

# Linear Analysis of Fast-Pairwise collective neutrino oscillations in Core-Collapse Supernovae

Based on the results of realistic Boltzmann Simulations

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Delfan Azari et al. (arXiv: 1902.07467)

# Fate of a star depends on its **Mass**



SN 1987A

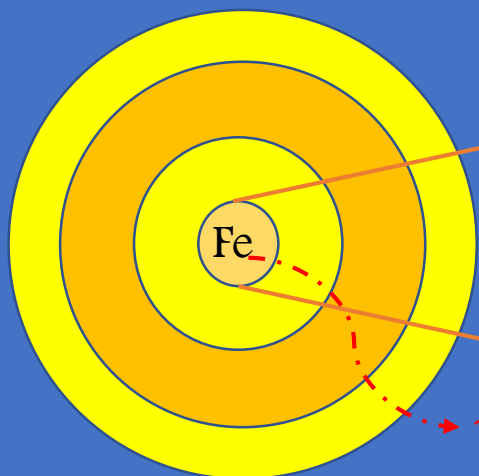
$$M_{\text{star}} \gtrsim 8 M_{\text{sun}}$$

Core-Collapse  
**Supernovae**

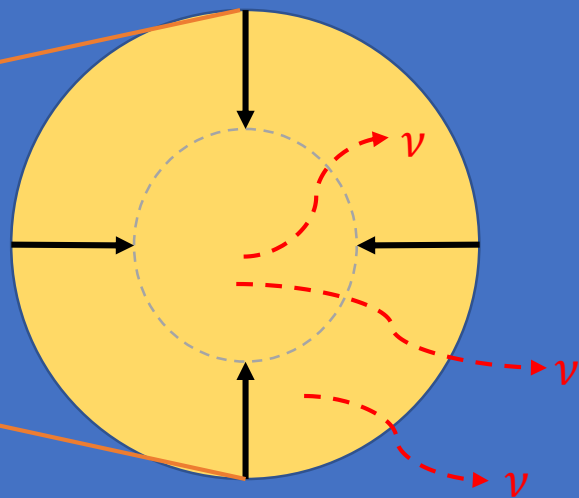


Cassiopeia A

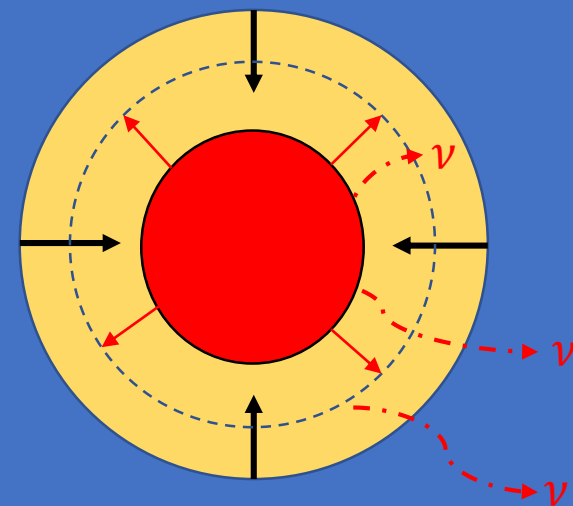
Core-Collapse



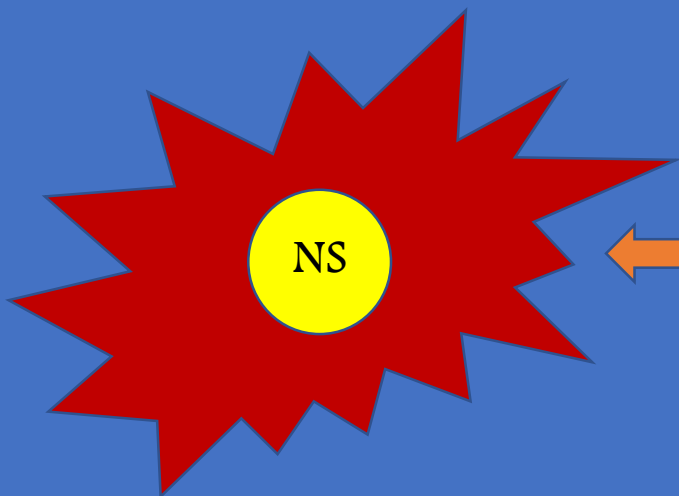
$\nu$  - trapping



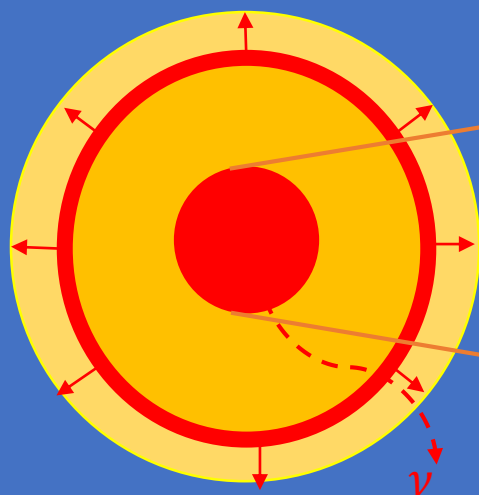
Core bounce



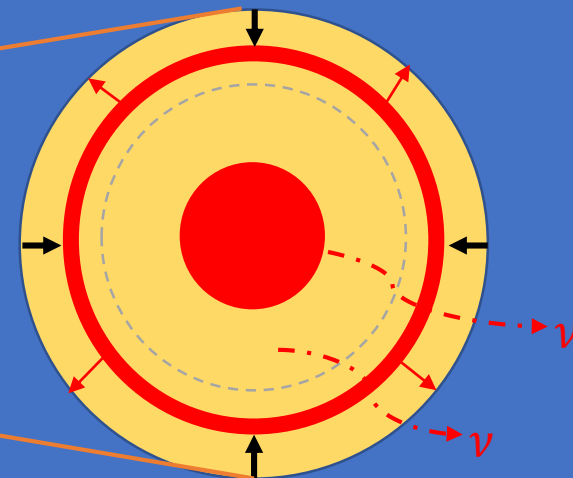
SN explosion and NS cooling



Shock in envelope & PNS cooling

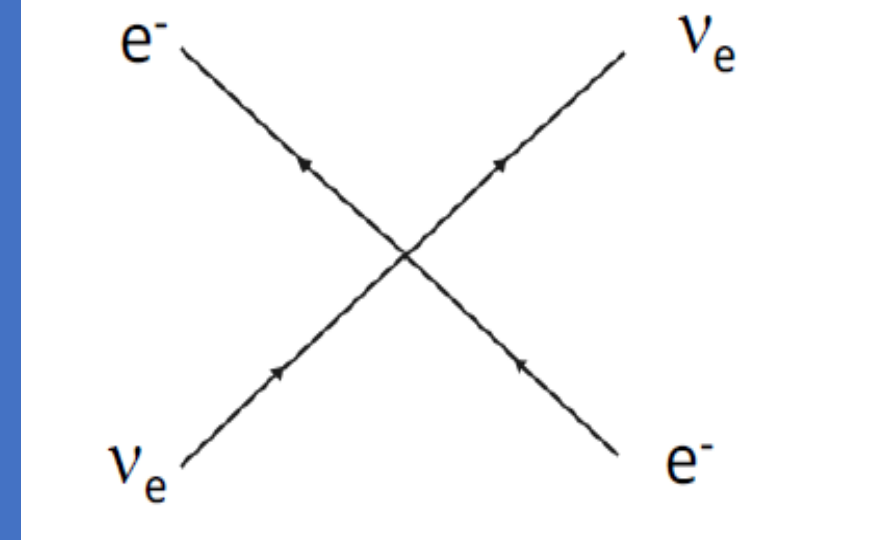


Shock propagation in core

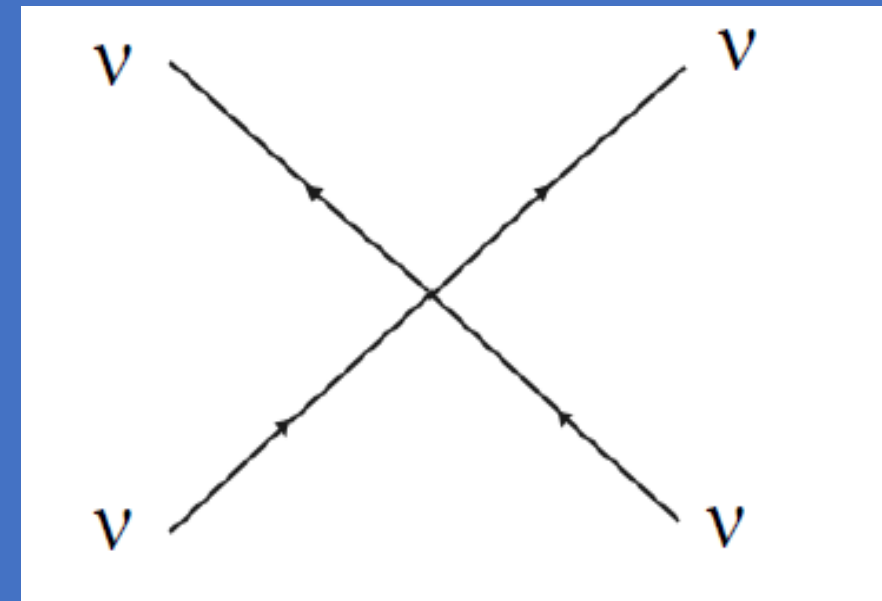


# Why neutrinos ?

- ✓ almost all of the binding energy of NS liberated in the gravitational collapse is emitted in the form of neutrinos and the kinetic energy of matter in the supernova explosion is just 1% of this energy.
- ✓ In the  $\nu$  – heating mechanism, a fraction of the electron–type neutrinos and anti–neutrinos are re–absorbed by the matter between the shock front and the so–called gain radius and deposit their energy to push the stagnated shock again.



Matter effect  $r > \sim 10^3$  km  
Wolfenstein PRD 17, 2369, 1978



$\nu - \nu$  self interaction  $r \sim 10^2$  km  
Duan et al., PRD 74, 105014, 2006

# What fast pairwise means?

- ✓ Collective pair conversion  $\nu_e \bar{\nu}_e \rightleftharpoons \nu_x \bar{\nu}_x$  by forward scattering maybe generic for SN neutrino transport
- ✓ Depending on the angular intensity, this rate is “Fast”

# Motivation

- ✓ Problems remaining;
- No successful explosion model for CCSNe  
and
- Collective neutrino oscillation has been ignored

✓ Why?

Because it is difficult !

# Motivation

✓ Problems remaining;

- No successful explosion model for CCSNe

and

- Collective neutrino oscillation has been ignored

✓ Our goal;

Study the collective neutrino oscillations and see whether they effect the explosion mechanism or no

# Basic Equations and Formulae

Equation of Motion:

$$(\partial_t + \mathbf{v} \cdot \nabla_r) \rho = i[\rho, H]$$

$$\rho = \frac{fv_e + fv_x}{2} + \frac{fv_e - fv_x}{2} \begin{pmatrix} S & S \\ S^* & -S \end{pmatrix},$$

$$H = \frac{M^2}{2E} + v^\mu \Lambda_\mu \frac{1}{2} \sigma_3 + \sqrt{2} G_F \int d\Gamma' v^\mu v'_\mu \rho'$$

$M^2$ : Mass-squared matrix

$v^\mu$ :  $(1, \mathbf{v})$

$\Lambda^\mu$ :  $\sqrt{2} G_F (n_e - n_{e+}) u^\mu$

$d\Gamma' = d\mathbf{v}' / 4\pi$

Kinetic Hamiltonian

Matter Hamiltonian

Neutrino-Neutrino Hamiltonian

Vacuum Oscillation

MSW Oscillation

Collective Oscillation



# Basic Equations and Formulae

Linearized Equation of Motion:

$$i(\partial_t + v \cdot \nabla_r) S_v = v^\mu (\Lambda_\mu + \Phi_\mu) S_v - \int \frac{d\mathbf{v}'}{4\pi} v^\mu v'_\mu G_{v'} S_{v'}$$

$$G_v = \sqrt{2} G_F \int_0^\infty \frac{dE E^2}{2\pi^2} [f_{\nu_e}(E, \mathbf{v}) - f_{\bar{\nu}_e}(E, \mathbf{v})]$$

$$\Phi^\mu \equiv \frac{d\mathbf{v}}{4\pi} G_v v^\mu$$

Assuming the solutions in the form of ;

$$S_v(t, r) = Q_v(\Omega, K) e^{-i(\Omega t - K \cdot r)}$$

$$v^\mu k_\mu Q_v = a^\mu$$

where ;  $a^\mu \equiv - \int \frac{d\mathbf{v}'}{4\pi} v^\mu v'_\mu G_{v'} Q_{v'}$   
 $k^\mu = K^\mu - \Lambda^\mu - \Phi^\mu$  with  $k^\mu = (\omega, \mathbf{k})$

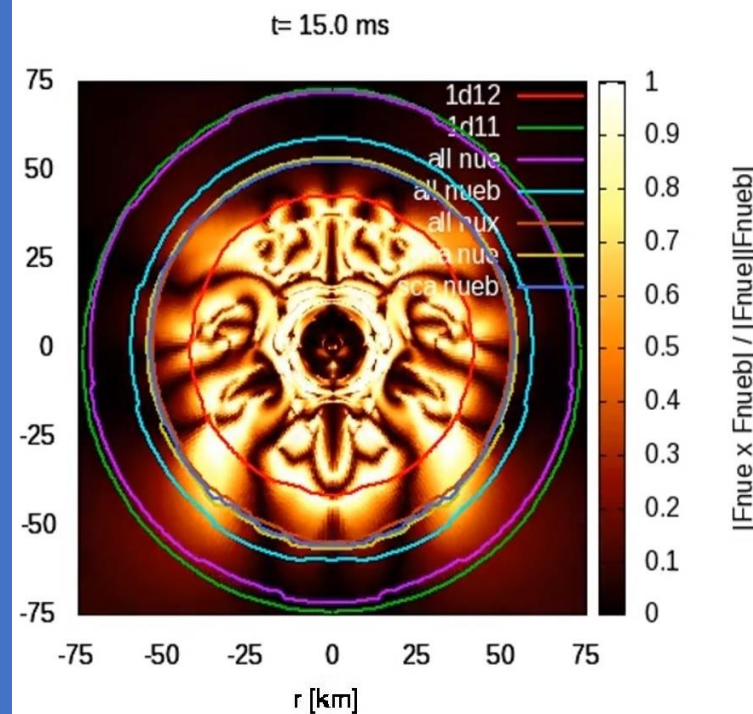
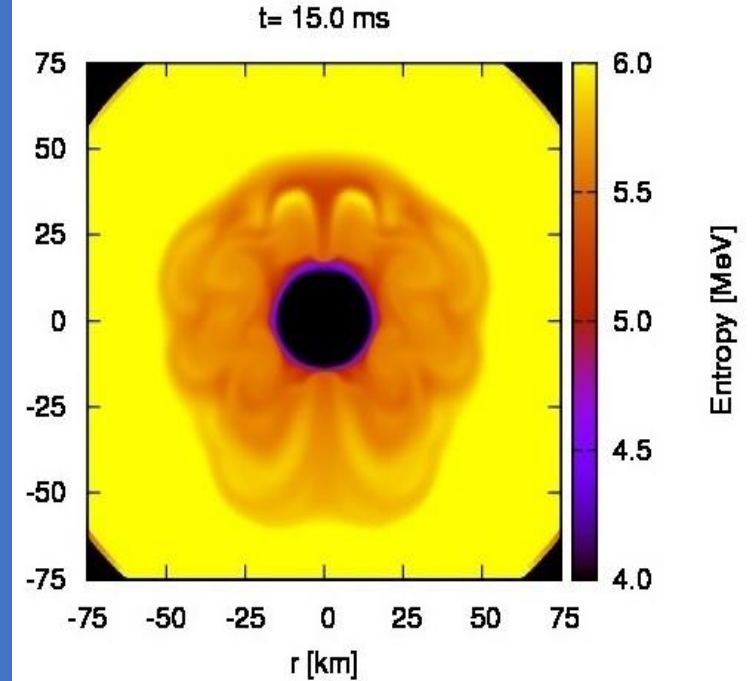
$$\Pi^{\mu\nu}(\omega, \mathbf{k}) a_\nu = 0$$

Polarization tensor  $\Pi^{\mu\nu} = \eta^{\mu\nu} + \int \frac{dV}{4\pi} G_V \frac{v^\mu v^\nu}{\omega - V \cdot k}$

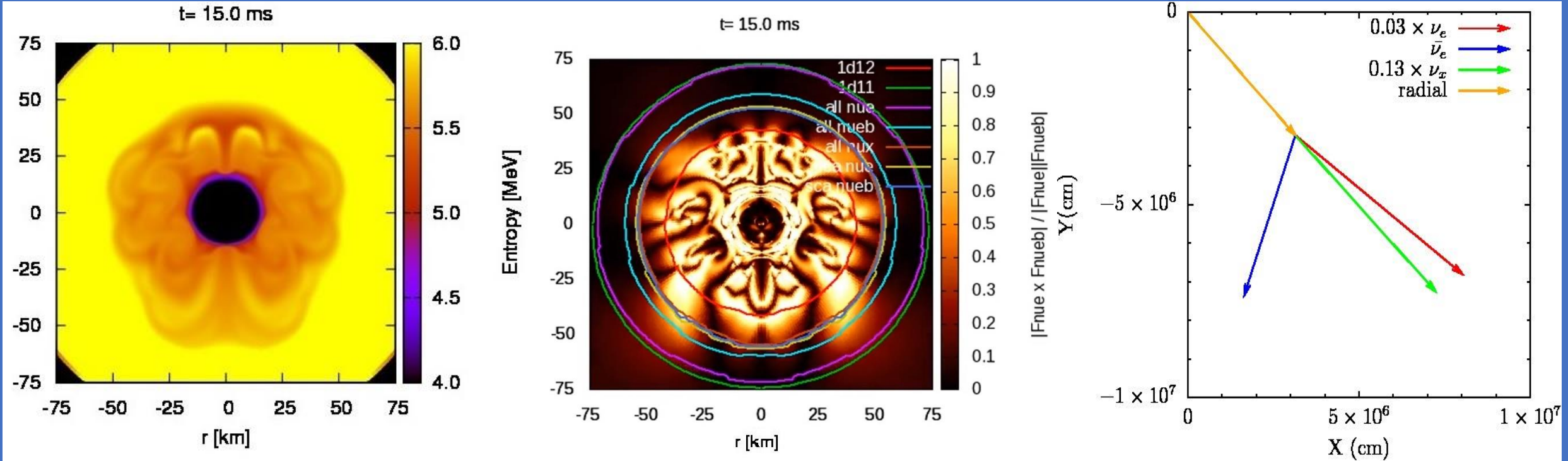
$$D(\omega, \mathbf{k}) \equiv \det[\Pi] = 0$$

# Background Numerical Model

- ✓ Results of the realistic 2D simulations on the K-Supercomputer Nagakura et al., ApJ 854, 136 (2018)
- ✓ For non-rotating progenitor model of  $M_{\text{star}} = 11.2 M_{\odot}$  Woosley et al., Reviews of Modern Physics 74, 1015 (2002)
- ✓ With the Boltzmann equation for neutrino transport being solved and special relativistic effect with a two energy grid technique. Nagakura et al., ApJS 214,16 (2014)
- ✓ Newtonian hydrodynamical equations & the Poisson equation for self-gravity were solved simultaneously.



# Method



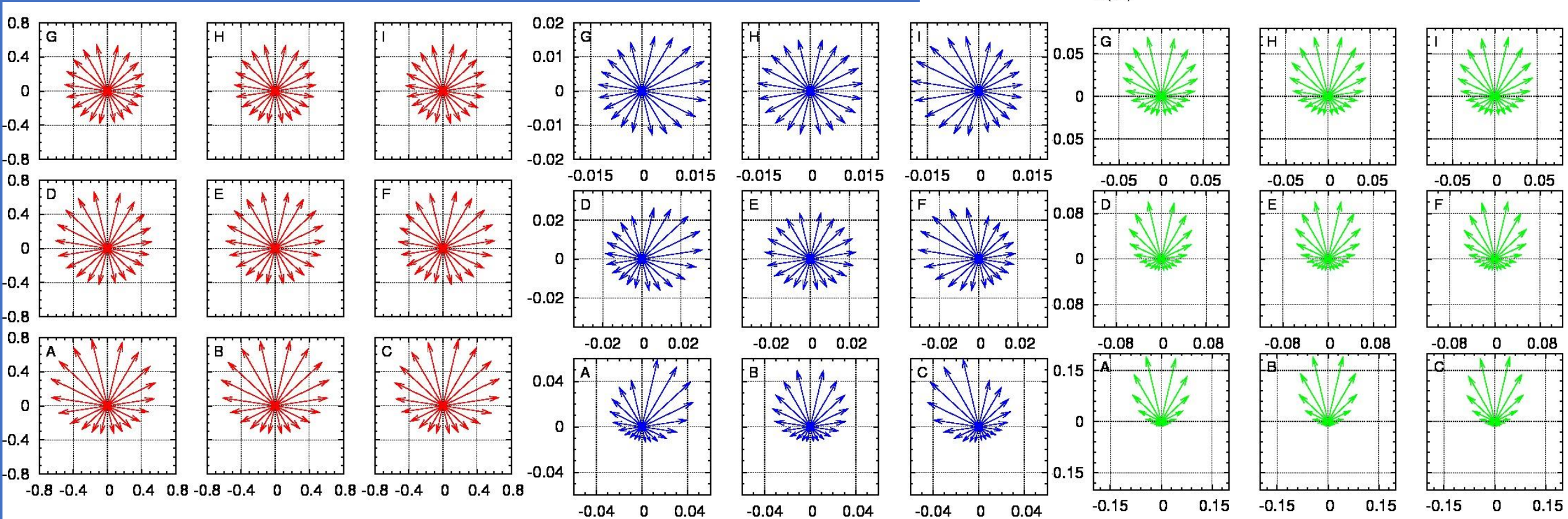
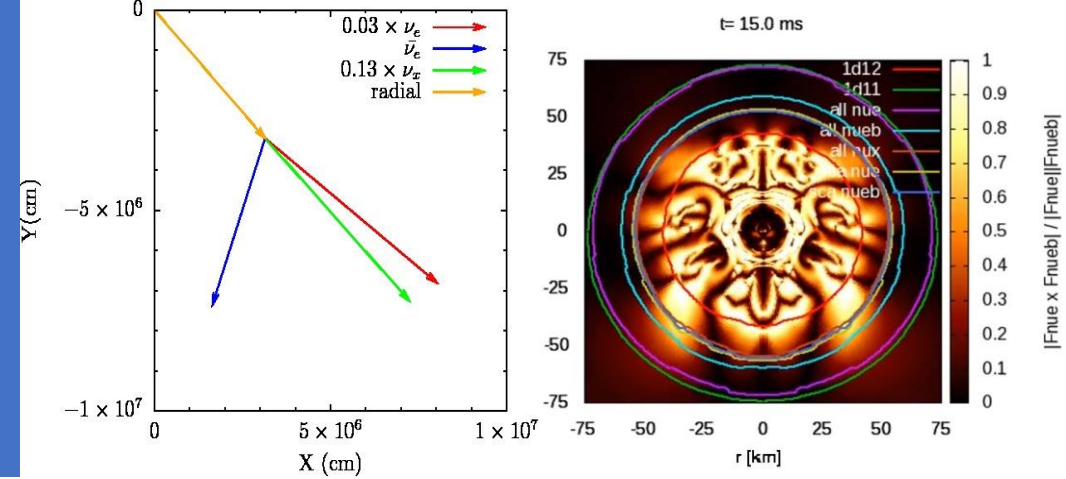
Entropy distributions at the central part of the core

Outer product of  $\nu_e$  and  $\bar{\nu}_e$   
(Misalignment between  $\nu_e$  and  $\bar{\nu}_e$ )

Energy integrated of fluxes of different neutrinos at the radial point at  $r = 44.8$  km

# Method

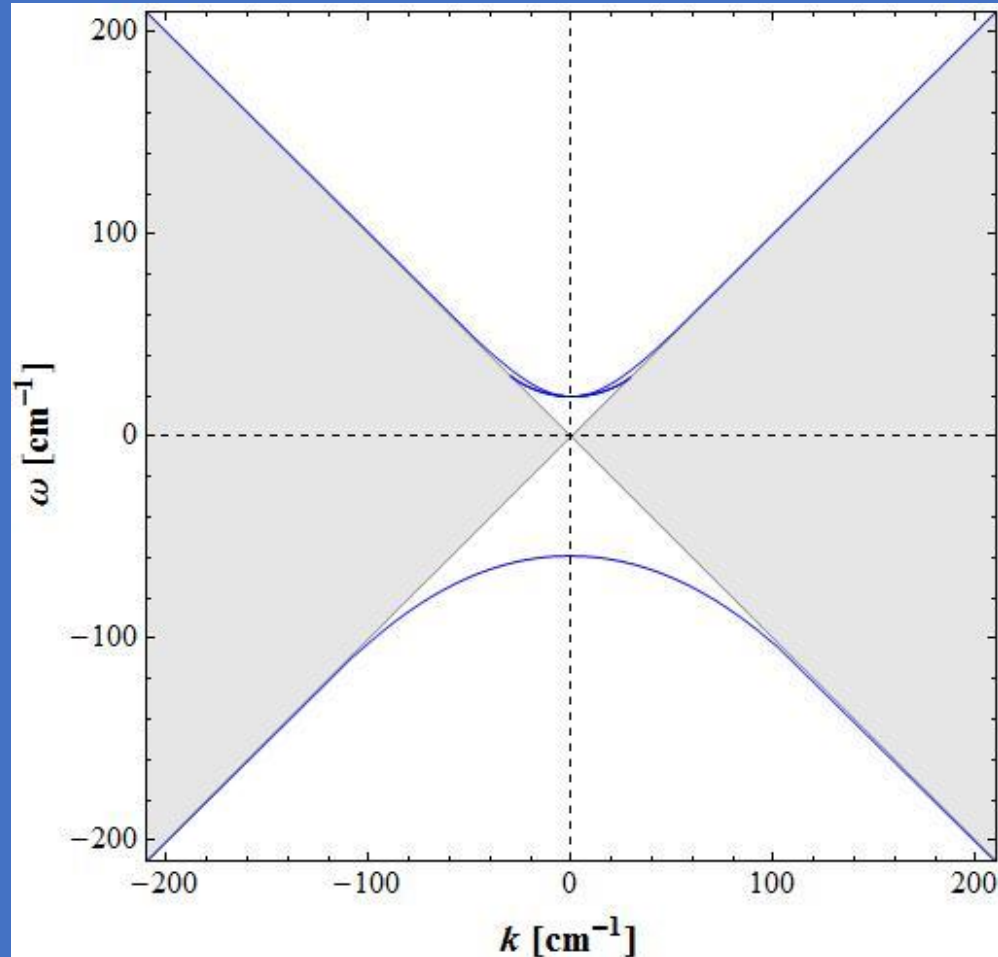
$r = 44.8$  km and  $t = 15.0$  ms post bounce



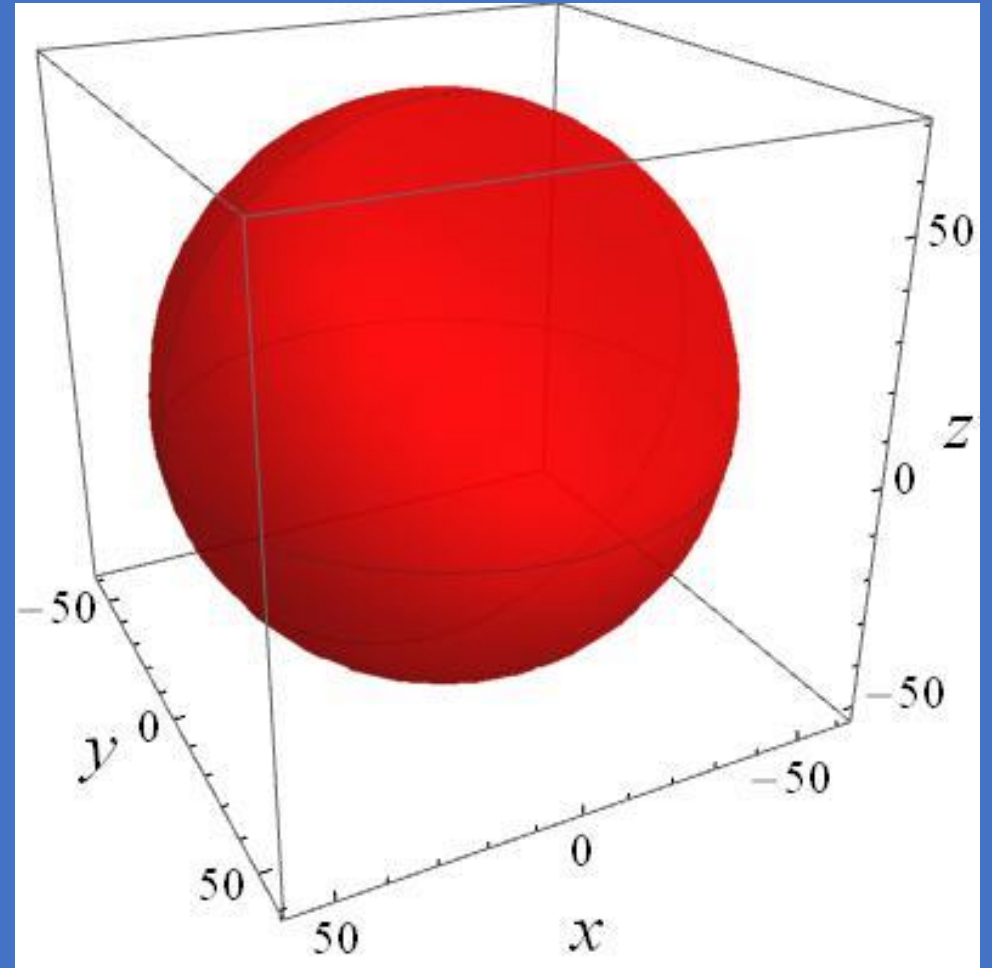
Angular distributions of  $\nu_e$ ,  $\bar{\nu}_e$  &  $\nu_x$  in meridional section at different azimuthal axis.

# Results – Realistic data

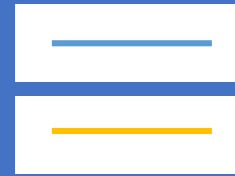
Dispersion relation



Rendering of the neutrino distribution



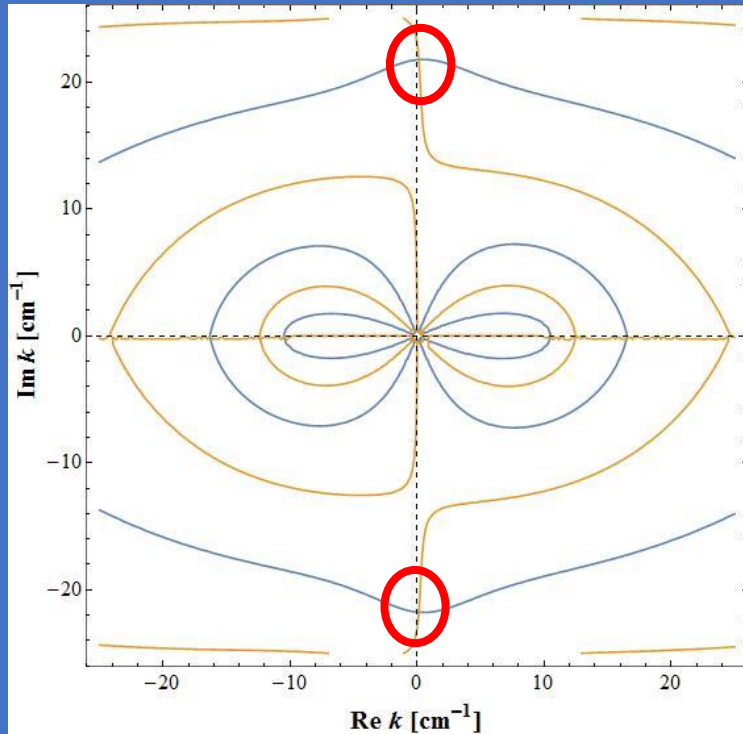
$$\det[\Pi] = 0$$



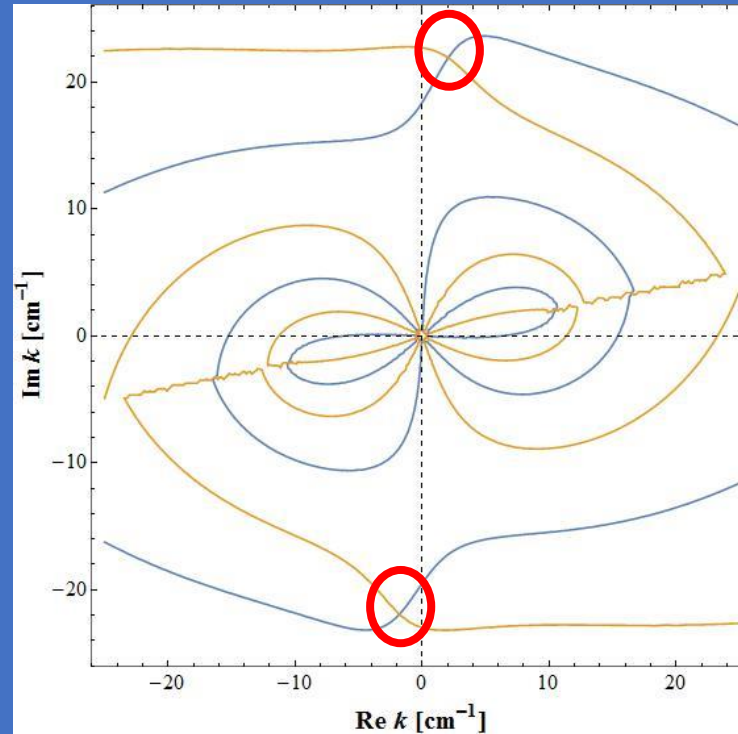
Real part of  $\det \Pi = 0$

Imaginary part of  $\det \Pi = 0$

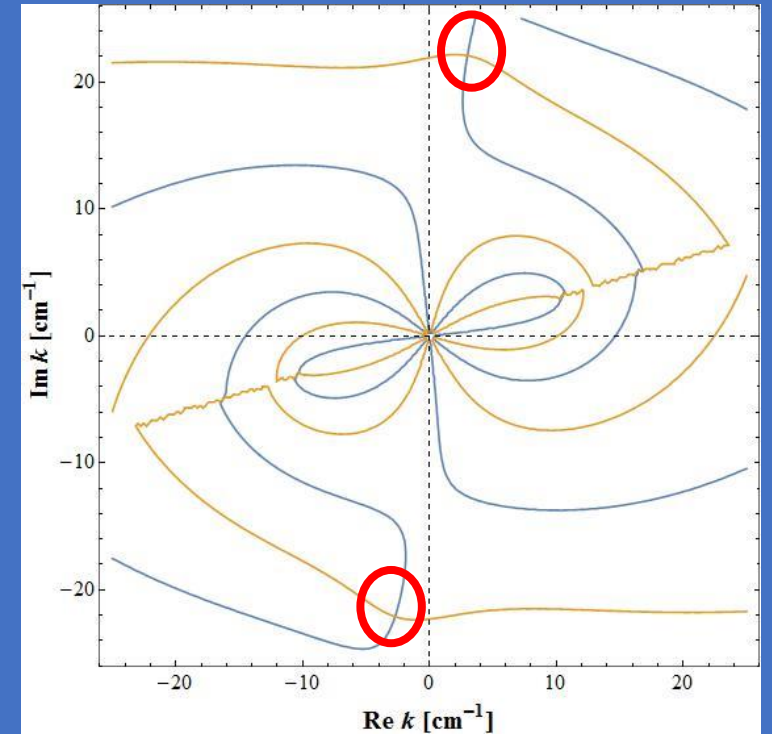
$$\omega = 10$$



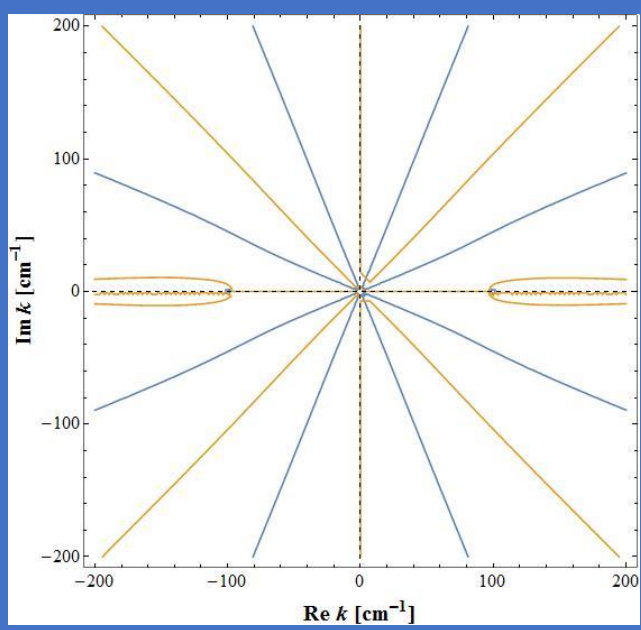
$$\omega = 10 + i 10$$



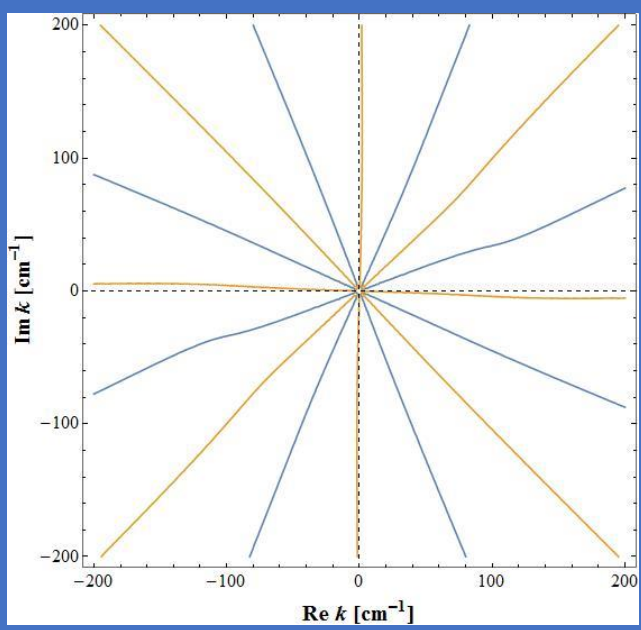
$$\omega = 10 + i 20$$



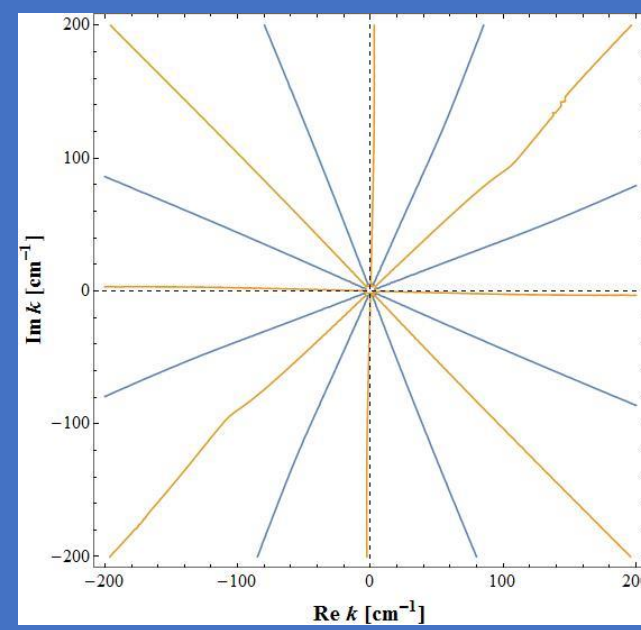
**NO** crossing in  $\text{Re } k$ , Means **NO** instability is expected



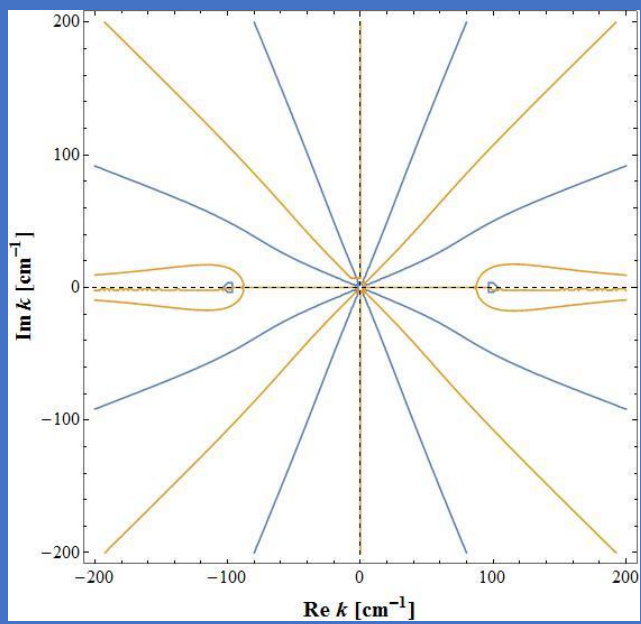
$$\omega = 100$$



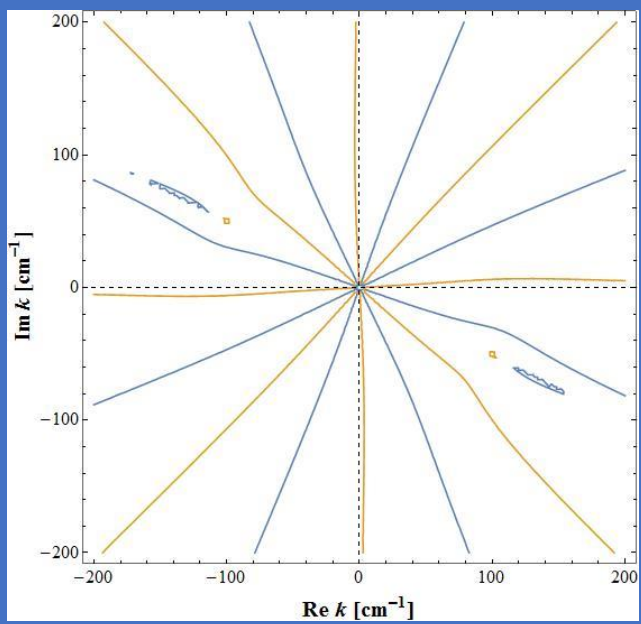
$$\omega = 100 + i 50$$



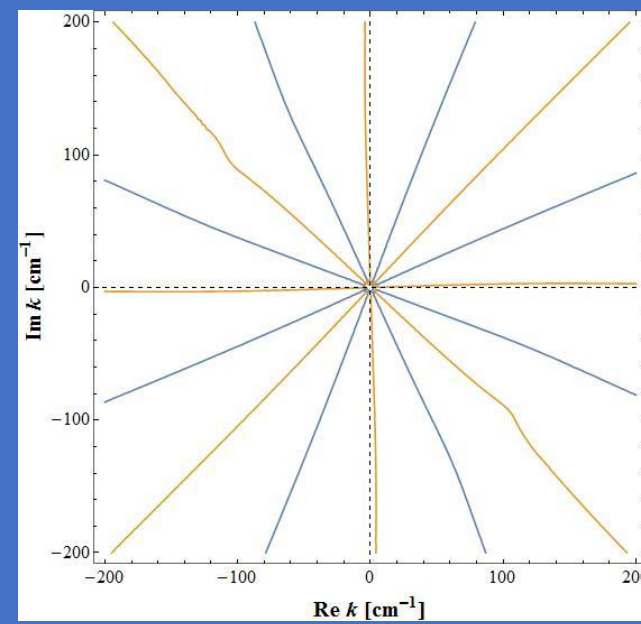
$$\omega = 100 + i 100$$



$$\omega = -100$$



$$\omega = -100 + i 50$$



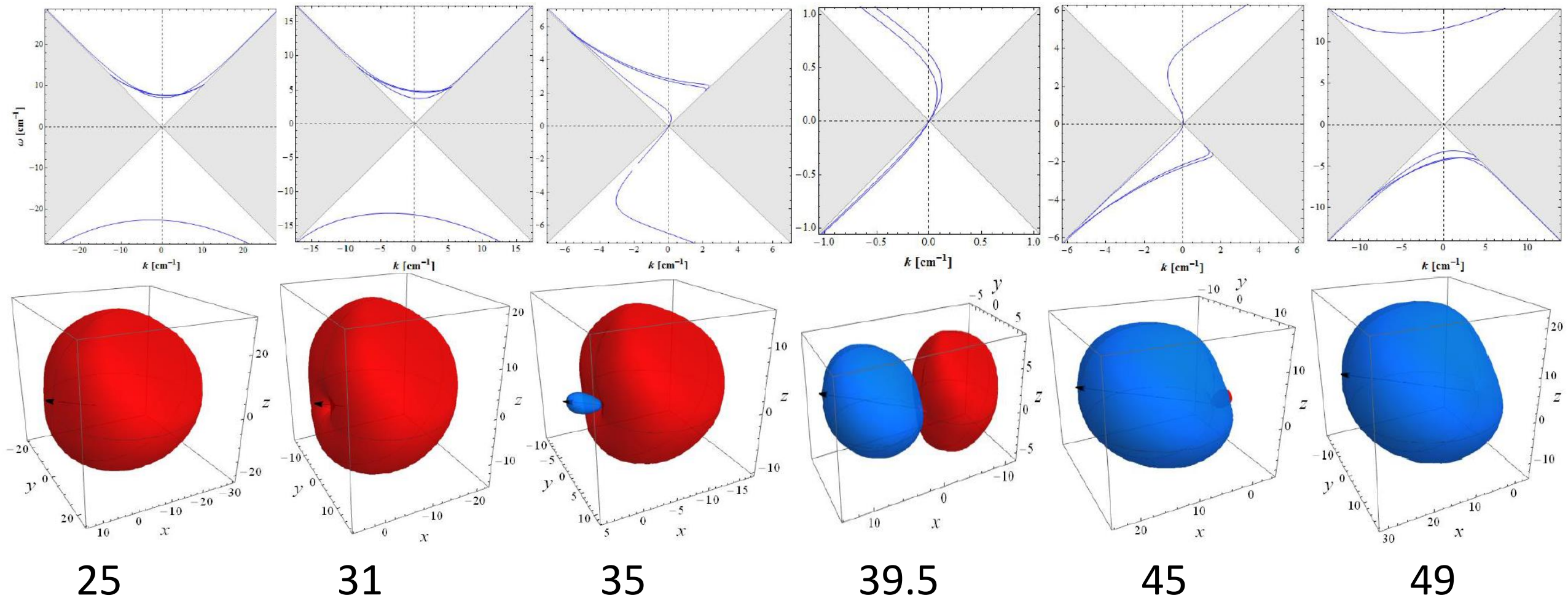
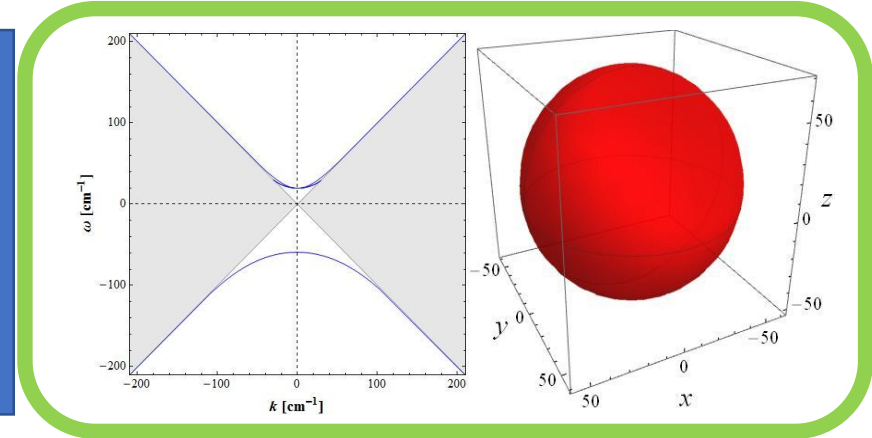
$$\omega = -100 + i 50$$

No oscillation so far ....

How & under what condition can  
we obtain the oscillation?

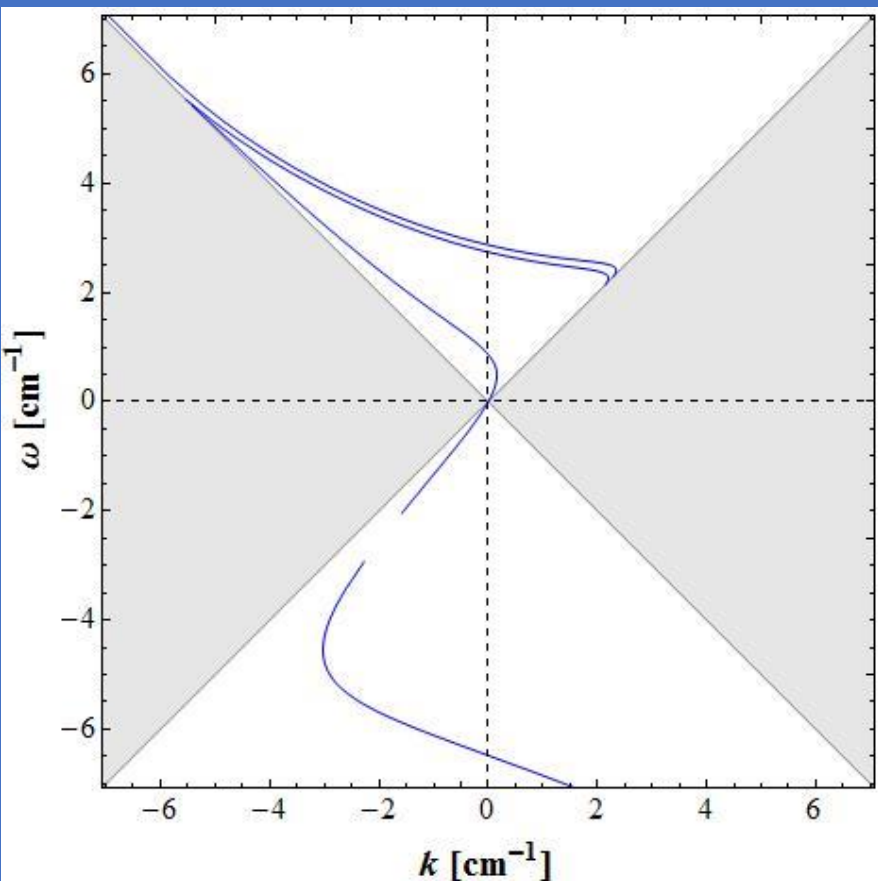


# Results– Scaled data ( $k$ – towards crossing )

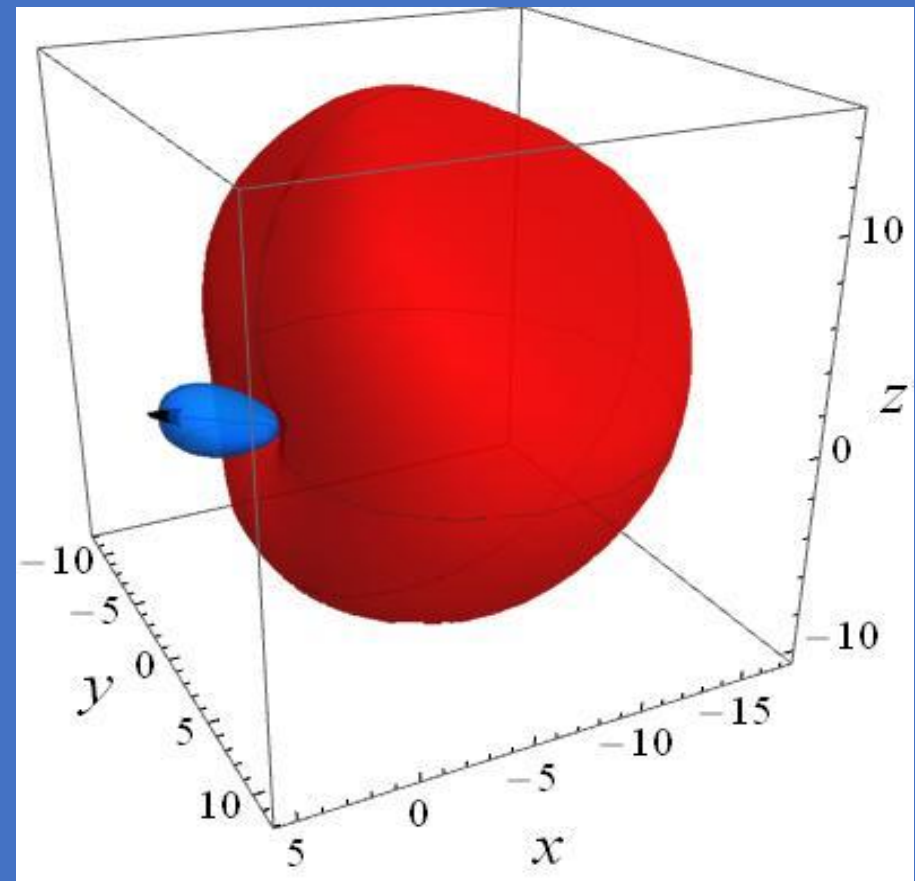


# Modified data

## Dispersion relation



## Neutrino distribution



 Solution

 Zone of avoidance

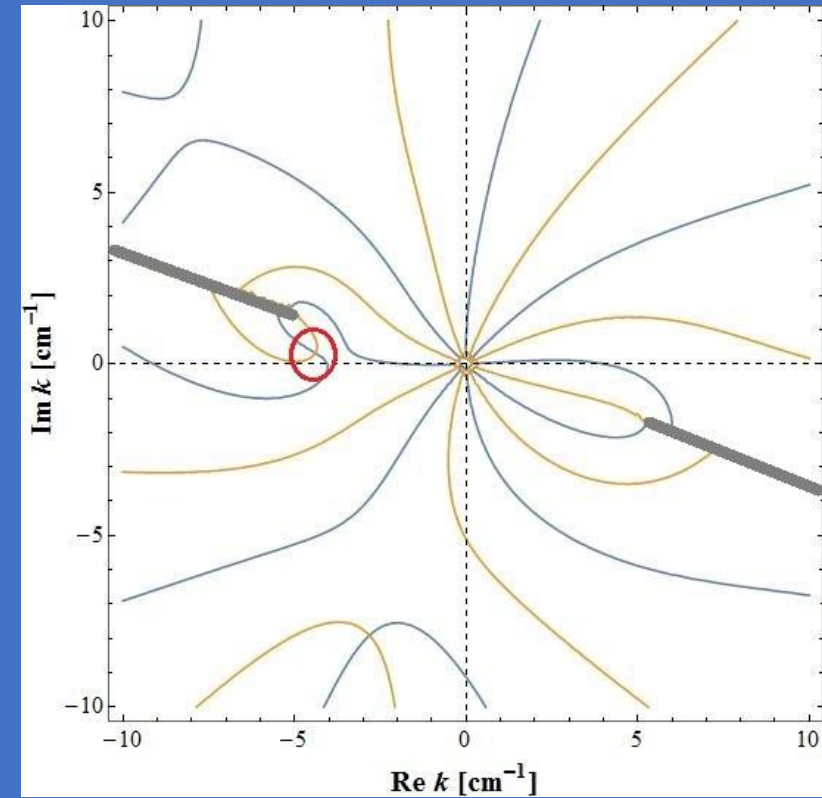
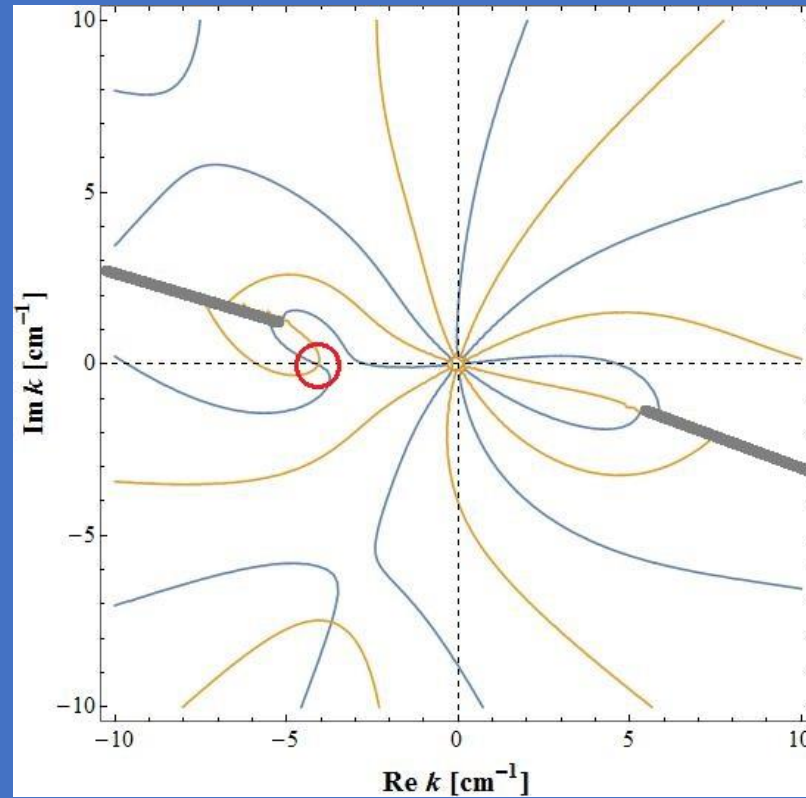
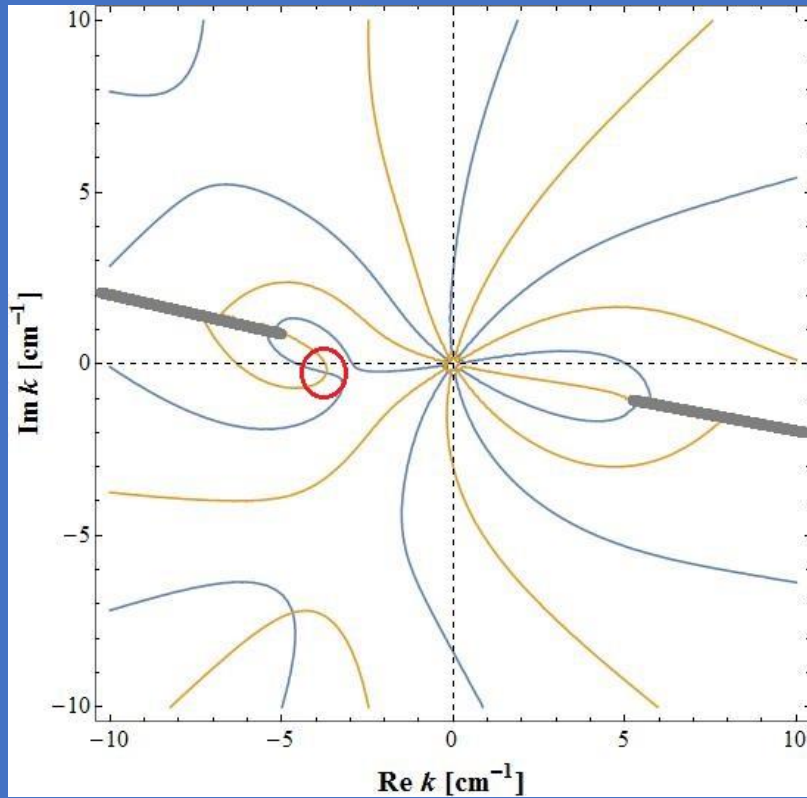
 Real part of  $\det \Pi = 0$

 Imaginary part of  $\det \Pi = 0$

$$\omega = -5 + i 1$$

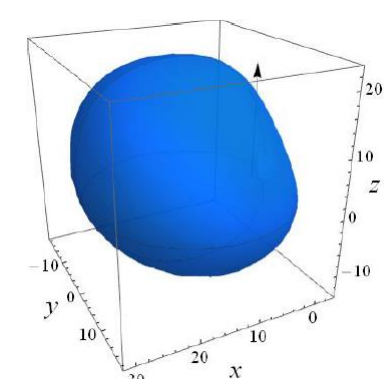
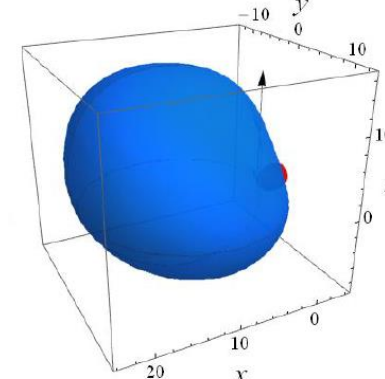
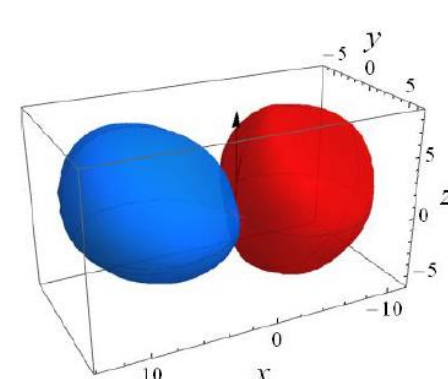
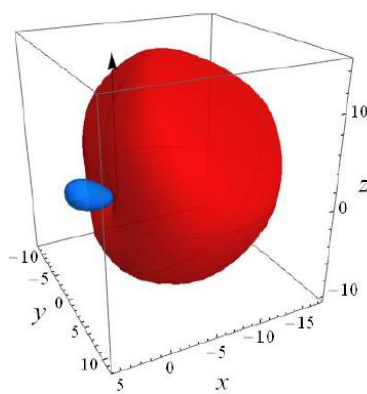
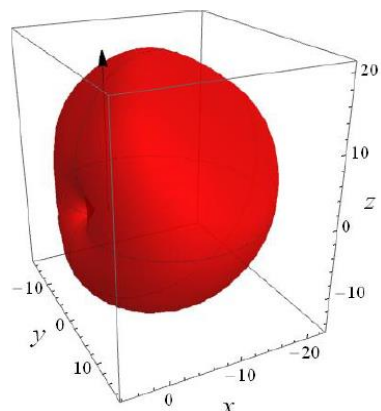
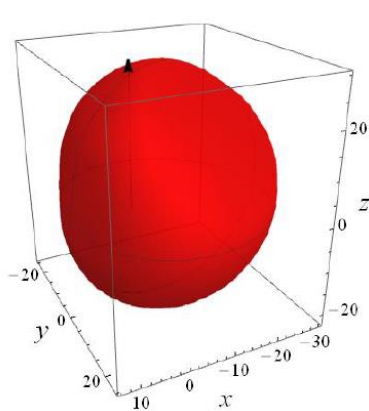
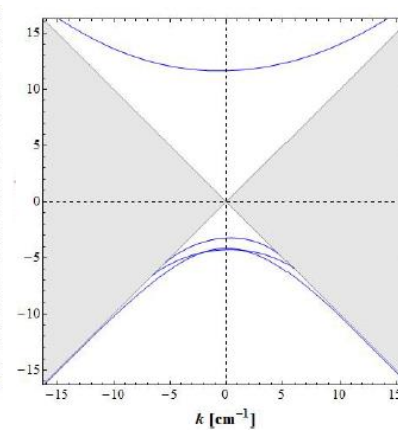
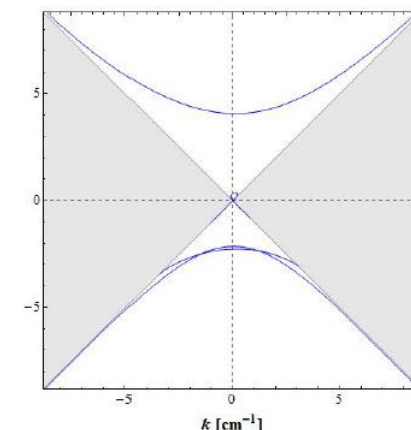
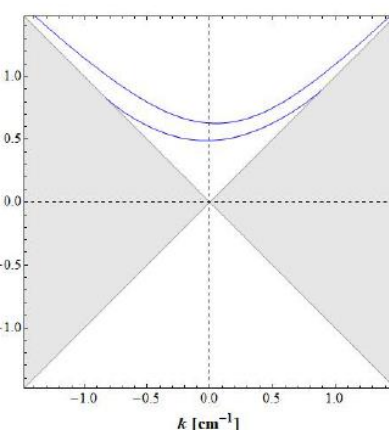
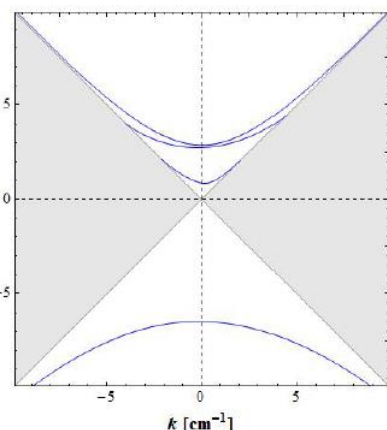
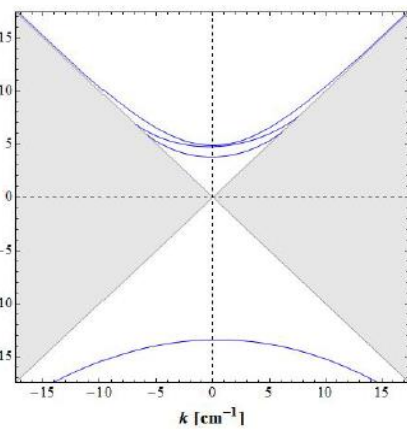
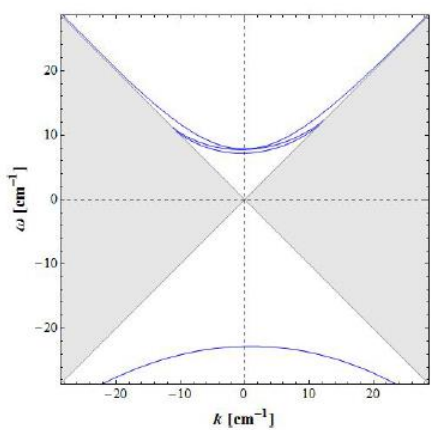
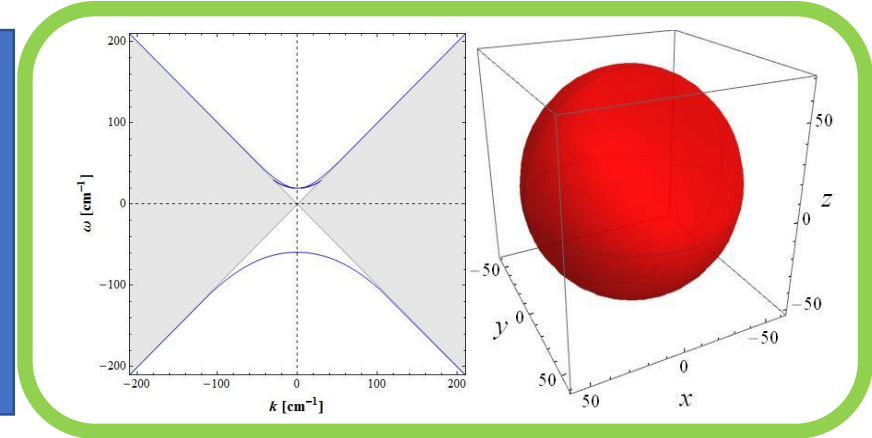
$$\omega = -5 + i 1.3$$

$$\omega = -5 + i 1.6$$



There **is** crossing in  $\text{Re } k$ , Means; instability **is** expected

# Results– Scaled data ( $k$ – towards $z$ –direction)



25

31

35

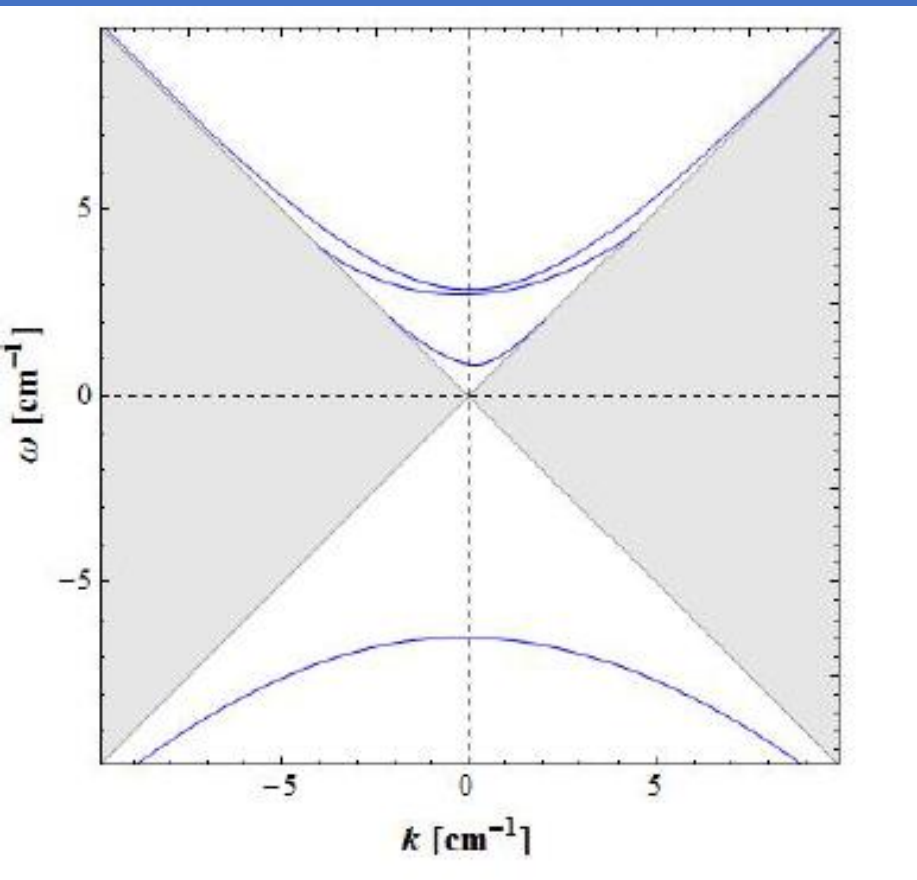
39.5

45

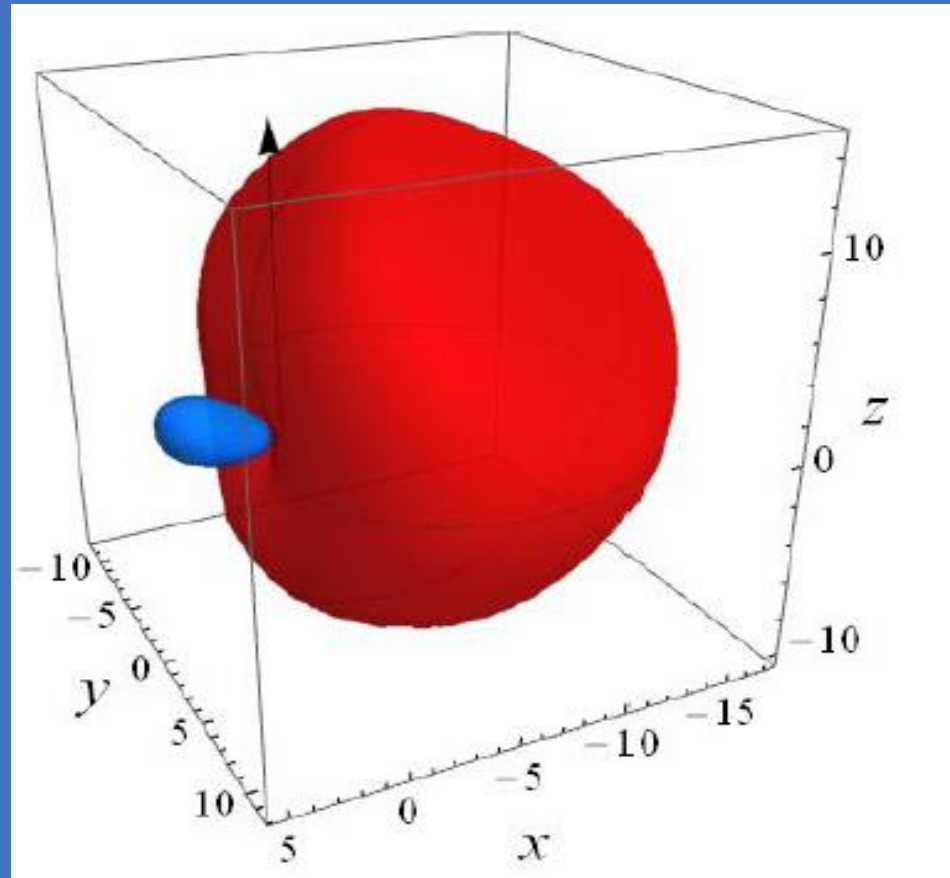
49

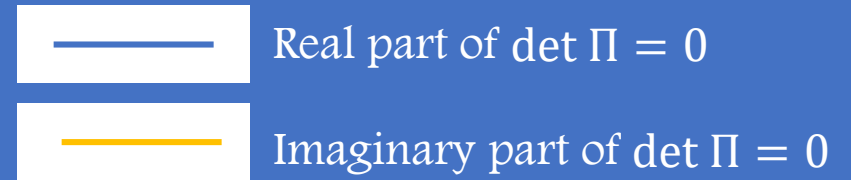
# Modified data

Dispersion relation



Neutrino distribution

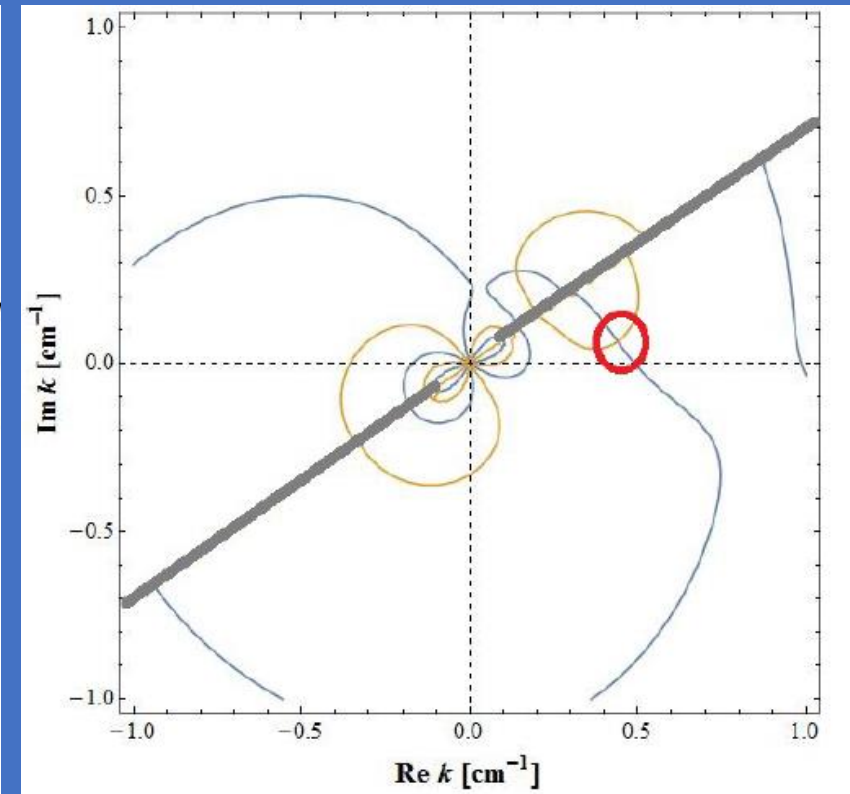
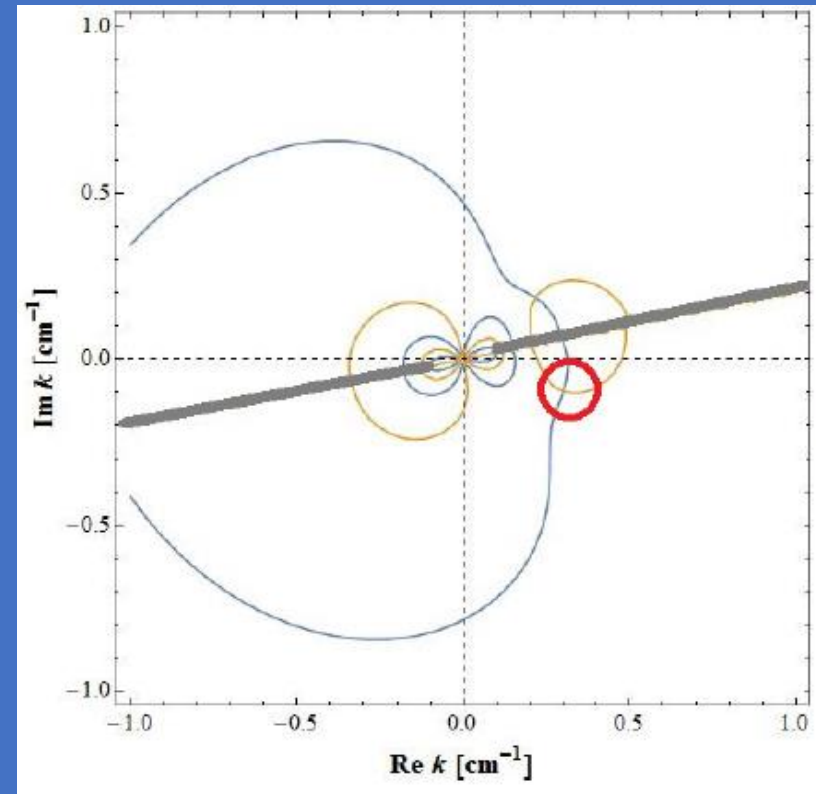
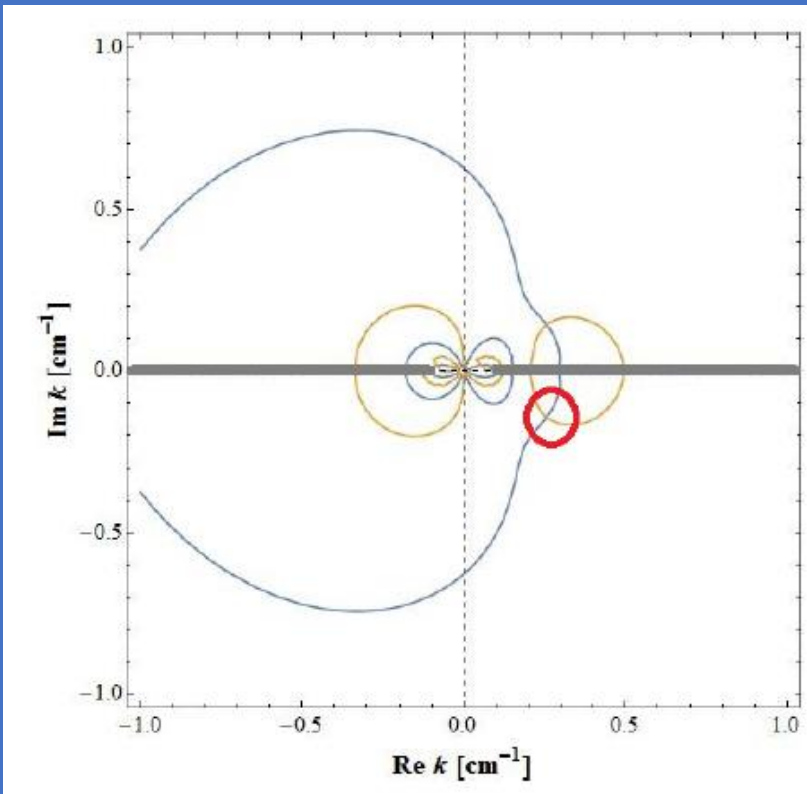




$\omega = 0.1 + i 0$

$\omega = 0.1 + i 0.02$

$\omega = 0.1 + i 0.07$



There **is** crossing in  $\text{Re } k$ , Means; instability **is** expected

# Summary **and** future works

- ✓ Fast-pairwise collective neutrino oscillations in core-collapse supernovae has been studied quantitatively.
- ✓ We confirm that there is no sign of instability which leads to the conversion in case of the realistic data analysis.
- ✓ The possible oscillation conditions in different radii and time steps has been studied.
- ✓ A systematic survey for all radii based on the angular distributions difference between the electron-neutrinos and anti-electron type neutrinos.
- ✓ As recently some data for the 3D simulations from my colleagues is available, the similar investigations might be a point of interest.
- ✓ This method might be applicable to other astrophysical objects such as NS-mergers disks, etc. We plan to investigate the conversion possibilities in these objects in near future.