Condition for the growth of the Rayleigh-Taylor instability at the relativistic jet interface

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What is a relativistic jet?

collimated bipolar outflow from gravitationally bounded object

- active galactic nuclei (AGN) jet: $\gamma \sim 10$
- microquasar jet: $v \sim 0.9c$
- Gamma-ray burst: $\gamma > 100$

Lorentz factor

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}}$$

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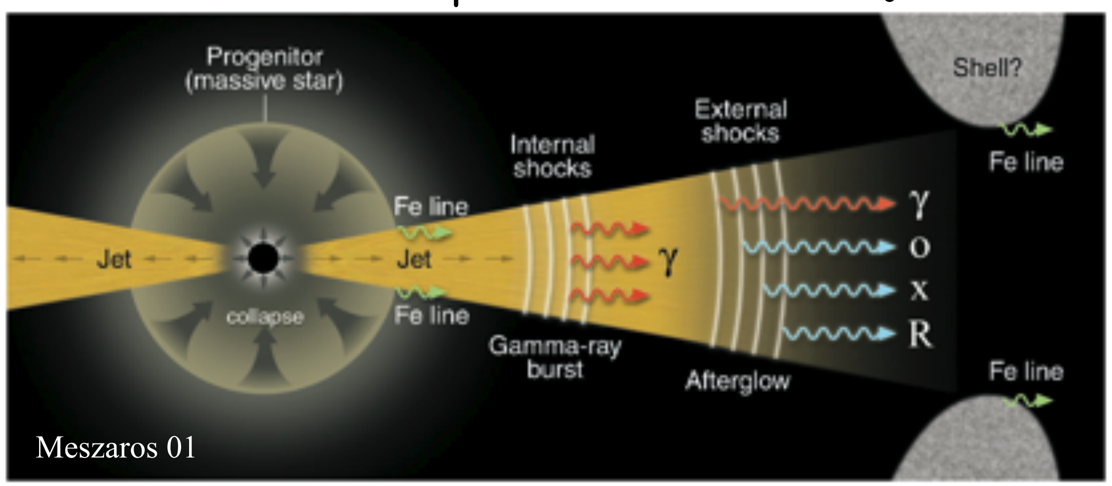
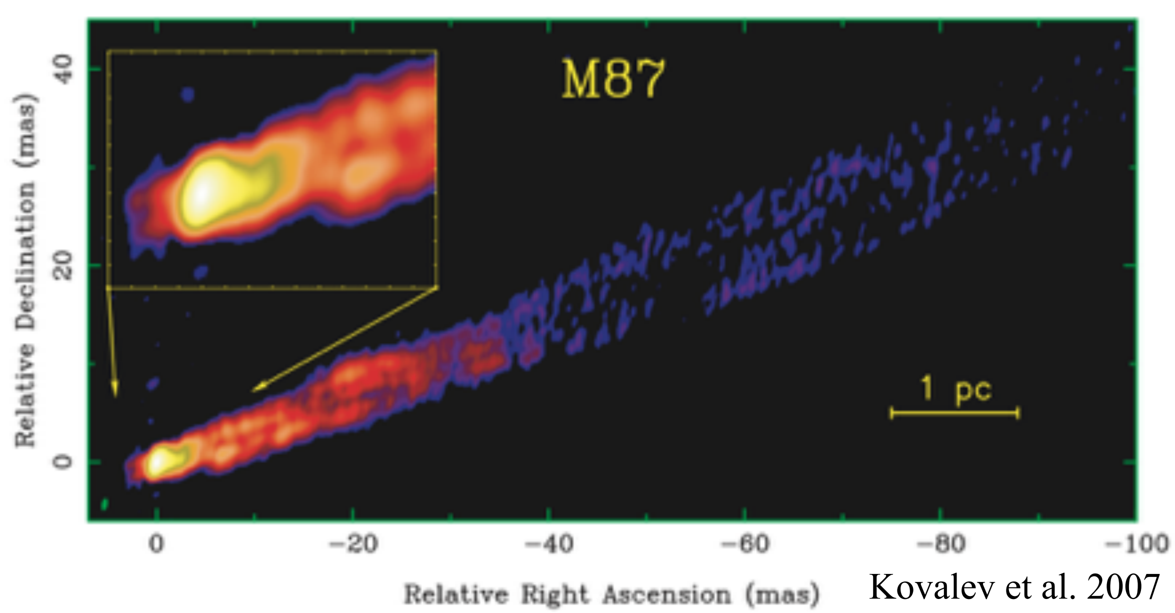
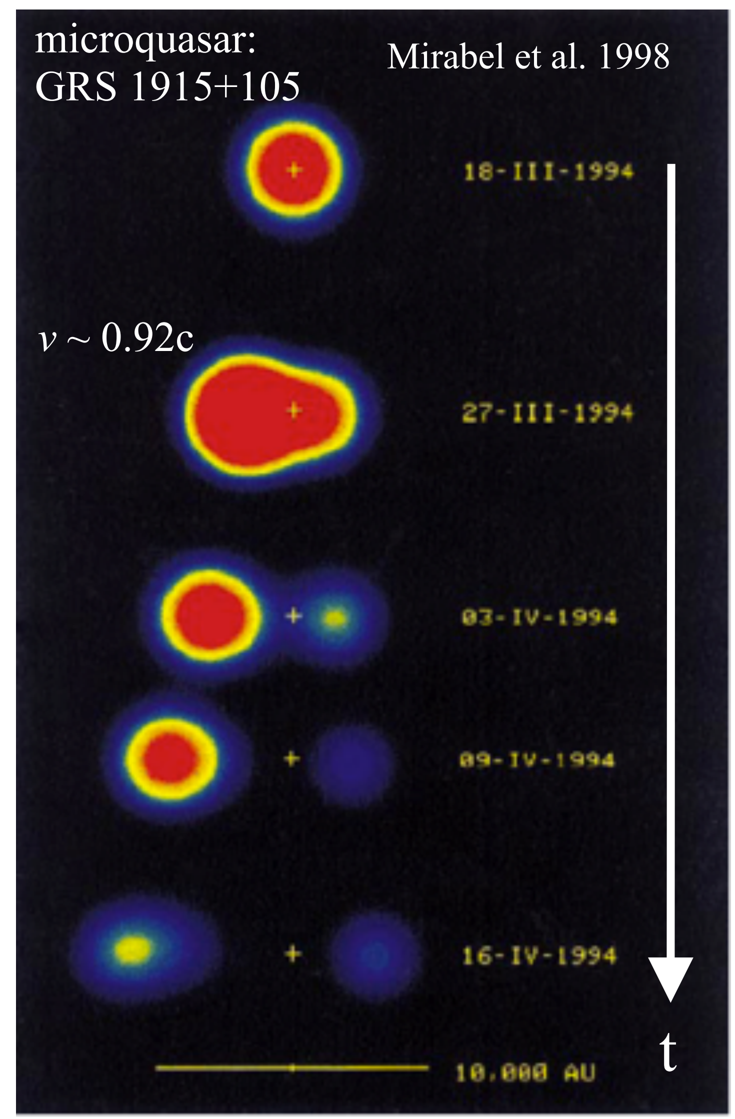
schematic picture of the GRB jet

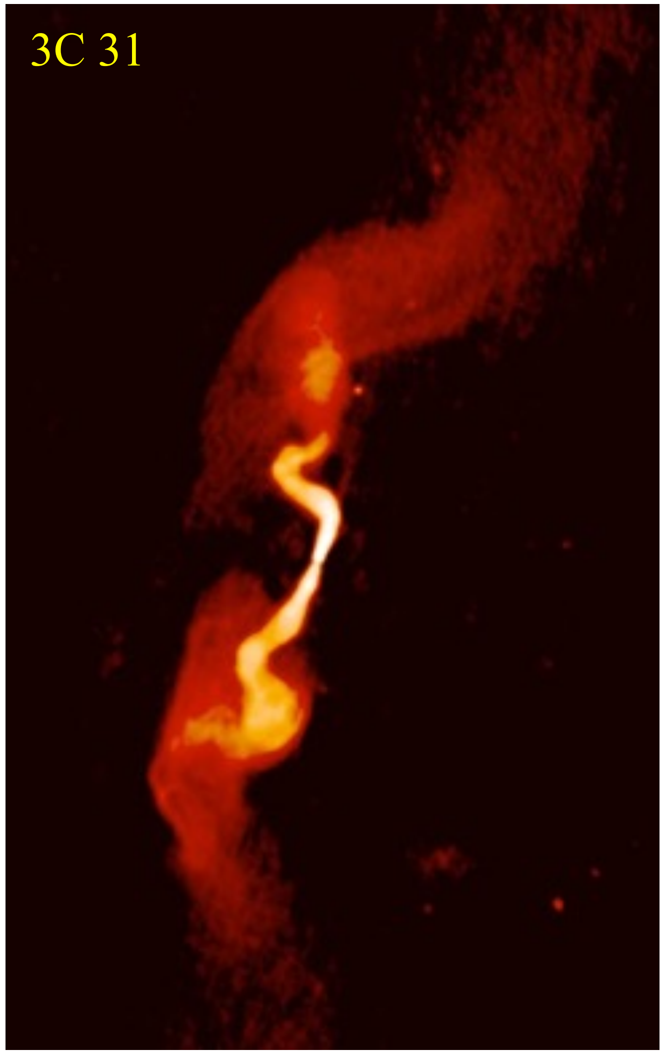




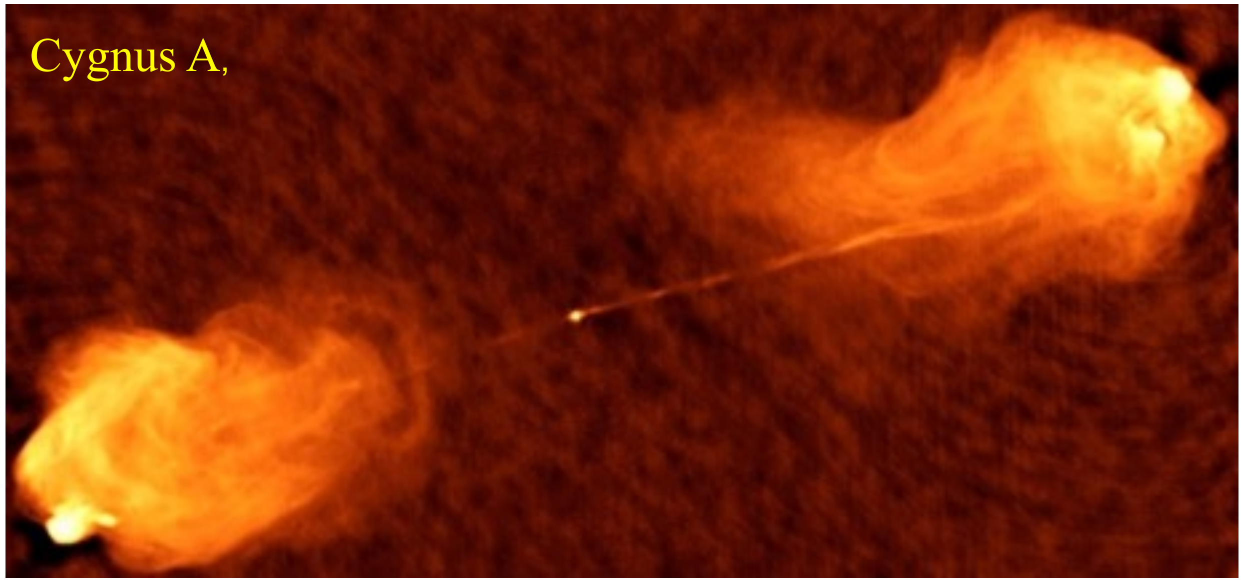
Kovalev et al. 2007



Morphological Dichotomy of the Jet



FR I



FR II

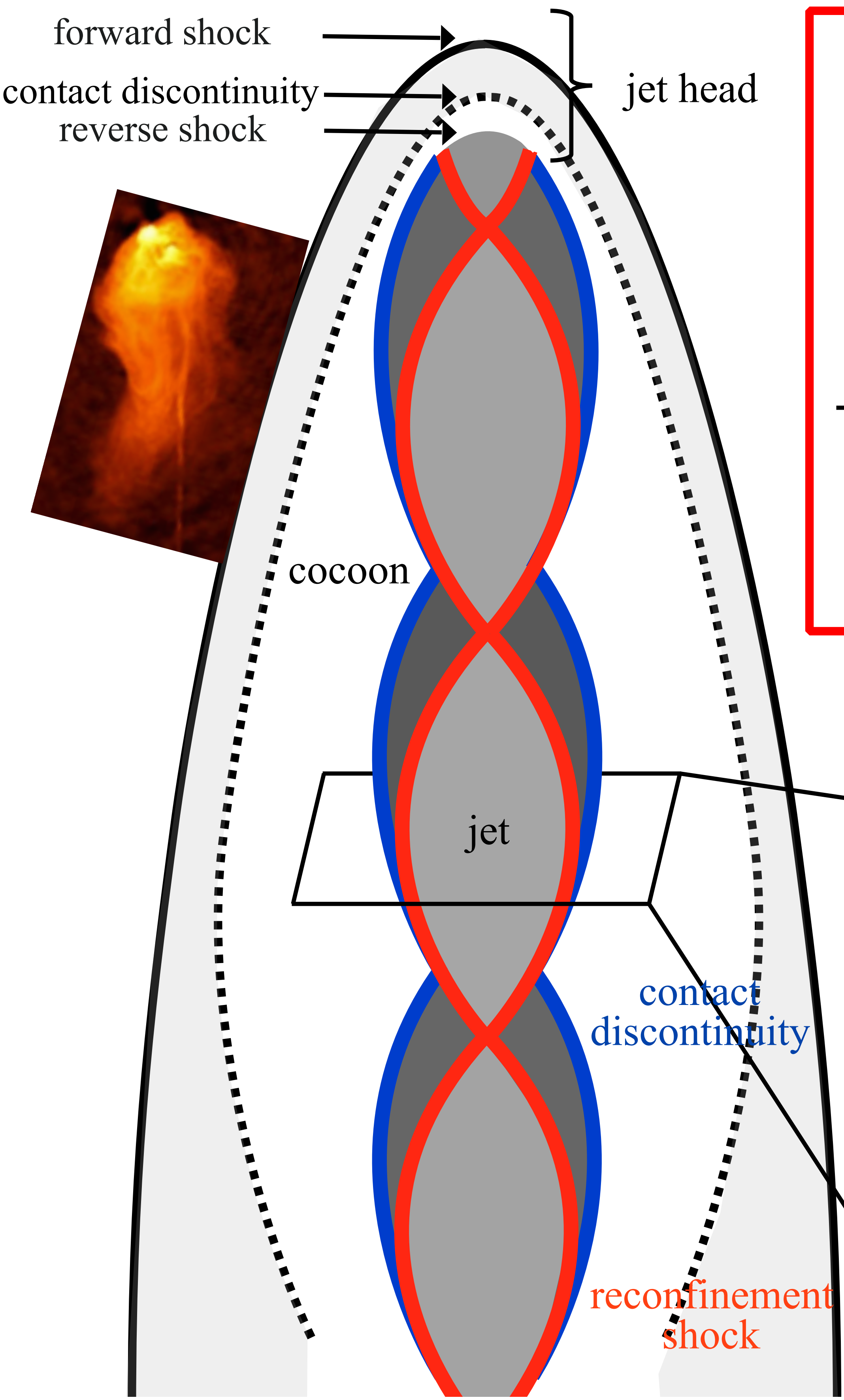
- Morphology is one of the most fundamental property of the relativistic jet.
- A morphological dichotomy between FR I and FR II
  - A complex combination of several intrinsic and external factors
- Instabilities play an important role in the morphology and stability of the jet through the interaction between the jet and external medium.

Why is stability of the jet interface so important?

- The stability of the jet interface is also related to the inhomogeneity of the jet and the evolution of the turbulence inside/outside the jet.
- They may affect on the radiation from the jet associated with the particle and/or photon acceleration.
- Multiple outflow layers inside the relativistic jet are essential to reproduce the typical observed spectra of GRBs (Ito et al. 2014)
- The development of the turbulence inside the jet is important point in order to discuss the mechanism of the particle acceleration in the context of the GRB (Asano & Terasawa 2015) and blazer (Asano & Hayashida 2015; Inoue & Tanaka 2016).

A promising mechanism to make the interface of the jet unstable is thought to be the Kelvin-Helmholtz instability. Many authors have investigated the growth of the Kelvin-Helmholtz instability. However, the growth of the Rayleigh-Taylor instability at the interface of the relativistic jet is not still understood well.

Modeling for the growth of RTI



initially hydrostatic equilibrium

$$-\frac{\partial P}{\partial y} = \gamma^2 \rho h g = \gamma^2 \left(\rho + \frac{\Gamma}{\Gamma - 1} \frac{P}{c^2} \right) g \qquad h = 1 + \frac{\Gamma}{\Gamma - 1} \frac{P}{\rho c^2}$$

if $y = 0$, then $P = P_0$

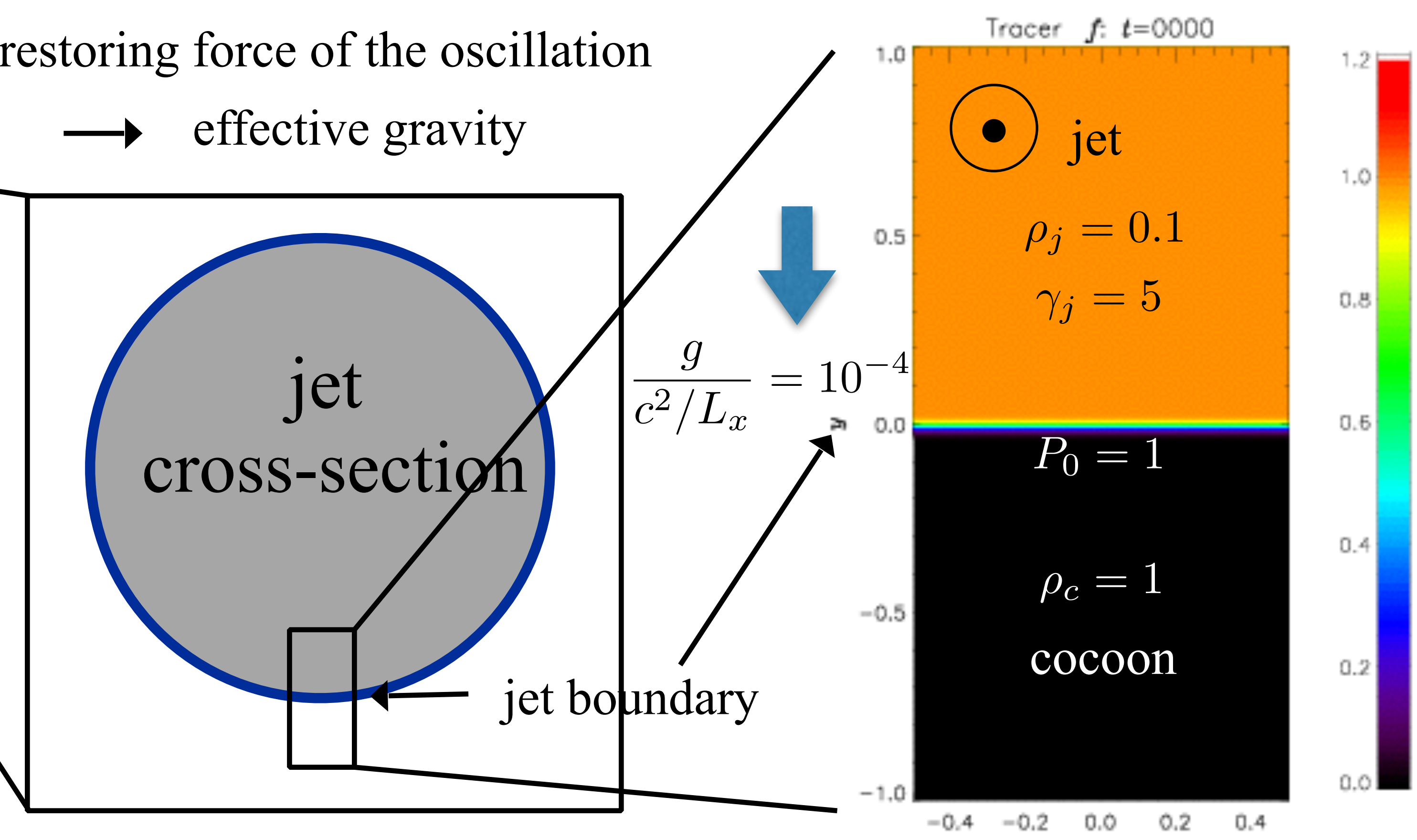
$$P = P_0 e^{-y/H} + \frac{\rho c^2 (\Gamma - 1)}{\Gamma} (e^{-y/H} - 1) \qquad \frac{1}{H} \equiv \frac{\Gamma}{\Gamma - 1} \frac{\gamma^2 g}{c^2}$$

initial perturbation

$$v_y = \frac{\delta v}{4} (1 + \cos(2\pi x))(1 + \cos(2\pi x/10)) \qquad \delta v = 10^{-4}$$

restoring force of the oscillation

$\longrightarrow$ effective gravity



in the rest frame of the decelerating jet interface

Estimation of the pressure scale height

Since the effective gravity has its origin in the radial oscillating motion of the jet, assuming the amplitude of the jet oscillation is roughly equal to the jet radius, the effective gravity is estimated as follows;

$$g \sim \frac{r_{jet}}{\tau_{osci}^2}$$

harmonic oscillator

$$a = \frac{\partial^2 r}{\partial t^2} = -\frac{r}{T^2}$$

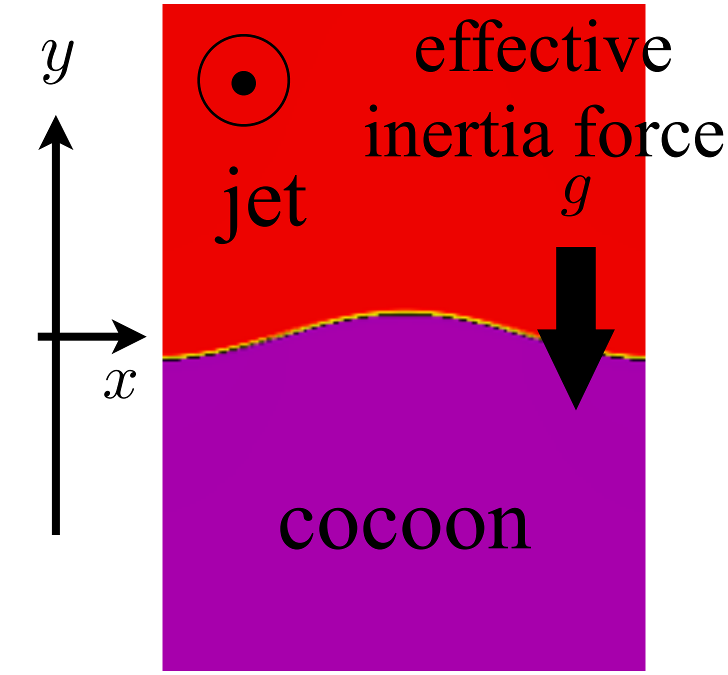
Here τ is the typical oscillation time of the jet and given by the propagation time of the sound wave over the typical oscillating radius of the jet (JM et al. 2012);

$$\tau_{osci} = \gamma_{jet} r_{jet} / C_s$$

Pressure scale height: $H \equiv \frac{\Gamma - 1}{\Gamma} \frac{c^2}{\gamma^2 g} \sim r_{jet}$

Since we neglect the impact of the curvature of the jet radius in this study, the position at y = H is far from the jet-cocoon interface (y = 0).

Derivation of linear growth of RTI



equation of motion:

$$\gamma^2 \rho h \left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} \right] + \nabla P + \cancel{\frac{\mathbf{v}}{c^2} \frac{\partial P}{\partial t}} - \gamma^2 \rho h \mathbf{g} = \mathbf{0}$$

assumption: $v_x, v_y \ll c_s$ $\longrightarrow$ incompressible fluid

temporal variation of the pressure is negligible

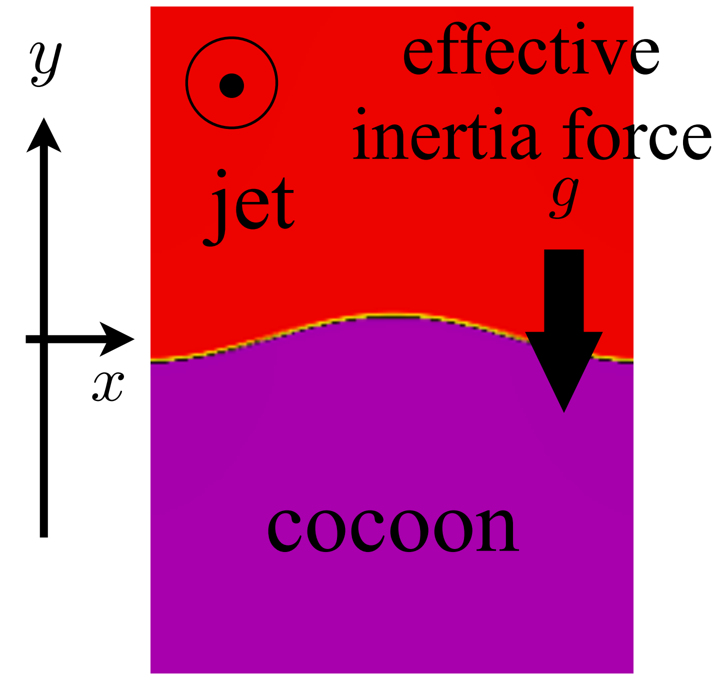
basic equations:

equation of motion: x	$\gamma^2 \rho h \left(\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} \right) = -\frac{\partial P}{\partial x}$
equation of motion: y	$\gamma^2 \rho h \left(\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} \right) = -\frac{\partial P}{\partial y} - \gamma^2 \rho h g$
continuity	$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0$
incompressible condition	$\frac{\partial}{\partial t}(\gamma\rho) + v_x \frac{\partial}{\partial x}(\gamma\rho) + v_y \frac{\partial}{\partial y}(\gamma\rho) = 0$
conservation of entropy	$\frac{\partial s}{\partial t} + v_x \frac{\partial s}{\partial x} + v_y \frac{\partial s}{\partial y} = 0$

Derivation of linear growth of RTI

equation of motion:

$$\gamma^2 \rho h \left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} \right] + \nabla P + \cancel{\frac{\mathbf{v}}{c^2}\frac{\partial P}{\partial t}} - \gamma^2 \rho h \mathbf{g} = \mathbf{0}$$



assumption: $v_x, v_y \ll c_s \longrightarrow$ incompressible fluid

temporal variation of the pressure is negligible

linearized equations:

equation of motion: x

$$\gamma^2 \rho h \frac{\partial v_x}{\partial t} = -\frac{\partial \delta P}{\partial x}$$

equation of motion: y

$$\gamma^2 \rho h \frac{\partial v_y}{\partial t} = -\frac{\partial \delta P}{\partial y} - \gamma^2 \left(\delta \rho + \frac{\Gamma}{\Gamma - 1} \frac{\delta P}{c^2} \right) g$$

continuity

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0$$

incompressible condition

$$\frac{\partial (\gamma \delta \rho)}{\partial t} = -v_y \frac{\partial (\gamma \rho)}{\partial y}$$

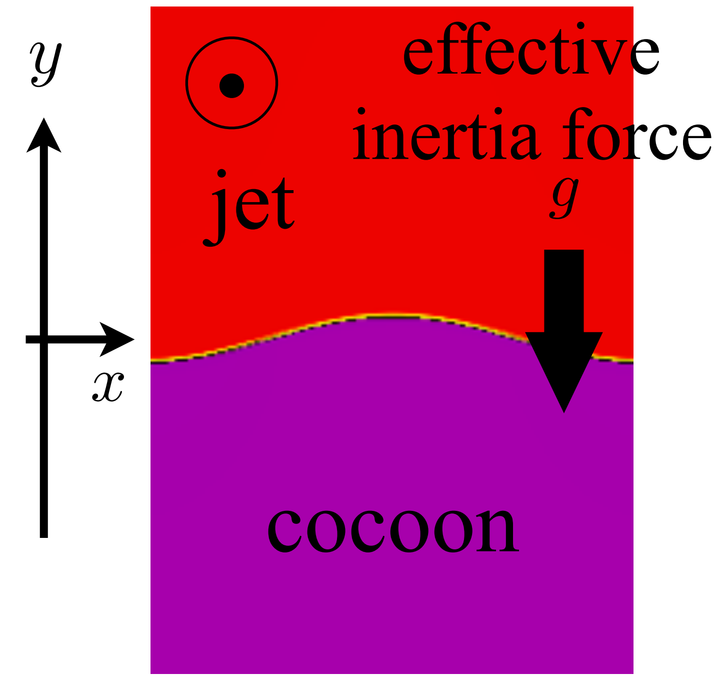
conservation of entropy

$$\frac{\partial}{\partial t} \left(\frac{\delta P}{P} - \Gamma \frac{\delta \rho}{\rho} \right) + v_y \left(\frac{1}{P} \frac{\partial P}{\partial y} - \Gamma \frac{1}{\rho} \frac{\partial \rho}{\partial y} \right) = 0$$

Derivation of linear growth of RTI

equation of motion:

$$\gamma^2 \rho h \left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} \right] + \nabla P + \cancel{\frac{\mathbf{v}}{c^2}\frac{\partial P}{\partial t}} - \gamma^2 \rho h \mathbf{g} = \mathbf{0}$$



assumption: $v_x, v_y \ll c_s \longrightarrow$ incompressible fluid

temporal variation of the pressure is negligible

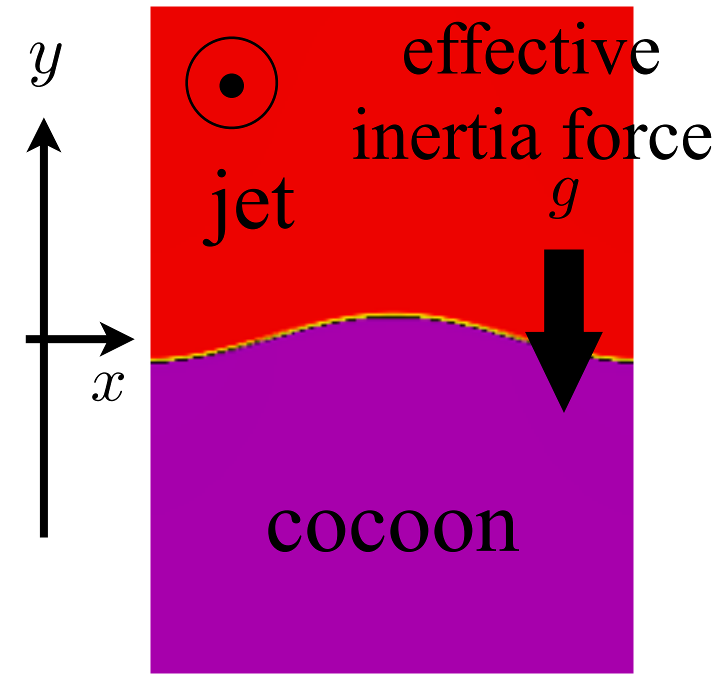
$$\underline{\delta\rho,\ \delta P,\ v_x, v_y \propto e^{i(kx - \omega t)}}$$

equation of motion: x	$i\omega\gamma^2 \rho h v_x = ik\delta P$
equation of motion: y	$i\omega\gamma^2 \rho h v_y = \dfrac{\partial \delta P}{\partial y} + \gamma^2 \left(\delta\rho + \dfrac{\Gamma}{\Gamma - 1}\dfrac{\delta P}{c^2} \right) g$
continuity	$ikv_x = -\dfrac{\partial v_y}{\partial y}$
incompressible condition	$i\omega\gamma\delta\rho = v_y \dfrac{\partial(\gamma\rho)}{\partial y}$
conservation of entropy	$\dfrac{\delta P}{P} = \Gamma \dfrac{\delta\rho}{\rho} - \dfrac{v_y}{i\omega}\left(\dfrac{\gamma^2 \rho h g}{P} + \Gamma \dfrac{1}{\rho}\dfrac{\partial \rho}{\partial y} \right)$

Derivation of linear growth of RTI

equation of motion:

$$\gamma^2 \rho h \left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} \right] + \nabla P + \cancel{\frac{\mathbf{v}}{c^2}\frac{\partial P}{\partial t}} - \gamma^2 \rho h \mathbf{g} = \mathbf{0}$$



assumption: $v_x, v_y \ll c_s$ $\longrightarrow$ incompressible fluid

temporal variation of the pressure is negligible

$$D_y = \frac{\partial}{\partial y}$$

$$D_y(\omega^2 \gamma^2 \rho h D_y v_y) - k^2 \left(\omega^2 - \frac{g}{H} \right) \gamma^2 \rho h v_y = k^2 \gamma \left(D_y(\gamma \rho) + D_y \gamma \frac{\Gamma^2}{\Gamma - 1} \frac{P}{c^2} \right) g v_y$$

Since the density and Lorentz factor are constant in the jet and cocoon regions,

using $\rho h = \left(\rho + \frac{\Gamma}{\Gamma - 1} \frac{P_0}{c^2} \right) e^{-y/H}$,

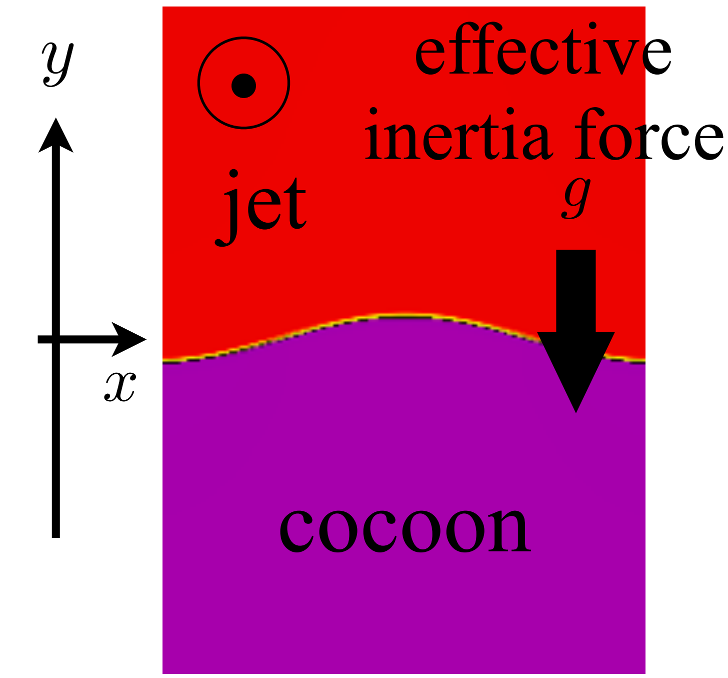
the differential equation for both jet and cocoon regions of the fluid reduces to

$$D_y^2 v_y - \frac{1}{H} D_y v_y - k^2 \left(1 - \frac{g}{H \omega^2} \right) v_y = 0$$

Derivation of linear growth of RTI

equation of motion:

$$\gamma^2 \rho h \left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v}\right] + \nabla P + \cancel{\frac{\mathbf{v}}{c^2}\frac{\partial P}{\partial t}} - \gamma^2 \rho h \mathbf{g} = \mathbf{0}$$



assumption: $v_x, v_y \ll c_s$ $\longrightarrow$ incompressible fluid

temporal variation of the pressure is negligible

$$D_y = \frac{\partial}{\partial y}$$

$$D_y^2 v_y - \frac{1}{H} D_y v_y - k^2 \left(1 - \frac{g}{H\omega^2}\right) v_y = 0$$

general solution: $v_y = A e^{\alpha_1 y} + B e^{\alpha_2 y}$

$$\alpha_1 = \frac{1}{2}\left(\frac{1}{H} + \sqrt{\frac{1}{H^2} + 4k^2\left(1 - \frac{g}{H\omega^2}\right)}\right) \quad , \quad \alpha_2 = \frac{1}{2}\left(\frac{1}{H} - \sqrt{\frac{1}{H^2} + 4k^2\left(1 - \frac{g}{H\omega^2}\right)}\right)$$

$$g/H \sim 1/\tau_{osci}^2 \;,\; g/H\omega^2 \ll 1$$

$$k \gg 1/H \sim 1/r_{jet}$$

$\Rightarrow$

boundary condition:

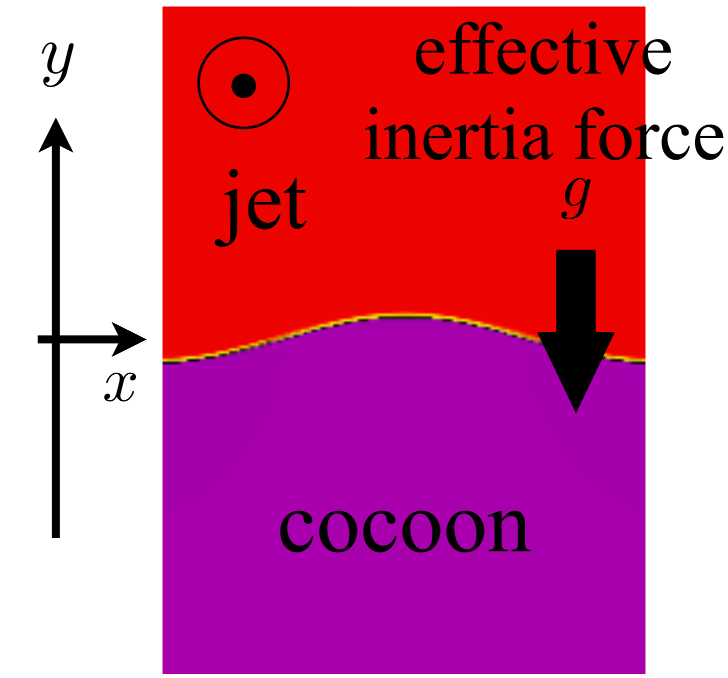
$v_y \to 0$ when $y \to \pm\infty$

$$v_y = \begin{cases} A e^{-ky}, & y > 0 \\ A e^{ky}, & y < 0 \end{cases}$$

Derivation of linear growth of RTI

equation of motion:

$$\gamma^2 \rho h \left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} \right] + \nabla P + \cancel{\frac{\mathbf{v}}{c^2}\frac{\partial P}{\partial t}} - \gamma^2 \rho h \mathbf{g} = \mathbf{0}$$



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$$D_y = \frac{\partial}{\partial y}$$

$$D_y(\omega^2 \gamma^2 \rho h D_y v_y) - k^2 \left(\omega^2 - \frac{g}{H} \right) \gamma^2 \rho h v_y = k^2 \gamma \left(D_y(\gamma\rho) + D_y \gamma \frac{\Gamma^2}{\Gamma - 1} \frac{P}{c^2} \right) g v_y$$

The dispersion relation is obtained by integrating the differential equation across the interface and dropping integrals of non-divergent terms.

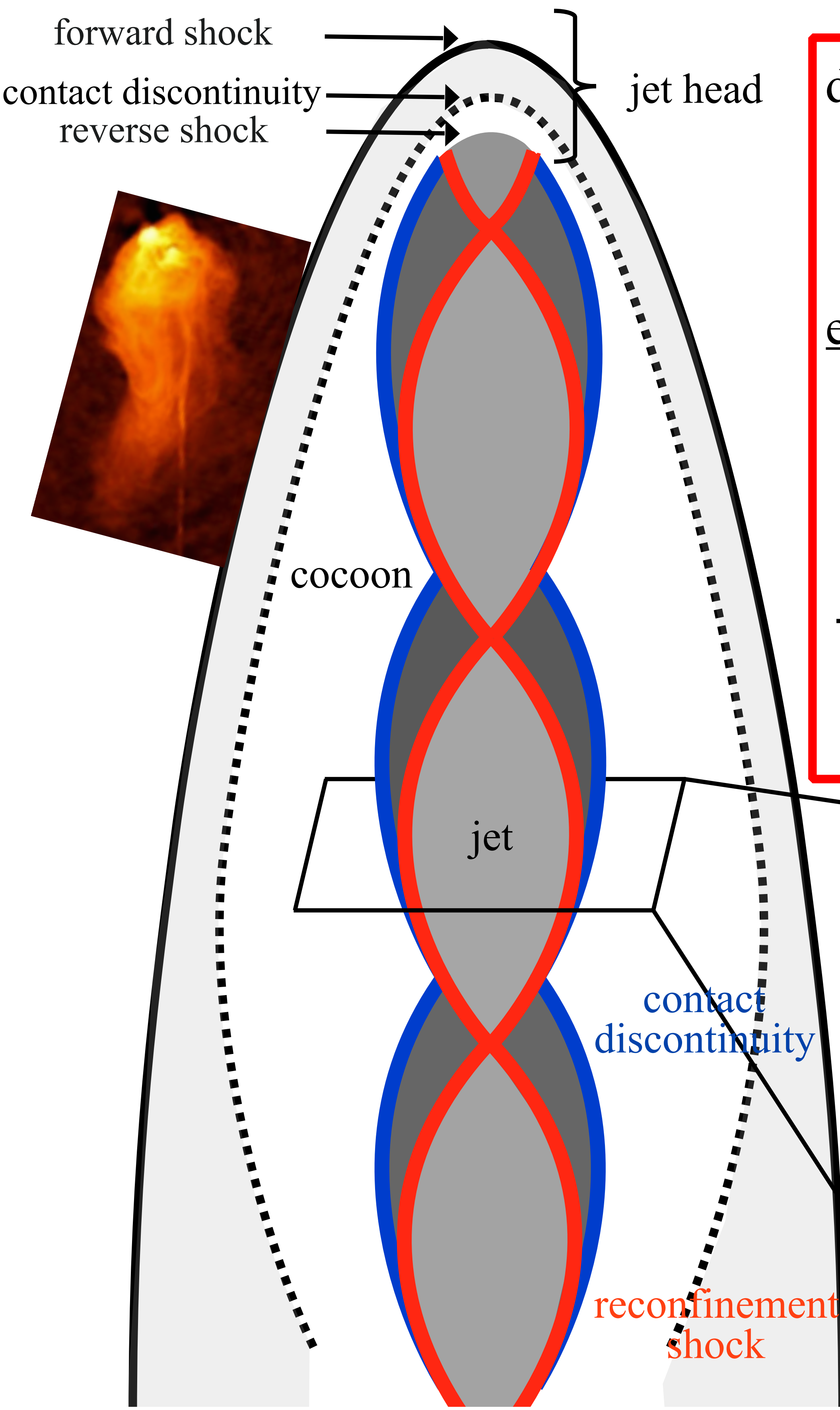
dispersion relation

$$\omega = i \sqrt{gk \frac{\gamma_j^2 \rho_j h'_j - \gamma_c^2 \rho_c h'_c}{\gamma_j^2 \rho_j h_j + \gamma_c^2 \rho_c h_c}} \qquad h' = 1 + \frac{\Gamma^2}{\Gamma - 1} \frac{P_0}{\rho c^2}$$

non-relativistic case

$$\omega = i \sqrt{gk \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}}$$

Modeling for the growth of RTI



dispersion relation: $h' = 1 + \frac{\Gamma^2}{\Gamma - 1}\frac{P_0}{\rho c^2}$

$$\omega = i\sqrt{gk\frac{\boxed{\gamma_j^2\rho_j h'_j - \gamma_c^2\rho_c h'_c}}{\gamma_j^2\rho_j h_j + \gamma_c^2\rho_c h_c}}$$

<u>estimation of inertia</u>

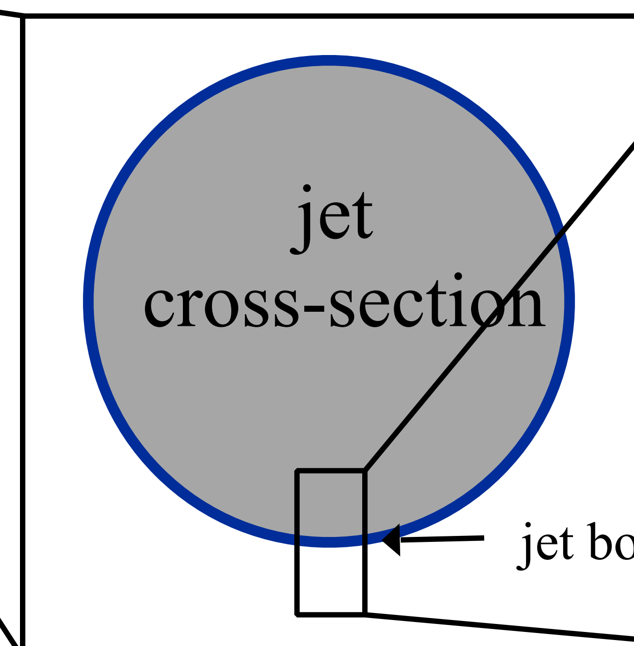
$$\gamma_j^2\rho_j h'_j \sim \boxed{\gamma_j^2} P_j$$

$$\gamma_c^2\rho_c h'_c \sim P_c$$

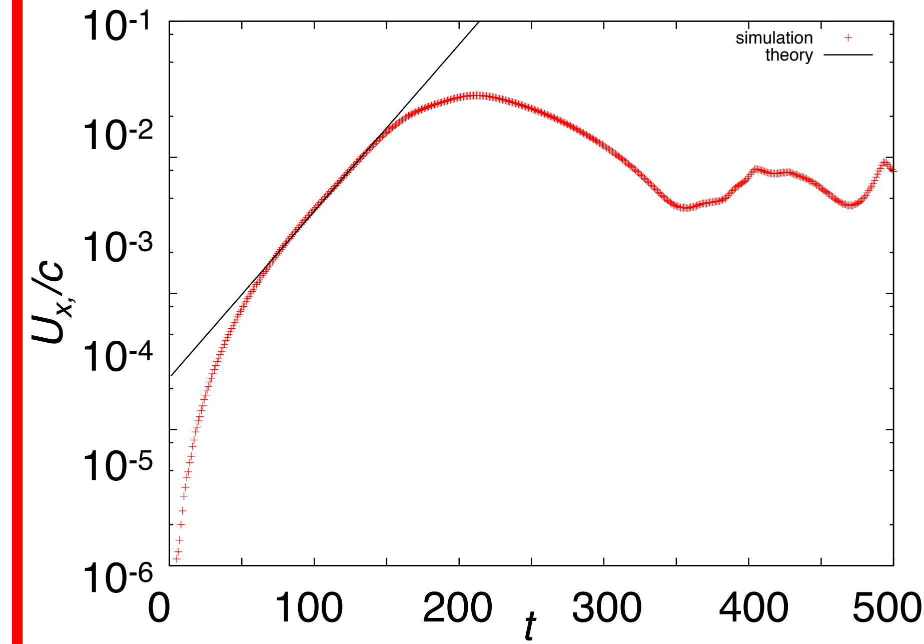
$$P_j \sim P_c$$

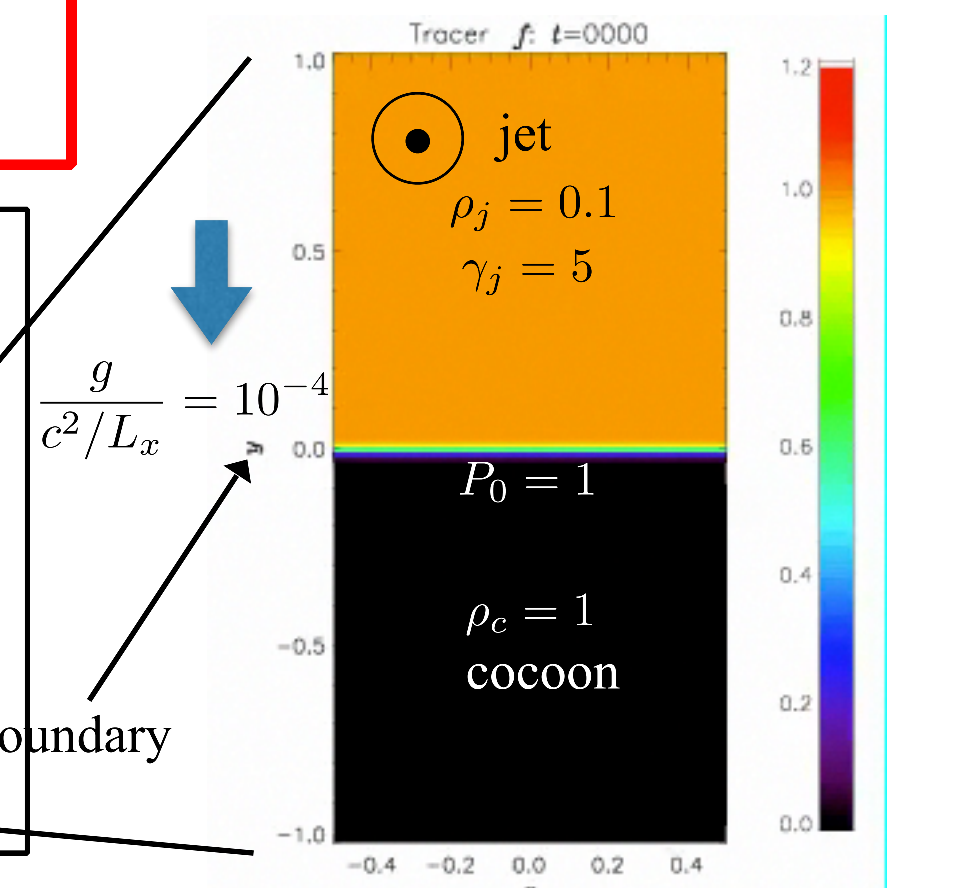
$$\rightarrow \quad \gamma_j^2\rho_j h'_j - \gamma_c^2\rho_c h'_c > 0$$

RTI grows at relativistic jet interface.



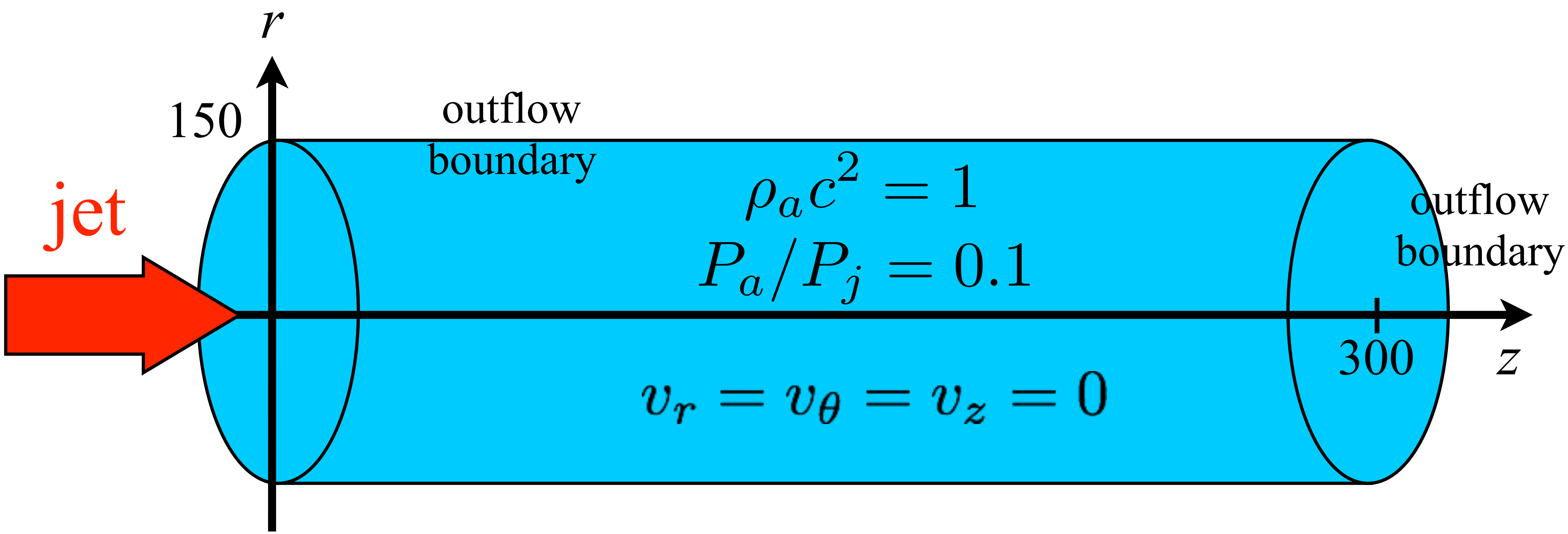
growth of the RTI





in the rest frame of the decelerating jet interface

Numerical Setting: 3D Toy Model



- cylindrical coordinate
- relativistic jet (z-direction)
- ideal gas
- numerical scheme: HLLC (Mignone & Bodo 05)
- uniform grid: $\Delta r = \Delta z = 0.1,\ \Delta\theta = 2\pi/160$

Jet models

Model	$\rho_j c^2$	P_j	h_j	γ_j	M_r	$\eta_{j,a}$	$\rho_a c^2$	P_a
H-	10^{-4}	10^{-3}	41	5	7	10^{-1}	1	10^{-4}
H+	10^{-2}	10^{-1}	41	5	7	10^{1}	1	10^{-2}
C-	10^{-4}	10^{-6}	1.04	31	272	10^{-1}	1	10^{-7}
C+	10^{-2}	10^{-4}	1.04	31	272	10^{1}	1	10^{-5}

■ two key parameters

- the effective inertia ratio of the jet to the ambient medium: $\eta_{j,a} = \dfrac{\gamma_j^2 \rho_j h_j}{\rho_a}$

Neglecting the multi-dimensional effect, the propagation velocity of the jet head through a cold ambient medium can be evaluated by the balancing the momentum flux of the jet and the ambient medium in the frame of the jet head (Marti+ 97, Mizuta+ 04):

$$v_h = \frac{\sqrt{\eta_{j,a}}}{1+\sqrt{\eta_{j,a}}} v_j$$

- dimension less specific enthalpy of the jet: h_j

Basic Equations

mass conservation

$$\frac{\partial}{\partial t}(\gamma\rho) + \nabla \cdot (\gamma\rho\mathbf{v}) = 0$$

momentum conservation

$$\frac{\partial}{\partial t}(\gamma^2\rho h\mathbf{v}) + \nabla \cdot (\gamma^2\rho h\mathbf{v}\mathbf{v} + P\mathbf{I}) = 0$$

energy conservation

$$\frac{\partial}{\partial t}(\gamma^2\rho h c^2 - P) + \nabla \cdot (\gamma^2\rho h c^2\mathbf{v}) = 0$$

specific enthalpy

$$h = 1 + \frac{\Gamma}{\Gamma - 1}\frac{P}{\rho c^2}$$

ratio of specific heats

$$\Gamma = \frac{4}{3}$$

Lorentz factor

$$\gamma = \frac{1}{\sqrt{1 - (v/c)^2}}$$

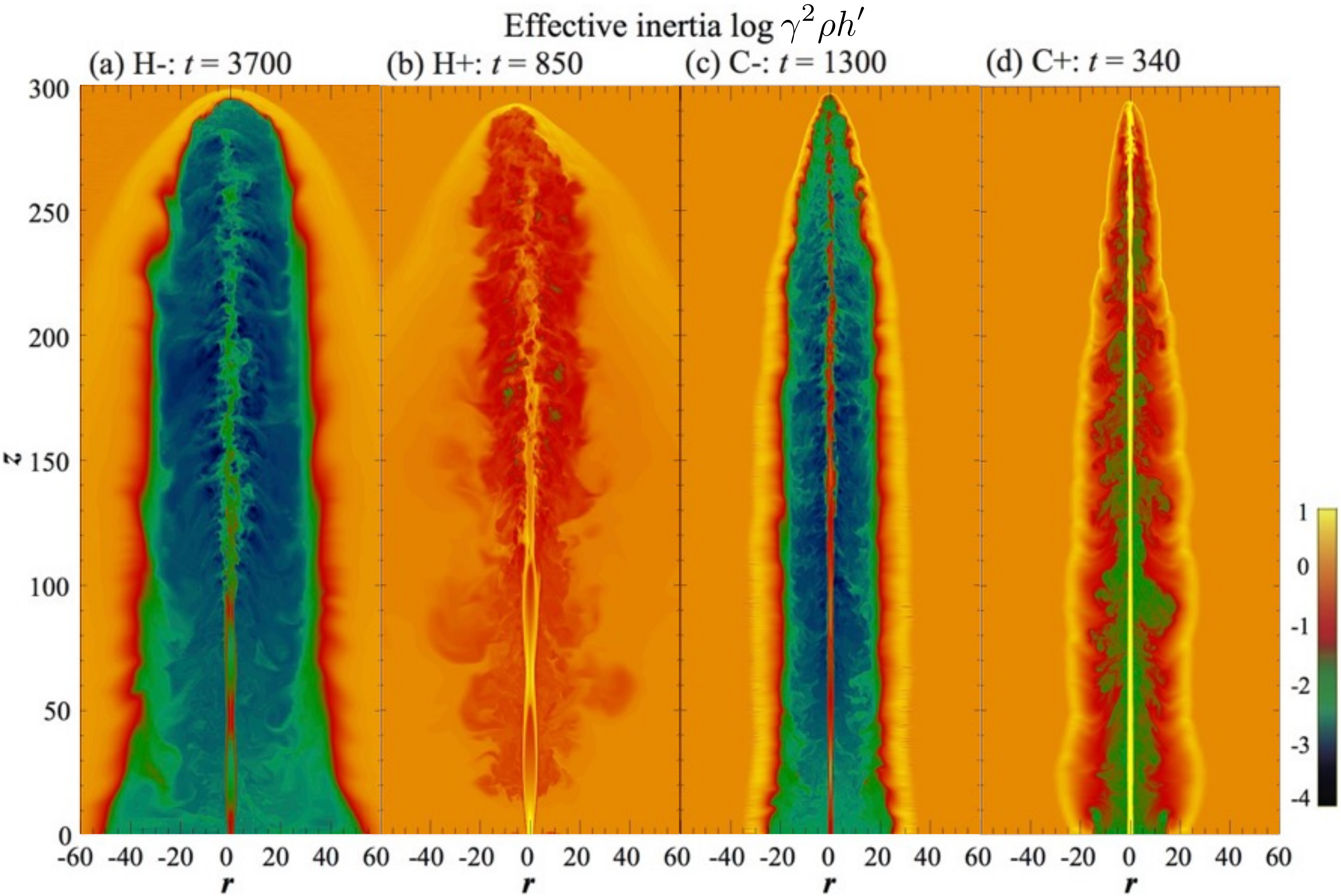
Result: Density

Only the jet component is shown.

The amplitude of the corrugated jet interface grows due to the oscillation-induced Rayleigh-Taylor and Richtmyer-Meshkov instabilities.

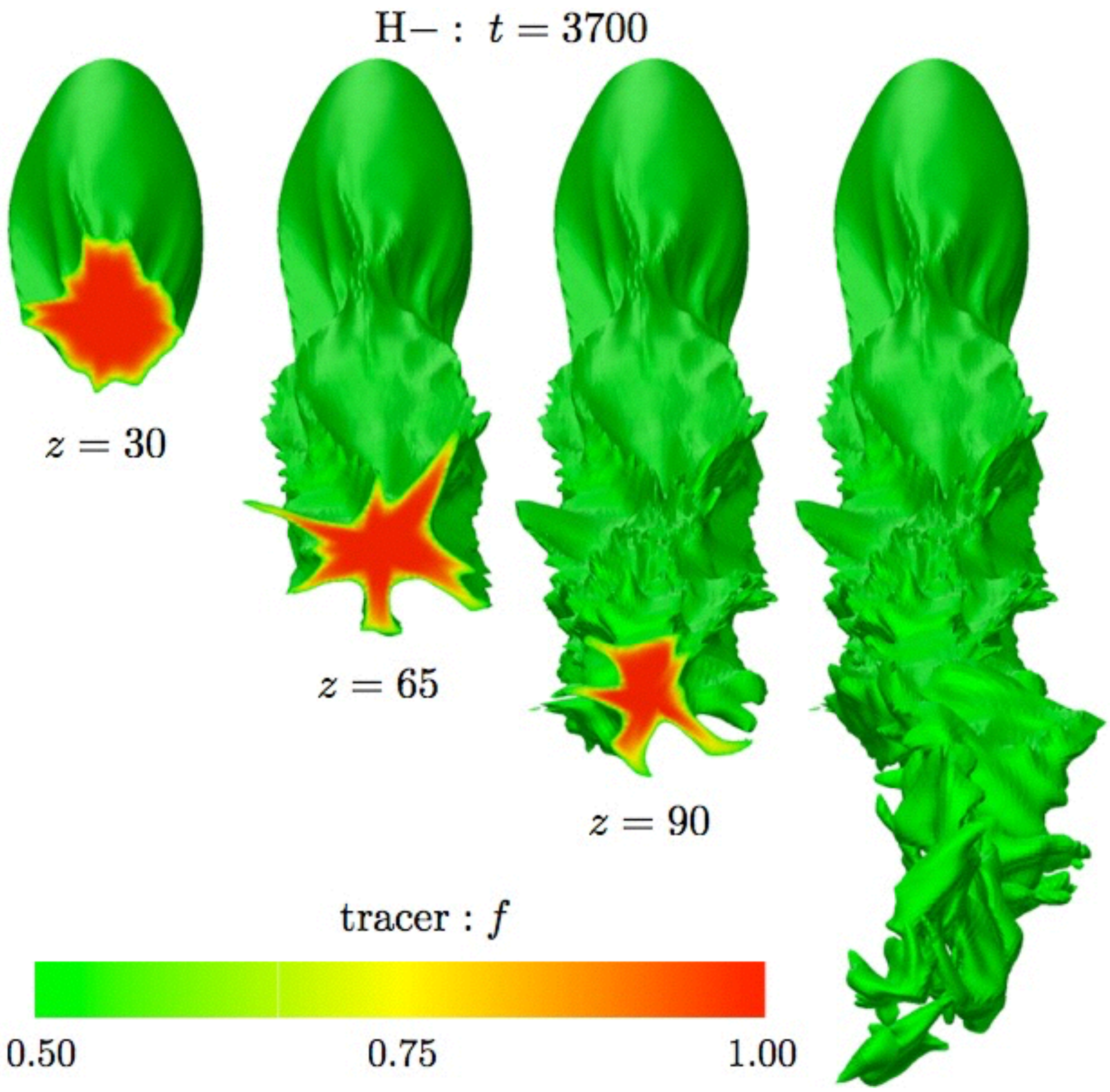
Since the relativistic jet is continuously injected into the calculation domain, standing reconfinement shocks are formed.

Distribution of the Modified Effective Inertia



As predicted analytically, the effective inertia of the jet becomes larger than the cocoon envelope for all the models.

3D Rendering of the Tracer



passive tracer: f

$f = 1$: jet material

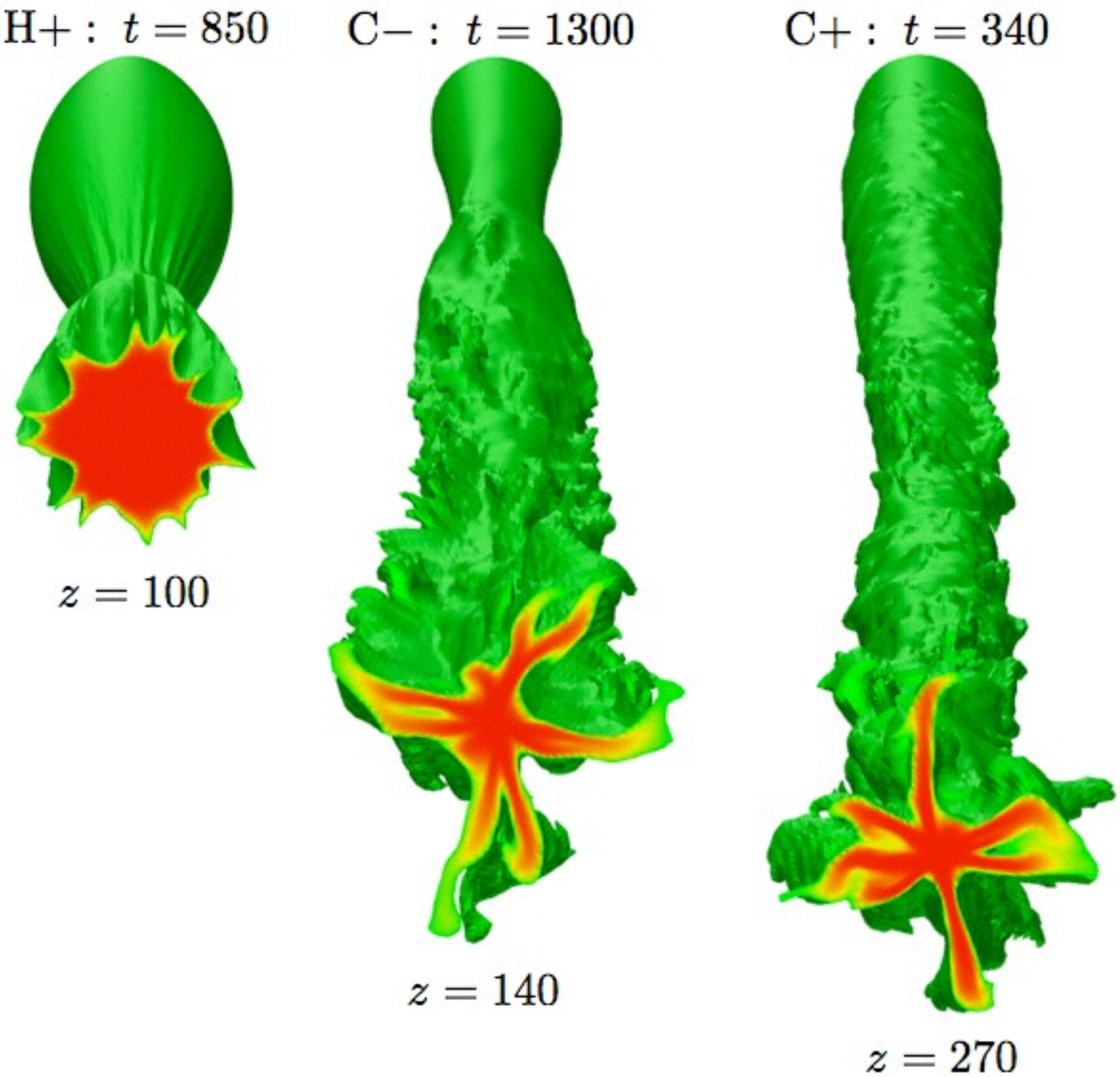
$f = 0$: ambient medium

$$\frac{\partial}{\partial t}(\gamma \rho f) + \nabla \cdot (\gamma \rho f \mathbf{v}) = 0$$

The oscillation-induced Rayleigh-Taylor instability is responsible for the distortion of the cross section at $z = 30$.

Finger-like structures appeared in the cross-section at z = 65 and 90 are outcome of both the Rayleigh-Taylor and Richtmyer-Meshkov instabilities.

Inherent Property of Relativistic Jet



- initial evolution
  - radial expansion (hot models)
  - radial contraction (cold models)

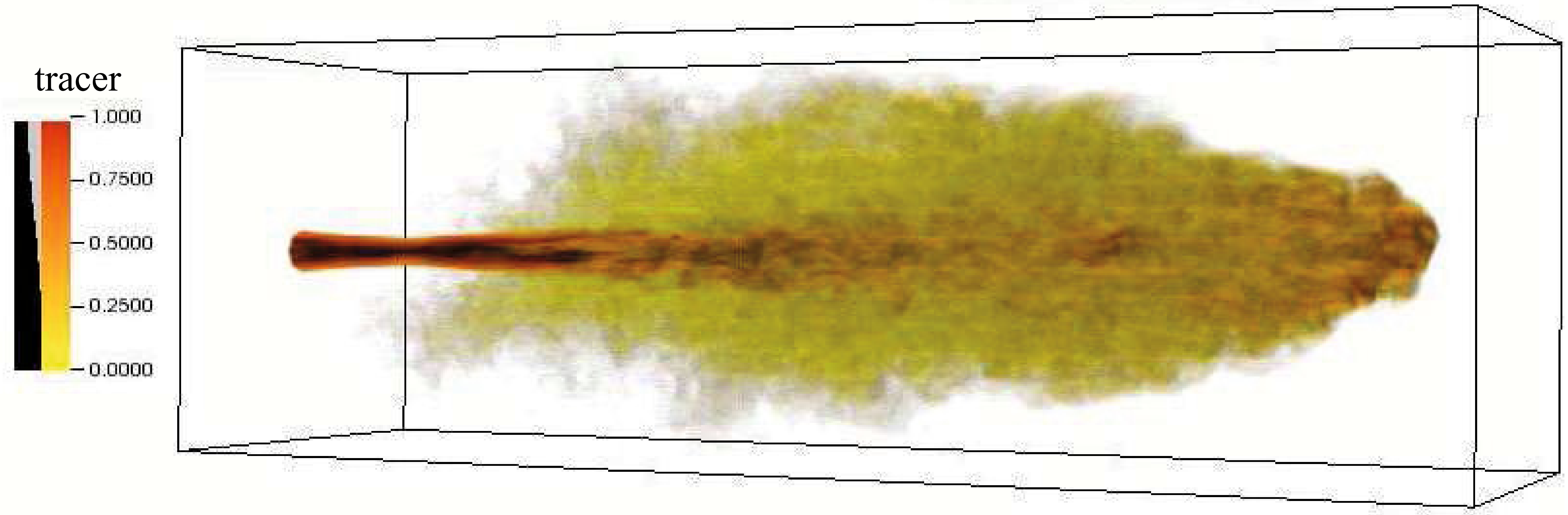
After initial stage, the cold jet also follows the same evolution path as the hot jet and thus excites the Rayleigh-Taylor and Richtmyer-Meshkov instabilites.

Although the relativistic jet shows a rich variety of the propagation dynamics depending on its launching condition, the oscillation-induced Rayleigh-Taylor instability and secondary Richtmyer-Meshkov instability grow commonly at the jet interface and then induce a lot of finger-like structures.

Kelvin-Helmholtz instability?

Rossi et al. 2008

3D pure hydro simulation for jet propagation

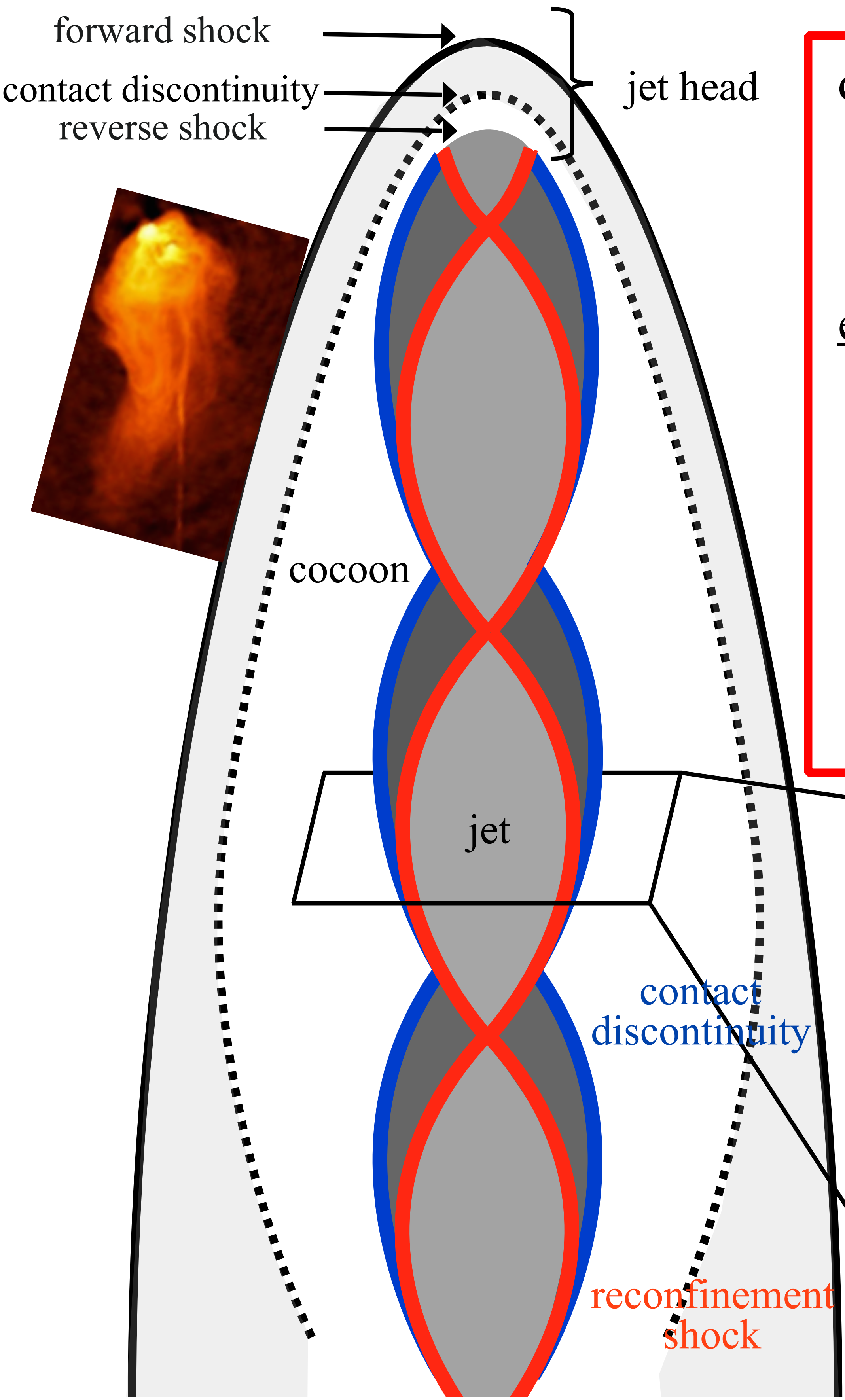


They claimed that the entrainment process takes place from the interaction between the jet beam and the cocoon, promoted by the development of Kelvin-Helmholtz instabilities at the beam interface.

Rayleigh-Taylor instability is also expected to grow at the interface of the jet.

Which instability is dominant??

Summary



dispersion relation

$$\omega = i\sqrt{gk\frac{\boxed{\gamma_j^2\rho_j h_j' - \gamma_c^2\rho_c h_c'}}{\gamma_j^2\rho_j h_j + \gamma_c^2\rho_c h_c}}$$

estimation of inertia

$$\gamma_j^2\rho_j h_j' \sim \boxed{\gamma_j^2} P_j$$

$$\gamma_c^2\rho_c h_c' \sim P_c$$

$$P_j \sim P_c$$

$$\gamma_j^2\rho_j h_j' - \gamma_c^2\rho_c h_c' > 0$$

RTI grows at relativistic jet interface.

growth of the RTI