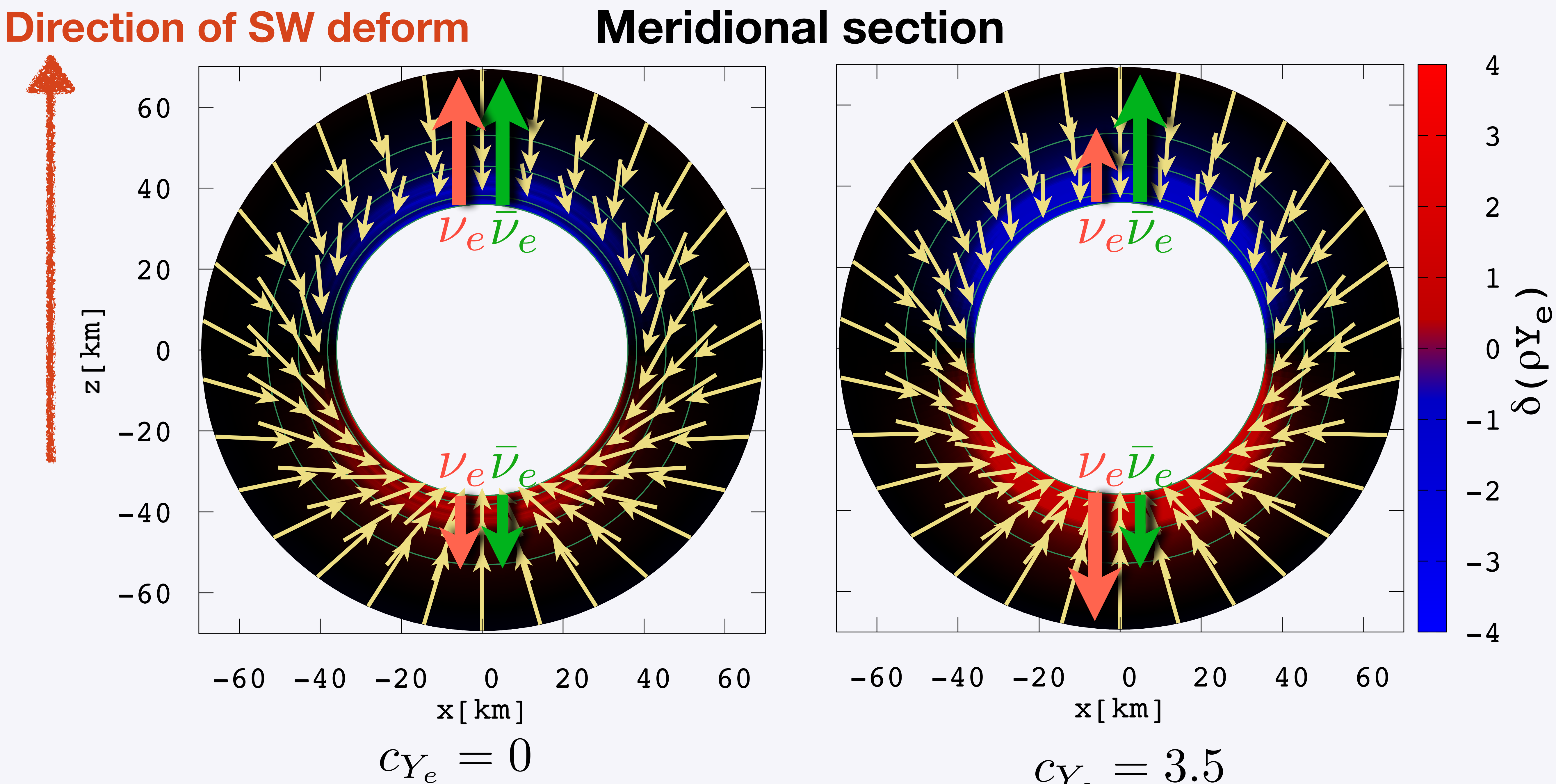
  → Resonance occur when $\omega \sim \omega_{\rm PNS}$

Model 2: Self-Sustained Structure



Inner b.c.: $\frac{\delta L_{\nu_e}}{L_0} = 4\frac{\delta T_{\nu_e}}{T_{\nu_e 0}} + c_{Y_e}\frac{\delta Y_e}{Y_{e0}}, \quad \frac{\delta L_{\bar{\nu}_e}}{L_0} = 4\frac{\delta T_{\bar{\nu}_e}}{T_{\bar{\nu}_e 0}} - c_{Y_e}\frac{\delta Y_e}{Y_{e0}}$

Model 2: Self-Sustained Structure



Inner b.c.: $\frac{\delta L_{\nu_e}}{L_0} = 4\frac{\delta T_{\nu_e}}{T_{\nu_e 0}} + c_{Y_e}\frac{\delta Y_e}{Y_{e0}}, \quad \frac{\delta L_{\bar{\nu}_e}}{L_0} = 4\frac{\delta T_{\bar{\nu}_e}}{T_{\bar{\nu}_e 0}} - c_{Y_e}\frac{\delta Y_e}{Y_{e0}}$

$$\delta r_{\rm sh} \leftrightarrow F_{\nu_e} + F_{\bar{\nu}_e}$$

Conclusion

Summary

- We have investigated the instability of the standing SW and the accretion flows downstream in the CCSNe by linear analysis.
  - ☑ Acoustic injection from the PNS enhances the instability especially when the neutrino luminosity is low.
  - ☑ On the other hand, the fluctuations of neutrino luminosity give slight effects on the instability (in linear regime).
  - ☑ The sum of flux of electron and anti-electron neutrinos is the key ingredient to the production of the self-sustained steady perturbed configuration.

Future work

- Since the background is spherically symmetric and no magnetic field, this analysis do not include these effects.
- Recently, steady shocked accretion flow with rotation and magnetic field can be calculated, linear analysis around these background is also important for comprehension of CCSNe mechanism.