Core-collapse Supernova Simulations with the Boltzmann-neutrino-transport

Akira Harada (UTAP)
Core-collapse supernovae

- Core-collapse Supernovae: explosive death of massive star
- Stellar core-collapse
  → explosion by released gravitational energy

SN1987A
©NASA, ESA/Hubble
Stellar core bounce

- Core-collapse by iron photodissociation/electron capture reactions
- Finally, the collapse is stopped by nuclear force
- The bounce shock is launched
- The energy of the shock is lost by photodissociation

→ The shock stalls
Neutrino heating mechanism

- How to revive the shock?
- The gravitational energy is contained in proto-neutron star.
- PNS evolves to be NS with emitting neutrinos
- The neutrino heating mechanism: emitted neutrinos heat the shock to revive.
The progress in CCSNe sim.

- Neutrinos rarely interacts
  → Boltzmann eq.
- Phase space dimension:
  ‣ 3 for spherical sym.
  ‣ 5 for axisym.
  ‣ 6 without sym.
- Approx. to reduce the cost

Microphysics is also important, but neglected here.
The progress in CCSNe sim.

- The gravity should be general relativistic
- Numerical relativity is the best, but difficult
- Newtonian, or approx.

Microphysics is also important, but neglected here.
The progress in CCSNe sim.

1D: fail to explode

\[ \nu \text{-transport} \quad \text{Full-Boltzmann} \quad \text{approx.} \quad \text{1D} \quad \text{2D} \quad \text{3D} \quad \text{Dimensionalilty} \]

Gravity

GR

approx.

Newton

1D

Trajectory of shock

\[ \rightarrow \text{shrinking} \]

Sumiyoshi+(2005)
The progress in CCSNe sim.

1D: fail to explode
2D: (sometimes) explode

Gravity

approx. GR
Newtonian

approx.
1D

2D

3D

ν -transport

Dimensionality

Full-Boltzmann

MPA (Mueller+ 2012)
The progress in CCSNe sim.

1D: fail to explode
2D: (sometimes) explode

- observed energy: $10^{51}$ erg, simulated energy: $10^{50}$ erg
- neutrino transport: not Boltzmann eq., but approx. eq.
- even qualitatively different results
  - Observed explosion is not yet reproduced

\(\nu\)-transport  

Dimensionality
The progress in CCSNe sim.

1D: fail to explode
2D: (sometimes) explode
3D: (sometimes) explode

\(\nu\)-transport

Gravity

1D

approx. GR
Newtonian

2D

approx. GR

3D

Full-Boltzmann

Princeton (Vartanyan+ 2018)

Dimensionality
Our work

1D: fail to explode
2D: (sometimes) explode
3D: (sometimes) explode

\( \nu \)-transport

Dimensionality

Gravity

Full-Boltzmann

approx. 1D
approx. 2D
approx. 3D

GR
approx. GR
Newtonian
Our work

Acceleration terms to track the PNS
PNS kick may be found (Nagakura in prep.)

- Boltzmann equation
  \[
  \frac{d\mathbf{x}}{dt} = \sum \left( \frac{dp_i}{dt} \right) \frac{\partial f}{\partial p_i}
  \]

- Newtonian Hydrodynamics
  \[
  \begin{align*}
  \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) &= 0 \\
  \frac{\partial \rho Y_e}{\partial t} + \nabla \cdot (\rho Y_e \mathbf{v}) &= \rho \Gamma \\
  \frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot (\rho \mathbf{v} \mathbf{v} + P \mathbf{I}) &= -\rho \nabla \Phi + M^i + \rho \ddot{\beta} \\
  \frac{\partial (\rho (e + \frac{1}{2} \mathbf{v}^2))}{\partial t} + \nabla \cdot \left( \rho \mathbf{v} (e + \frac{1}{2} \mathbf{v}^2 + \frac{P}{\rho}) \right) &= -\rho \mathbf{v} \cdot \nabla \Phi + Q + \rho \mathbf{v} \cdot \ddot{\beta}
  \end{align*}
  \]

- Newtonian Gravity
  \[
  \Delta \Phi = 4\pi G \rho
  \]
Our work

LSEOS
Explode

FS EOS
Fail

Nagakura+(2018)

- The Boltzmann-radiation-hydrodynamics code
- There are several EOS models
  - EOS comparison paper: LS VS FS
- The simulation with LS EOS shows shock revival, but probably due to an artifact of the single-nuclear approximation
- Detailed analysis will appear (Harada in prep.)
Rotation

- Both positive and negative effects on shock revival
- Neutrino distributions are distorted
- (Thanks to the Boltzmann solver,) The accuracy of approximation is checked.

- Presented in Harada+ (2019)
Setup

- 11.2 M☉ progenitor of Woosley+ (2002)
- Shellular rotation (almost the fastest according to current stellar evolution theory)

\[ \Omega(r) = \frac{1 \text{ rad/s}}{1 + (r/10^8 \text{ cm})^2} \]

- Furusawa-Shen equation of state
- Neutrino reactions
  \[
  \begin{align*}
  \nu_e + n & \leftrightarrow e^- + p & \nu + A & \leftrightarrow \nu + A \\
  \bar{\nu}_e + p & \leftrightarrow e^+ + n & \nu + e^- & \leftrightarrow \nu + e^- \\
  \nu_e + A & \leftrightarrow e^- + A' & \nu + \bar{\nu} & \leftrightarrow e^- + e^+ \\
  \nu + N & \leftrightarrow \nu + N & N + N & \leftrightarrow N + N + \nu + \bar{\nu}
  \end{align*}
  \]
- Notation: \( \nu_x = \nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau \)
Entropy distribution

- Time evolution until ~200 ms after bounce.
Shock evolution

- Postbounce evolution until ~200 ms
- The difference between rotating & non-rotating model
Neutrino ang. distribution

- Distribution functions at ~10 ms after bounce.

1 MeV
4 MeV
19 MeV

~60 km
~170 km
Neutrino ang. distribution

- Distribution functions at ~10 ms after bounce.

1 MeV
4 MeV
19 MeV

~170 km
Moment formalism

- Boltzmann equation

\[ p^\alpha \frac{\partial f}{\partial x^\alpha} \bigg|_{p^i} - \Gamma^i_{\alpha\beta} p^\alpha p^\beta \frac{\partial f}{\partial p^i} \bigg|_{x^\alpha} = (-p^\alpha \hat{u}_\alpha) S_{\text{rad}} \]

zero-th moment

\[ \int d\Omega \frac{\partial E}{\partial t} + \frac{\partial F^i}{\partial x^i} = S_0 \]

first moment

\[ \int d\Omega \frac{\partial F^i}{\partial t} + \frac{\partial P^{ij}}{\partial x^j} = S_1^i \]

\[ E \sim \int d\Omega p^0 f \quad F^i \sim \int d\Omega p^i f \quad P^{ij} \sim \int d\Omega p^i p^j f \]
Eddington tensor

- Evaluation of M1-closure scheme-Eddington tensor

\[ E^{ij} = \begin{cases} 
\frac{P^{ij}}{E} & \text{Boltzmann-Eddington tensor} \\
\frac{3\chi - 1}{2} \delta^{ij} + \frac{1 - \chi}{2} \frac{F^i F^j}{F^2} & \text{M1-Eddington tensor} 
\end{cases} \]

\[
\frac{w}{\chi} = \frac{3 + 4\tilde{F}^2}{5 + 2\sqrt{4 - 3\tilde{F}^2}}, \quad \tilde{F} = \frac{|F|}{E}
\]

Eddington fac.  Flux fac.
Eddington factor

- Eddington tensor at ~10 ms after bounce
- spatial distribution of eigenvalues
- ~20% error in M1-closure scheme
Eddington factor

- Eddington tensor at \( \sim 10 \) ms after bounce
- Comparison between Boltzmann- and M1-Edd. factors
- Information which distinguish these situations may improve the accuracy

\[
\begin{array}{c}
\text{Boltzmann} \\
\text{M1} \\
\end{array}
\]

- Prolateness of distribution
- M1: estimated from deviation

radius \( r \) [km]
Future prospects

1D: fail to explode
2D: (sometimes) explode
3D: (sometimes) explode

Current work

ν -transport

Gravity

Full-Boltzmann

Dimensionality

GR
approx. GR
Newtonian
Future prospects

1D: fail to explode
2D: (sometimes) explode
3D: (sometimes) explode

Current work
3D-Boltzmann

\(\nu\)-transport

\text{Dimensionality}
Future prospects

1D: fail to explode
2D: (sometimes) explode
3D: (sometimes) explode

Current work

3D-Boltzmann
GR-Boltzmann

\( \nu \) -transport

Dimensionality

Gravity

- GR
- approx. GR
- Newtonian

GR-Boltzmann
- Numerical rel.
- GR-hydro
- Boltzmann in curved spacetime
Future prospects

1D: fail to explode
2D: (sometimes) explode
3D: (sometimes) explode

Current work
3D-Boltzmann
GR-Boltzmann

Final goal

$\nu$-transport

Dimensionality
Summary

- Simulations for the neutrino heating mechanism of CCSNe have been performed.
- The Boltzmann-radiation-hydrodynamics code is one of the most sophisticated code.
- Unique feature is obtained by using the Boltzmann code.